Zonal Flow Generation:

$\underline{k}$-space and configuration space perspectives.

P.H. Diamond

I.) Brief Overview
(from plasma perspective)

II.) $\underline{k}$-space view point → <u>energetics</u>

- inverse cascade "blocking" (Rhines)

* - modulational interaction

III.) Real space viewpoint → { <u>momentum</u>; <u>spatial structure</u> }

a/a' Charney-Drazin theorems

<u>cf.</u>

Reviews: P.D., et. al.; P.P.C.F. '05 (on Wiki)
K. Itoh, et.al.; PoP '06

Paper: P. D., et.al. P.P.C.F. '08 (on Wiki)

What is a zonal flow?

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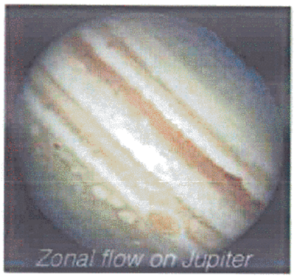
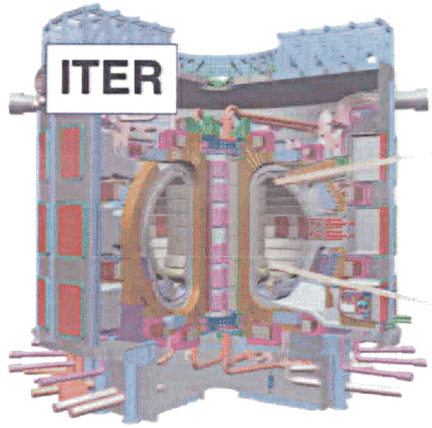
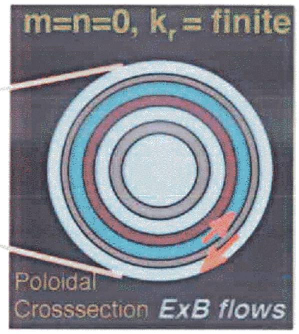






(cartoon)

Drift Waves → Drift waves + Zonal flows

Paradigm Change

ZFs are "mode", but:

1. Turbulence driven
2. No linear instability
3. No direct radial transport

minimal: inertia damping transport

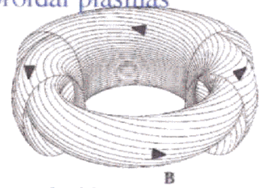
Models

Ω_i: large B_T — Lorentz Dominant

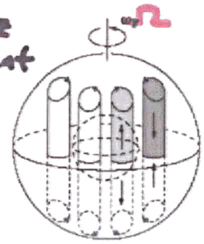
Coriolis Dominant

Toroidal plasmas

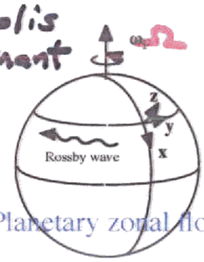
→ radius
→ θ
→ φ



Drift waves



Thermal Rossby waves, ...



Planetary zonal flow

Rossby waves

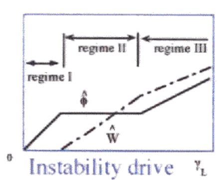
W

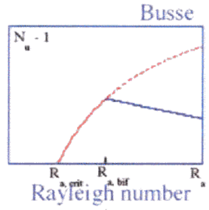
$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + [\phi, \nabla_\perp^2\phi] = d_\parallel(\phi - n) + \mu_c\nabla_\perp^4\phi$$
$$\frac{\partial}{\partial t}n + [\phi, n] = d_\parallel(\phi - n) - \frac{\partial\phi}{\partial y} + D_c\nabla_\perp^2 n$$

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + [\phi, \nabla_\perp^2\phi] = \frac{\partial}{\partial y}(h\phi - RT) + \nabla_\perp^4\phi$$
$$\frac{\partial}{\partial t}T + [\phi, T] = -\frac{1}{P}\frac{\partial\phi}{\partial y} + \frac{1}{P}\nabla_\perp^2 T$$

$$\frac{\partial}{\partial t}(\nabla_\perp^2\psi - \psi) + [\psi, \nabla_\perp^2\psi] - \frac{\partial\psi}{\partial y} = 0$$

Q.G. Eqn,
→ 'Hasegawa-Mima' Eqn.





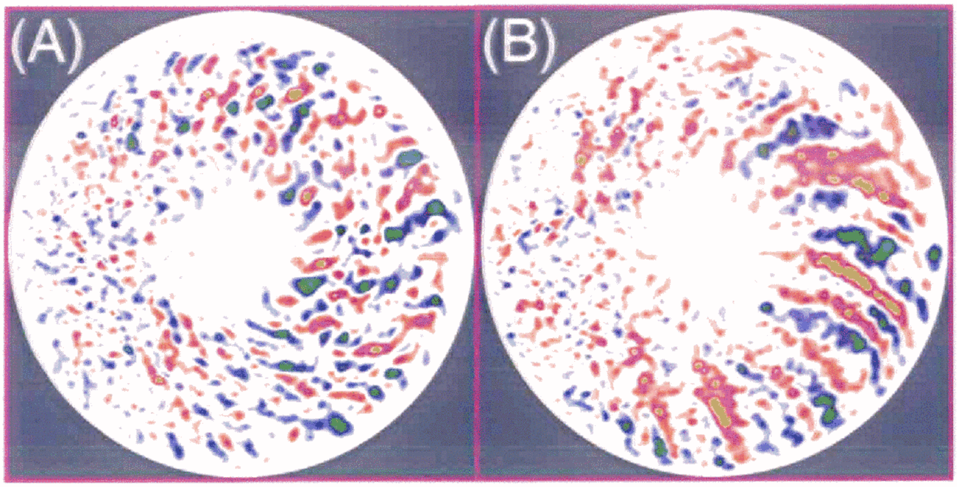
$V \sim \rho_i \frac{c_s}{L}$

Asymmetry: ion-electron diamagnetic drift

$V \sim \left|\rho_R^2 \frac{\partial\omega_{F,z}}{\partial x}\right| \sim 50\ \mathrm{m\ sec}^{-1}$

East-west asymmetry

Gyrokinetic Simulations of Plasma Microinstabilities: turbulence decorrelation by zonal flows



With Flow **Without Flow**

Turbulence reduction via sheared plasma flow (A), compared to case with flow suppressed (B). [Z. Lin *et al.*, **Science** 281, 1835 (1998)]

Basic Physics of a zonal flow

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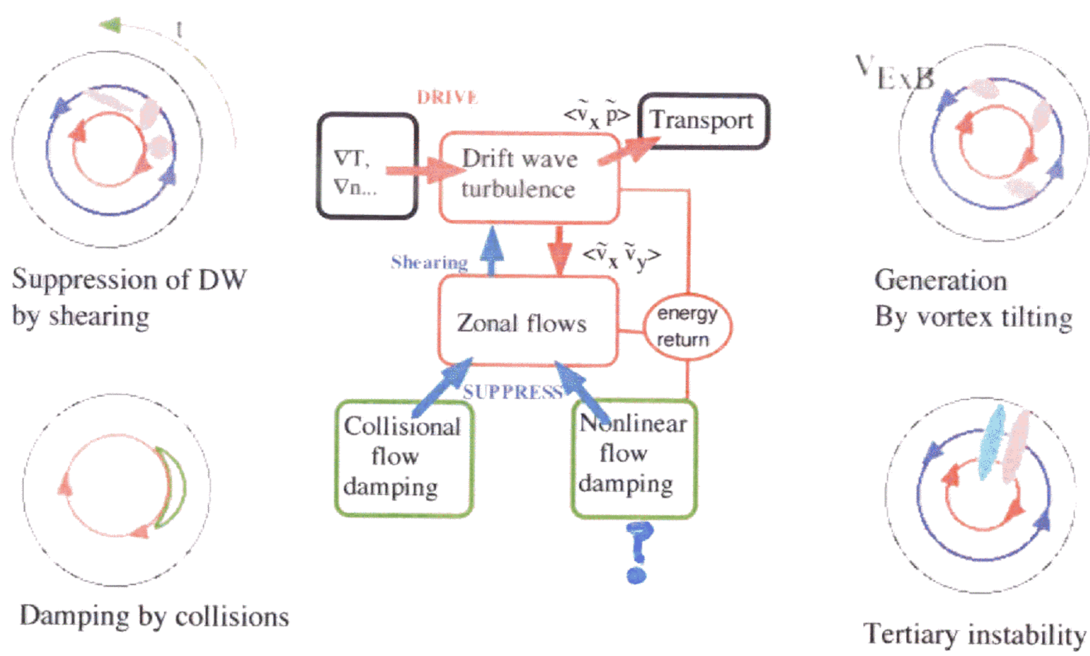

Linear Damping of Zonal Flows

{ Key regulator → drag

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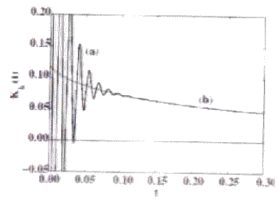
Neoclassical process

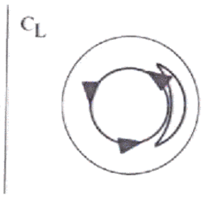


Rosenbluth-Hinton undamped flow - survives for $\tau > \tau_{transport}$

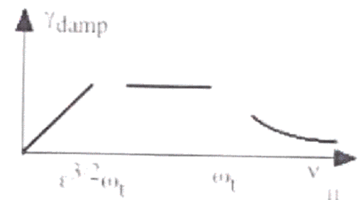
$$\frac{\phi_q(t)}{\phi_q(0)} = \frac{1}{1 + 1.6\varepsilon^{-1/2}q^2}$$

Role of bananas

Frictional damping



Banana-plateau transition



$\gamma_{damp} \simeq \nu_{ii}/\varepsilon \Rightarrow \chi_i \propto \nu_{ii}$ even in "collisionless" regime

Screening effect if $q_r\rho_p \sim O(1)$

Growth Mechanism

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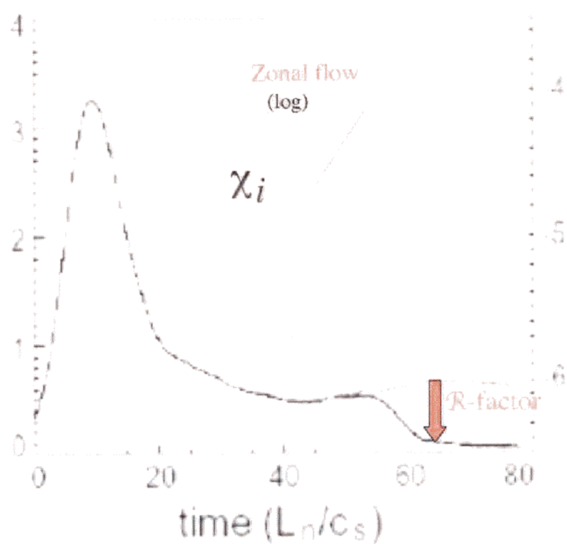
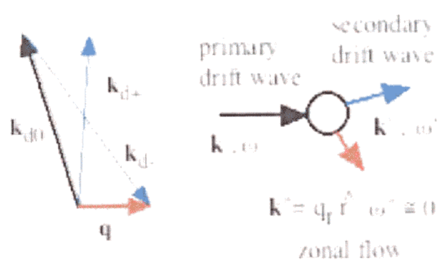
ZFs by

modulational instability





Numerical experiment indicates instability of finite amplitude gas of drift waves to zonal shears

Impt: Z.F. not unique saturation mechanism

Generation Mechanisms:

→ inverse cascade 'blocking'

→ modulational interaction

and implications ....

• Origin of Zonal Flows → $\left\{\begin{array}{l}\text{Rhines} \\ \text{Mechanism}\end{array}\right.$

(the classic)

→ important contrast

MFE applications

broad dynamic range → 3-5 dkds
"Re" > 1
{ waves weakly dispersive

} wave turbulence + ⋮

GFD / Planetary Atmospheres

huge dynamic range
Re >>> 1

{ waves strongly dispersive

} 2D turbulence + Rossby waves + zonal flows / jets + ⋮

∴

MFE ⟺ zonal flow emerges as unstable modulation of wave ensemble

GFD ⟺ zonal flow emerges from 'blocking' of inverse cascade by dispersion

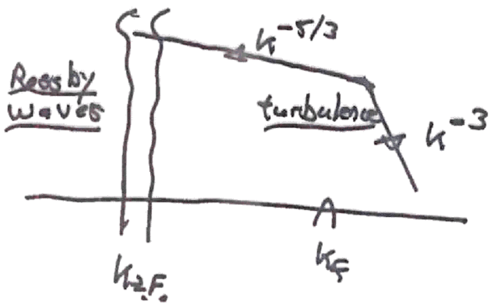
i.e. for each $\underline{k}$ → $\left\{\begin{array}{l}\omega_{\underline{k}} \\ \Delta\omega_{\underline{k}} \sim 1/\tau_{c\underline{k}}\end{array}\right.$

$$\langle \phi(t)\phi(t')\rangle_{\underline{k}} = |\phi_{\underline{k}}|^2 e^{-i\omega_{\underline{k}}(t-t')} e^{-\Delta\omega_{\underline{k}}(t-t')}$$

then: $\omega_k = \beta k_x / k^2$

$1/\tau_{ck} \cong k \tilde{V}_k \qquad V_k^2 \sim \epsilon^{2/3} k^{-2/3}$

⇒



Turbulence: $1/\tau_{ck} \gg \omega_k$ → "eddys"

waves: $1/\tau_{ck} \ll \omega_k$ → "waves"

cross-over ⇒ $\boxed{\text{Rhines scale } k_R \sim \beta^{3/5}/\epsilon^{1/5}}$ ↔ {characteristic Z.F. scale

Why?:

→ for $k < k_R$
- wave dynamics
- difficult to match $\omega_k + \omega_{k'} + \omega_{k''} = 0$

↔ dispersion ...

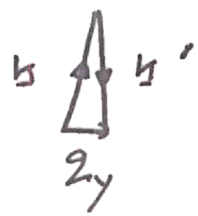
so

∴ energy flux barrier at $k = k_R$ due dispersion blocking of transfer ?

→ but resolve: 3 waves → 2 waves + 1 Z.F.

Z.F.: $k_x = 0$, $\omega_q = 0$ } allows easy match

⇒



→ thin, isoceles triads carry energy for $k \lesssim k_R$

→ zonal flows appear at k_R

→ spectrum changes..... slope..... {energy accumul. in flow

?

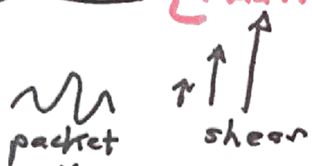
→ β_R effects, emergence of Z.F.'s must ultimately alter self-similar inverse cascade feeding....

⇒ does large scale damping leave footprint on inv. cascade?

Origin of Zonal Flows → Modulation $\left\{\begin{array}{l}\text{Wave turb.}\\ \text{+Ad. Th.}\end{array}\right.$

- back-of-envelope
  - → wave energy
  - → adiabatic invariant

$\mathcal{E} = N\omega$



refraction → Reynolds work

$$\frac{d\mathcal{E}}{dt} = N V_{gx}\frac{dk_x}{dt} = N V_{gx}(-k_y V_y')$$

where: $\frac{d}{dt}(\mathcal{E} + V_y^2) = $ dissp'n...

Key: adiabatic invariant → models of stresses.

- slightly larger envelope:

$$\frac{d\mathcal{E}}{dt} = (\langle N\rangle + \tilde{N})V_{gx}\left(-k_y(\langle v_y\rangle' + \tilde{V}_y')\right)$$

⇒

$$\frac{d\langle\mathcal{E}\rangle}{dt} = -k_y\langle\tilde{V}_y'\tilde{N}\rangle V_{g,x}$$

For $\tilde{N}$: WKE $\left\{\begin{array}{l}\text{evolves adiabatic}\\ \text{invariant — wave population density}\end{array}\right.$

$$\frac{\partial N}{\partial t} + (\underline{V}_g + \underline{V})\cdot\underline{\nabla} N - \frac{\partial}{\partial\underline{x}}(\omega + \underline{k}\cdot\underline{V}_E)\cdot\frac{\partial N}{\partial\underline{k}}$$

$$= \gamma_{\underline{k}} N + C(N)$$

Mean field approach .....

quasilinear calculation ⇒

$$-k_y\langle\tilde{V}_y'\tilde{N}\rangle = -D_k\frac{\partial\langle N\rangle}{\partial k_x}$$

key process:

$$D_k = \sum_{\underline{q}}\left[\frac{k_y^2|\tilde{V}'_{\underline{q},y}|^2\tilde{\tau}_{k,\underline{q}}}{(1+\tilde{\tau}^2_{k,\underline{q}}q^2V^2_{gx})}\right]$$

→ k_x-diffusivity
→ random refraction ⇔ chaotic rays

n.b. as in usual QLT, requires only stochastic rays, not stochastic flow shears

and

$$\frac{d}{dt}\left\{\int d\underline{k}\,\langle\mathcal{E}_{\underline{k}}\rangle + \int d\underline{q}\,|\tilde{V}_{\underline{q}}|^2\right\} = 0, \text{ to dissp'n.}$$

⇒ Pot. Enst. density spectrum

$$\frac{d\langle\mathcal{E}_{\underline{k}}\rangle}{dt} = \frac{2k_x\rho_s^2}{(1+k_\perp^2\rho_s^2)^2}D_k\frac{\partial\langle\Omega_k\rangle}{\partial k_x}$$

(usual): $\partial\langle\Omega_k\rangle/\partial k_x < 0$ → $\langle\mathcal{E}_k\rangle < 0$
(no population inv.). → flows excited from waves

$\partial\langle\Omega_k\rangle/\partial k_x > 0$ → flows damped.

– converting to Z.F. growth:

$$\gamma_Z = -2q^2 c_s^2 \sum_{k} \frac{k_\theta^2 \rho_s^2}{(1+k_\perp^2\rho_s^2)^2} \left\{ \tau_{k,Z} \Big/ (1+\tau^2_{kZ}\Omega^2) \right\} k_x \frac{\partial \langle N \rangle}{\partial k_x}$$

→ Regulation Feedback – shearing

mean field $\langle N \rangle$ eqn:

$$\frac{\partial \langle N \rangle}{\partial t} = \gamma_k N + C(N) + \frac{\partial}{\partial k_x} D_k \frac{\partial \langle N \rangle}{\partial k_x}$$

spectral transport to high k_x ⇒ damping ⟵⟶ ZF mediated

with coupling to:

$$\frac{\partial |\phi_Z|^2}{\partial t} = \gamma_Z(\langle N \rangle)|\phi_Z|^2 - \nu |\phi_Z|^2$$

+ transport, i.e. $\gamma_k = \gamma_k[\nabla P]$

$$Q = -\chi\left[\langle N \rangle, |\phi_Z|^2\right]\nabla P$$

etc.

Motivating the theory...

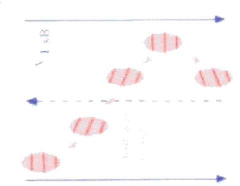
Close Relationship: DW + ZF and Vlasov Plasma

(i) DW + ZF :

$$\frac{dx}{dt} = v_{gx}(\mathbf{k}) \qquad \frac{dk_x}{dt} = -\frac{\partial}{\partial x}(k_y \tilde{V}_Z)$$
$$\frac{dy}{dt} = v_{gy}(\mathbf{k}) \qquad \frac{dk_y}{dt} = 0$$

⇧ 'Ray' Trapping

$\Omega = q_x v_{gx}$



symmetry!

$$\frac{\partial}{\partial t}\tilde{V}_Z + \gamma_{damp}\tilde{V}_Z = -\frac{\partial}{\partial x}\langle \tilde{V}_x \tilde{V}_y \rangle$$

$$\frac{\partial}{\partial t}N + v_{gx}\frac{\partial}{\partial x}N - \frac{\partial}{\partial x}(k_y V_Z)\frac{\partial N}{\partial k_x} = \gamma_k N + C_k(N)$$

(ii) 1D Vlasov Plasma :

$$\frac{dx}{dt} = v \qquad \frac{dv}{dt} = \frac{e}{m}\tilde{E}$$

⇧ Particle Trapping

$\omega = kv$

$$\frac{\partial \tilde{E}}{\partial x} = 4\pi n_0 e \int dv\, f$$

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{e}{m}\tilde{E}\frac{\partial f}{\partial v} = C(f)$$

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Note: Conservation energy between ZF and DW

RPA equations

DW $$\frac{\partial}{\partial t}\left|\tilde{V}_{DW}\right|^2+\sum_k\left(\gamma_{L,k}+C_k(N)\right)\left|\tilde{V}_{DW,k}\right|^2=\frac{2}{B^2}\sum_{q_x}\int d^2k\,\frac{q_x^2k_\theta^2k_x\left|V_{ZF,q}\right|^2}{\left(1+k_\perp^2\rho_s^2\right)^2}R(k,q_x)\frac{\partial\langle N\rangle}{\partial k_x}$$

ZF $$\left(\frac{\partial}{\partial t}+\gamma_{damp}\right)\left|V_{ZF}\right|^2=-\frac{2}{B^2}\sum_{q_x}\int d^2k\,\frac{q_x^2k_\theta^2k_x\left|V_{ZF,q}\right|^2}{\left(1+k_\perp^2\rho_s^2\right)^2}R(k,q_x)\frac{\partial\langle N\rangle}{\partial k_x}$$

Coherent equations — Pump (S)

DW $$\frac{dP^2}{d\tau}=P^2-2PZS\cos(\Psi)$$ (S: beat wave)

ZF $$\frac{dZ^2}{d\tau}=-\frac{\gamma_{damp}}{\gamma_L}Z^2+2PZS\cos(\Psi)$$

$$\boxed{\frac{\partial}{\partial t}W_d\Big|_{\text{by ZF}}=-\frac{\partial}{\partial t}W_{ZF}\Big|_{\text{by DW}}}$$

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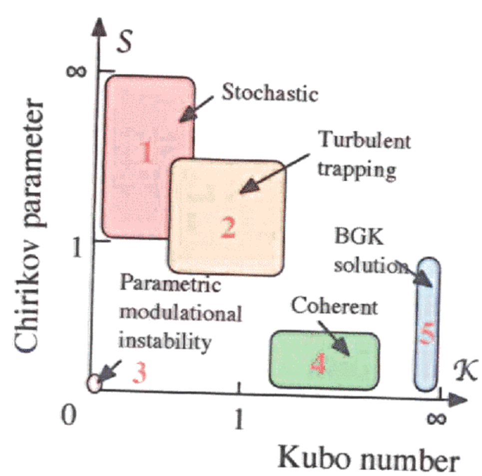
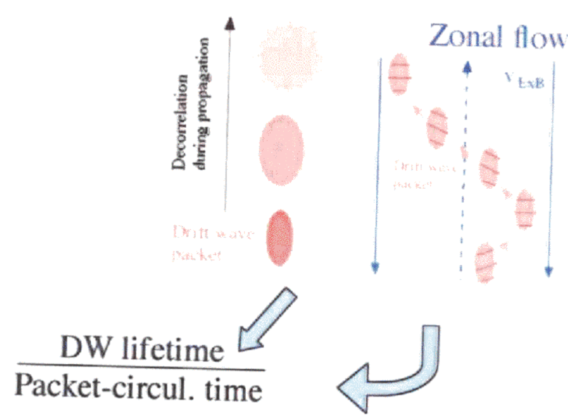




Regime	Keywords
1	k_r Diffusion
2	Turbulent trapping
3	Single wave modulation
4	Reductive perturbation
5	DW trapping in ZF

References

Zakharov, PD, Itoh, Kim, Krommes
Balescu, Itoh
Sagdeev, Hasegawa, Chen, Zonca
Taniuti, Weiland, Champeaux
Kaw, Smolyakov, PD

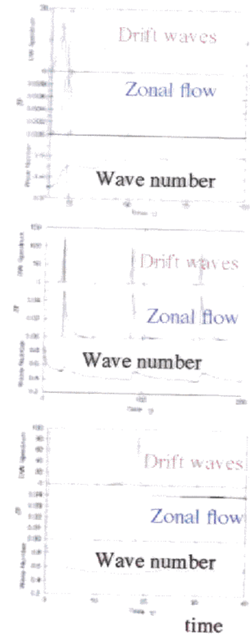
Self-regulating System Dynamics

Simplified Predator-Prey model

DW $\frac{\partial}{\partial t}\langle N\rangle = \gamma_L \langle N\rangle - \gamma_2 \langle N\rangle^2 - \alpha \langle U^2\rangle\langle N\rangle$

ZF $\frac{\partial}{\partial t}\langle U^2\rangle = -\gamma_{damp}\langle U^2\rangle + \alpha\langle U^2\rangle\langle N\rangle$

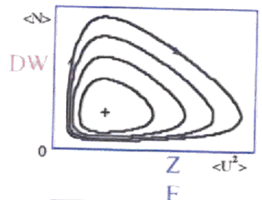
Stable fixed point



Cyclic bursts

$\gamma_2 \to 0$

(No self damping)

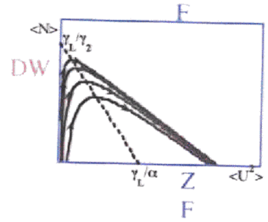


Single burst
(Dimits shift)

$\gamma_{damp} \to 0$

(No ZF friction)

transport quenched



Self-regulation: Co-existence of ZF and DW

$$\begin{cases} \frac{\partial}{\partial t} W_d = \gamma\left[\nabla p_0, \cdots\right] W_d - \alpha W_d W_{ZF} \\ \frac{\partial}{\partial t} W_{ZF} = \gamma_{damp}\left[\cdots\right] W_{ZF} + \alpha W_d W_{ZF} \end{cases}$$

W_d : drift wave energy

W_{ZF} : zonal flow energy

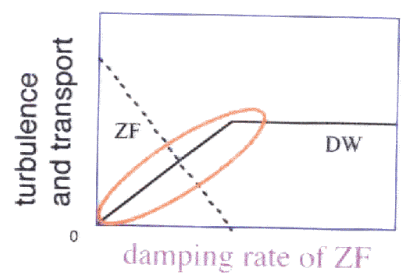
(v_{ii}, q, ε geometry) (+ rf, etc.)

$$W_d \sim \frac{\gamma_{damp}}{\alpha}$$

Transport coefficient

$$\chi_i \sim \frac{\gamma_{damp}}{\omega_{eff}}\chi_{gB} \Rightarrow \chi_i = \mathcal{R}\chi_{gB}$$

"$\mathcal{R}$ – Factor"



e.g. Garbet, Lin

Confinement enhancement
Includes other reduction effects
(i.e., cross phase)

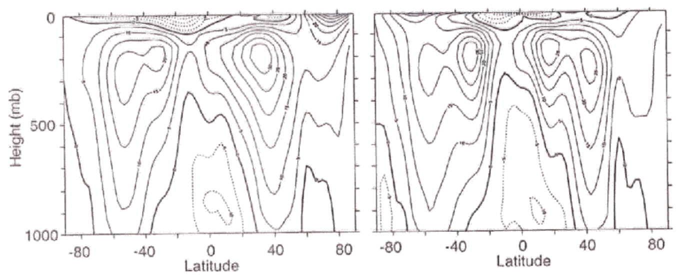
5.

II.) Atmospheric Jets (c.f. G. Vallis, '06)

→ persistent feature in atmospheric wind pattern

westward mid-latitude jet

eastward sub-tropical jet



subtropical:
- $\nabla T)_{\text{merid.}}$ driven

mid-latitude:
- less structure, shear

(*) → How does dipole shear form?
- symmetry breaking?
- structure?

2.

→ Minimalist Understanding

- subtropical excitation ⇒ {wave radiation

- outgoing waves ⇒ $\phi \sim e^{ik_y y} \to 0$ as $y \to \infty$

$\omega \to \omega - i\nu$

$\delta k_y = i\nu / V_{gr_y}$ $\qquad V_{gr_y} = \dfrac{2\beta k_x k_y}{(k^2)^2} > 0$

$$\boxed{k_x k_y > 0}$$

- but momentum flux

$$\langle \tilde{v}_y \tilde{v}_x \rangle = -\sum_{k} k_y k_x |\phi_k|^2 < 0$$

Key Point: Energy Radiation ↔ Momentum Convergence

- Rossby waves "backward"

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→ cont'd

$\boxed{\text{Energy Radiation} \leftrightarrow \text{Momentum Convergence}}$

- i.e.

- form dipole via:

- momentum influx locally boosts eastward flow ⇒ sub tropical jet

- resulting momentum deficit generates westward mid-latitude jet

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→ Collisionless Saturation of Zonal Flows

- if all collisional drag, diffusion → 0

$$\partial_t \left\{ \langle v_\theta \rangle - P_{\theta\, ps} \right\} = \langle \tilde{v}_r \tilde{n} \rangle - \frac{1}{\langle u \rangle'} \partial_r \langle \tilde{v}_r \tilde{u}^2 \rangle$$

⇒ stationarity:

$$\langle \tilde{v}_r \tilde{u}^2 \rangle \sim \int dr \, \langle \tilde{v}_r \tilde{n} \rangle \langle u \rangle'$$

potential enstrophy flux

production by Γ_0, $\langle u \rangle'$

- calculating $\langle \tilde{v}_r \tilde{u}^2 \rangle$ non-trivial (c.f. Gurcan, P.D., Hahm '06)

→ $\langle \tilde{u}^2 \rangle$ not even close to passive tracer

→ $\langle \tilde{v}_r \tilde{n}^2 \rangle \neq \langle \tilde{v}_r \tilde{\omega}^2 \rangle$

- What of ZF KH Instability?

$\langle u \rangle' \to 0$ is signature ....

→ Potential Enstrophy Flux

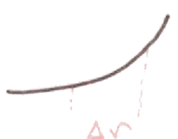
– novel feature : spreading of specific quantity

– origin Z.F. $<$ $\langle \tilde{q}^2 \rangle \updownarrow d\langle u \rangle / dr$ $\Rightarrow$ {Reynolds Force

∴ ⇒ transport $\langle \tilde{q}^2 \rangle$ must alter flow (akin J_h in dynamo)

⇒ for levels a la' MLT, (with L_ϵ) NOT SMALL

– jumps in $\langle \tilde{v}_r \tilde{q}^2 \rangle$ ⇒ {Shear Layers



$$\Delta r \, \frac{d\langle u \rangle}{dr} \left(\partial_t \langle V_\theta \rangle \right) \approx \langle \tilde{v}_r \tilde{q}^2 \rangle \Big|_{x_-}^{x_+}$$

– feedback loop

a.) seed shear → $\Delta \langle \tilde{v}_r \tilde{q}^2 \rangle \rightarrow \partial_t \langle V_\theta \rangle \neq 0$

b.) $\langle V_\theta \rangle' \rightarrow$ enhanced $\Delta \langle \tilde{v}_r \tilde{q}^2 \rangle$

– especially relevant to edge

→ Zonal Flow Structure

– stationary, standard regime

$$\boxed{\langle V_\theta \rangle = \frac{1}{\nu} \left\{ \Gamma_0 - \frac{1}{\partial\langle u \rangle/\partial r} \left(D_0 \langle (\nabla \tilde{q})^2 \rangle + \partial_r \langle \tilde{v}_r \tilde{q}^2 \rangle \right) \right\}}$$

– exact result ; in terms macroscopics

– flow structure ↗ dissipation profile ↘ enstrophy spreading

– Zonal Flow Shear :

$$\langle V_\theta \rangle' \cong -\frac{\nu'}{\nu^2} \Gamma_0 - \frac{1}{\nu \langle u \rangle'} \left\{ D_0 \langle (\nabla \tilde{q})^2 \rangle + \partial_r \langle \tilde{v}_r \tilde{q}^2 \rangle \right\}'$$

– shear ↔ ν', $D_0 \langle (\nabla \tilde{q})^2 \rangle'$, spreading

– $\langle V_\theta \rangle'$ up → $\langle \tilde{v}_r \tilde{n} \rangle$ drops → fixed Γ_0 demands $D_0 \, \partial\langle n \rangle / \partial r$

∴ collisional {particle heat transport critical for flow dynamics near marginal.

14.

→ "No-Slip" Momentum Theorem (H-W)

$$\partial_t \left\{ \langle V_\theta \rangle \overset{P_{rel}}{-} \left(-\frac{\langle \tilde{u}^2 \rangle}{d\langle u \rangle / dr} \right) \right\} + \overset{drag}{\nu} \langle V_\theta \rangle$$

$$= \underset{\text{driving flux}}{\langle \tilde{v}_r \tilde{n} \rangle} - \left(\frac{d\langle u \rangle}{dr} \right)^{-1} \left\{ \underset{\text{diffusion}}{D_0 \langle (\nabla \tilde{u})^2 \rangle} + \underset{\text{transport of Pot. Enstr.}}{\partial_r \langle \tilde{v}_r \tilde{u}^2 \rangle} \right\}$$

— no Reynolds modelling ....

— Similar QG, but:

$$\langle \tilde{v}_r \tilde{n} \rangle = \underset{\text{fixed net flux}}{\Gamma_0} + \underset{\text{collisional flux}}{D_0 \, \partial \langle n \rangle / \partial r}$$

is driver.

collisional flux → negligible but for ITB, Dimits

- Pseudomomentum $\sim \langle \tilde{u}^2 \rangle$

independent $k_\parallel^2 D_\parallel / \omega_k$, etc.

no restrictions ....

13.

→ Momentum Theorem

- observe: $\begin{cases} \text{3D system} \\ \text{Conserved PV} \end{cases}$ but: ↔ 2D dynamics

$$\begin{cases} \boxed{u = \nabla^2 \phi - n} \\ \dfrac{du}{dt} = D_0 \nabla^2 u \end{cases}$$

2D evolution eqn.

$$\frac{d\langle u \rangle}{dr} = \frac{d}{dr} \langle \nabla^2 \phi \rangle - \frac{d\langle n \rangle}{dr}$$

— Potential Enstrophy Balance:

$$\partial_t \langle \tilde{u}^2 \rangle + \frac{\partial}{\partial r} \langle \tilde{v}_r \tilde{u}^2 \rangle = \overset{\text{PV flux}}{-\langle \tilde{v}_r \tilde{u} \rangle} \, d\langle u \rangle / dr - D_0 \langle (\nabla \tilde{u})^2 \rangle$$

(as before)

but:

$$\partial_t \langle V_\theta \rangle = \underset{\text{only vorticity flux evolves zonal flow}}{\langle \tilde{v}_r \nabla^2 \tilde{\phi} \rangle} - \nu \langle V_\theta \rangle$$

∴

— now momentum conservation coupled to transport .....

12.

II.) Zonal Flow Momentum - DWT

→ H-W system ($\perp$ $A_r = 1$)

$$\frac{dn}{dt} = -D_{\parallel} \nabla_{\parallel}^2 (\phi - n) + D_0 \nabla^2 n$$

$$\frac{d}{dt} \nabla^2 \phi = -D_{\parallel} \nabla_{\parallel}^2 (\phi - n) + D_0 \nabla^2 \nabla^2 \phi$$

→ minimal relevant system

- 3D, parallel dissipation
- drift wave instabilities
- finite $\langle \tilde{v}_r \tilde{n} \rangle$ → transport

→ simple but detailed, i.e. $k_{\parallel}^2 D_{\parallel} / \omega_* \gtrless 1$, $\langle n \phi \rangle \neq 0$, damped modes .....

→ Zonal Flow structure in H-W ?

11.

→ Meaning ?

- Fluid: 2 coupled component < zonal flow / quasi-particle flow

- $\partial_t P_{rel} = \cdots$ ⇒ "No slip"
  except by preferential damping or excitation of one component

- absent $\tilde{F}$, D ⇒ can't accelerate $\langle v_x \rangle$ with stationary turbulence

→ Stationarity ⇒ Dipole

$$\boxed{\langle v_x \rangle = \frac{1}{\gamma \beta^*} \left\{ \langle \tilde{F} \tilde{\omega} \rangle - \mu \langle (\nabla \tilde{\omega})^2 \rangle - \partial_y \langle \tilde{v}_y \tilde{\omega}^2 \rangle \right\}}$$

forcing region $\sim \langle \tilde{F} \tilde{\omega} \rangle / \gamma \beta^*$ → Eastward jet

viscous damping region ("beach") $\sim -\frac{\mu \langle (\nabla \tilde{\omega})^2 \rangle}{\gamma \beta^*}$ → Westward jet

(10.)

→ Extended Charney-Drazin Theorem

(Charney + Drazin '61 ; Rhines + Holland '79)

$$\partial_t \Big\{ \underbrace{\langle V_x \rangle}_{\text{flow}} - \Big(- \underbrace{\frac{\langle \tilde{\omega}^2 \rangle}{\beta^*}}_{\text{pseudomomentum}} \Big) \Big\} + \nu \langle V_x \rangle$$

$$= \underbrace{\frac{\langle \tilde{f} \tilde{\omega} \rangle}{\beta^*}}_{\text{forcing}} - \underbrace{\frac{\mu \langle (\nabla \tilde{\omega})^2 \rangle}{\beta^*}}_{\text{viscous damping}} - \underbrace{\frac{1}{\beta^*} \partial_y \langle \tilde{v}_y \tilde{\omega}^2 \rangle}_{\text{enstrophy spreading}}$$

→ Pseudomomentum ~ Wave Momentum Density (WMD)

- { enstrophy → intensity
  β → orientation

- not tied to weak nonlinearity

→ β effect ⇒ zonal acceleration w/o net momentum input

(9.)

→ Some Theory

- zonal mean flow

$$\partial_t \langle V_x \rangle = -\partial_y \langle \tilde{v}_y \tilde{v}_x \rangle - \nu \langle V_x \rangle$$
$$= \langle \tilde{v}_y \tilde{\omega} \rangle - \nu \langle V_x \rangle$$

Reynolds Force ↔ Vorticity Flux (Taylor, '15)

- Vorticity Flux ↔ $\boxed{\text{Enstrophy Balance}}$

$$\partial_t \langle \tilde{\omega}^2 \rangle + \partial_y \langle \tilde{v}_y \tilde{\omega}^2 \rangle + \beta \langle \tilde{v}_y \tilde{\omega} \rangle$$
$$= \langle \tilde{f} \tilde{\omega} \rangle - \mu \langle (\nabla \tilde{\omega})^2 \rangle$$

i.e. Reynolds Force → Production via { Vorticity Flux, $\nabla \langle \omega \rangle$ }

∴

- Zonal Momentum Linked to Enstrophy Balance

Configuration Space Approach?

Coming Attractions:

S. Tobias on

"Jet Formation in MHD"

→ Tachocline discussion, next week.