What We Can Learn from being Free and Tensionless

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Based on works with:
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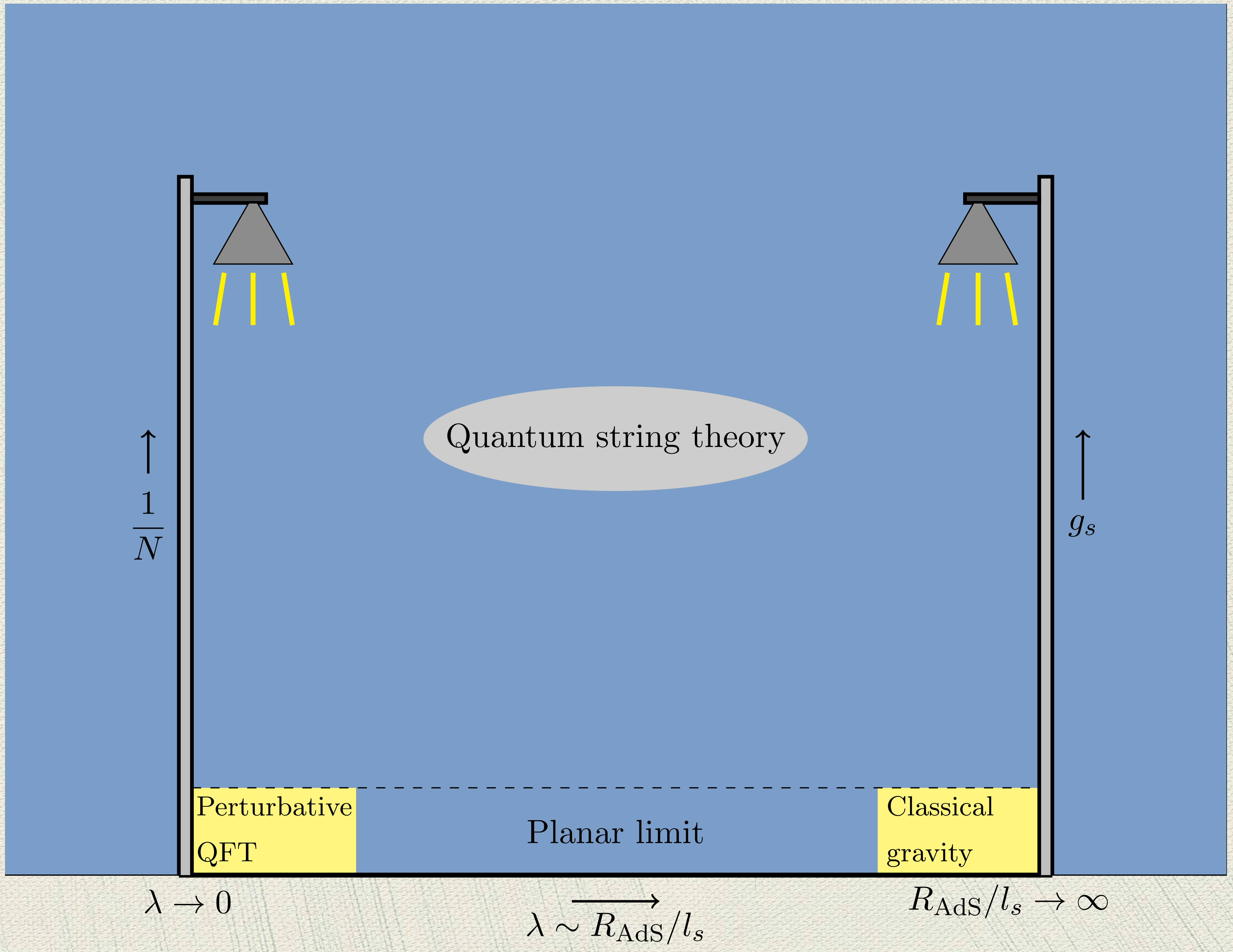
The Other Lamp Post



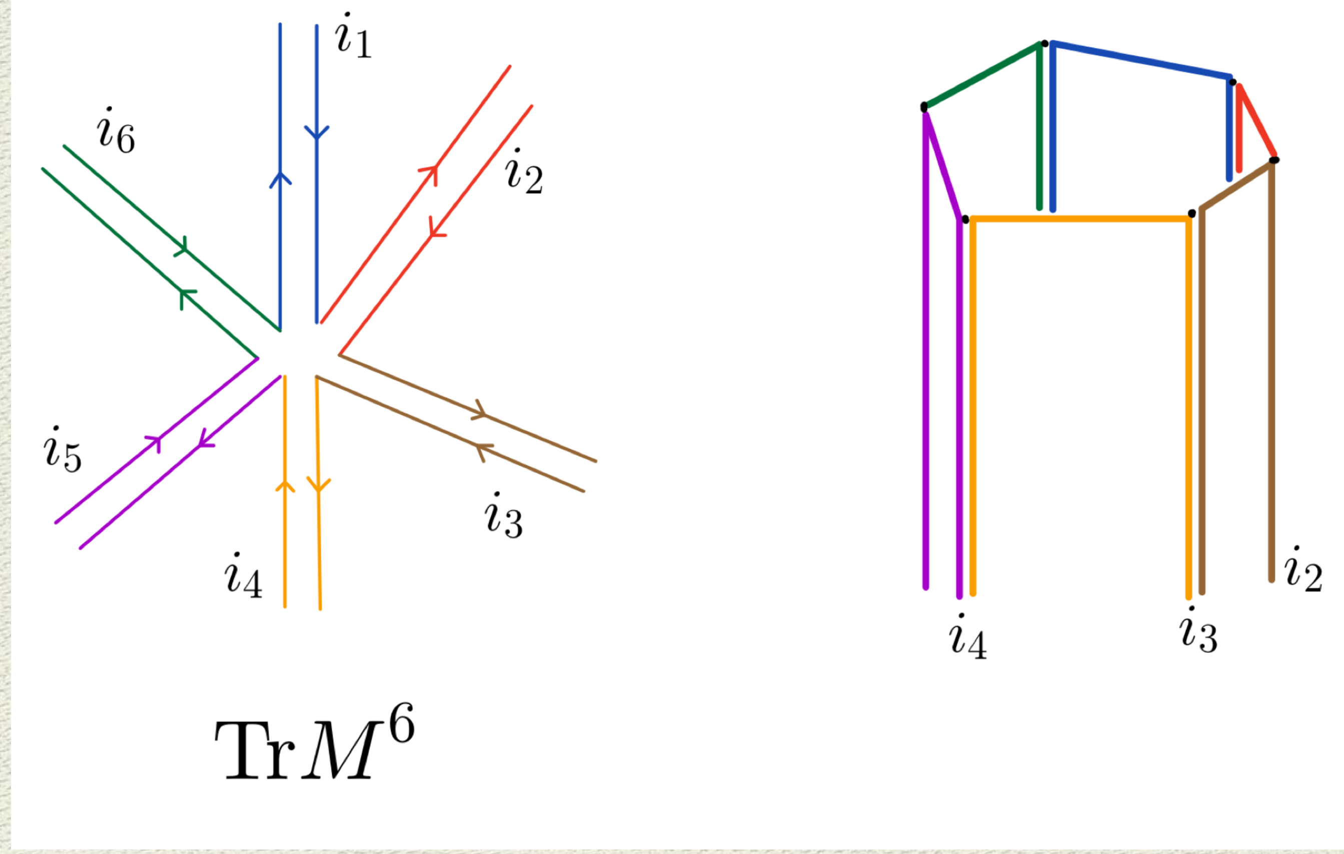
- Focus on the corner where we understand the QFT but not necessarily the bulk.
- Free (perturbative) QFT as $\lambda \to 0$
  $\leftrightarrow$ 'Tensionless' limit of dual string theory.
  [Sundborg, Sezgin-Sundell, Witten…]
- Finite # of Feynman diagrams, of any genus.
  Power series in λ given by Wick contractions.
- Test Cases:
  Matrix Model $\leftrightarrow$ Top. A/B-Model String.
  $(T^4)^N/S_N \leftrightarrow AdS_3 \times S^3 \times T^4$.
  Pert. $\mathcal{N} = 4$ SYM $\leftrightarrow AdS_5 \times S^5$.

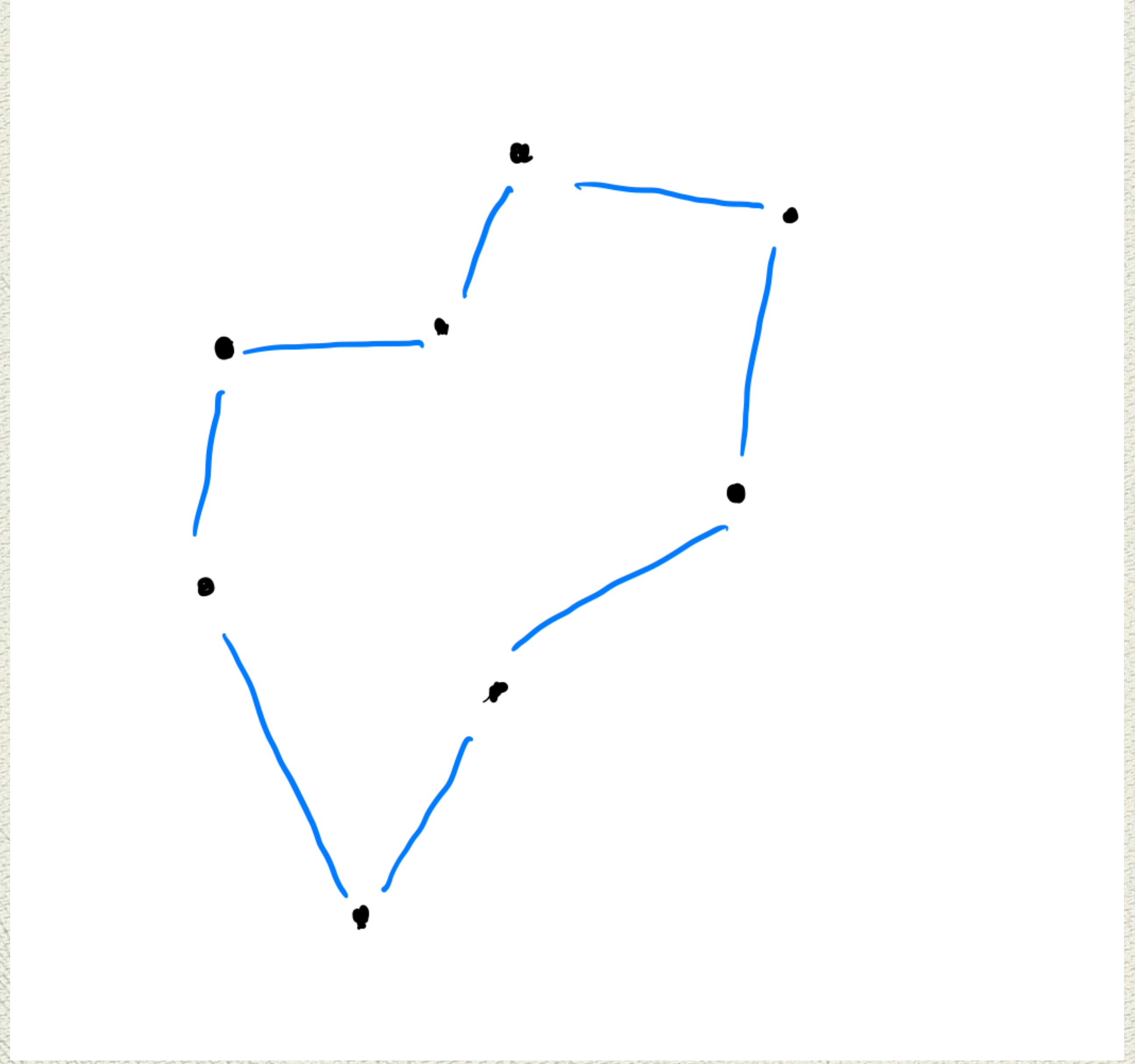
Gleaning General Lessons

- What might be general features in the free/tensionless limit?
- Can we build a universal bridge between perturbative gauge theories and dual string theories: explicitly understand how large N theories reassemble into worldsheet theories (a la 'tHooft)?
  - String Bits on the worldsheet.
  - Gross-Mende like behaviour.
  - Open-Closed-Open Triality.
  - Higher Spin/Extended Symmetry.
  - A Worldsheet-Spacetime Correspondence.
  - Bulk spacetime as a construct (even in the closed string description)?

Bit from QuBit?

- Gauge theory operator built out of individual qubits (cf. spin chain) realise a string bit picture [BMN, Thorn, Klebanov-Susskind].
- String Bits glued together by gauge constraint to form a semi-rigid string - fixed # of beads connected by rods.
- Precise dictionary between Feynman diagram and string worldsheet through Strebel construction of $\mathscr{M}_{g,n}$.
- A given correlator has support only on a finite number of points on moduli space - specific allowed shapes.
- Delta functions on $\mathscr{M}_{g,n}$ characteristic of a topological string.



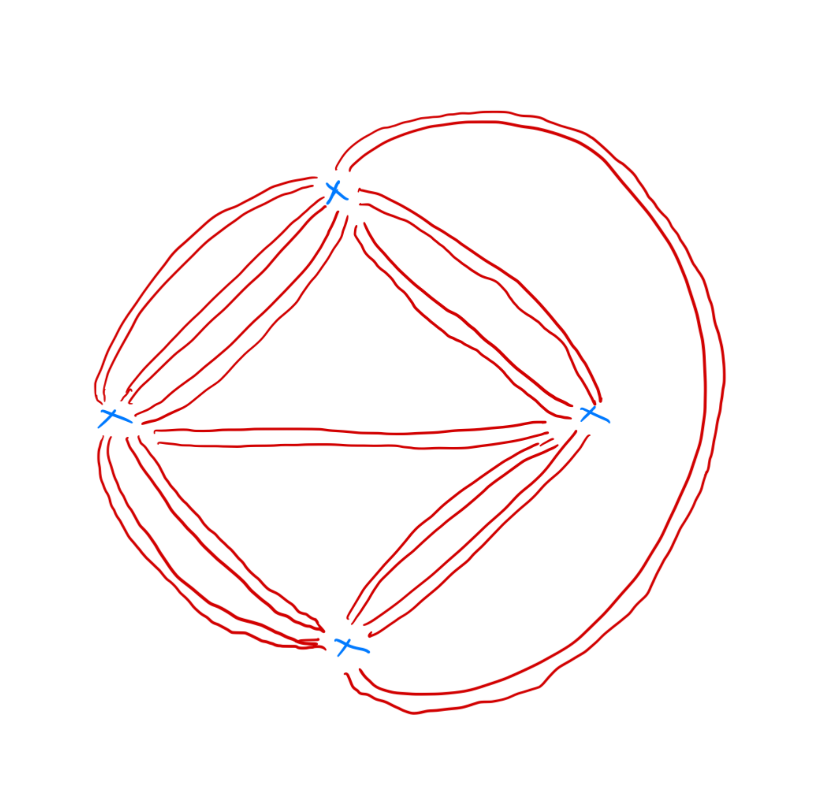


Bridge from Fields to Strings

- Explain how individual Feynman diagrams translate to specific world sheets - points on $\mathcal{M}_{g,n}$.
- Pattern seen in our test cases: Matrix Models, Free Symmetric Product Orbifold CFT (and very likely, Pert. Yang-Mills).
- Feyn. Diags. $\leftrightarrow$ Permutations $\leftrightarrow$ Branched Coverings $\leftrightarrow$ Special points on $\mathcal{M}_{g,n}$.
- Matches with worldsheet amplitude - delta functions on $\mathcal{M}_{g,n}$. Weight also seems to be the natural geometric one (Nambu-Goto).

Feynman diagrams ↔ Permutations

- Feynman diagrams for a correlator enumerate Wick contractions. View in terms of counting permutations.



- Matrix Model: $N^n \langle \prod_{i=1}^{n} \mathrm{Tr} M^{k_i} \rangle_{conn}^{(g)} = N^{2-2g} \sum_{\alpha \in S_k} \delta(1, \alpha \cdot \beta \cdot \gamma) \mathbf{1}$.

α, β, γ are conjugacy classes in S_k, ($k = \sum k_i$)

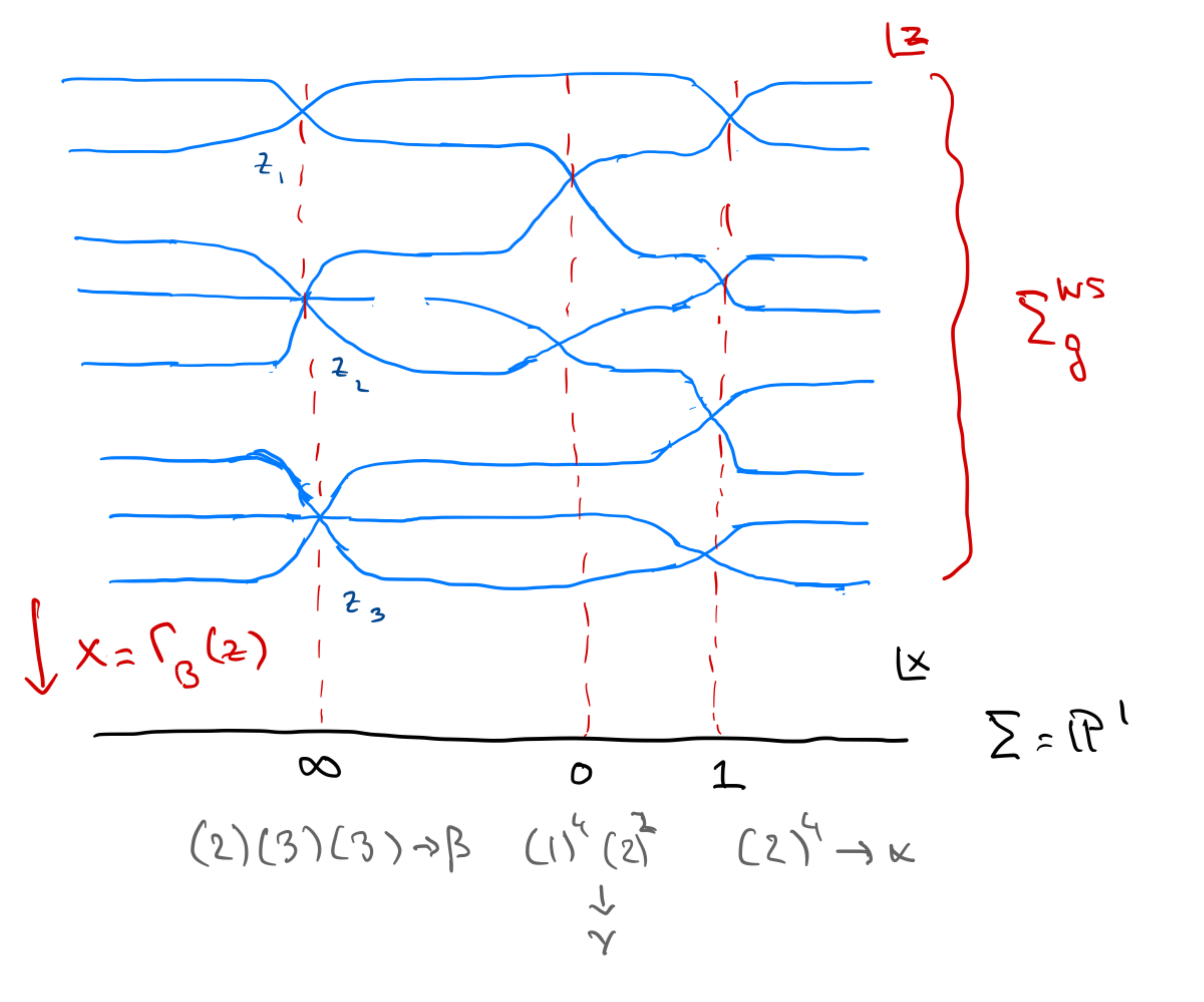
- β indicates the cyclic structure of edges around each vertex $(k_1) \ldots (k_n)$;
- $\alpha \in (2)^{k/2}$ labels different Wick contractions of edges;
- $\gamma = \beta^{-1} \alpha^{-1}$ labels edges around a face.

[Itzykson et.al., de Mello Koch-Ramgoolam]

- Symm. Orbfld. CFT: $\langle \prod_{i=1}^{n} \sigma_{k_i}(x_i) \rangle_{conn}^{(g)}$ labelled by n distinct single cycles (k_i). Correlator is a sum over permutations that sew together these cycles.

Permutations ↔ Branched Coverings

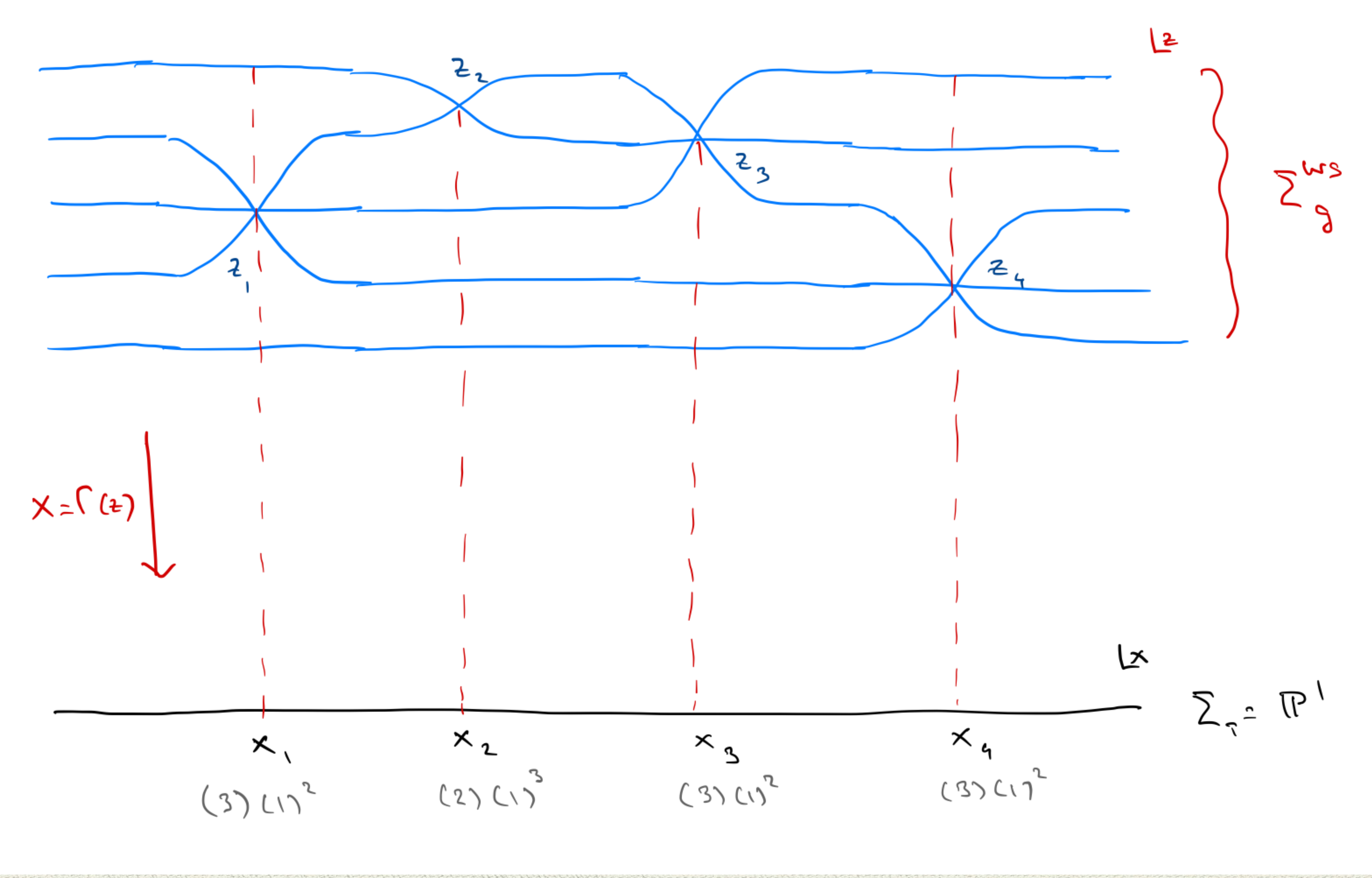
- Enumerating such permutations is the same as counting branched covers.
- Each permutation in a conjugacy class associated to a branch point with definite ramification (cycle structure).
- For Matrix correlators, (α, β, γ) associated with three branch points of Σ_g over $\mathbb{P}^1$ $(\alpha \cdot \beta \cdot \gamma = 1)$.
- `Belyi maps' - only admissible at arithmetic points on $\mathscr{M}_{g,n}$.



[A contribution to $\langle \mathrm{Tr}M^2\, (\mathrm{Tr}M^3)^2 \rangle$]

Permutations ↔ Branched Coverings

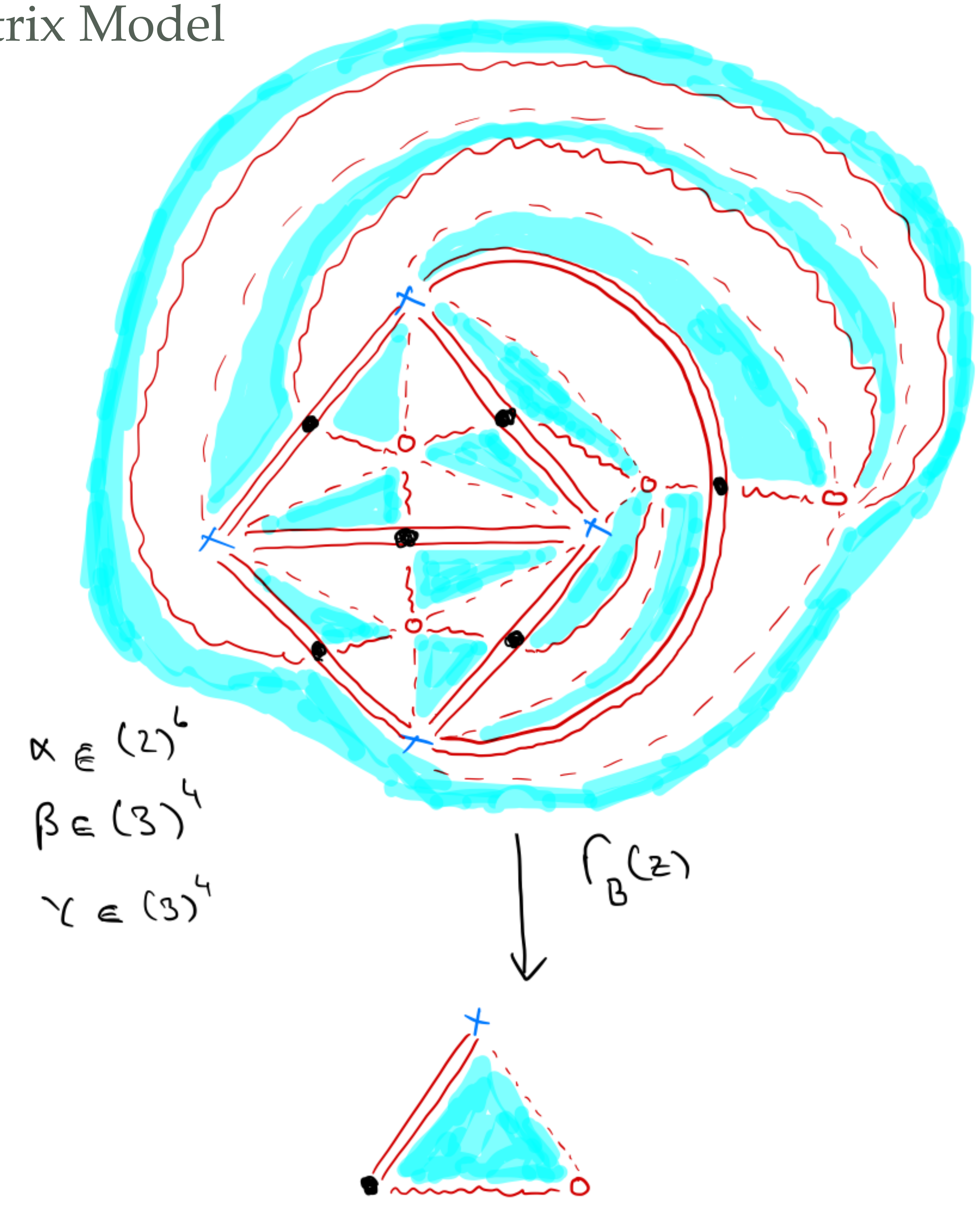
- This connection also underlies the Lunin-Mathur computation of symm. orbfld. CFT correlators.
- Lift to covering space geometrises permutations.
- x_i-dependence of correlators now means n points w/ branching k_i.
- Admissible only at discrete points on $\mathcal{M}_{g,n}$ (depend on x_i).



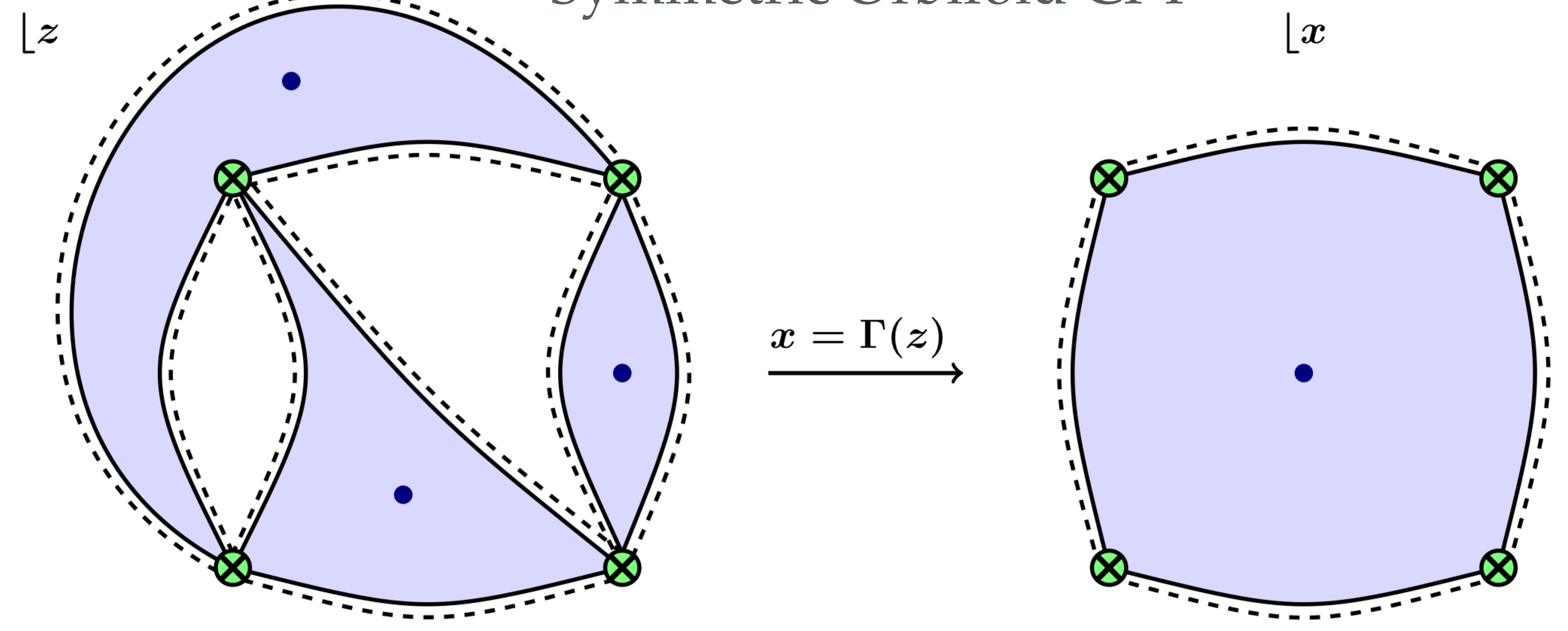
[A contribution to $\langle \sigma_3(x_1)\sigma_2(x_2)\sigma_3(x_3)\sigma_3(x_4)\dots\rangle$]

Feynman diagrams ↔ Branched Covers

Matrix Model



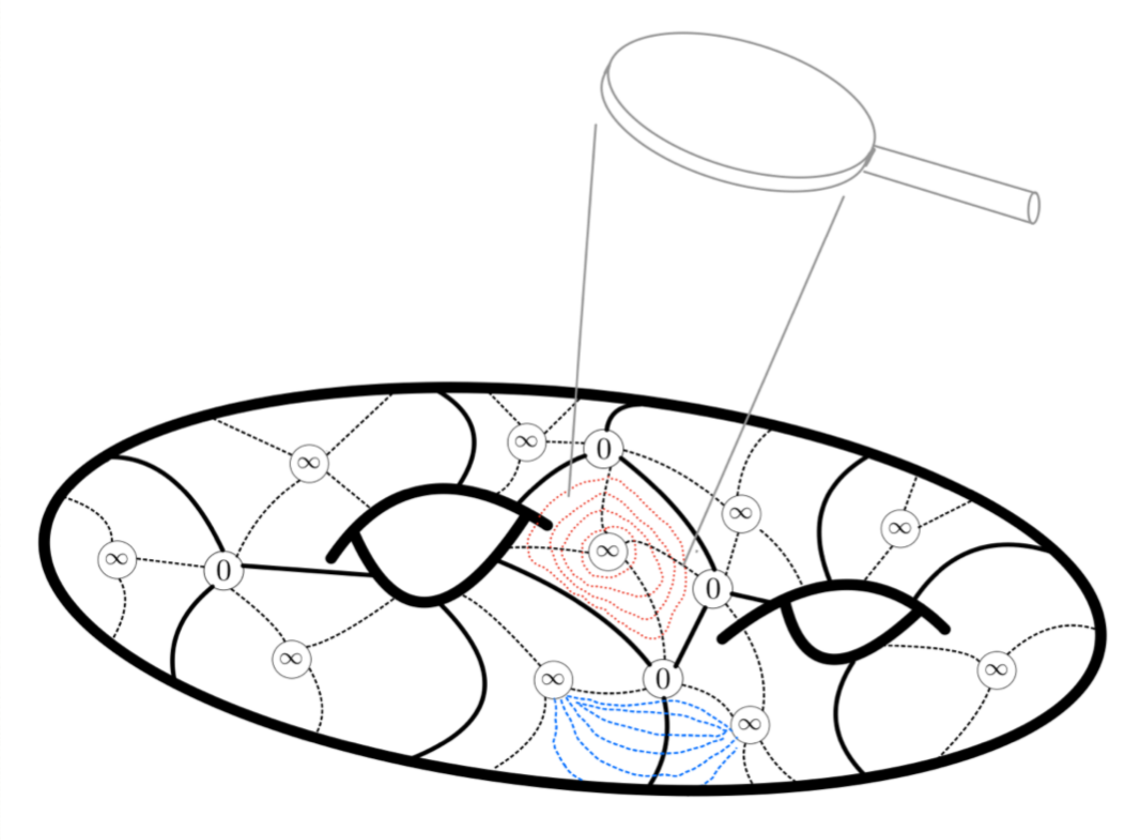
Symmetric Orbifold CFT

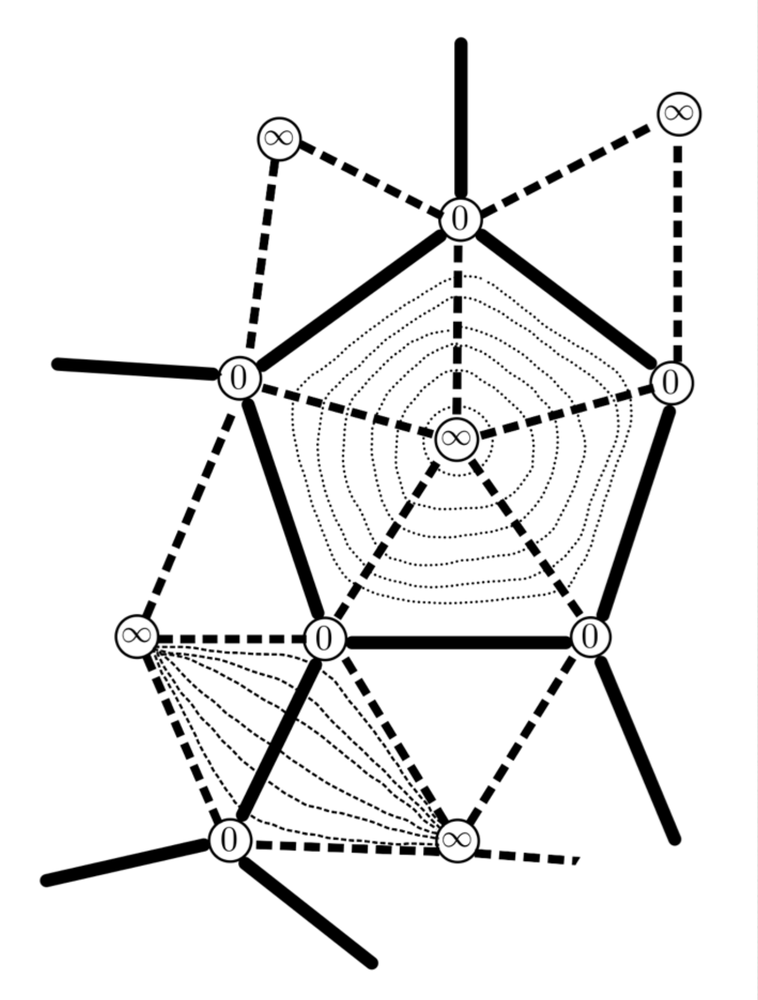


- Feynman diagrams directly pictured as covering space worldsheets pulled back from target space.
  [cf. Pakman, Rastelli, Razamat]
- 1-1 correspondence between branched covers and distinct Feynman diagrams.

The Strebel Construction

- The point on $\mathscr{M}_{g,n}$ associated to a Feynman diagram given by an explicit gluing construction of strips - string bit worldsheets.





- Relies on the parametrisation of $\mathscr{M}_{g,n}$ by a unique quadratic (Strebel) differential $\phi_S(z)dz^2$. Has n double poles at marked points.
- It's `horizontal trajectories' - $\phi_S(z(t))\left(\frac{dz(t)}{dt}\right)^2 > 0$ - foliate $\Sigma_{g,n}$ into n disk faces each with pole at z_i, separated by a critical graph. Vertices of this Strebel graph are zeroes a_m of ϕ_S.
- Real Strebel lengths $l_{km} = \int_{a_k}^{a_m} \sqrt{\phi_S(z)}dz$ and critical graph topology parametrises $\mathscr{M}_{g,n}$.

Strebel Construction (Contd.)

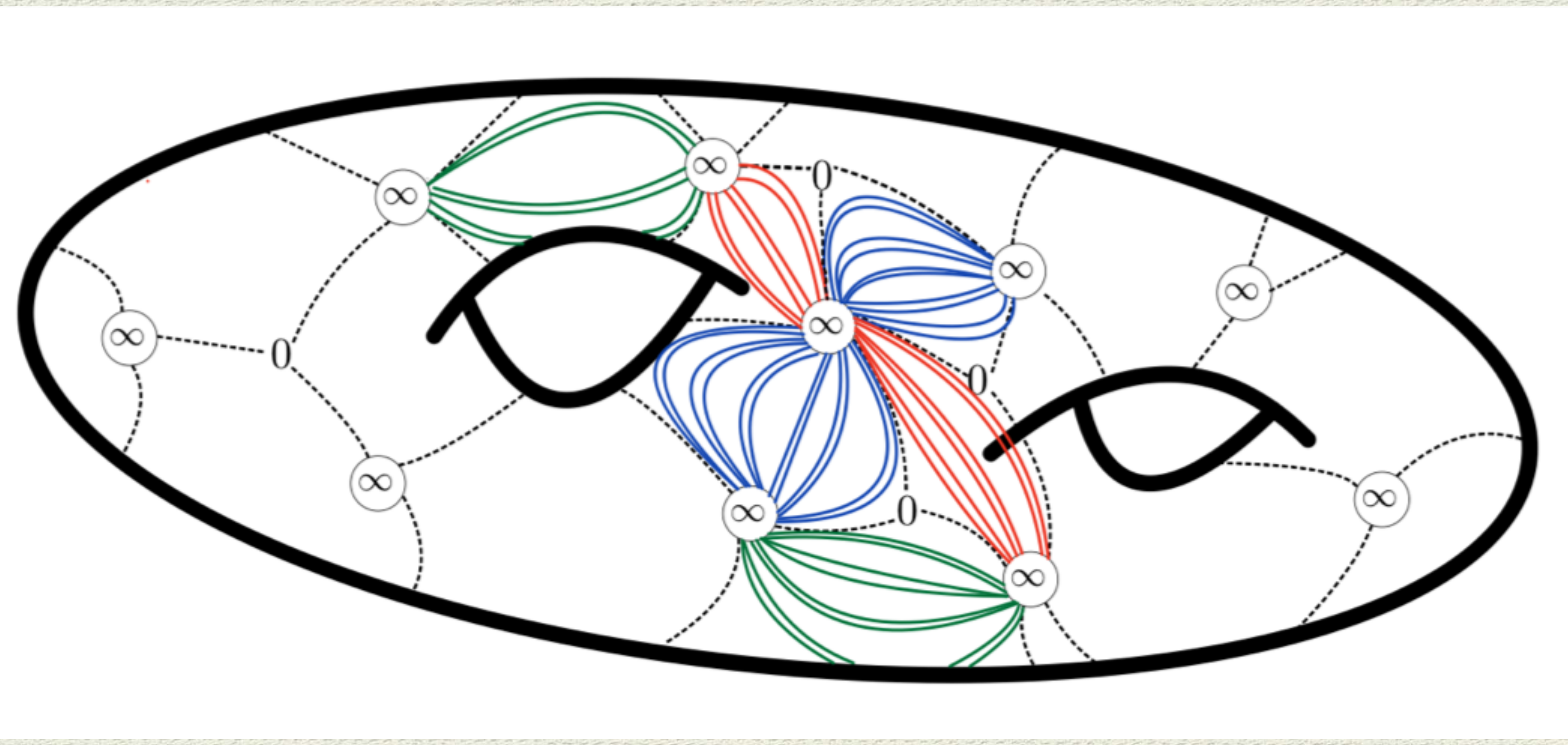
- \`Vertical trajectories' - $\phi_S(z(t))\left(\frac{dz(t)}{dt}\right)^2 < 0$, begin and end on poles z_i. Generates strips, glued at centres of faces (poles) and vertices of Strebel graph (zeroes) correspond to Feynman propagators.

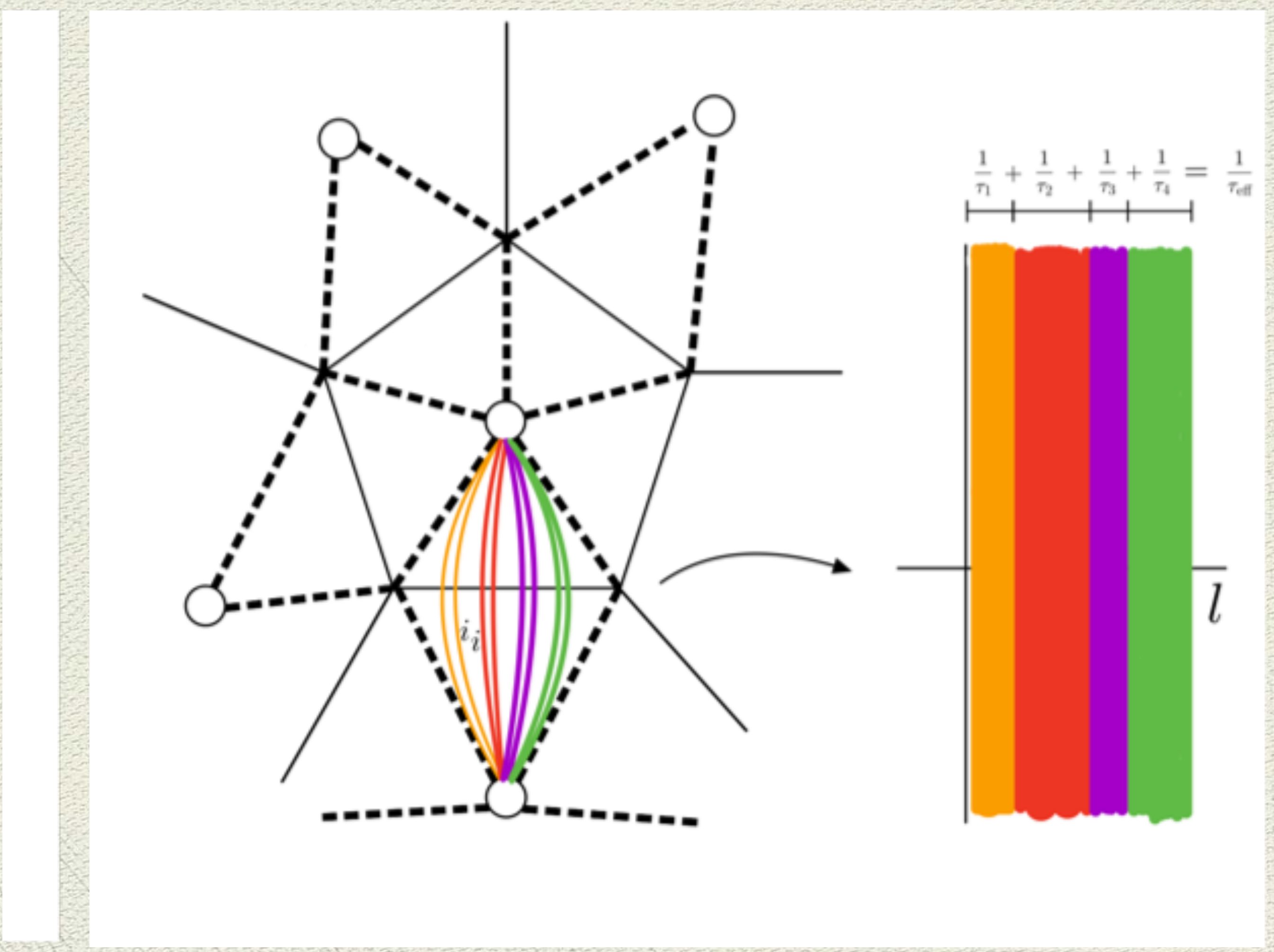
- Feynman graph of corresponding worldsheet is dual to the Strebel graph. [Strebel graph another open string description - Triality!]

- Integer Strebel lengths (# of wick contractions in matrix model) ↔ arithmetic points on $\mathcal{M}_{g,n}$. [Razamat, Mulase-Penkava, R.G.]

- Gross-Mende like limit ($k_i \to \infty$) in symm. orbfld. Also picks out the same arithmetic points. [Gaberdiel-R.G.-Knighton-Maity]

- Nambu-Goto weight in \`Strebel gauge' $e^{-\int_\Sigma d^2z|\phi_S(z)|}$ gives right Liouville weight in this limit. $\phi_S(z) \approx S[\Gamma(z)]$ - Schwarzian.





Bridge from Strings to Fields

- This picture of large N Feynman diagrams strikingly predicts that the dual worldsheet amplitudes should have (a presentation with) δ-function support on points of $\mathscr{M}_{g,n}$.
- Realised precisely in tensionless worldsheet theory of $AdS_3 \times S^3 \times T^4$. [Eberhardt-Gaberdiel-R.G.]
- Correlators of $\mathfrak{sl}(2,\mathbb{R})_1$ spectrally flowed representations $\mathscr{V}^{k_i}_{j=1/2}(x_i, z_i)$ are zero unless there exists $x = \Gamma(z)$ s.t. $x \approx x_i + (z - z_i)^k + \ldots$ in the vicinity of each z_i.
- The δ-fn. behaviour transparent in a twistorial free field description of worldsheet. [Dei-Gaberdiel-R.G.-Knighton]
- Liouville weight associated with the non-zero points also matches with Lunin-Mathur computation.

Belyi Maps from Strings

- Dual to the Hermitian Matrix model proposed [R.G.-Mazenc '22]: A-model Topological String theory on Kazama-Suzuki coset $\mathfrak{sl}(2,\mathbb{R})_1/\mathfrak{u}(1)$ (with momentum deformation).
- Physical cohomology given by operators $\mathscr{Y}^{(k_i)}_{j=1/2}(z_i)$ in $\mathfrak{sl}(2,\mathbb{R})_1$ spectrally flowed representations - exactly like in the AdS_3 case. [Mukhi-Vafa-Frenkel; Ashok-Murthy-Troost]
- Ward Identity arguments again imply δ-fn. support on covering maps. But now no x_i dependence. All $\mathscr{Y}^{(k_i)}_{j=1/2}(z_i)$ mapped to ∞ on target space cigar. Branching $(k_1)\ldots(k_n) \rightarrow \beta$.
- Need $(k = \sum_i k_i)/2$ momentum (2) insertions for mom. conservation. Branching $(2)^{k/2} \rightarrow \alpha$.

[R.G.-Mazenc (to appear)]

- Together with permutation γ corresponding to scattering, structure exactly that of Belyi maps.

Generalising….

- Can this picture of discrete covering maps generalise to higher dim. e.g. $\mathcal{N} = 4$ SYM?

- Proposal for a worldsheet dual in terms of holomorphic ambitwistor fields $(\lambda^{\alpha}(z), \mu^{\dot{\beta}}(z), ..)$.

[Gaberdiel-R.G.]

- Position $x^{\alpha\dot{\beta}}(z,\bar{z}) = \dfrac{\lambda^{\alpha}(z)\hat{\mu}^{\dot{\beta}}(\bar{z}) - \hat{\lambda}^{\alpha}(\bar{z})\mu^{\dot{\beta}}(z)}{\langle\hat{\lambda}(\bar{z})\lambda(z)\rangle}$ not necessarily holomorphic.

- Assuming holomorphic covering maps of twistors (not proven), natural way in which Strebel differential gives Feynman propagator weights for two point functions $\langle\Phi^{w}(x_i)\bar{\Phi}^{w}(x_j)\rangle$.

- Strebel area $A_S = \int d^2z\,|\phi_S(z)| \propto 2wL = \dfrac{w}{2\pi}\ln\left(\dfrac{x_{ij}^2}{\epsilon^2}\right)$ and $e^{-2\pi A_S} = \epsilon^{2w}\left(\dfrac{1}{x_{ij}^2}\right)^{w}$ - Feynman propagator!

  Additive areas give multiplicative propagators. [Bhat-R.G.-Maity-Radhakrishnan]

Thanks for your attention