Critical spin liquid with spinon Fermi surface in 2+1 dimensions

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Ref : SL, Phys. Rev. B 78, 085129 (2008)

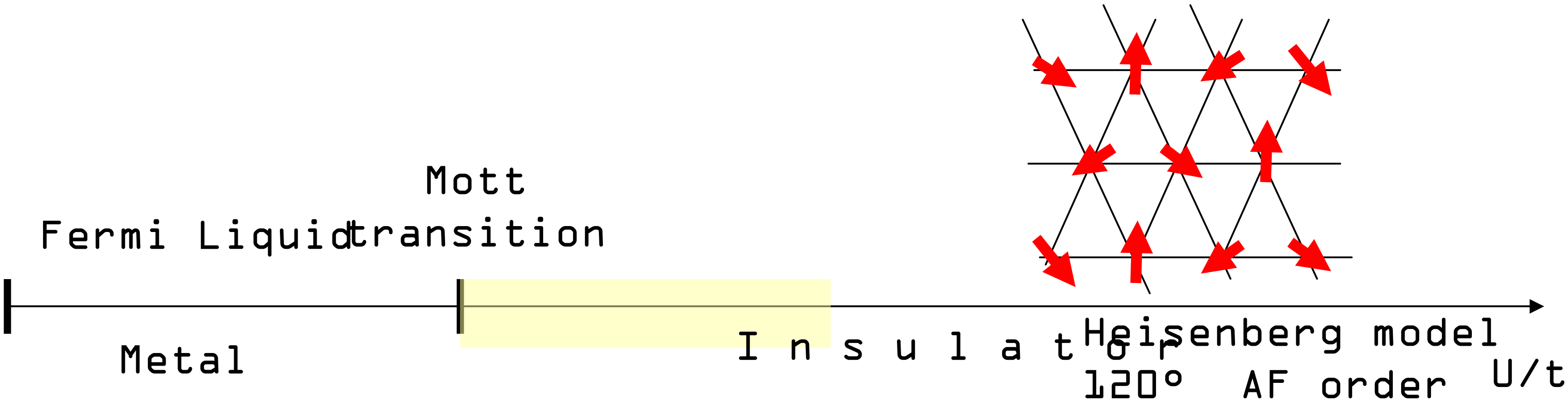
SL, arXiv:0905.4532

Outline

- Spin Liquid from the Hubbard model : slave-particle approaches
- Stability of deconfinement phase
- Low energy effective theory
  - Minimal theory
  - Genus expansion
  - Non-renormalization of gauge coupling
- Conclusion

Hubbard Model : parent model of many phases (Metal, SC, AF, Spin Liquid, …)

$$H = -t \sum_{<i,j>} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i + \dots$$



$$H_{eff} = J \sum_{<i,j>} S_i \cdot S_j + \dots .$$

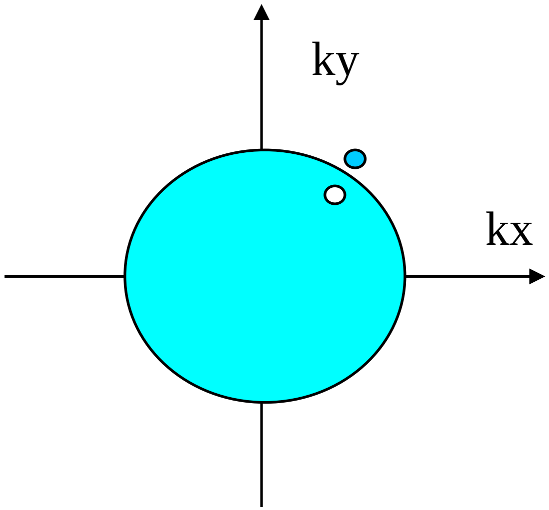
Charge fluctuations / geometrical frustration may disrupt spins from ordering even at T=0 near the metal-insulator transition.

Spinon Fermi surface coupled with a U(1) gauge field

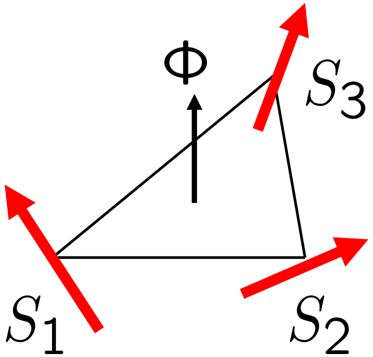
$$\vec{S}_r = f^\dagger_{r\alpha}\vec{\sigma}_{\alpha\beta}f_{r\beta}$$ Spinon : EM charge 0, spin 1/2

Gauge redundancy : $f_{r\alpha} \longrightarrow f_{r\alpha}e^{i\theta_r}$

1) neutral spinon
(Fermi surface)



2) spin chirality :
compact U(1) gauge field

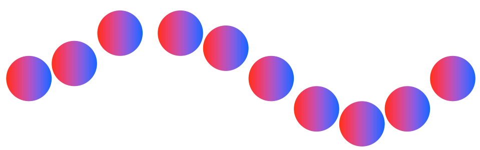


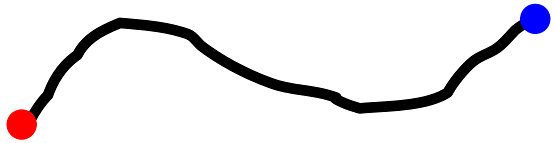
$$\nabla \times a \sim \sin\Phi \sim S_1 \cdot (S_2 \times S_3)$$

Comments on slave-particle approaches (cont'd)

- Spinon and gauge field become useful d.o.f. only if the gauge theory is in deconfinement phase (dynamical issue)
- Bare gauge coupling is infinite (constraint) : nonetheless deconfinement is possible at low energy

Eg)





String of point-like dipolar particles

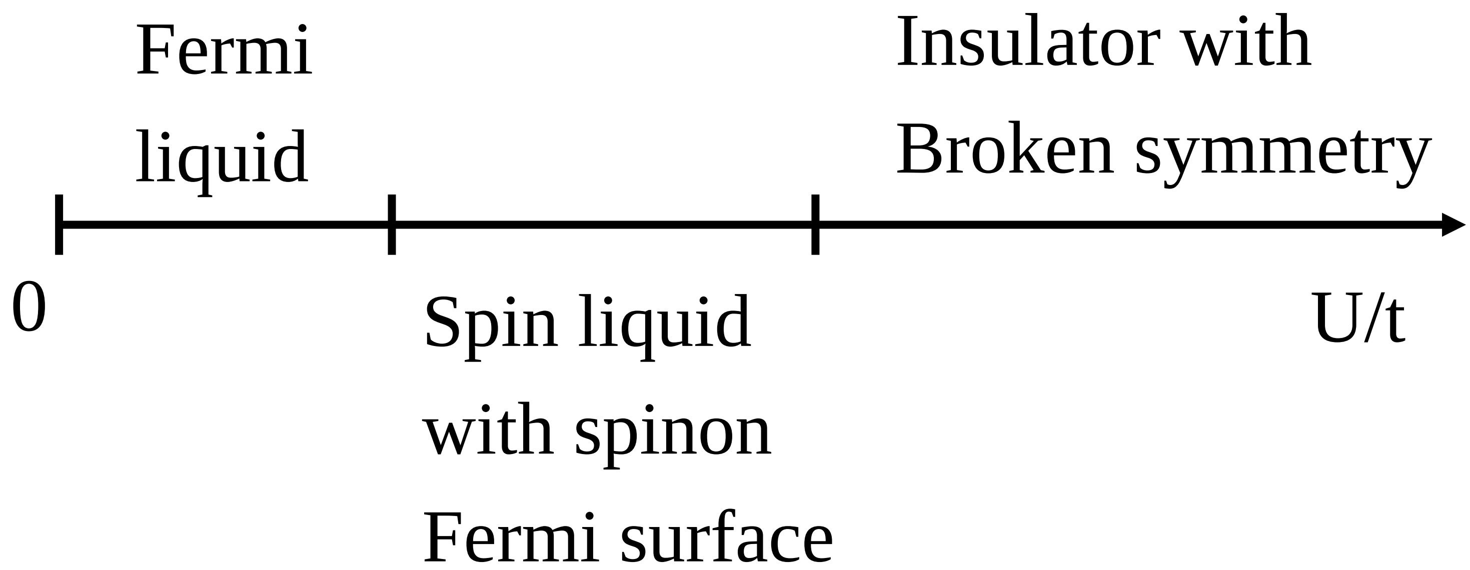
= ± charges attached to electric flux line

If strings are highly fluctuating, charges are deconfined! [Wen]

Comments on slave-particle approaches

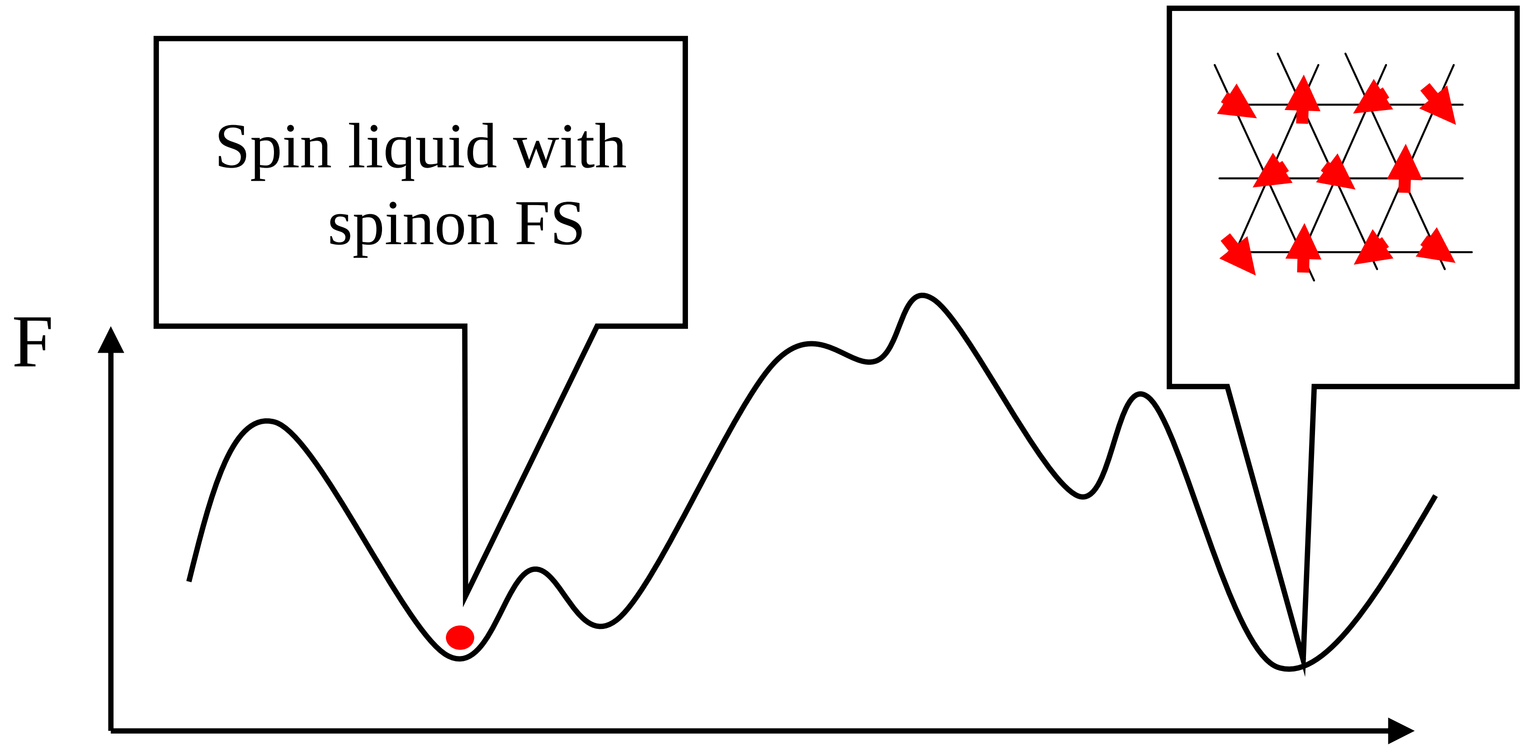
- Description, not derivation of state
- Choice of gauge group and statistics of slave-particle is not unique (except for a few known cases) : different states
- Strategy : find a locally stable saddle point and identify soft modes and compare the predictions of the low energy effective theory with experiments.

Mean-field phase diagram



[Motrunich, SL and Patrick Lee (05)]

Landscape of the Hubbard model



Instead of asking whether the spinon Fermi surface is realized in a particular model/material, I will address the following questions:

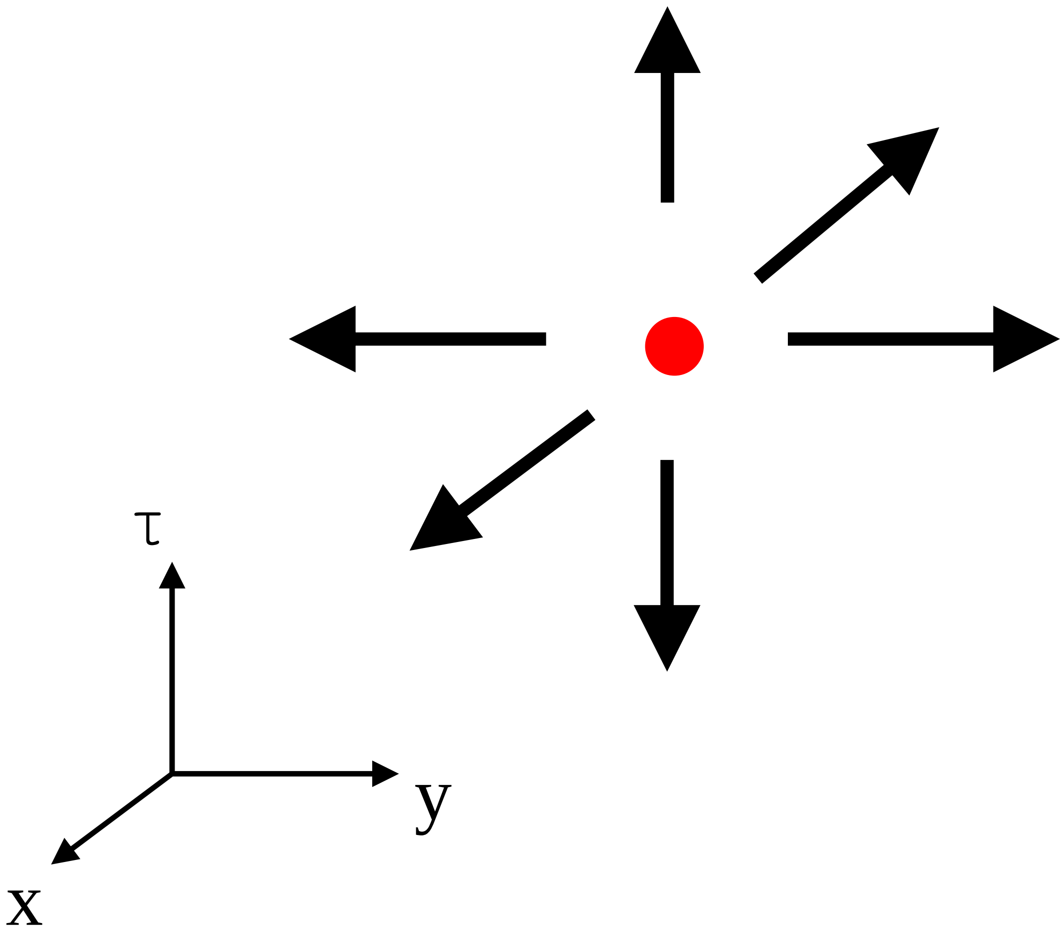
- Does the spinon Fermi surface describe a stable phase of matter ?

- What is the effective theory ?

Non-perturbative stability :

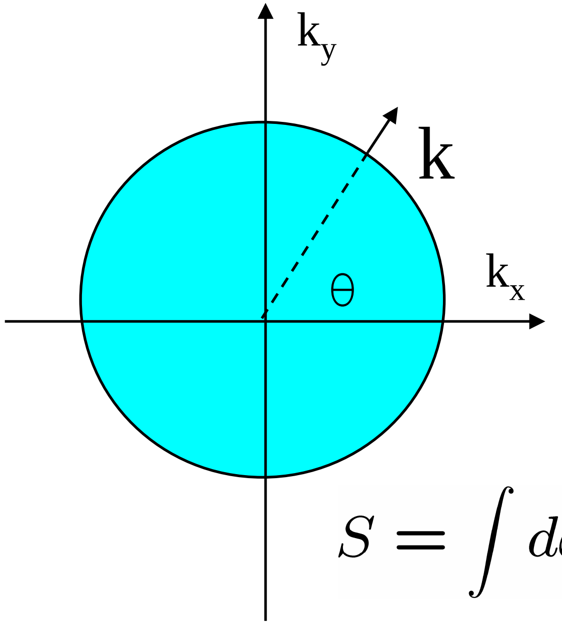
Is instanton irrelevant ?

Instanton : a source of 2π flux in space-time



$$\int_S dA = 2\pi$$

Fermions moving around an instanton background : non-interacting fermions



2d Fermi surface

= Infinite set of 1d chiral fermions

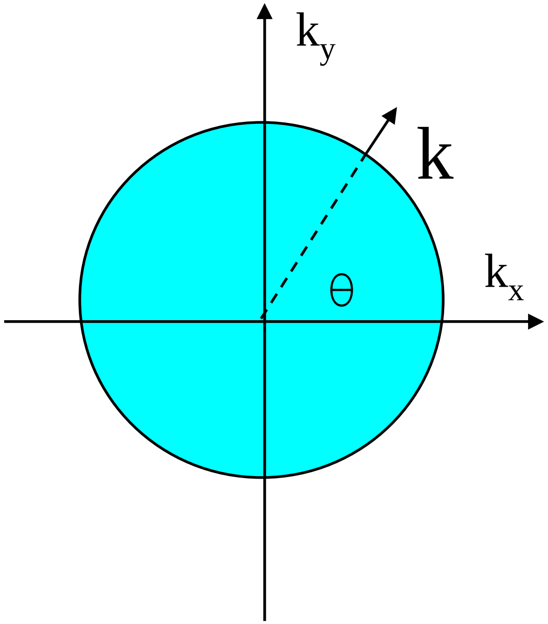
$$S = \int d\omega dk d\theta \quad (i\omega + k)\, \psi^*(\omega, k, \theta)\psi(\omega, k, \theta)$$

Scale invariance :

$$\begin{aligned} \omega &= b^{-1}\ \omega' \\ k &= b^{-1}\ k' \\ \psi &= b^{3/2}\ \psi' \end{aligned}$$

Scaling dimension of instanton ?

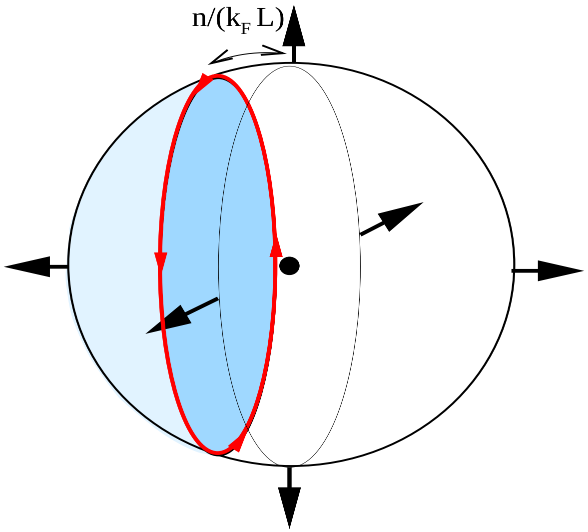
From momentum space to 1+1D real space

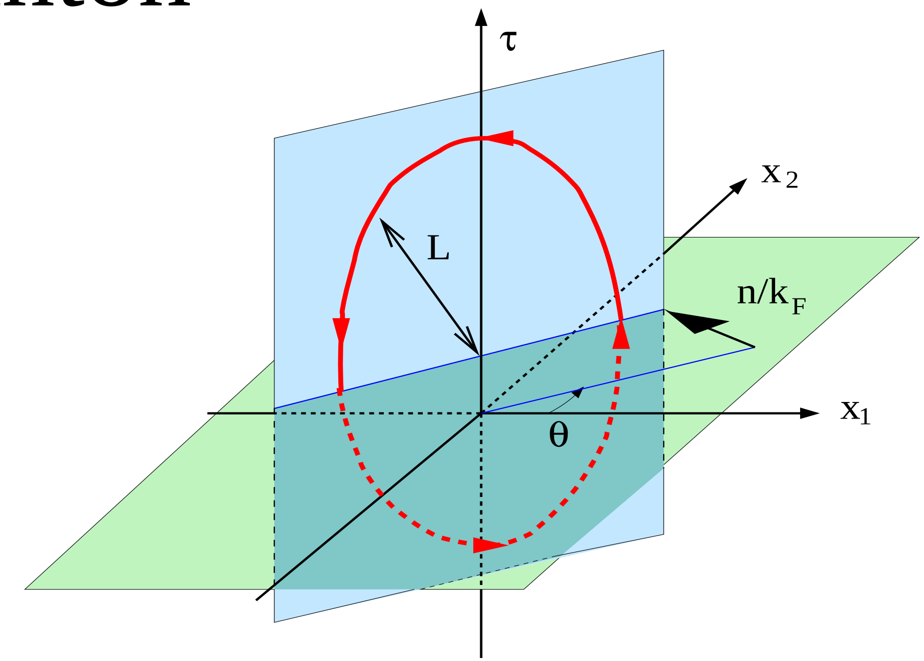


$$
\begin{aligned}
\omega &\longrightarrow \tau, \\
k &\longrightarrow x, \\
\theta &\longrightarrow n
\end{aligned}
$$

$$\psi_n(\tau, x) = \int d\omega dk d\theta \; e^{i(\omega\tau + kx + n\theta)} \psi(\omega, k, \theta)$$

Fermions in the background of an instanton





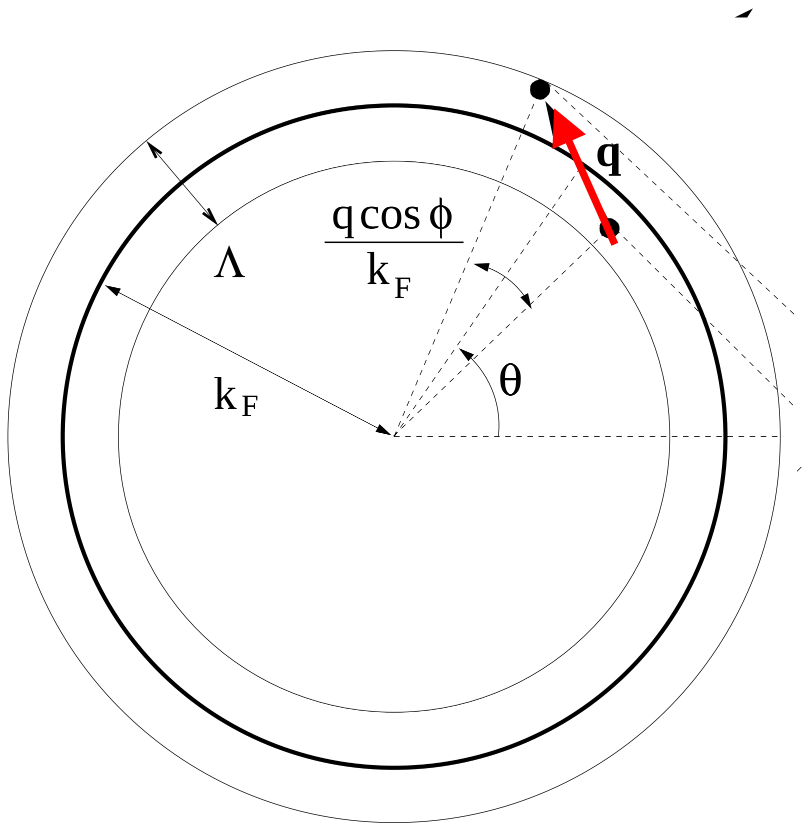
$$\mathcal{L} = \sum_n \psi_n^*(\tau,x)[\partial_\tau - i(\partial_x - i a_\theta^s(\tau,x))]\psi_n(\tau,x)$$

Twist angle : $$\Phi_n(L) = sgn(n)\pi\left(1 - \frac{|n|}{\sqrt{(Lk_F)^2+n^2}}\right)$$

Non-interacting fermions

- Instanton twists boundary conditions of the n-th angular momentum mode by π if $L >> n/k_F$
- At scale L, roughly L angular momentum modes are twisted
- Each fermion contributes the scaling dimension 1/8 to an instanton operator
- Instanton has an infinite scaling dimension and is strongly suppressed at long distances (instanton action grows as L ln L)

Interacting fermions

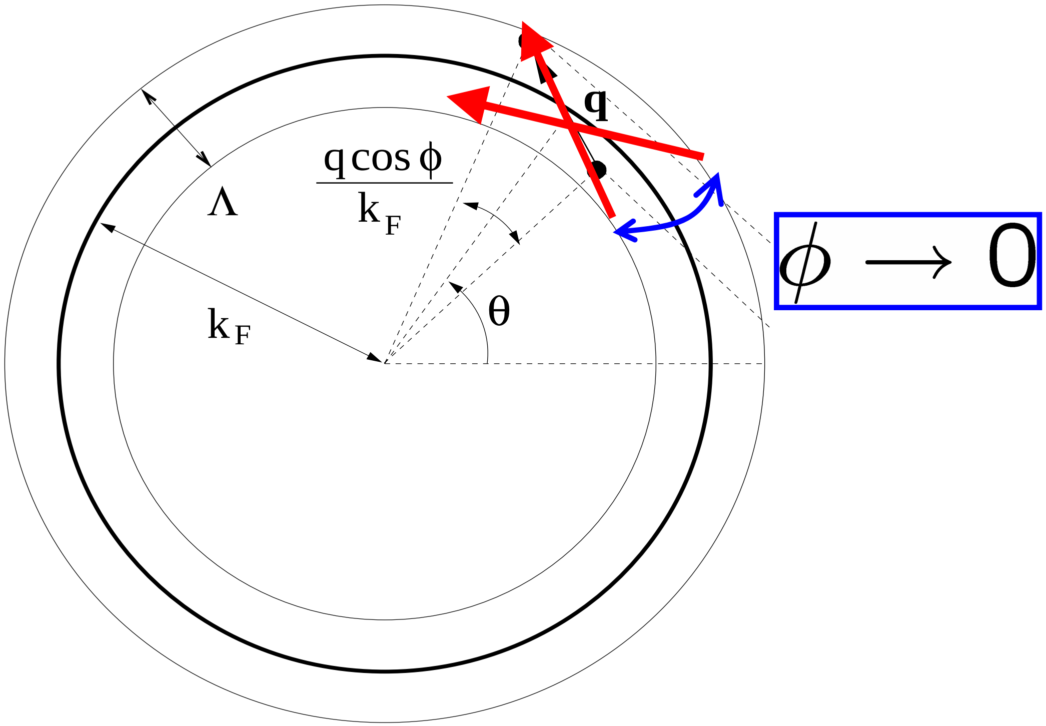


Fermions at different angles are coupled with each other.

One can not just add the contributions of fermions at different angles.

One can still show that the scaling dimension of instanton is infinte.

Locality in the angular direction



At low energies, fermions with angle θ couples only with the gauge field whose momentum is tangential to the Fermi surface at that angle.

As a result, modes at different angles are essentially decoupled.

Decompactification of θ

At low energies, momentum of gauge field is scaled down, but the circumference of the FS is fixed

→ gauge fluctuation becomes more and more ineffective in changing the motion of fermion

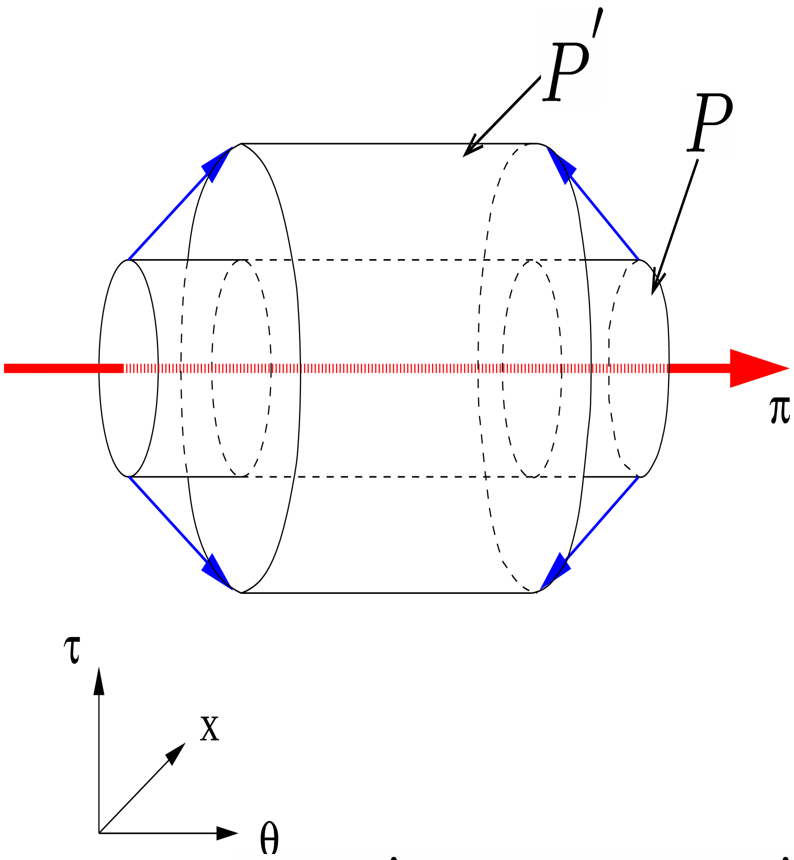
→ circumference of FS looks larger at lower energy

→ angular variable acquires a positive anomalous dimension

→ angular variable becomes decompactified in the low energy limit : $[-\infty,\infty]$

→ instanton becomes a π-vortex along the extended angular direction

Vortex-state correspondence

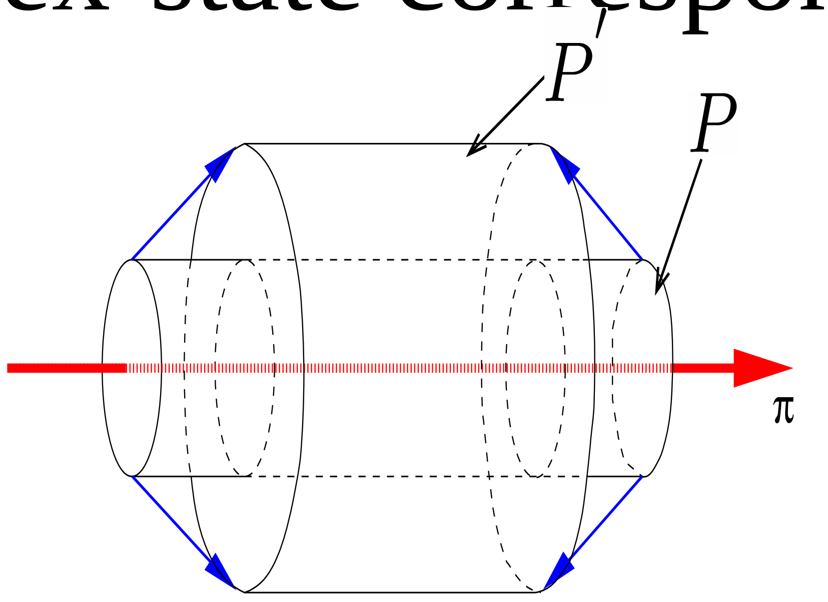


Instanton : vortex along θ

A vortex defines a quantum state on the surface that encloses the vortex

$$\Psi[\psi'(\tau,x,\theta),\psi^{*\prime}(\tau,x,\theta),a_q'(\tau,x,\theta)]\Big|_{(\tau,x,\theta)\in\mathcal{P}}$$
$$=\int D\psi D\psi^* Da_q e^{-S[\psi,\psi^*,a]}$$

Vortex-state correspondence



Scaling dimension of instanton : energy of the state defined on the cylinder (radial quantization)

Since the theory is local in θ, any extended object should have an infinite energy $\rightarrow$ scaling dimension is infinite!

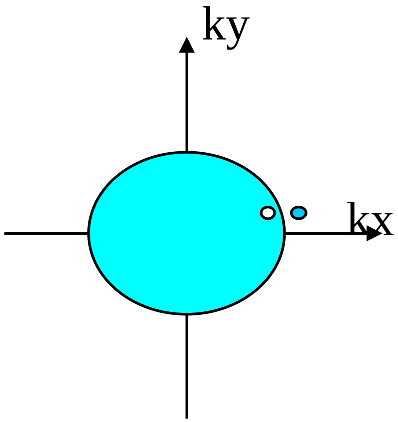
Summary I

- The spinon Fermi surface coupled with a compact U(1) gauge field is stable against confinement for **ANY** nonzero spinon flavor

- The deconfined (Coulomb) phase can be stable in physical case with N=2

Low energy effective theory

Fermi surface coupled with U(1) gauge field



Fermi surface
of spinon

Non-compact U(1) gauge field

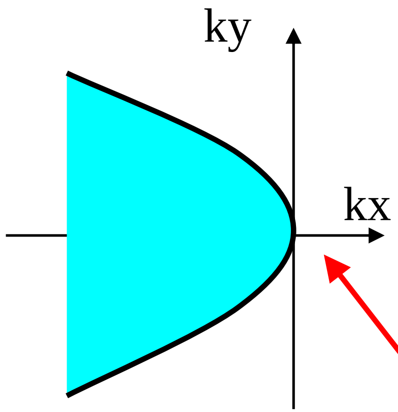
QED3 with nonzero chemical potential

This theory arises in various different contexts :

- high temperature superconductors,
- half-filled Landau level in the fractional quantum Hall state
- spin liquid in frustrated spin systems

  (κ-(BEDT-TTF)2Cu2(CN)3 ?)

Focus on one area near the Fermi surface (Chiral Fermi Surface)



fermions near one patch

Critical boson : emergent U(1) gauge field

$$\mathcal{L} = \sum_j \psi_j^* (\partial_\tau - i v_x \partial_x - v_y \partial_y^2) \psi_j + e \sum_j a \psi_j^* \psi_j + a \left[-\partial_\tau^2 - \partial_x^2 - \partial_y^2 \right] a$$

Backscattering not included here : important effect to be taken into account.

Non-Fermi liquid state in 2+1D

No dimensionless parameter except for the fermion flavor N=2

Generalize to a large N

→ What is the nature of the theory ?

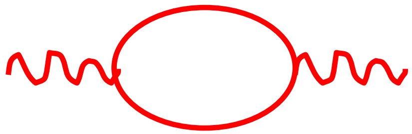
In relativistic theories, large N usually makes all quantum corrections perturbatively small.

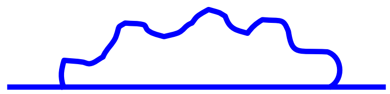
With FS, this is NOT true!

- Softness of the Fermi surface
- Strong IR quantum fluctuations

→ non-trivial QFT

One-loop quantum correction





$$
\begin{aligned}
\Gamma \;=\; & \sum_j \int dk \left[i\frac{c}{N}\,\mathrm{sgn}(k_0)|k_0|^{2/3} + ik_0 + k_x + k_y^2 \right] \psi_j^*(k)\psi_j(k) \\
& + \int dk \left[\gamma \frac{|k_0|}{|k_y|} + k_0^2 + k_x^2 + k_y^2 \right] a^*(k)a(k) \\
& + \frac{e}{\sqrt{N}} \sum_j \int dkdq \;\; a(q)\psi_j^*(k+q)\psi_j(k)
\end{aligned}
$$

Irrelevant!

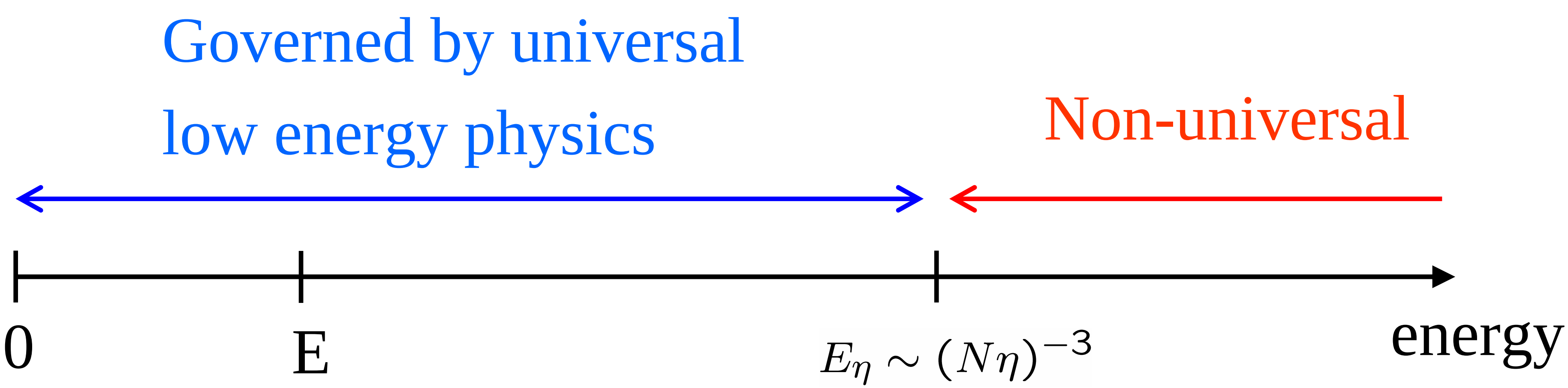
$$
\begin{aligned}
k_0 &= b^{-3} k_0^{'} \\
k_x &= b^{-2}\, k_x^{'} \\
k_y &= b^{-1}\, k_y^{'}
\end{aligned}
$$

- If we drop the local frequency dependent terms, theory becomes completely local in time (no propagating mode).

→ we should keep an irrelevant term so that it generates the marginal self energy before it dies off in the low energy limit

Minimal theory

$$\mathcal{L} = \sum_j \psi_j^*(\eta\partial_\tau - iv_x\partial_x - v_y\partial_y^2)\psi_j + \frac{e}{\sqrt{N}}\sum_j a\psi_j^*\psi_j + a(-\partial_y^2)a$$

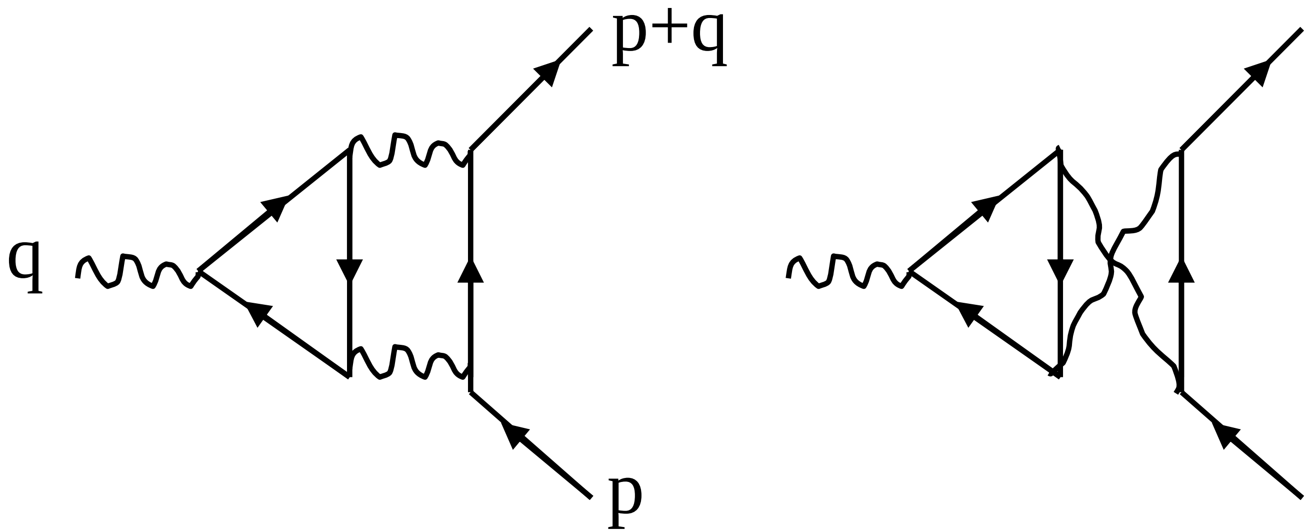


Universal field theory limit : Fix our energy scale E

$E_\eta \to \infty$

Then take the large N limit

Strong quantum fluctuations near FS



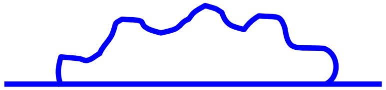
(external fermions on FS; generic nonzero and finite q)

Naïve counting : $N^{-3/2}$ $N^{-3/2}$

(every vertex : $N^{-1/2}$, every loop of fermion : N)

Actual order : $N^{-1/2}$ $N^{-3/2}$

The reason why the naïve counting fails : Fermion propagator becomes O(N) on the FS



+ ....

0 in the low energy limit

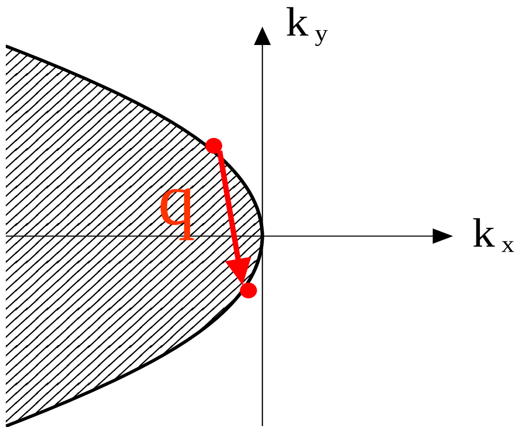
0 on the FS

$$g(k) = \frac{1}{i\eta k_0 + i\frac{c}{N}\,\mathrm{sgn}(k_0)|k_0|^{2/3} + k_x + k_y^2}$$

If there are ways for virtual particle-hole excitations to stay on the FS, the amplitude is enhanced : quantum corrections are enhanced due to the infinitely many soft modes near the FS.

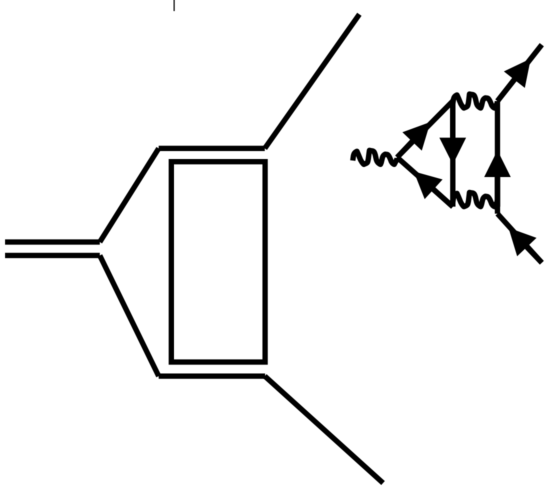
Double line representation

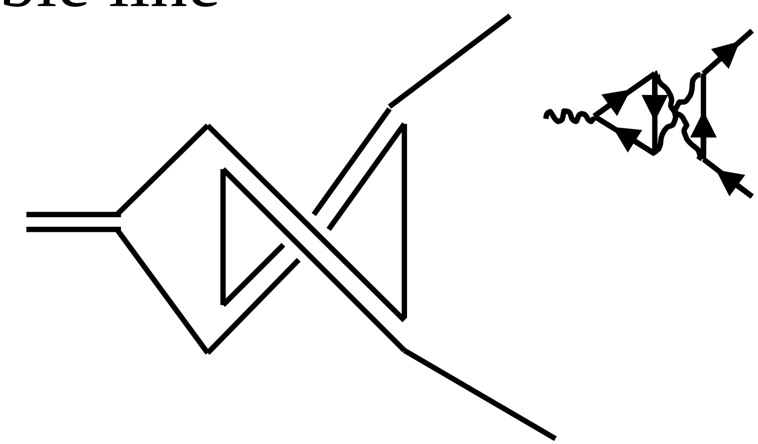
fermion momentum on FS is parameterized by an angle: k_θ



boson momentum can be uniquely decomposed into two fermion momenta on the FS : $q = k_\theta - k_{\theta'}$

→ represent a boson propagator as a double line



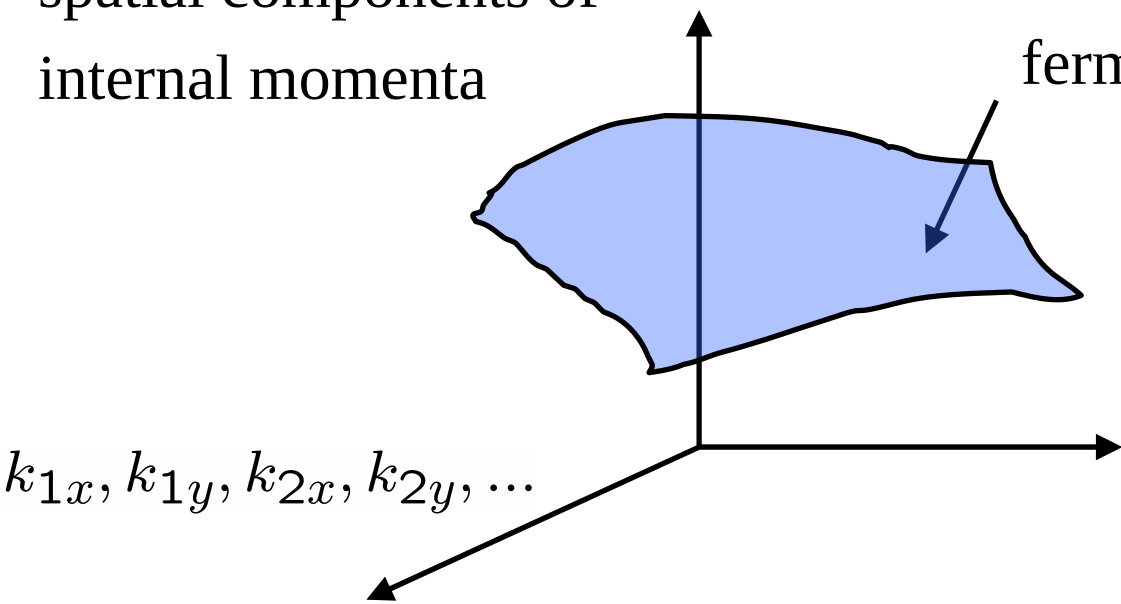


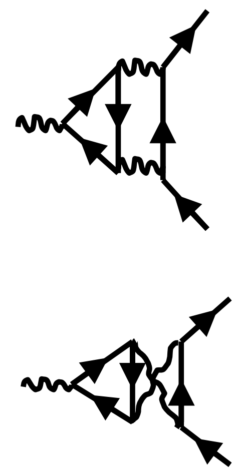
closed loops determine in how many way virtual particle-hole excitations can stay on the FS

Singular manifold

2L-dimensional space
spatial components of
internal momenta

m-dimensional subspace
on which all internal
fermions are on the FS





m=1

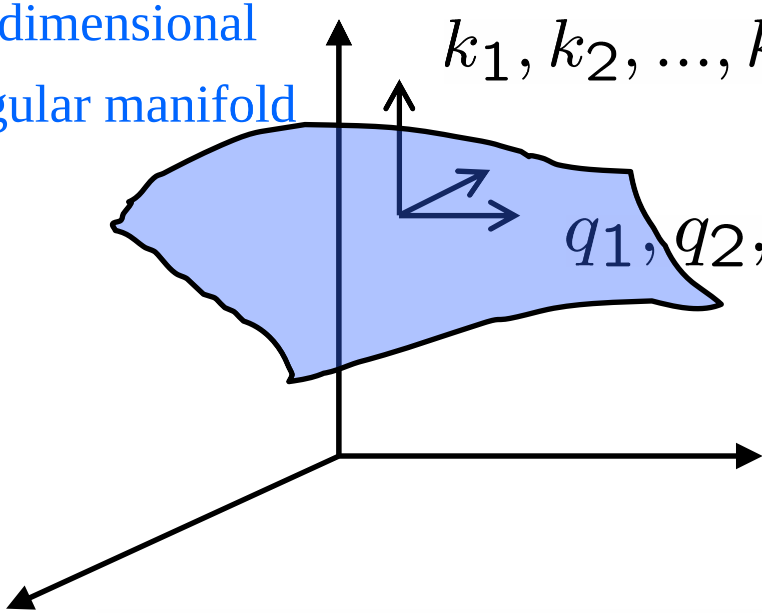
m=0

# of closed loops =

dimension of manifold in the space of internal momenta
in which all fermions remain on the FS

Singular manifold

m-dimensional
singular manifold



$k_1, k_2, \ldots, k_{2L-m}$: integration perpendicular to the singular manifold

$q_1, q_2, \ldots, q_m$: integration along the singular manifold

$$\int dq_1 dq_2 \ldots dq_m \int dk_1 dk_2 \ldots dk_{2L-m} \Pi_{i=1}^{I_f} \left[\frac{1}{\alpha_j^i k_j + i \frac{1}{N} f(\omega_i)} \right]$$

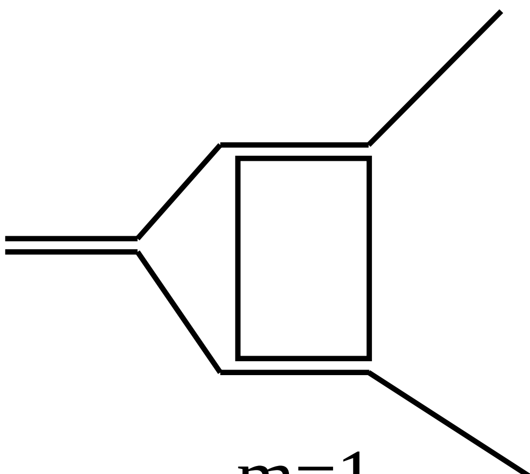
$$\sim \int dq_1 dq_2 \ldots dq_m O(N^{I_f-(2L-m)})$$

Correct power counting : $N^{-\frac{V}{2}+L_f+[I_f-2L+m]}$

(V : # of vertices; L_f : # of fermion loops;

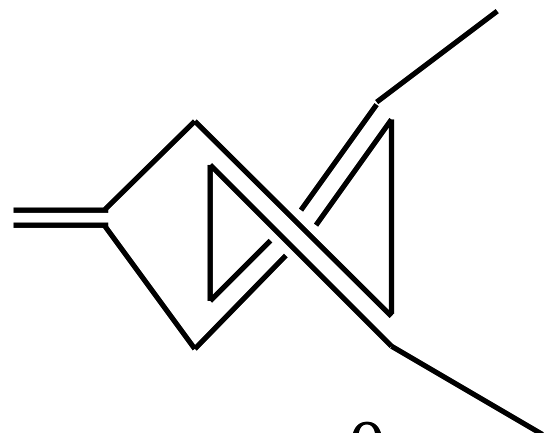
I_f : # of fermion propagators; L : # of total loops;

m : # of single line loop in double line rep.)



m=1

$N^{-1/2}$



m=0

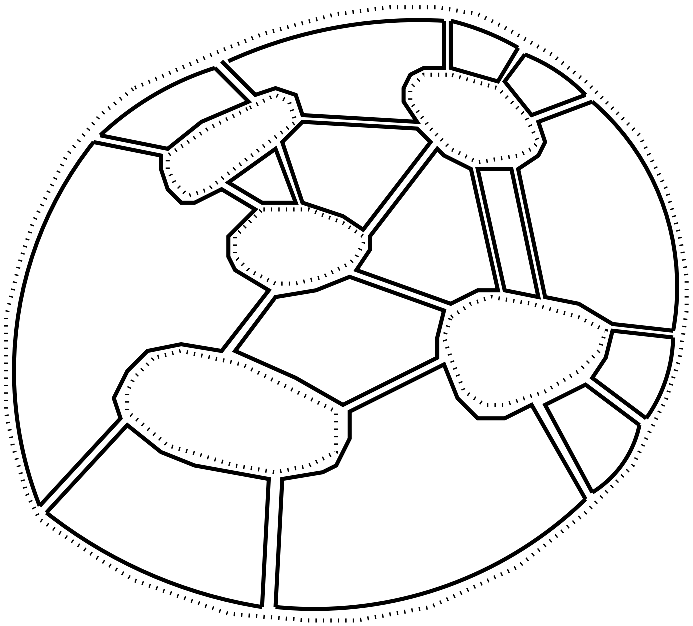
$N^{-3/2}$

Correct power counting : $N^{-\frac{V}{2}+N+[I_f-2L+m]}$

- For vacuum diagrams this becomes N^{-2g}

  (g : genus of the 2d surface on which Feynman graph is drawn without crossing in the double line representation)
- Planar diagram exactly follows this power
- Non-planar diagram may deviate from this nominal power
  - Cancellation due to even-odd symmetry
  - Log N corrections

Planar vacuum diagram

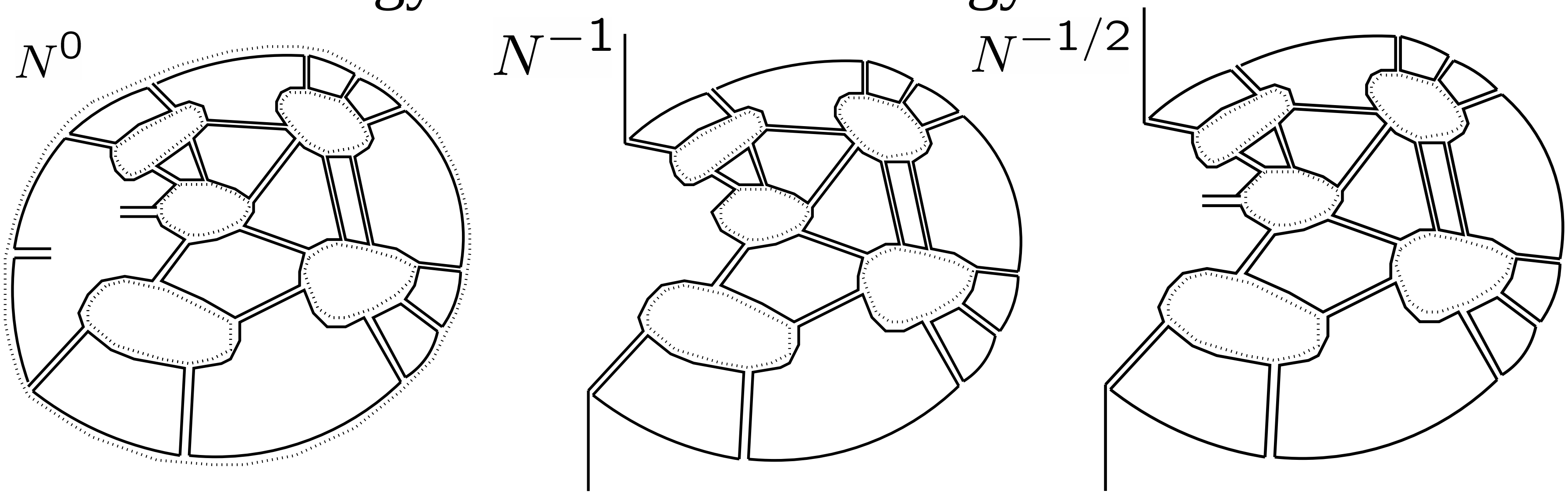


Planar diagram ~ N^0

- Infinitely many planar diagrams of the order of N^0

Leading order quantum corrections

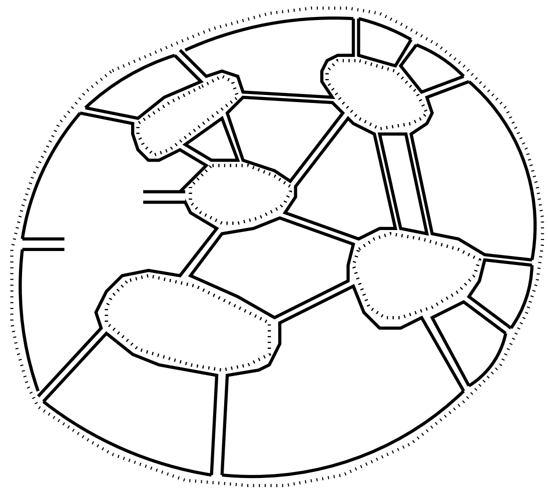
boson self energy | fermion self energy | vertex corrections



- There are infinitely many planar diagrams that determines the leading frequency dependence of fermion spectral function
- Fermions on FS remains strongly coupled even in the large N limit
- This is consistent with the assumption (motivated from the one-loop correction) that leads to this conclusion.

Non-renormalization of boson propagator beyond one-loop

N^0



All planar boson self energy graphs vanish beyond the one-loop due to kinematic constraint (chirality)

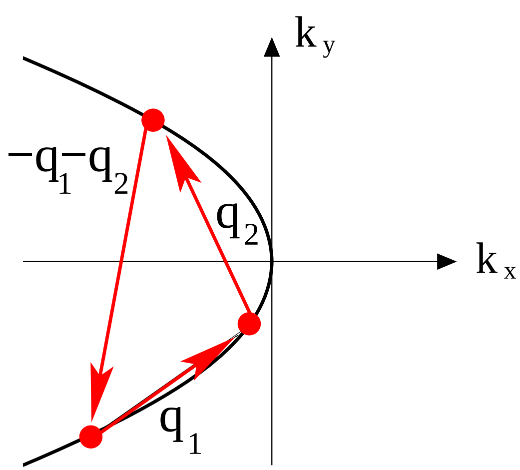
One-loop beta function is zero

→ The theory remains stable in the large N limit even though the theory is strongly coupled

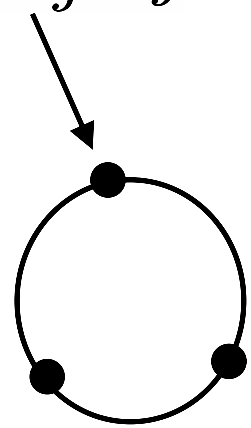
Correlation functions of gauge invariant operators

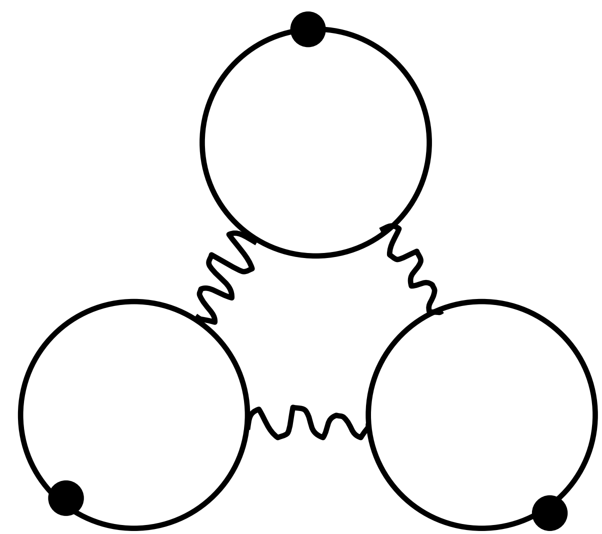
When external momenta connect two points on the Fermi surface, correlation function is strongly enhanced

→ infinitely many planar diagrams to be included.



$$\rho = \psi_j^* \psi_j$$





Summary II

- Fermi surface coupled with U(1) gauge field in 2+1D in the large N limit
  - Low dimensionality and the infinitely many gapless modes enhance IR quantum fluctuations : all planar diagrams are important even at $N=\infty$
  - Generic feature of 2+1D NFL
- It is expected that weakly coupled description is a string theory in a curved 1+1D space (likely to be highly curved, $\lambda\sim1$) – yet to be understood (matrix model under construction)

Hubbard Model : parent model of many phases (metal, SC, AF, Spin Liquid, …)