HOW FAST IS MAGNETIC RECONNECTION?

- MOTIVATION
- DEFINITIONS
- RECONNECTION IN WEAKLY IONIZED GAS

AXEL BRANDENBURG

FABIAN HEITSCH (NOW!)

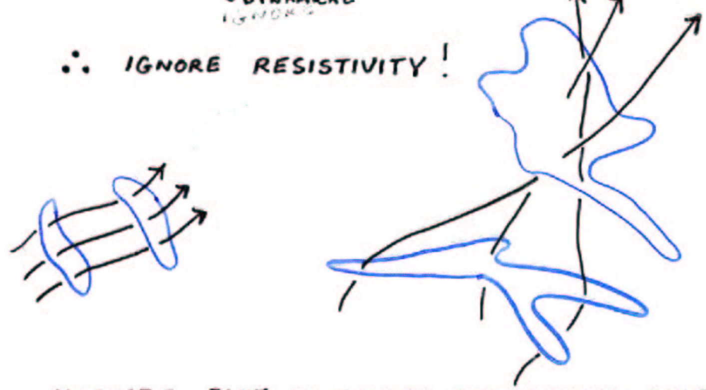
ASTRO-PH

0205103

DILEMMA:

$$\frac{\tau_{\text{RESISTIVE}}}{\tau_{\text{DYNAMICAL}}} \sim 10^{15} - 10^{21}$$ FOR INTERSTELLAR MEDIUM

∴ IGNORE RESISTIVITY!



MAGNETIC FLUX IS FROZEN TO COMOVING SURFACES;
FIELD PROGRESSIVELY COMPLEXIFIES

DYNAMO PROBLEM:

AMPLIFY WEAK FIELDS BY FLUID MOTIONS, BUT
LENGTH IS PROPORTIONAL TO STRENGTH

GALAXIES: $B_{\text{COSMOLOGICAL}} / B_{\text{GALACTIC}} \sim 10^{-12}$

HOW TO GENERATE A LARGE SCALE FIELD?

OHMIC DIFFUSION IS TOO SLOW TO DESTROY THE SMALL SCALE FIELD

MAGNETIC RECONNECTION:

- A HYBRID OF RESISTIVE & DYNAMICAL PROCESSES
- CREATES BOUNDARY LAYERS WITHIN WHICH

  $\tau_{RESISTIVE} \approx \tau_{DYNAMICAL}$
- IN SLOW RECONNECTION, GLOBAL RATE OF FIELD DESTRUCTION DEPENDS ON RESISTIVITY
- IN FAST RECONNECTION, FIELD IS DESTROYED AT A RATE INDEPENDENT OF RESISTIVITY

CANONICAL WISDOM:

RECONNECTION IN ASTROPHYSICS MUST BE FAST!

BASIC EQUATIONS:

$$\frac{\partial \vec{B}}{\partial t} = -c\,\vec{\nabla}\times\vec{E}$$

$$\vec{E} = -\frac{\vec{v}\times\vec{B}}{c} + \eta\vec{J} \qquad \vec{v} \equiv \text{PLASMA VELOCITY}$$

$$\vec{J} = \frac{c}{4\pi}\vec{\nabla}\times\vec{B}$$

COMBINE INTO MAGNETIC INDUCTION EQN:

$$\frac{\partial \vec{B}}{\partial t} = \underbrace{\vec{\nabla}\times(\vec{v}\times\vec{B})}_{\text{ADVECTION}} + \underbrace{\lambda_\Omega \nabla^2\vec{B}}_{\text{OHMIC DIFFUSION}} \qquad \lambda_\Omega \equiv \frac{\eta c^2}{4\pi}$$

ADVECTION $\tau_{ADV} \sim \frac{L}{V}$

OHMIC DIFFUSION $\tau_\Omega \sim \frac{L^2}{\lambda_\Omega}$

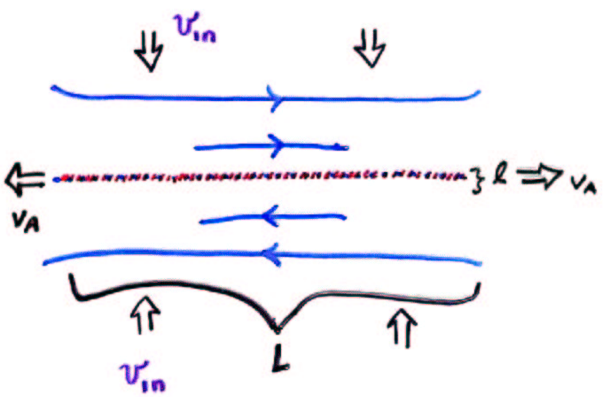
LUNDQUIST NUMBER:

SET V = MAGNETIC SIGNAL SPEED $V_A \equiv \frac{B}{\sqrt{4\pi\rho}}$

$$S \equiv \frac{\tau_\Omega}{\tau_{ADV}} = \frac{LV_A}{\lambda_\Omega}$$

$$S \sim 10^{15} - 10^{21} \text{ IN GALAXIES}$$

SWEET- PARKER RECONNECTION THEORY:



STEADY STATE

CONSERVATION OF MASS:

$$L v_{in} = \ell v_A$$

STEADY STATE E:

$$\frac{v_{in} B}{c} = \eta J \sim \frac{\eta c}{4\pi} \frac{B}{\ell}$$

COMBINE TO GIVE:

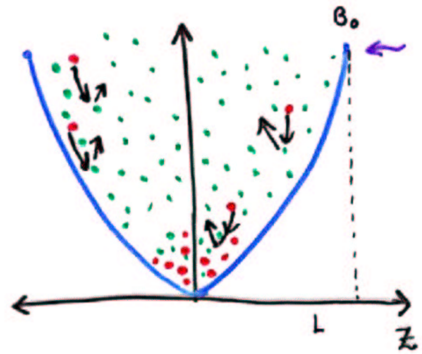
$$\ell = \left(\frac{\lambda_\Omega L}{v_A}\right)^{1/2} = L S^{-1/2}$$

SLOW RECONNECTION

$$v_{in} = v_A S^{-1/2}$$

ROBUST THEORY DESPITE CARTOONISH PRESENTATION

RECONNECTION IN WEAKLY IONIZED GAS



MAGNETIC PRESSURE $\frac{B^2}{8\pi}$ WITH A NULL AT $z = 0$

IONS ARE ACCELERATED DOWN THE MAGNETIC PRESSURE GRADIENT, AND DECELERATED BY COLLISIONS WITH NEUTRALS AND BY THEIR OWN PRESSURE GRADIENT

NEAR THE ORIGIN, B IS RESISTIVELY DESTROYED AND THE ION BUILDUP IS LIMITED BY RECOMBINATION

HOW FAST IS FIELD BROUGHT IN TO MAINTAIN $B = \pm B_0$ AT $z = \pm L$?

STEADY STATE EQUATIONS:

CONTINUITY:

$$\frac{d}{dz}\rho_i u_i = \underbrace{\zeta\rho_n}_{\text{IONIZATION}} - \underbrace{\alpha\rho_i^2}_{\text{RECOMBINATION}}$$

MOMENTUM:

$$\frac{d}{dz}\left(P_i + \frac{B^2}{8\pi}\right) = -\rho_i \nu_{in} u_i$$

PRESSURE GRADIENT BALANCES DRAG FORCE

INDUCTION:

$$\underbrace{u_i B}_{\text{INDUCTIVE TERM}} - \underbrace{\lambda_\Omega \frac{dB}{dz}}_{\text{RESISTIVE TERM}} = -cE = \text{CONSTANT}$$

SOLVE ON $0 < z < L$

$B(0) = 0, \quad B(L) = B_0$

$u_i(0) = 0, \quad \rho_i(L) = \rho_{io} \equiv (\zeta\rho_n/\alpha)^{1/2}$

4 CONDITIONS, 3 EQUATIONS, 1 EIGENVALUE: E

THE LIMIT OF INSTANTANEOUS RECOMBINATION/ ZERO ION PRESSURE

$$u_i = \frac{-B}{4\pi\rho_i\nu_{in}}\frac{\partial B}{\partial z}$$

LET $B = B_0 b$, $\rho_i = \rho_{io}$

$$\lambda_{AD} \equiv \frac{B_0^2}{4\pi\rho_i\nu_{in}}$$

INDUCTION EQN. BECOMES

$$(\lambda_{AD} b^2 + \lambda_\Omega)\frac{db}{dz} = \frac{cE}{B_0}$$

INTEGRATE & APPLY BOUNDARY CONDITIONS

$$\frac{cE}{B_0} = \frac{\lambda_{AD}}{3L}\left(1 + \frac{3\lambda_\Omega}{\lambda_{AD}}\right)$$

$$v_{in} = -\frac{cE}{B_0}$$

INDEPENDENT OF λ_Ω AS $\lambda_\Omega \to 0$ (FAST RECONNECTION)

NATURE OF THE FAST RECONNECTION SOLUTION

INDUCTIVE REGION:

$$b^2 \gg \frac{\lambda_\Omega}{\lambda_{AD}}$$

$$b \sim \left(\frac{z}{L}\right)^{1/3}$$

$$u_i \sim -\frac{\lambda_{AD}}{3L}\left(\frac{L}{z}\right)^{1/3}$$

RESISTIVE REGION:

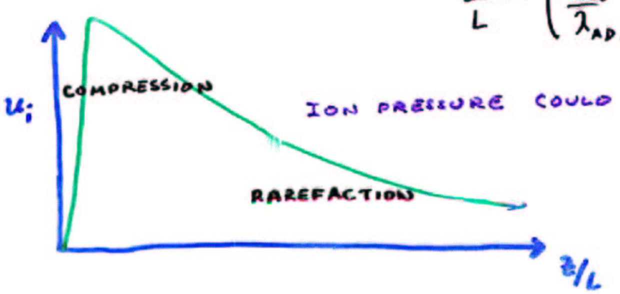
$$b^2 \ll \frac{\lambda_\Omega}{\lambda_{AD}}$$

$$b \sim \frac{\lambda_{AD}}{3\lambda_\Omega}\frac{z}{L}$$

$$u_i \sim -\left(\frac{\lambda_{AD}}{3\lambda_\Omega}\right)^2 \frac{\lambda_{AD} z}{L^2}$$

WIDTH OF RESISTIVE LAYER:

$$\frac{z_\ell}{L} \sim \left(3\frac{\lambda_\Omega}{\lambda_{AD}}\right)^{3/2}$$



ION PRESSURE COULD BE IMPORTANT

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PARAMETER SPACE CHARACTERIZED BY TIMESCALES

OHMIC TIMESCALE $\tau_\Omega \equiv L^2/\lambda_{AD}$

$$\tau_\Omega = 6.54\ 10^{16} L_{pc}^2 \frac{T^{3/2}}{\left[1 + 1.2\ 10^{-11} T^2 \chi_i^{-1}\right]} \quad \mathrm{yr}$$

AMBIPOLAR TIMESCALE $\tau_{AD} \equiv L^2/\lambda_\Omega$

$$\tau_{AD} = 9.40\ 10^{-3} L_{pc}^2 \left(\frac{n_n}{B}\right)^2 \frac{\mu_i \mu_n}{\mu_i + \mu_n} \chi_i \quad \mathrm{yr}$$

RECOMBINATION TIMESCALE τ_i

DISSOCIATIVE RECOMBINATION

$$\tau_i = \frac{2.30\ 10^{-3}}{T^{3/4} \chi_i n_n} \quad \mathrm{yr}$$

RECOMBINATION ON GRAINS

$$\tau_i = 1.30\ 10^4 \frac{\mu_i^{1/2} T^{1/2}}{n_n} \quad \mathrm{yr}$$

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PARAMETERS FOR DIFFUSE & MOLECULAR CLOUDS

DIFFUSE CLOUDS

$n_n = 10^2, \quad T = 80\,K, \quad x_i = 10^{-4} \quad B_0 = 10^{-5}\,G$

$\tau_\Omega = 1.10\ 10^{19}\ L_{pc}^2\ yr$

$\tau_{AD} = 9.40\ 10^{7}\ L_{pc}^2\ yr$

$\tau_i = 3.4\ 10^{3}\ yr$

$\beta_i \equiv \frac{16\pi P_i}{B_0^2} = 4.6\ 10^{-5}$

MOLECULAR CLOUDS

$n_n = 10^4, \quad T = 10\,K, \quad x_i = 10^{-7} \quad B_0 = 5\ 10^{-5}\,G$

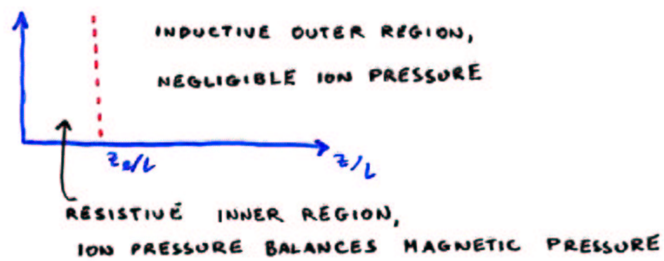
$\tau_\Omega = 4.87\ 10^{17}\ L_{pc}^2\ yr$

$\tau_{AD} = 8.65\ 10^{7}\ L_{pc}^2\ yr$

$\tau_i = 7.6\ yr$

$\beta_i = 2.3\ 10^{-8}$

ANALYTICAL ESTIMATES



OUTER SOLUTION

$$b = \left(\frac{z + z_*}{L + z_*}\right)^{1/3}$$

$$\frac{cE}{B_0} = \frac{\lambda_{AD}}{3(L + z_*)}$$

z_* A CONSTANT OF INTEGRATION WHICH PARAMETERIZES E

INNER SOLUTION

$$u_i = -\frac{z}{\tau_d}$$

$$\rho_i \approx \rho_{im}$$

4 QUANTITIES TO SOLVE FOR:

$$z_*,\ z_\ell,\ \tau_d,\ \rho_{im}$$

4 RELATIONS

(1) CONTINUITY EQN AT $z=0$:

$$\frac{\rho_m}{\rho_{io}} = \frac{\tau_i}{2\tau_d} + \sqrt{1+\left(\frac{\tau_i}{2\tau_d}\right)^2} \sim \frac{\tau_i}{\tau_d}$$

(2) INDUCTIVE & RESISTIVE CONTRIBUTIONS TO E ARE EQUAL AT $z=z_\ell$

$$\frac{z_\ell}{\tau_d} \sim \frac{\lambda_R}{z_\ell}$$

(3) ION PRESSURE BALANCES MAGNETIC PRESSURE AT $z=z_\ell$

$$\beta\left(\frac{\rho_{im}}{\rho_{io}}\right)^\gamma \sim \left(\frac{z_\ell+z_*}{L+z_*}\right)^{2/3}$$

(4) MASS FLUX IS CONTINUOUS AT $z=z_\ell$

$$\frac{z_\ell}{\tau_d}\rho_{im} = \frac{\rho_{io}\lambda_{AD}}{3(L+z_*)^{2/3}(z_\ell+z_*)^{1/3}}$$

CRANK...

$$z_* = z_*(Z)$$

$$Z \equiv \beta^{3/\gamma}\frac{\tau_i\tau_R}{9\tau_{AD}^2}$$

WHEN $Z \ll 1$,

$$\frac{z_*}{L} \sim Z^{\frac{3\gamma}{6+2\gamma}} = Z^{15/28} \quad \text{FOR } \gamma = 5/3$$

WHEN $Z \gg 1$,

$$\frac{z_*}{L} \sim Z^{1/2}$$

FAST RECONNECTION REQUIRES $Z < 1$

LARGE Z LIMIT

$$v_{in} \rightarrow \left(\frac{\lambda_\Omega}{\beta^{3/4}\tau_i}\right)^{1/2}$$

$$\frac{v_{in}}{v_{SP}} = \left(\frac{L_{SP}}{\beta^{3/4}\tau_i V_A}\right)^{1/2} \sim \left(\frac{\tau_A}{\beta^{3/4}\tau_i}\right)^{1/2} \gg 1$$

$$z_\ell \rightarrow \left(\beta^{1/4}\tau_i \lambda_\Omega\right)^{1/2}$$

$$\frac{z_\ell}{z_{SP}} \rightarrow \left(\beta^{1/4}\frac{\tau_i}{\tau_A}\right)^{1/2} \ll 1$$

THE LARGE Z LIMIT IS FORMALLY SLOW,

BUT FAST IN PRACTICE

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NUMERICAL RESULTS

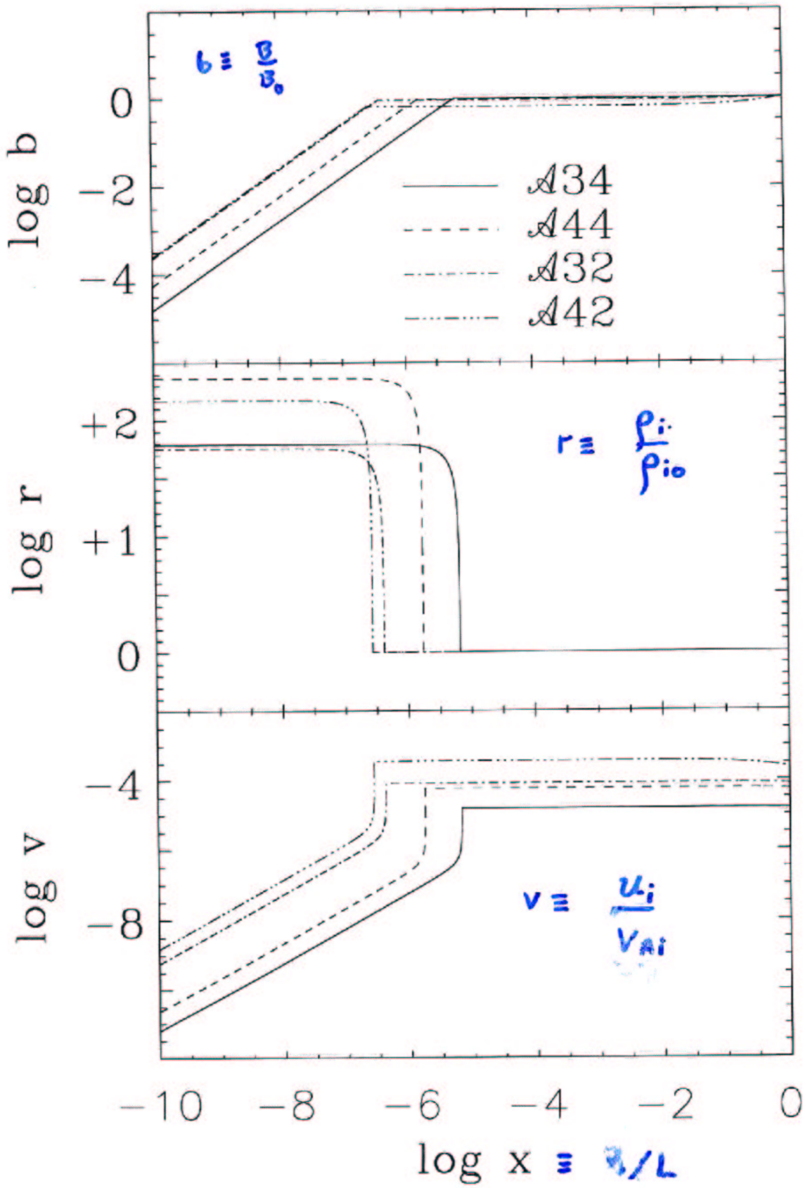
SIMULATION PARAMETERS

Model	β	τ_i	τ_{AD}	τ_A	τ_Ω	Z/τ_Ω
$\mathcal{A}34$	10^{-3}	10^4	10^4	10	*	4.4×10^{-11}
$\mathcal{A}44$	10^{-4}	10^4	10^4	10	*	7.0×10^{-13}
$\mathcal{A}32$	10^{-3}	10^2	10^4	10	*	4.4×10^{-13}
$\mathcal{A}42$	10^{-4}	10^2	10^4	10	*	7.0×10^{-15}

NOTE.—Time scales (in years) and β for the simulations. For all runs of type $\mathcal{A}$ we have $10^6 \leq \tau_\Omega \leq 10^{14}$, denoted by *. See §2.2.6 for a discussion of the physical regimes covered. The first digit in the model name indicates β, the second τ_i.

$$7\times 10^{-9} \leq Z \leq 4.4\ 10^3$$

LAYER PROPERTIES



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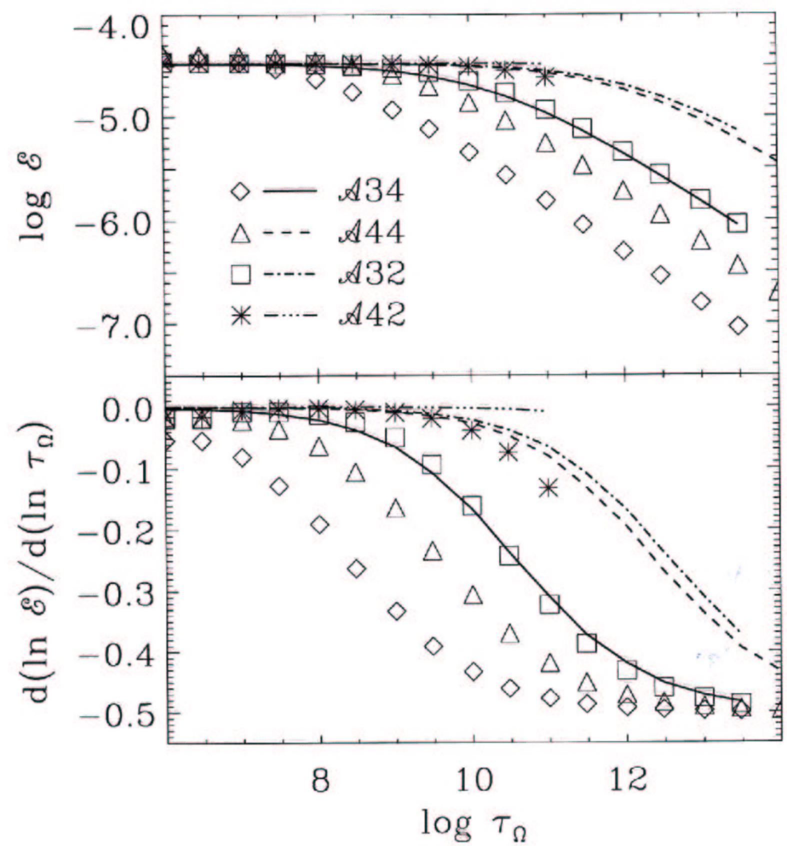
ELECTRIC FIELD VS τ_Ω -

SIMULATIONS & PREDICTIONS



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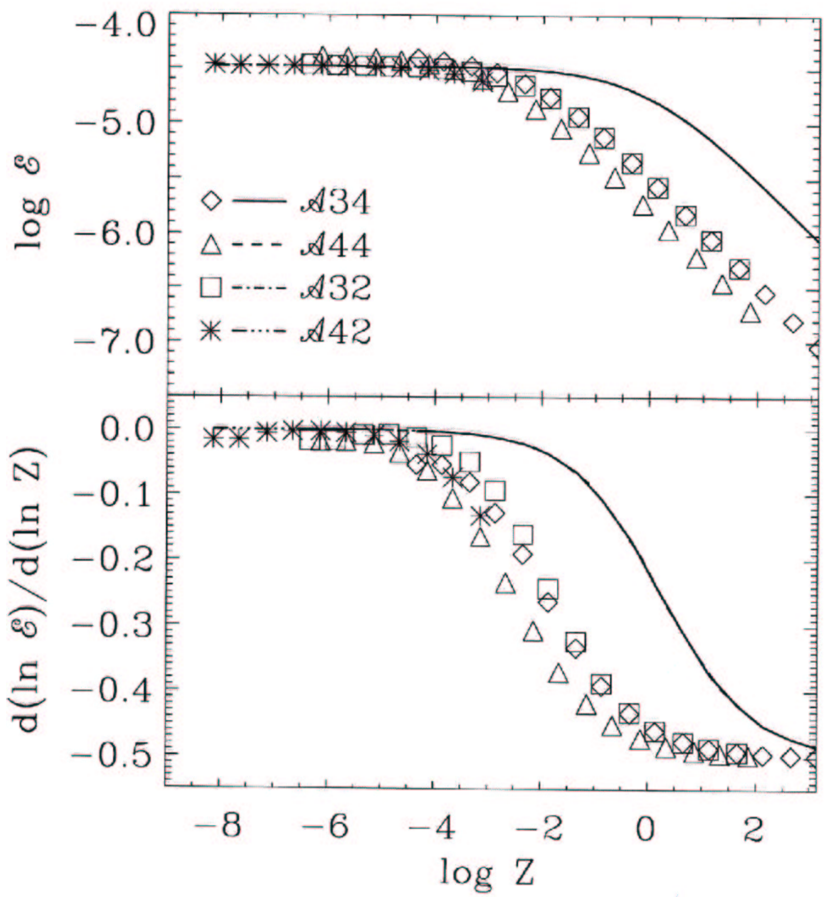
ELECTRIC FIELD IS WELL CORRELATED WITH Z PARAMETER



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THE CHOICE OF L

(1) ION-NEUTRAL DECOUPLING SCALE L_{AD}

$$L_{AD} \equiv \frac{\lambda_{AD}}{v} = \frac{v_A}{v}\left(\tau_{ni}\, v_A\right)$$

WHERE

$$v = \max\left(\left(v_A^2 + v_s^2\right)^{1/2},\ v_{fluid}\right)$$

MUST HAVE $\boxed{L < L_{AD}}$

(2) IONS MUST BE COLLISIONAL

$$L > L_{col} \equiv \frac{v_{si}}{\nu_{in}} > \frac{u_i}{\nu_{in}}$$

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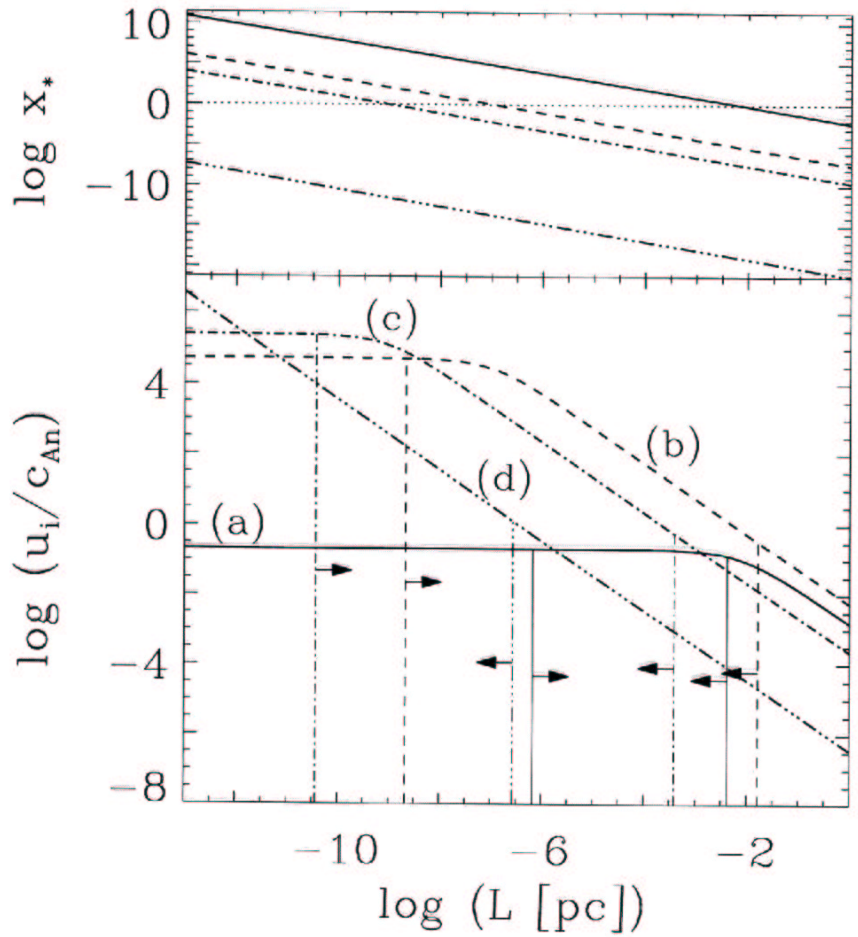
FIDUCIAL PARAMETERS FOR 4 ENVIRONMENTS

quantity	(a)	(b)	(c)	(d)
n_n [cm^{-3}]	10^2	10^4	10^6	10^{13}
μ_n	1.3	2.2	2.2	2.2
μ_i	12	29	29	40
x_i	10^{-4}	10^{-7}	10^{-8}	10^{-12}
B_0 [G]	10^{-5}	5×10^{-5}	2×10^{-4}	1
T_n [K]	80	10	30	500
c_{si} [cm/s]	3.0×10^4	6.9×10^3	1.2×10^4	4.1×10^4
c_{Ai} [cm/s]	6.3×10^6	6.4×10^7	8.1×10^7	1.1×10^{10}
c_{sn} [cm/s]	9.2×10^4	2.5×10^4	4.3×10^4	1.8×10^5
c_{An} [cm/s]	1.9×10^5	7.4×10^4	3.0×10^4	4.7×10^4
τ_i [yr]	3.4×10^3	7.6×10^0	2.9×10^{-1}	1.6×10^{-7}
β	4.6×10^{-5}	2.3×10^{-8}	4.3×10^{-8}	2.9×10^{-11}
r_{gyro} [cm]	2.6×10^6	2.9×10^5	1.3×10^5	1.2×10^2
L_{AD} [pc]	4.2×10^{-3}	1.6×10^{-2}	3.9×10^{-4}	2.7×10^{-7}
L_{col} [pc]	6.7×10^{-7}	2.1×10^{-9}	3.6×10^{-11}	1.2×10^{-17}

Table 1: Parameters for (a) diffuse clouds, (b) dense clouds, (c) cores and (d) protoplanetary disks. $\gamma = c_p/c_v = 5/3$. Molecular weights μ, sound speed c_s and Alfvén velocity c_A refer to the ions or neutrals, depending on their index i or n. Plasma β refers to the ions. The ion gyroradius $r_{gyro} = c\sqrt{kT\mu_i m_H}/eB$.

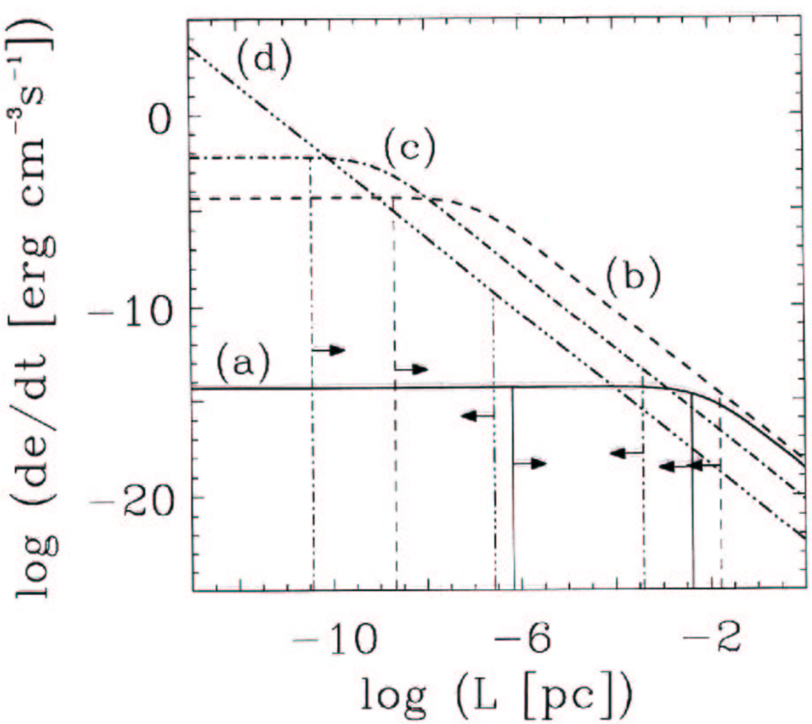
RECONNECTION RATES vs L

(a) ≡ DIFFUSE CLOUDS
(b) ≡ MOLECULAR CLOUDS
(c) ≡ CLOUD CORES
(d) ≡ PROTOPLANETARY DISKS



LARGE OHMIC HEATING RATES IN THE RESISTIVE LAYER

DO THEY QUENCH THE RECONNECTION PROCESS?



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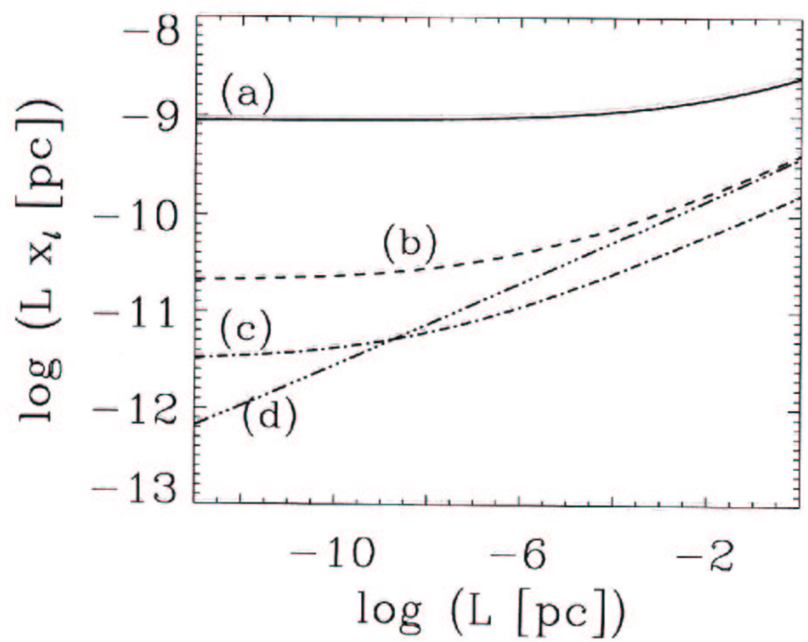
RESISTIVE LAYERS ARE EXTREMELY THIN, WITH LOW COLUMN DENSITIES & LOW EMISSION MEASURES



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EFFECTS BEYOND THE MODEL

- INTENSE OHMIC HEATING: INCREASE τ, SLOW DOWN RECONNECTION
- INSTABILITY OF ELECTRON DISTRIBUTION FUNCTION: DRIVE FLUCTUATIONS, ENHANCE RESISTIVITY
- SELF-IONIZATION OF LAYER BY DRIFT OR RANDOM ELECTRON MOTIONS (TURNS WHOLE THING OFF?)
- SHEARED FIELD INSTEAD OF NULL LAYER:
  1-D FLOW IS DECELERATED BY MAGNETIC PRESSURE
  PREVIOUS WORK ON TEARING MODES SUGGESTS RECONNECTION RATE IS SLOW

BUT, WITH THESE CAVEATS,

CONCLUSIONS

- AMBIPOLAR DRIFT CAN STEEPEN MAGNETIC NULL LAYERS TO NEAR SINGULARITY
- WHEN ION PRESSURE IS IGNORABLE, MAGNETIC MERGING PROCEEDS AT A RATE INDEPENDENT OF THE RESISTIVITY
- ION PRESSURE SLOWS THE RECONNECTION RATE TO THE PARKER-SWEET RATE, WITH THE FLOW TIME REPLACED BY THE IONIZATION TIME, REDUCED BY A POWER OF β.
- THE RECONNECTION RATE IS DETERMINED BY THE PARAMETER

$$\mathcal{Z} \equiv \frac{\beta^{3/8}\,\tau_i\,\tau_\Omega}{9\,\tau_{AD}^2}$$

  VALIDATED BY SIMULATIONS OVER 8 ORDERS OF MAGNITUDE IN τ_Ω
- DUE TO SHORT ION LIFETIMES, EVEN SLOW RECONNECTION IS FAST.