many body
quantum chaos

condensed
matter

SYK

holography


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Correlated Systems with Multicomponent Local Hilbert Spaces

Coordinators: George Jackeli, Natasha Perkins, Lucile Savary, and Oskar Vafek

KITP and the program coordinators will be delivering remote talk sessions to this program's participants.

Theorists often idealize descriptions of solids via simple models, e.g. Ising or Hubbard, with a very small local Hilbert space. Typically, magnetic models involve s=1/2 spins on a lattice, and conductors are usually modeled by one or two bands. Similarly, many systems exist where magnetic and fermionic degrees of freedom are coupled, e.g. Kondo systems and itinerant magnets, but the minimal physics is usually simple, and the models are often considered in extreme limits. What happens when the local Hilbert space is larger and one must consider multiple entangled degrees of freedom? The focus of attention of the condensed matter community has gradually been shifting towards physical systems where the latter appears to be true. While some of these systems have been known for decades, a number of experimental and theoretical discoveries of strongly correlated phenomena have caused a notable revival of interest in the field. For example, the interplay of magnetism and topological band structures, and twisted bilayer graphene have proven to be very fertile areas.

This program will address the following questions: What entangled phases can one generate out of spin-orbital models? Are the natural/physical interactions different in band touching systems? What phases can result from the coupling of two strongly interacting systems, magnetic and conducting? How much of this physics exists in magic-angle twisted bilayer graphene and other moiré systems? Can such systems realize models akin to the SYK model which itself involves the coupling of N orbitals? What are the prospects for studying these systems with numerics?

here: phenomenological approach to correlated systems w/o microscopic background

SYK model

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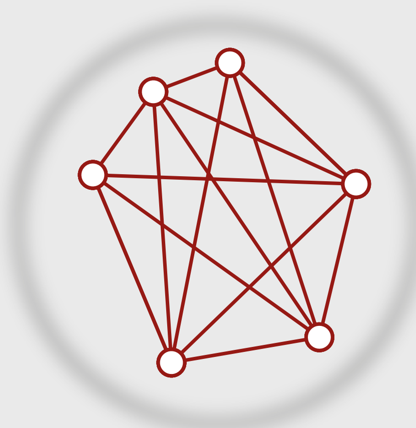
here: phenomenological approach to correlated systems w/o microscopic background

A new approach to interactions

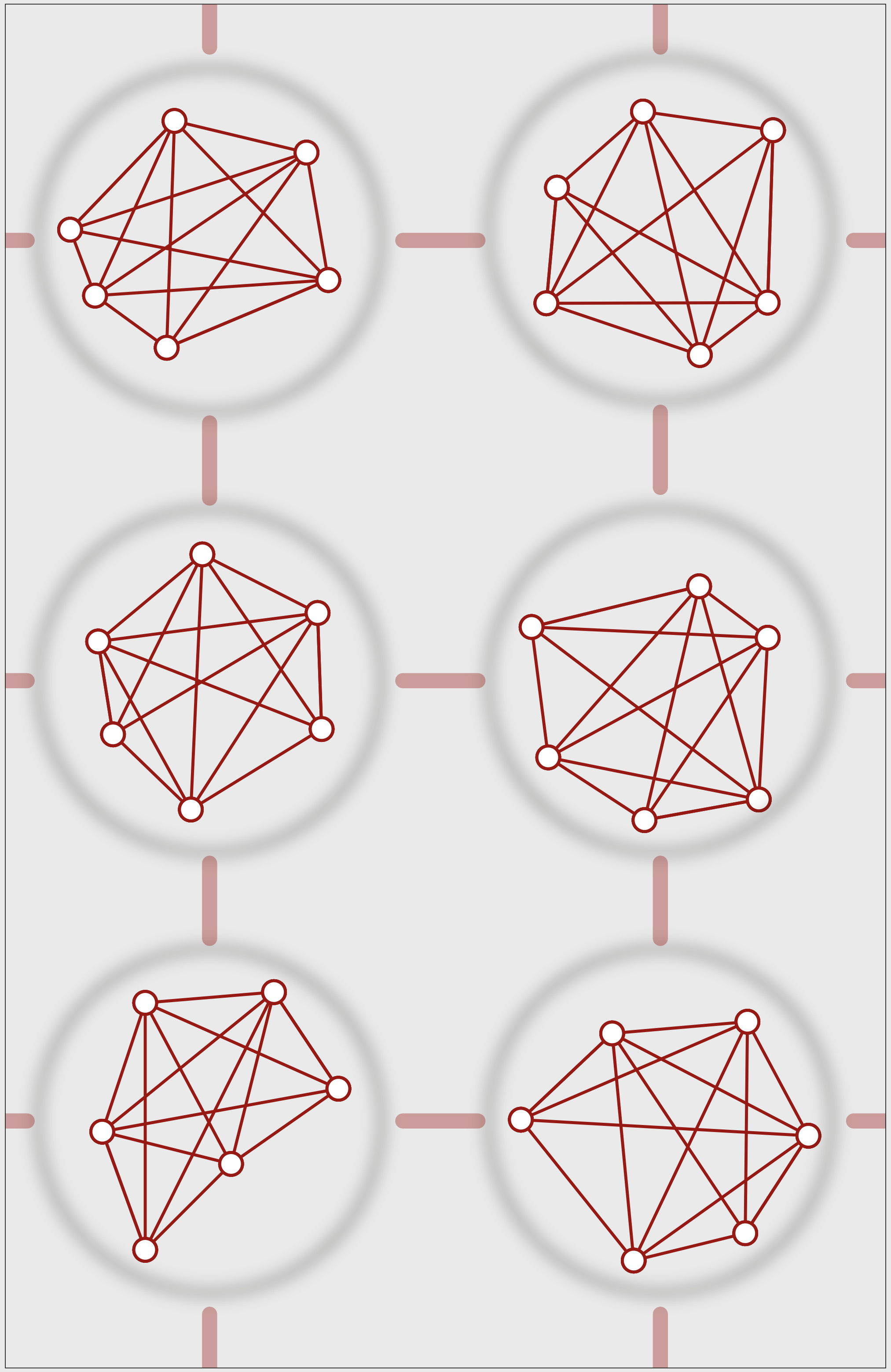


1.) $1 \ll N < \infty$ d.o.f. on each quantum cell

2.) asymptotically strong cellular interactions

3.) local maximum entropy/mean field/ disorder/entanglement principle

A new approach to interactions



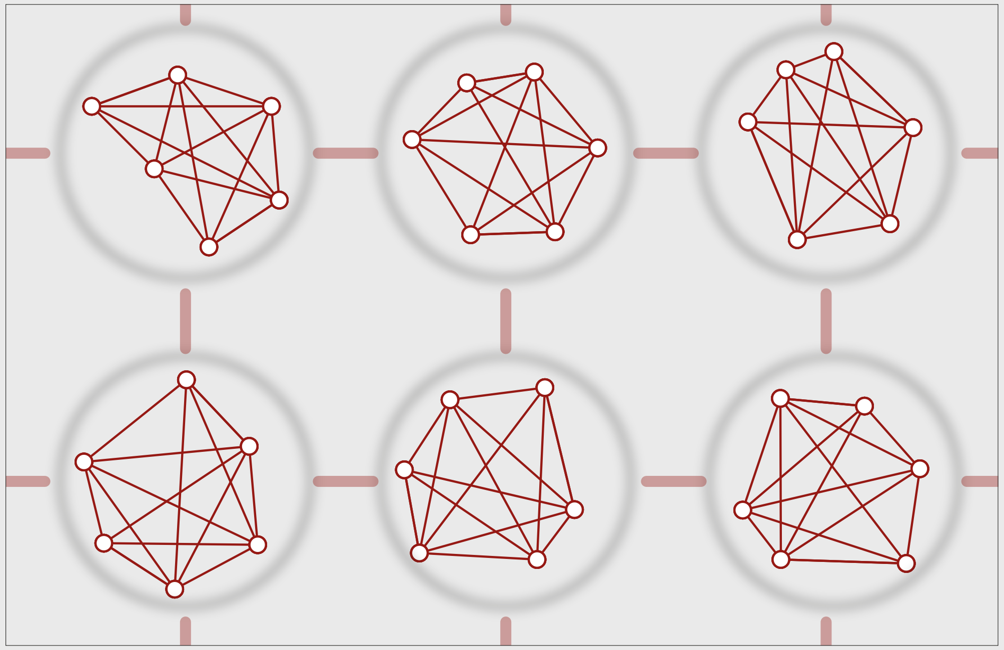
1.) $1 \ll N < \infty$ d.o.f. on each quantum cell

2.) asymptotically strong cellular interactions

3.) local maximum entropy/mean field/ disorder/entanglement principle

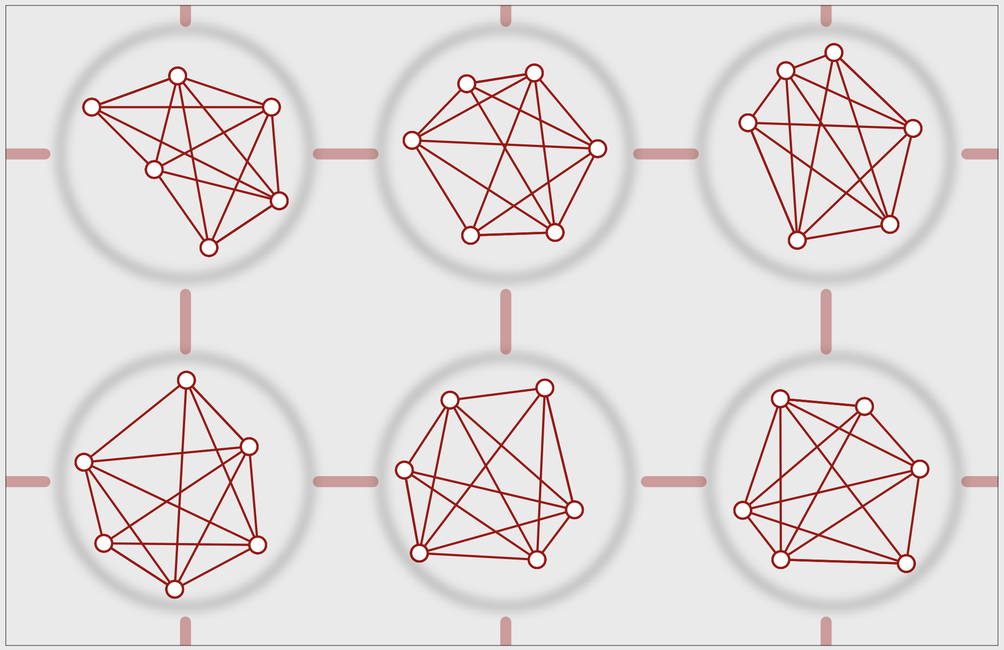
4.) extension to arrays

Applications



Phys. Rev. Lett. **123**, 106601 (2019)

Applications



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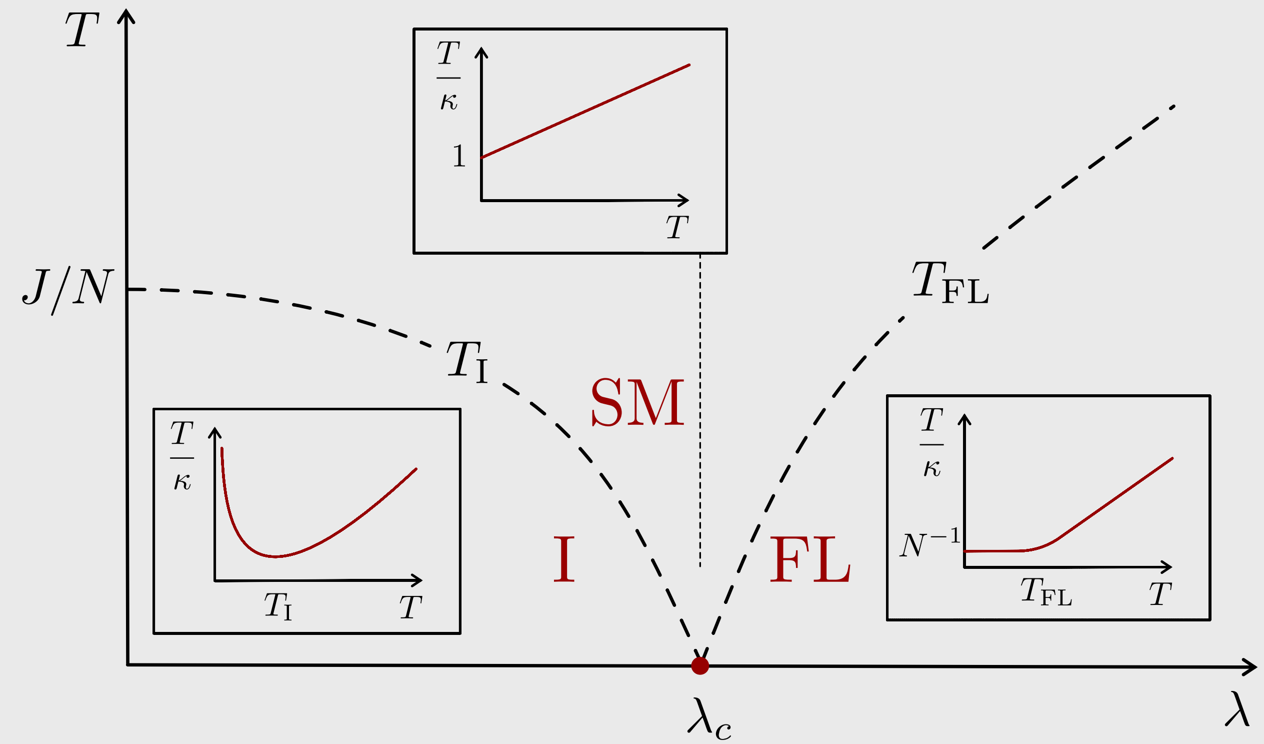
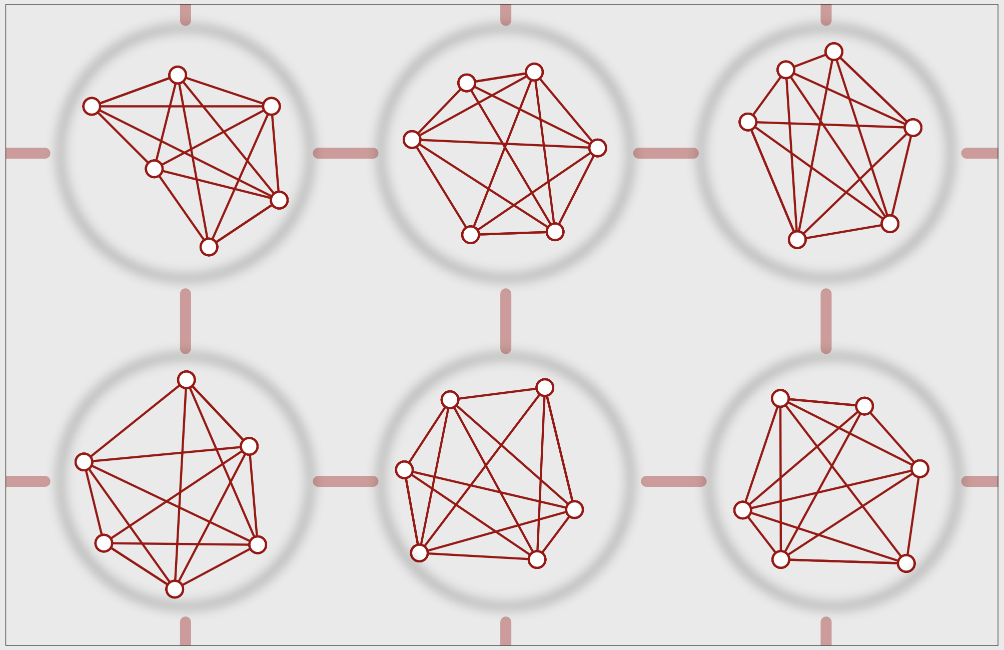
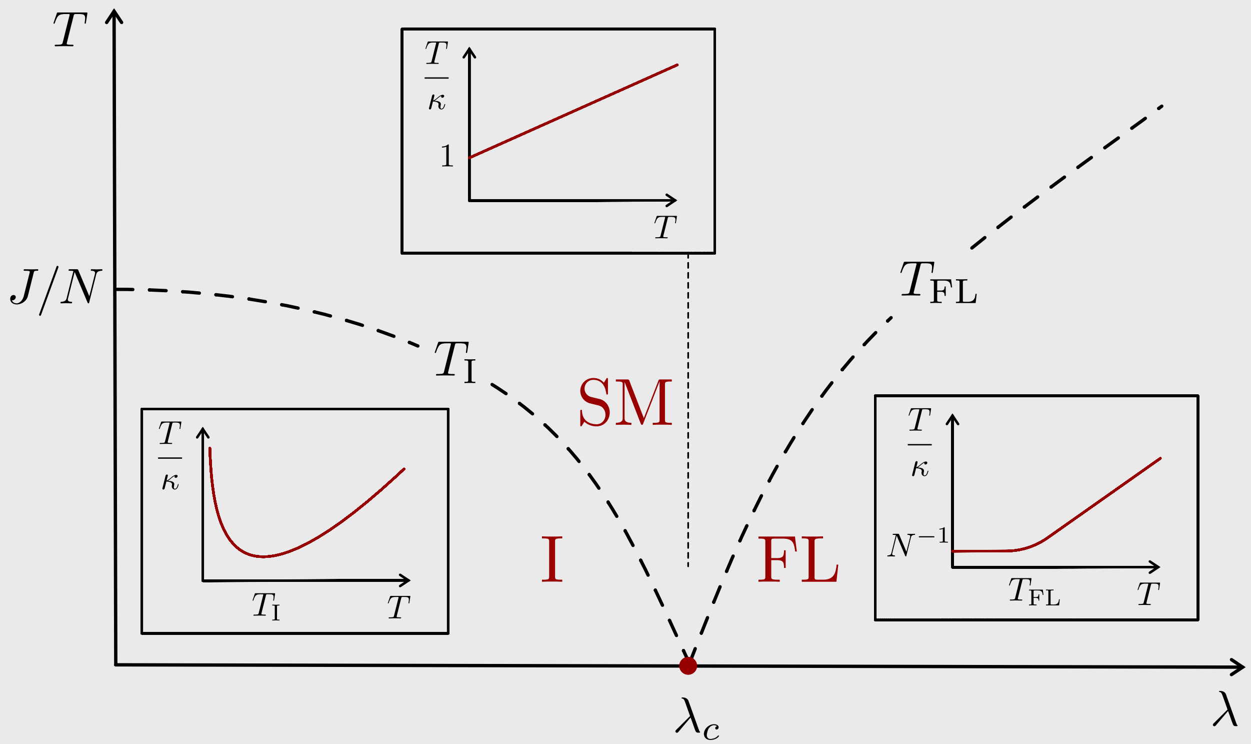


Applications





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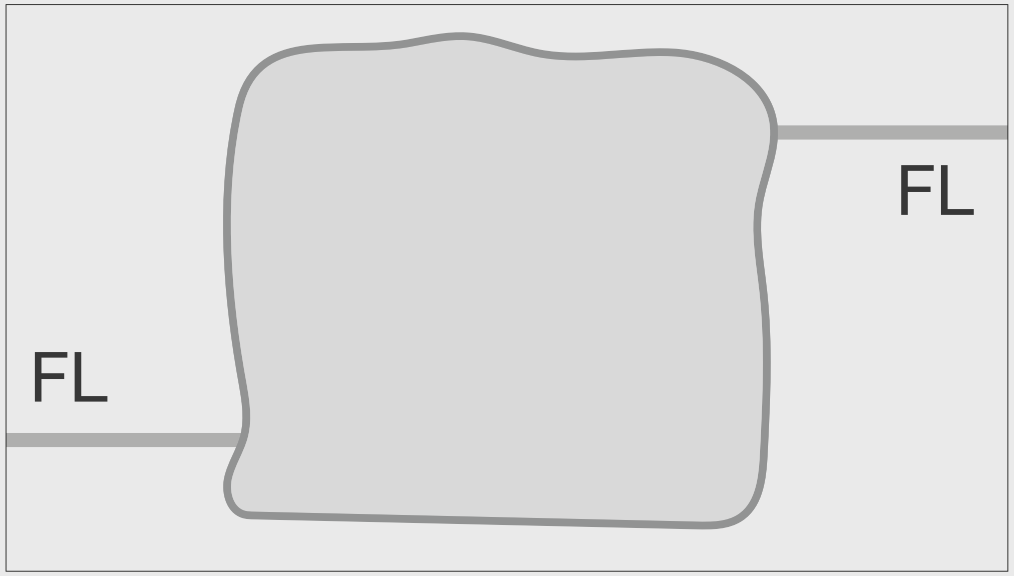

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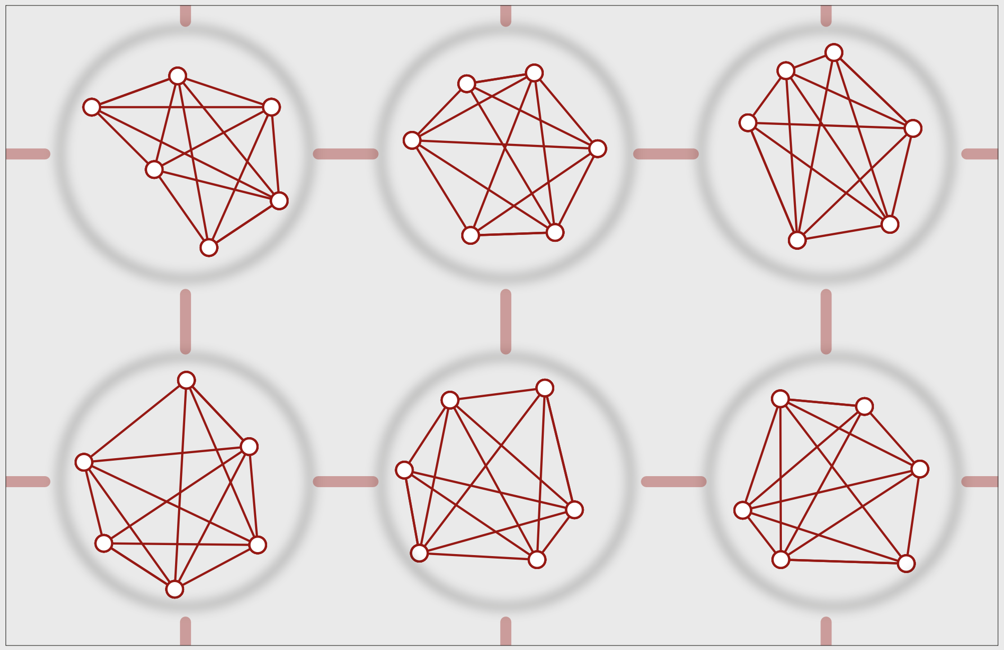

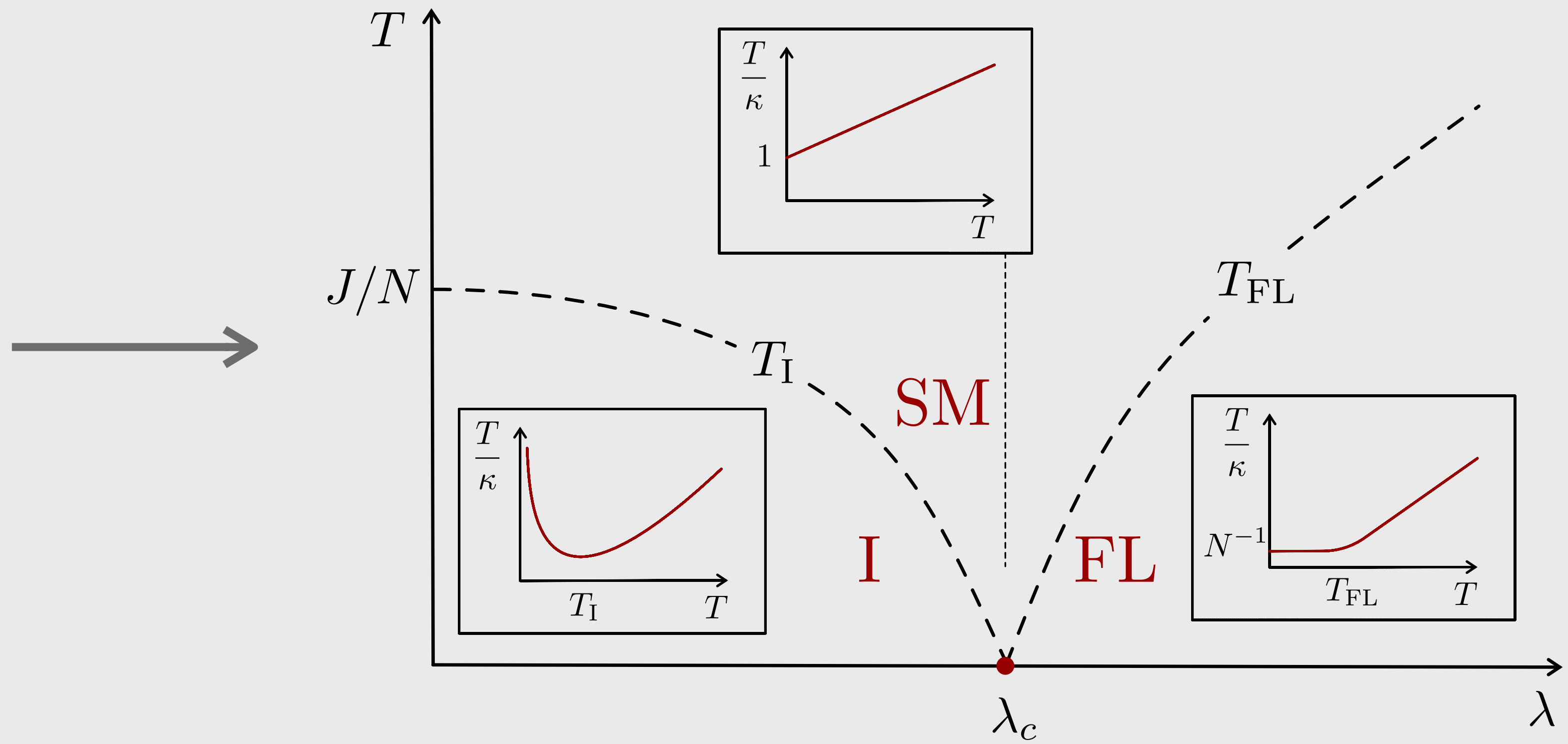

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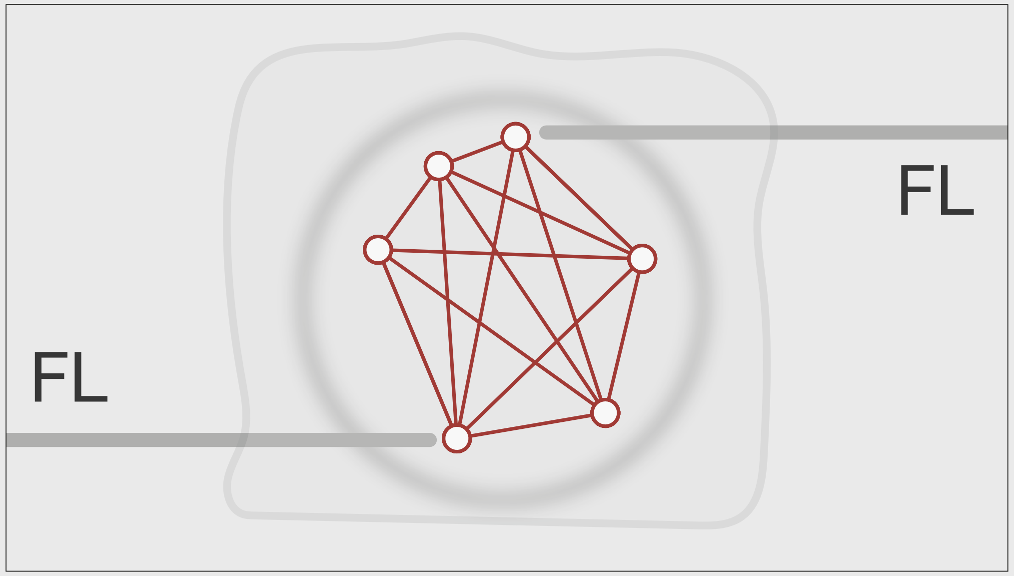

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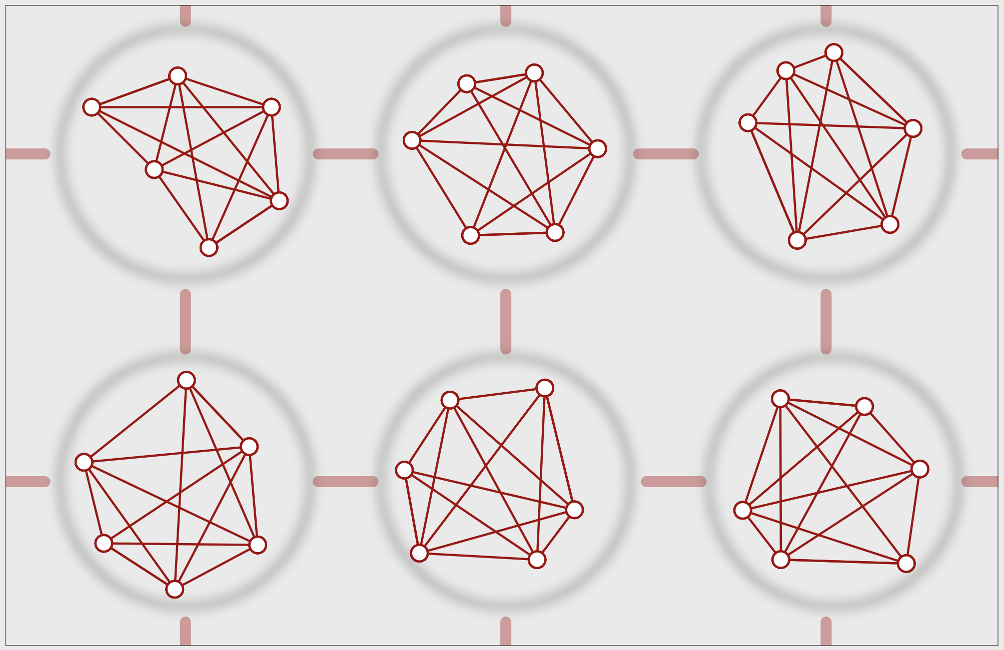

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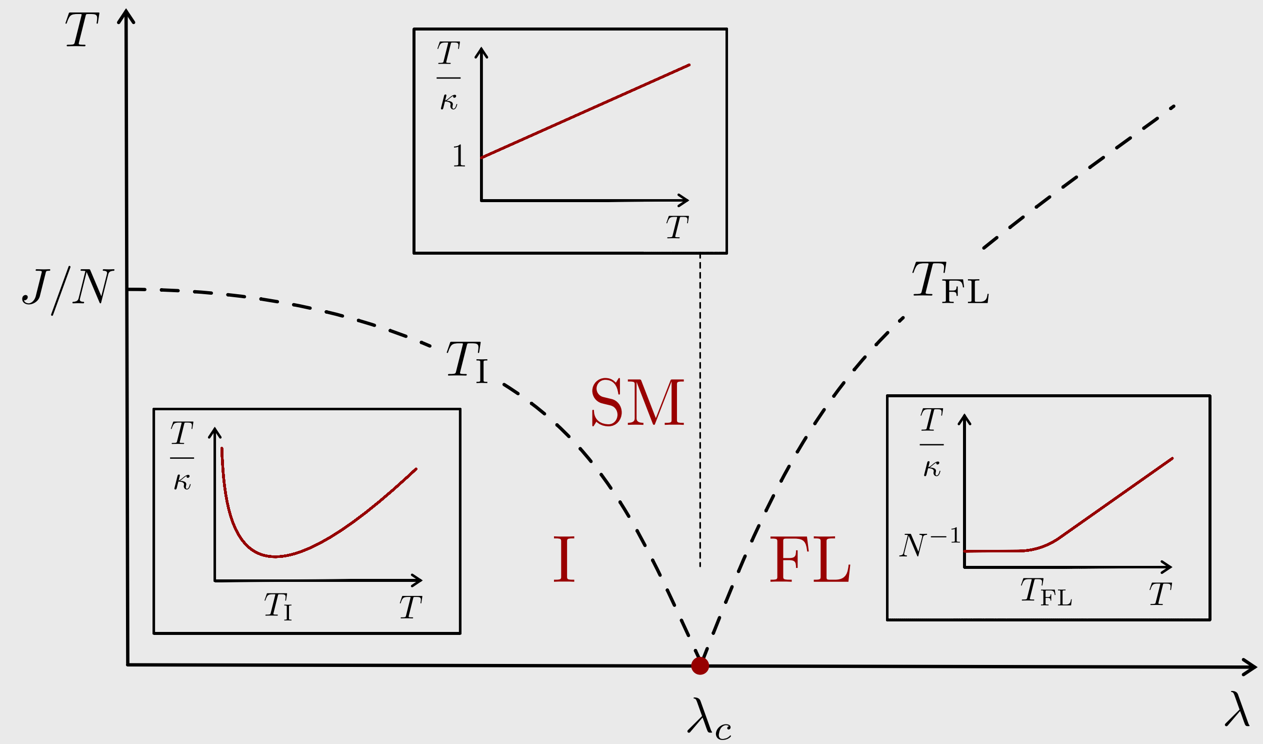

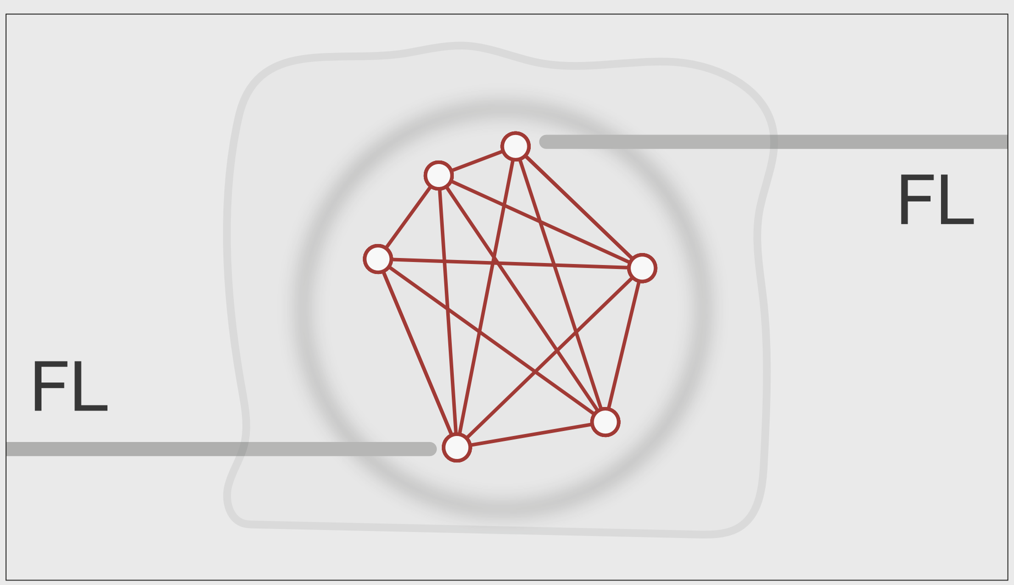

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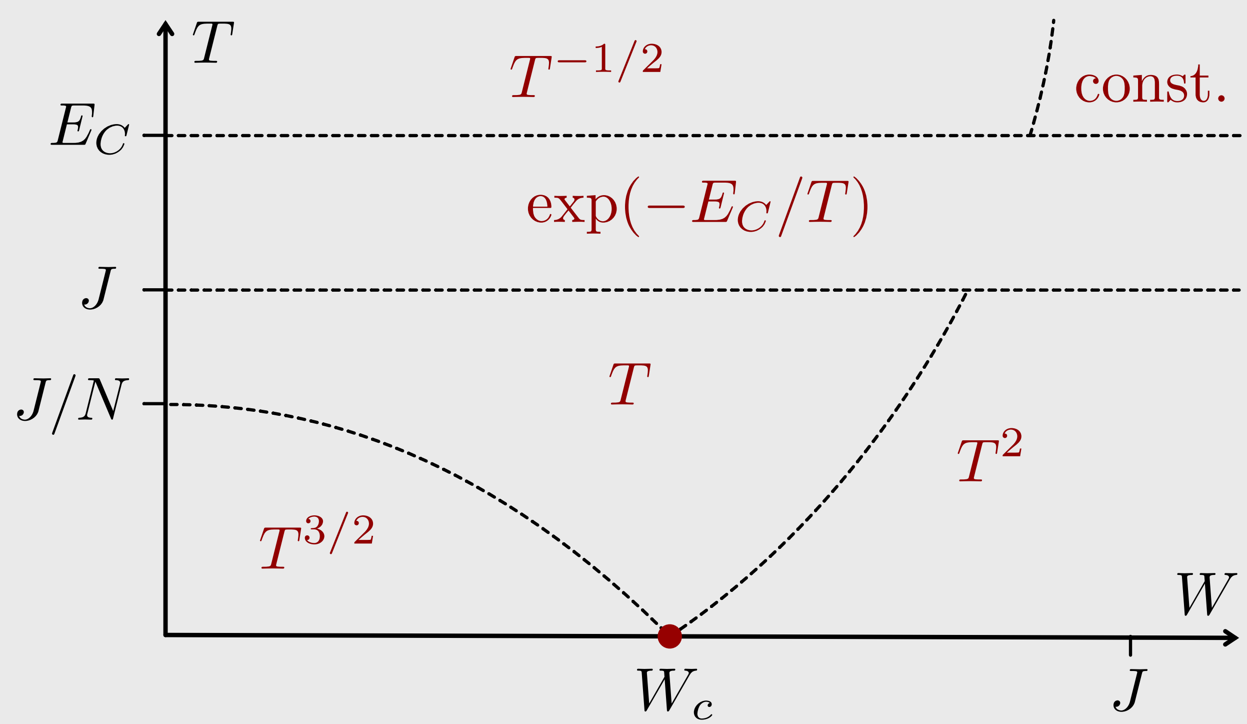

Correlated quantum matter via the SYK paradigm

Correlated20, Oct. 1st 2020

Alexander Altland, Dmitry Bagrets (Cologne),
Alex Kamenev (Minnesota)

SYK model & conformal symmetry breaking

quantum fluctuations

granular extension

Nucl. Phys. B **911**, 191 (2016)
Nucl. Phys. B **921**, 727 (2017)
Phys. Rev. Lett. **123**, 226801 (2019)
Phys. Rev. Lett. **123**, 106601 (2019)

$$\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, c_i^\dagger c_j^\dagger c_k c_l$$

max. entropy/random

invention in nuclear physics

Volume 34B, number 4 PHYSICS LETTERS 1 March 1971

TWO-BODY RANDOM HAMILTONIAN AND LEVEL DENSITY

O. BOHIGAS and J. FLORES *
Institut de Physique Nucléaire, Division de Physique Théorique ‡, 91 - *Orsay - France*

Received 22 December 1970

Volume 33B, number 7 PHYSICS LETTERS 7 December 1970

VALIDITY OF RANDOM MATRIX THEORIES FOR MANY-PARTICLE SYSTEMS *

J. B. FRENCH
Department of Physics and Astronomy, University of Rochester, Rochester, New York, USA

and

S. S. M. WONG
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and Department of Physics and Astronomy, University of Rochester, Rochester, New York, USA

Received 19 October 1970

rediscovery in cond-mat

VOLUME 69, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1992

Universal Quantum-Critical Dynamics of Two-Dimensional Antiferromagnets

Subir Sachdev and Jinwu Ye

Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
and Center for Theoretical Physics, P.O. Box 6666, Yale University, New Haven, Connecticut 06511

(Received 13 April 1992)

The universal dynamic and static properties of two-dimensional antiferromagnets in the vicinity of a zero-temperature phase transition from long-range magnetic order to a quantum-disordered phase are studied. Random antiferromagnets with both Néel and spin-glass long-range magnetic order are considered. Explicit quantum-critical dynamic scaling functions are computed in a $1/N$ expansion to two-loop level for certain nonrandom, frustrated square-lattice antiferromagnets. Implications for neutron scattering experiments on the doped cuprates are noted.

PACS numbers: 75.10.Jm, 05.30.Fk, 75.50.Ee

Sachdev-Ye-Kitaev Model (15)

A model of *N* randomly interacting *Majorana* fermions

$$\boxed{\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l\,, \qquad \{\chi_i, \chi_j\} = 2\delta_{ij}}$$

SYK model

where the interaction constants are static and random,

$$\langle |J_{ijkl}|^2 \rangle = \frac{6J^2}{N^3}$$

high energy scale

SYK model & conformal symmetry breaking

Strong interactions:

$$\boxed{\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l}$$

‘infinite range’, strong, chaotic: amenable to large *N* mean field methods

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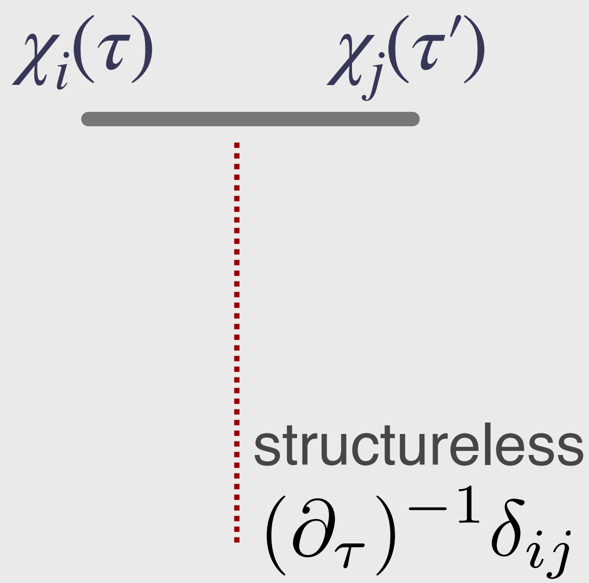
first assault: diagrammatic expansion of Majorana propagator



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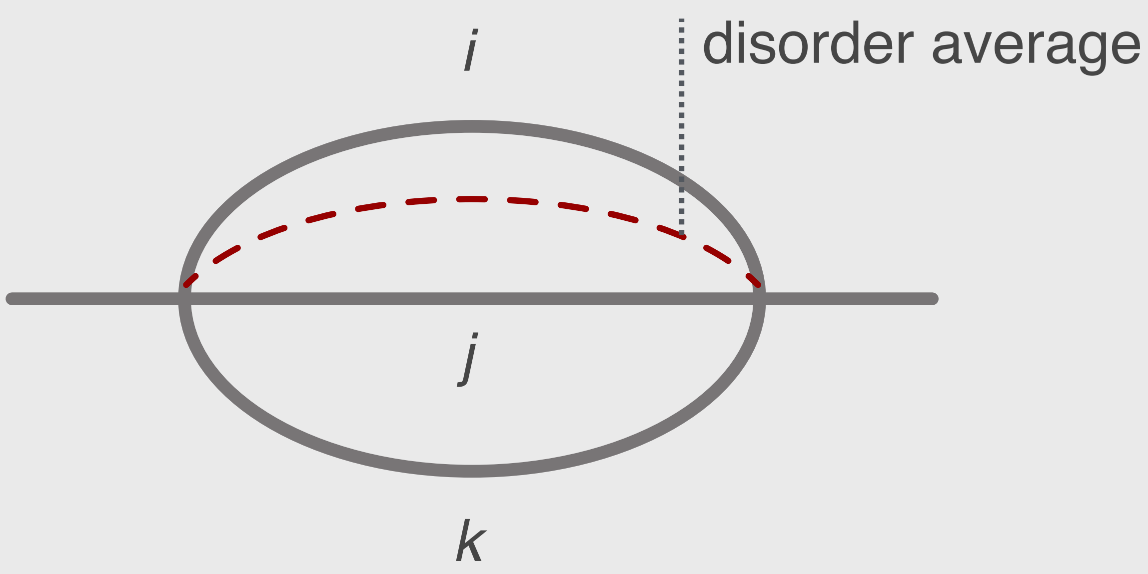

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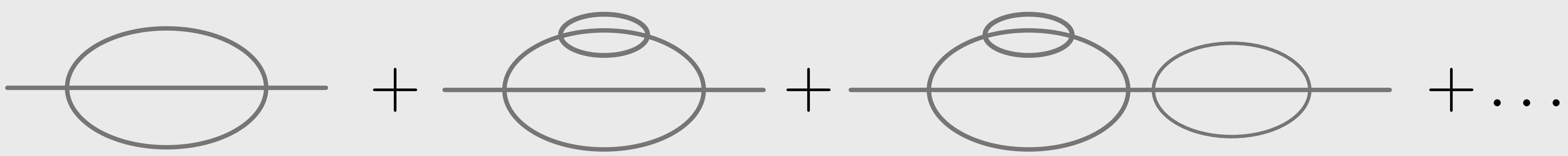

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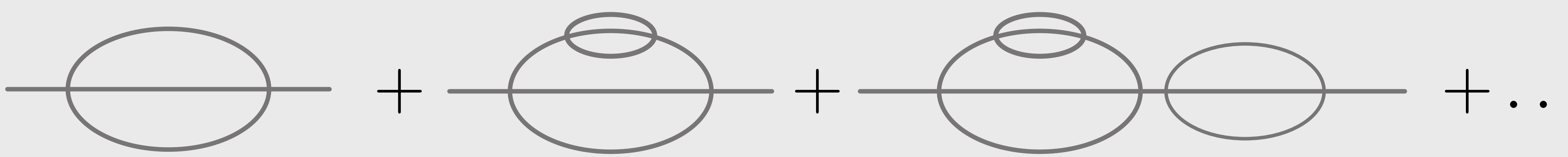
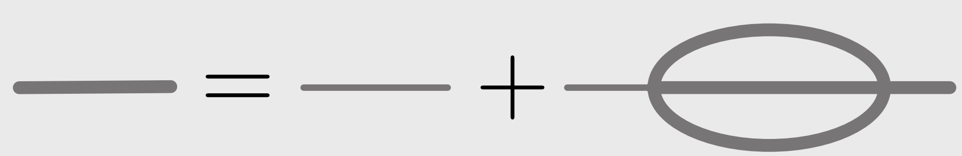
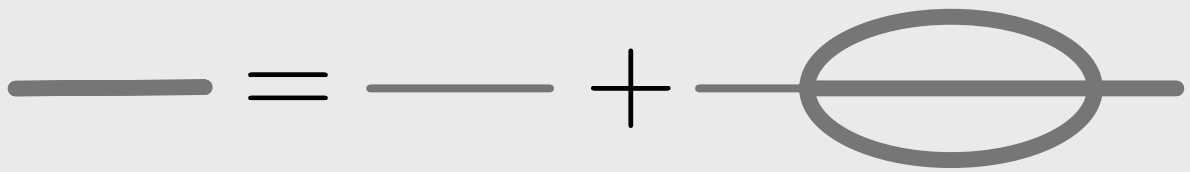




solution of mean field equations



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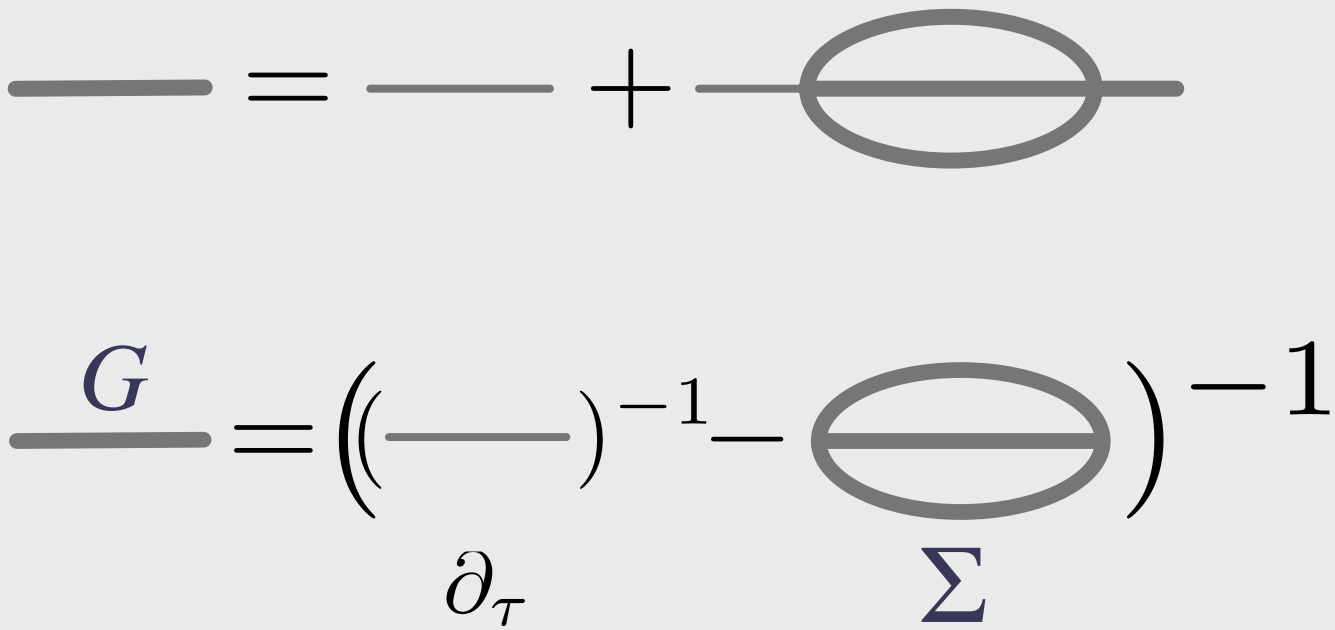
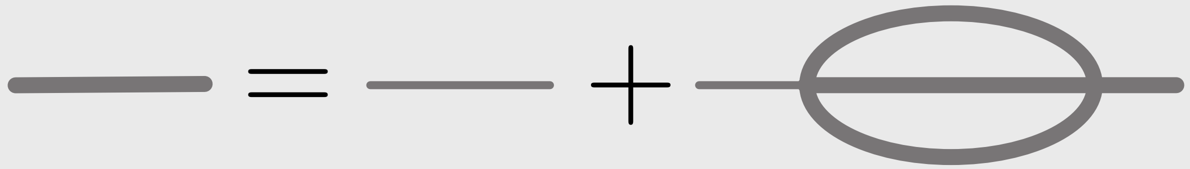
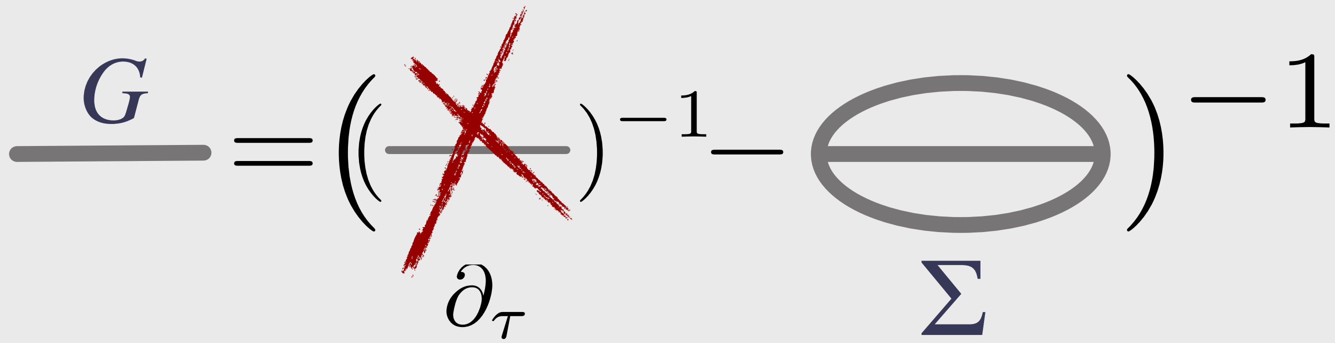


$$G = (\partial_\tau - \Sigma)^{-1}, \qquad \Sigma = J[G]^3$$

solution of mean field equations





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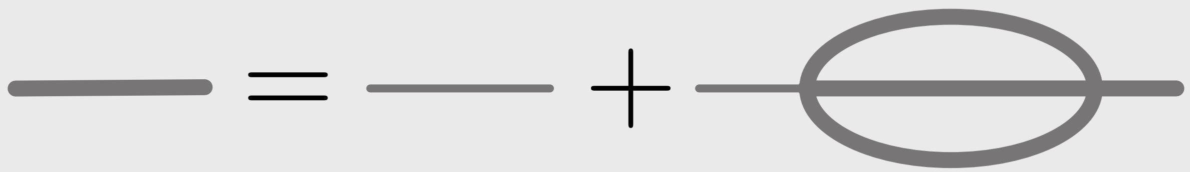

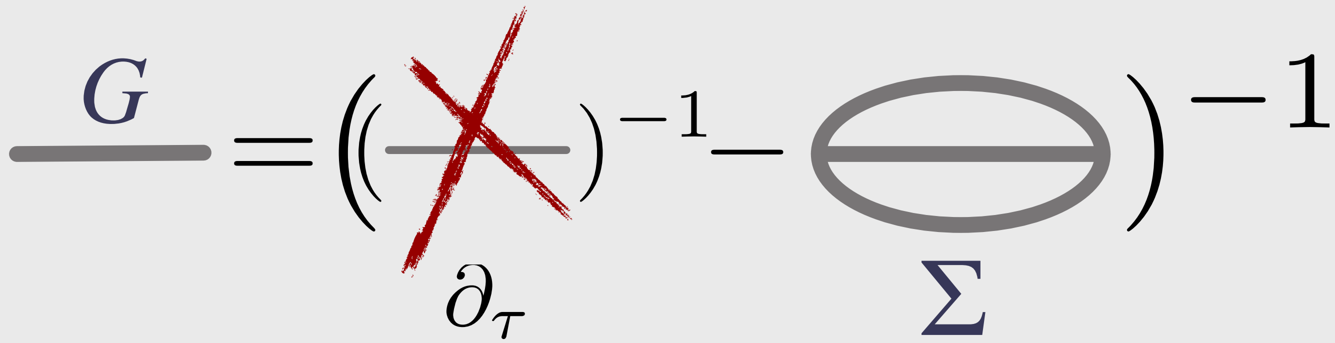

$$G = (\cancel{\partial_\tau} - \Sigma)^{-1}, \qquad \Sigma = J[G]^3$$

solutions $(\partial_\tau \ll J)$

numerical factor

$$\boxed{\begin{aligned} G(\tau,\tau') &= -\frac{b}{J^{1/2}}\,\frac{\mathrm{sgn}(\tau-\tau')}{|\tau-\tau'|^{1/2}} \\ \Sigma(\tau,\tau') &= -\frac{b^3}{J^{1/2}}\,\frac{\mathrm{sgn}(\tau-\tau')}{|\tau-\tau'|^{3/2}} \end{aligned}}$$

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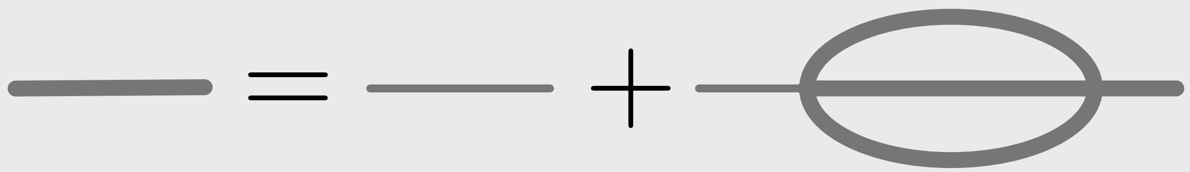

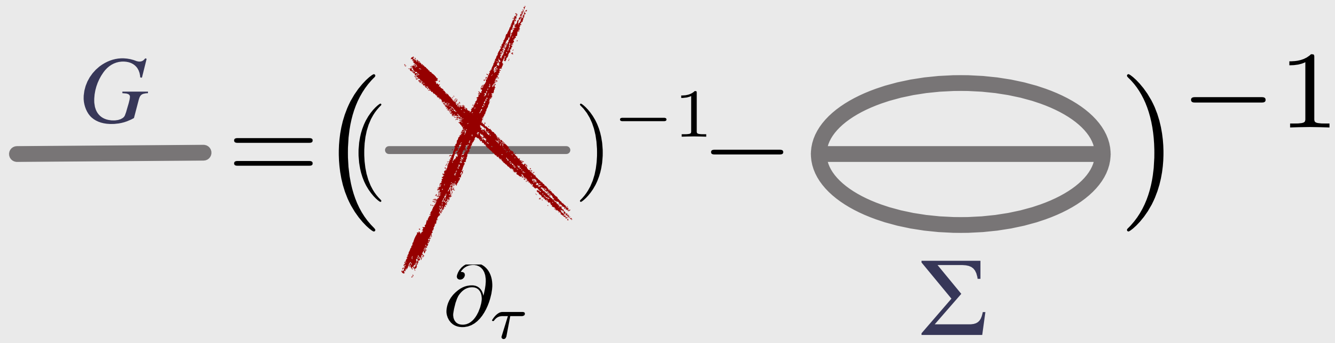

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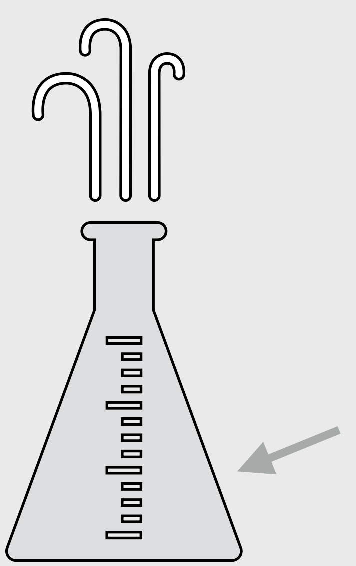


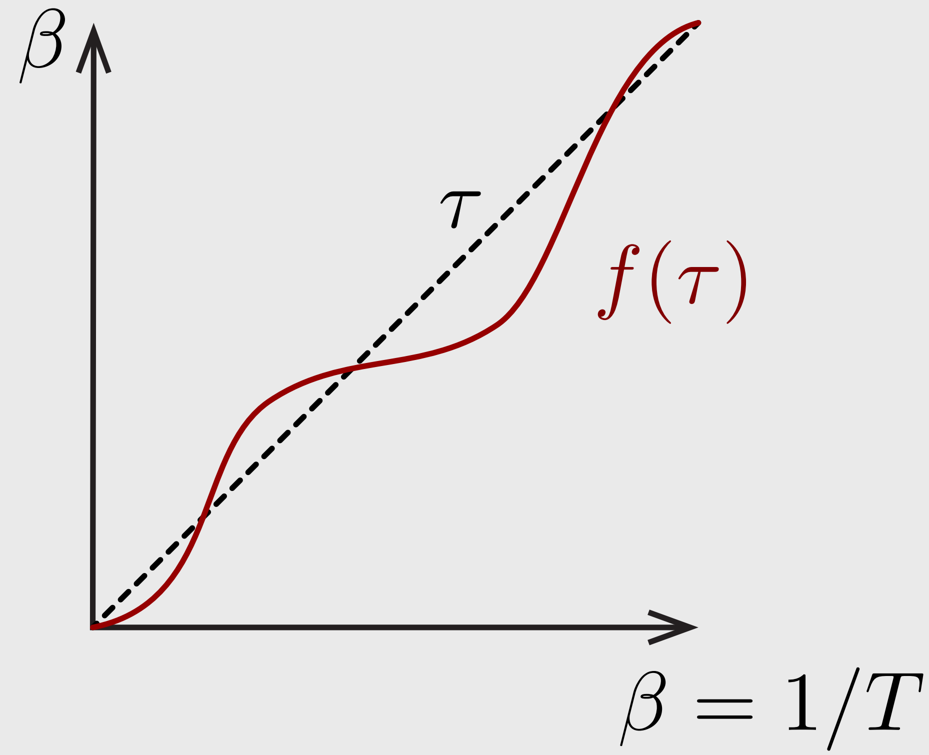
non-FL

Symmetries

action $S[\eta] = \int_0^\beta d\tau(\eta\partial_\tau\eta + \eta^4)$

interaction invariant under reparameterization of time

$$f : S^1 \to S^1, \tau \mapsto f(\tau),$$
$$f \in \mathrm{Diff}(S^1)$$



and transformation of fields

$$\eta(\tau) \to \eta_f(f) \equiv \left(\frac{df}{d\tau}\right)^{1/4} \eta(\tau(f))$$

$$\boxed{S[\eta] \simeq S[\eta_f]}$$

broken by ∂_τ

Elements of the diffeomorphism manifold describe reparameterizations of time. Infinitesimally: generated by Virasoro algebra. Weakly broken by time derivatives — problem has NCFT_1 symmetry (Maldacena and Stanford, 15).

Symmetry of the mean field

$$G(\tau,\tau') \sim \frac{\operatorname{sgn}(\tau-\tau')}{|\tau-\tau'|^{1/2}}$$

invariant under conformal transformations $\tau \to \dfrac{a\tau+b}{d\tau+c}$

each $f \in \mathrm{Diff}(S^2)/\mathrm{SL}(2,R)$ generates new solution

$$G_f(\tau,\tau') = f'(\tau)^{1/4} f'(\tau')^{1/4} G(f(\tau), G(f(\tau'))$$

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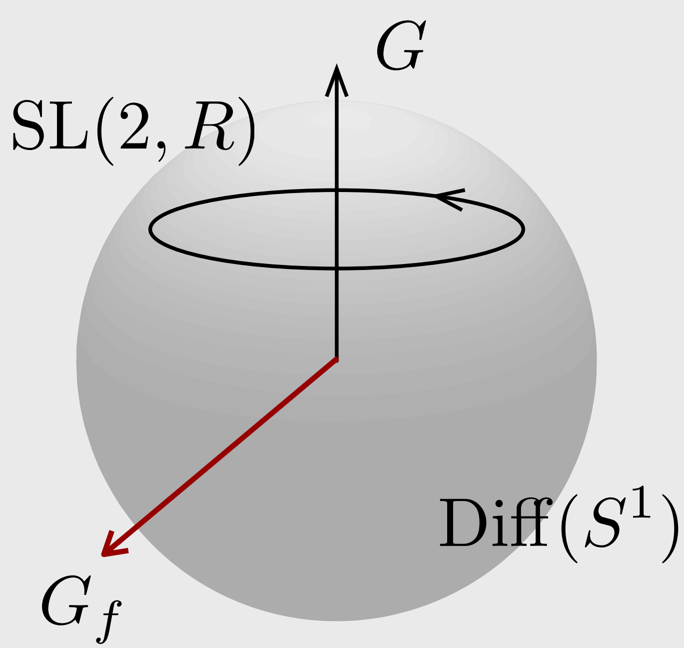

emergence of infinite dimensional
Goldstone mode manifold

$$\boxed{\mathrm{Diff}(S^1)/\mathrm{SL}(2,R)}$$

Quantum fluctuations

	AF spin chain	SYK

	AF spin chain	SYK
μ representation	quantum partition sum	$G\Sigma$ action

	AF spin chain	SYK

$$S[\Sigma, G] = -\frac{N}{2}\left[\mathrm{tr}\ln(\partial_\tau + \Sigma) + \frac{J}{4}\int_0^\beta d\tau d\tau' \left((G_{\tau,\tau'})^4 + \Sigma_{\tau,\tau'}G_{\tau',\tau}\right)\right]$$

Sachdev, Ye 91

	AF spin chain	SYK
μ representation	quantum partition sum	$G\Sigma$ action
mean field	Neel state, $\langle m \rangle$	Green function, $\langle G \rangle$

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	$\langle m(x) m(y) \rangle$ non-trivial	$\langle G(\tau) G(\tau') \rangle$
	anomalous dimensions	anomalous dimensions

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Liouville Quantum mechanics

reparameterization: $f'(\tau) = \exp(\phi(\tau))$

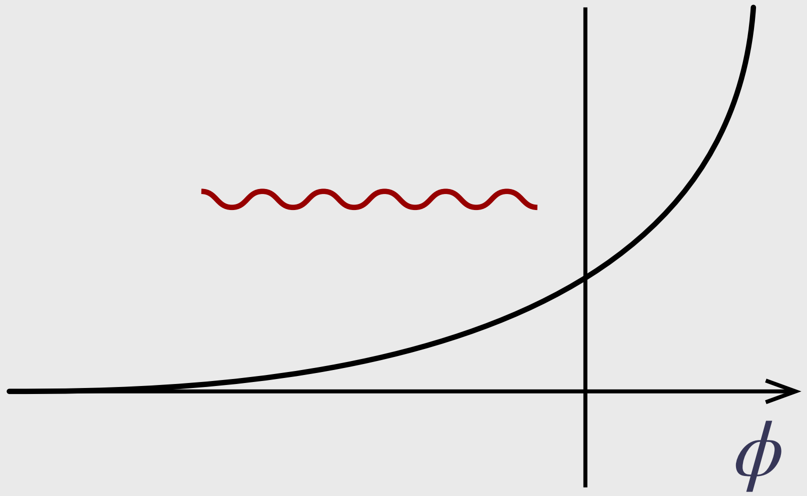
$$\boxed{Z = \int D\phi\, e^{-S[\phi]}, \qquad S[\phi] = M \int d\tau \left(\frac{1}{2} \partial_\tau \phi^2 + 2e^{\phi} \right)}$$

$$M = \frac{b^2}{32J} N \log(N)$$

time scale at which fluctuations become strong

effect of low energy Goldstone mode fluctuations encapsulated in Liouville QM. Universal feature (Shelton, Tsvelik 98): operator correlation functions decay as

$$\langle \mathcal{O}(\tau)\mathcal{O}(\tau') \rangle \sim |\tau - \tau'|^{-3/2}$$



Sanity check I: Green function

path integral representation of Green function

$$G_f(\tau,\tau') = -\frac{b}{\sqrt{\pi}J^{1/2}} \left\langle e^{\frac{1}{4}(\phi(\tau_1)+\phi(\tau_2))} \int_0^\infty \frac{d\alpha}{\sqrt{\alpha}}\, e^{-\alpha \int_{\tau_1}^{\tau_2} ds\, e^{\phi(s)}} \right\rangle_\phi$$

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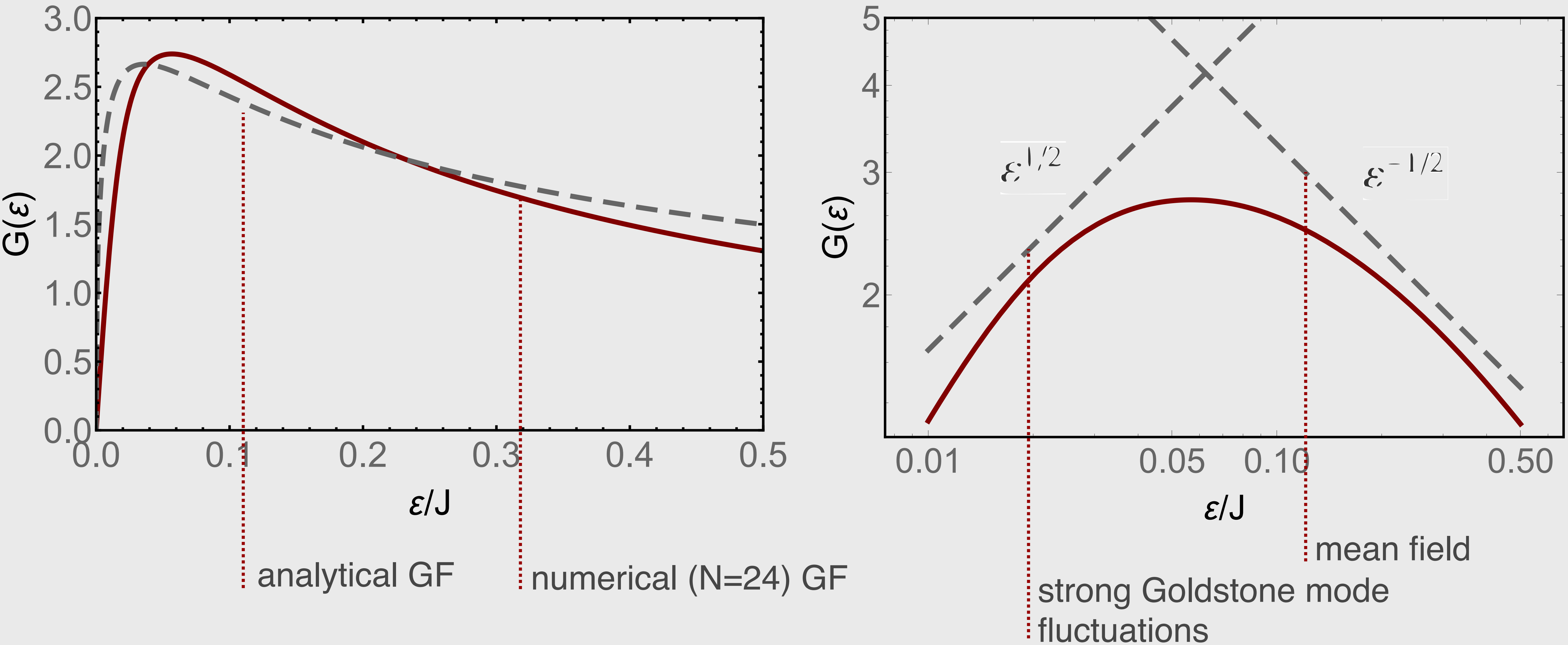
$$\Big\downarrow$$

$$G(\epsilon) = -\frac{ib}{\sqrt{J}}\left(\frac{2}{\pi M}\right)^{1/2}\int_0^{+\infty} dk\,\frac{k\sinh(2\pi k)}{2\pi^2}\,\Gamma^2\left(\tfrac{1}{4}+ik\right)\Gamma^2\left(\tfrac{1}{4}-ik\right)\,\frac{2\epsilon}{E_k^2+\epsilon^2},$$

$$E_k = k^2/2M$$

Sanity check I: Green function

SYK Green function beyond mean field: resurrection of full symmetry at small energies

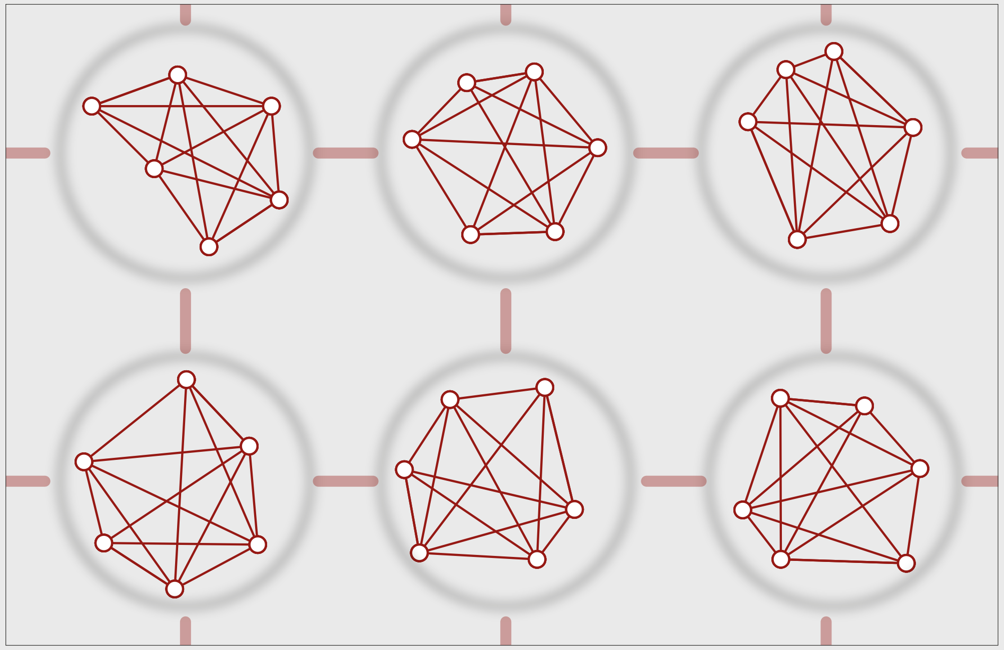


Green function crossover

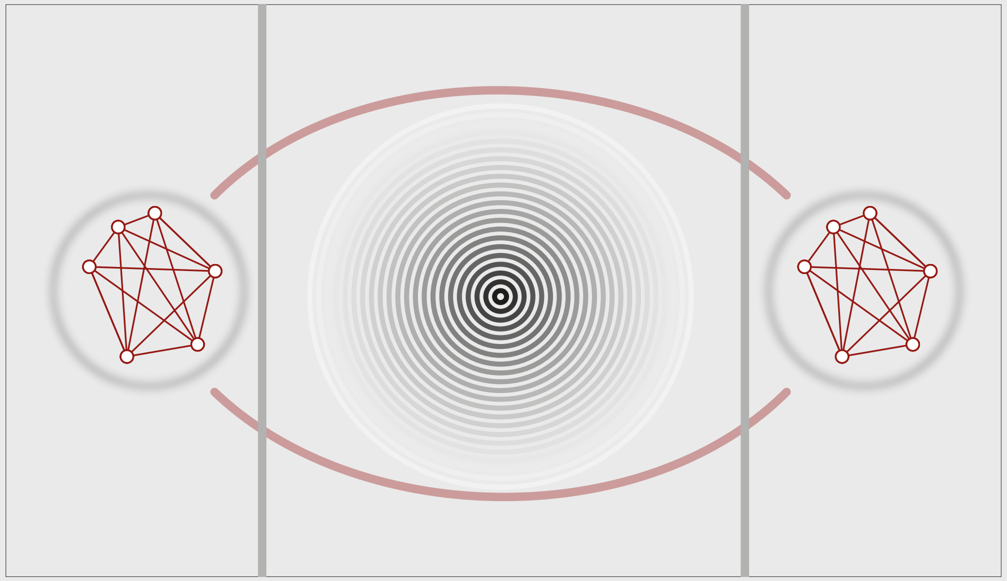
$$G(\tau - \tau') \sim \begin{cases} \dfrac{1}{|\tau - \tau'|^{1/2}}, & |\tau - \tau'| \lesssim M, \\ \dfrac{1}{|\tau - \tau'|^{3/2}}, & |\tau - \tau'| \gtrsim M, \end{cases}$$

$$[\eta] \sim \begin{cases} \tau^{-1/4}, & |\tau - \tau'| \lesssim M, \\ \tau^{-3/4}, & |\tau - \tau'| \gtrsim M, \end{cases}$$

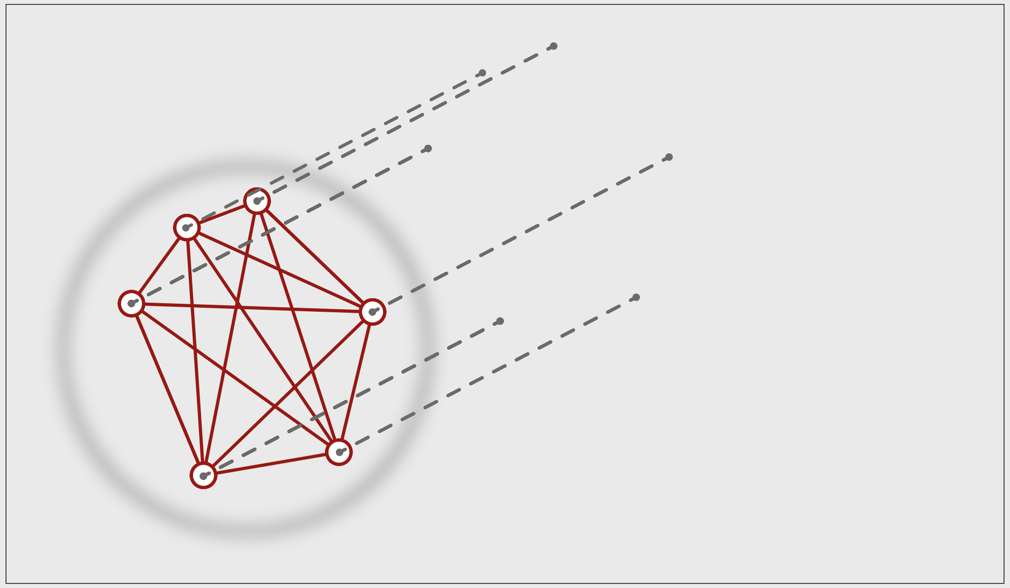
applications



modeling correlated quantum matter as SYK arrays



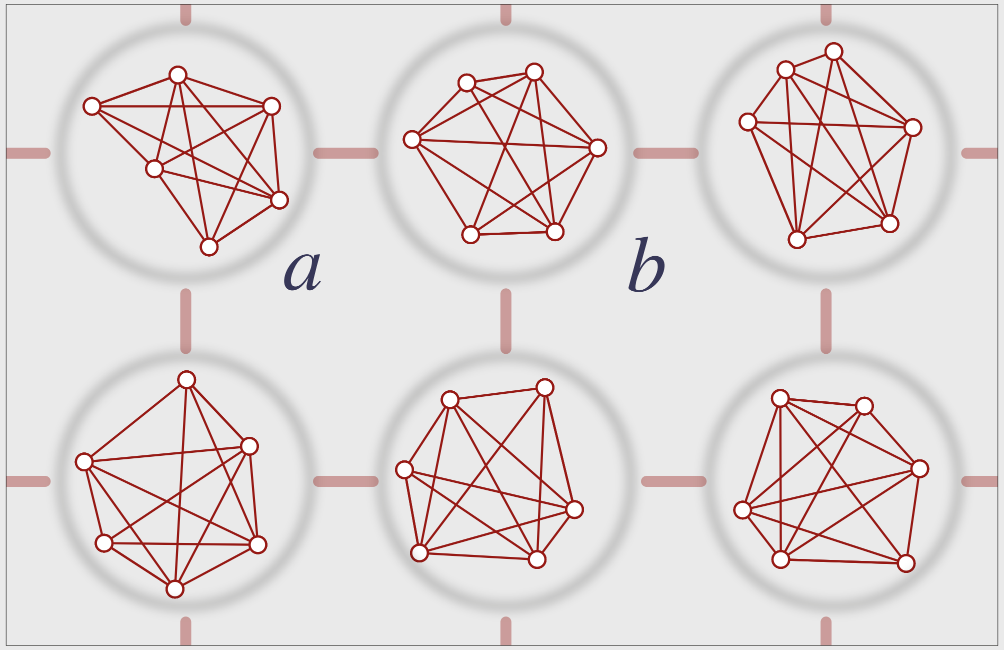
understanding AdS/CFT correspondence



understanding the quantum mechanics of chaotic Fock spaces

granular extension & quantum criticality

granular quantum matter from SYK



$$\hat{H} = \sum_a \hat{H}_{\mathrm{SYK}}(\eta^a) + \hat{H}_{\mathrm{T}}$$

$$\hat{H}_{\mathrm{T}} = \frac{i}{2} \sum_{\langle a,b \rangle} V_{ab}^{ij} \eta_i^a \eta_j^b$$

$$\langle (V_{ab}^{ij})^2 \rangle = \frac{v^2}{N}$$

mean field physics (Song *et al. 17)*

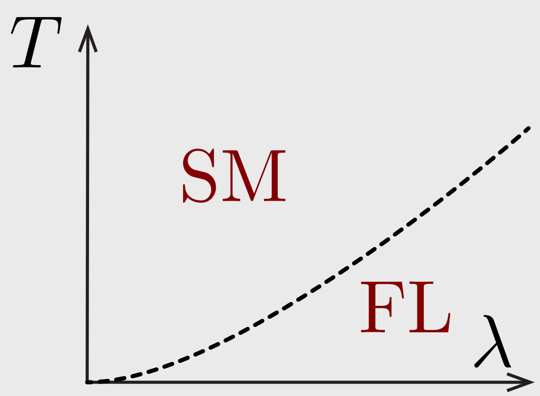
for $N \to \infty$: $G(\tau,\tau') \sim \langle \eta(\tau)\eta(\tau') \rangle_{\mathrm{SYK}} \sim \dfrac{1}{|\tau-\tau'|^{1/2}} \Rightarrow [\eta] = \tau^{-1/4}$

$$[S_{\mathrm{T}}] = \left[\int d\tau \eta\eta\right] = \tau^{1-2\times 1/4} = \tau^{1/2}$$ relevant perturbation

inter-dot tunneling induces crossover between

NFL/strange metal phase at high temperatures (thermal conductance $\kappa = \mathrm{const.}$)

FL/metallic phase at low temperatures ($\kappa \sim T$)



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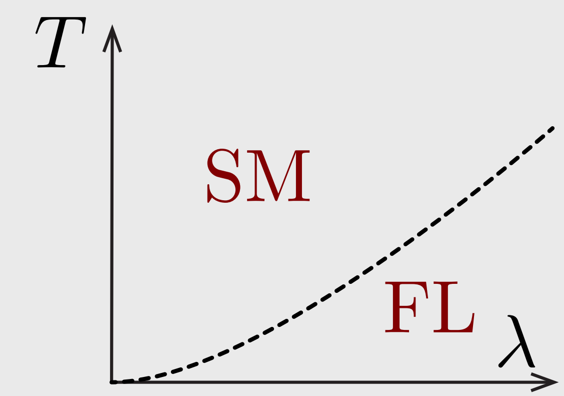
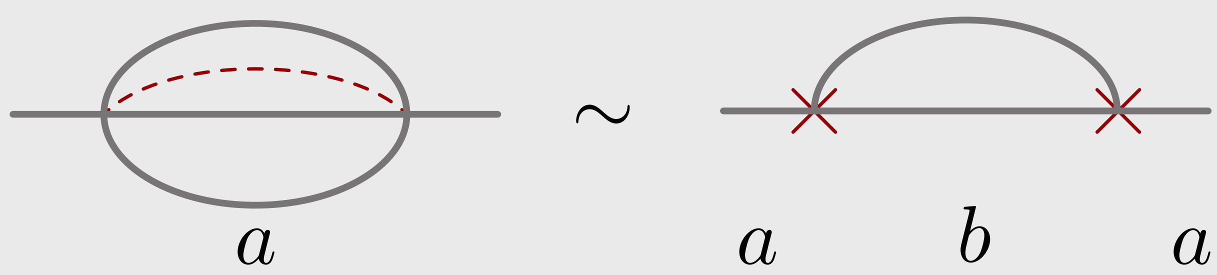


crossover temperature



$$J \int_0^\beta G^3 \sim v^2 \int_0^\beta G \quad \Rightarrow \quad \boxed{T_{\mathrm{FL}} \sim \frac{v^2}{J}}$$

physics of mesoscopic granules (*N* finite)

2nd energy scale $T_\mathrm{F} = M^{-1}$ causes competition upon lowering temperature

$$[S_\mathrm{T}] = \left[\int d\tau \eta\eta\right]$$

$$[G] = [\eta\eta] = \begin{cases} t^{-1/2} & , T > T_\mathrm{F}, \\ t^{-3/2} & , T < T_\mathrm{F} \end{cases}$$

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T

T_{FL}

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relevant perturbation

T_{F}

metallic phase

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irrelevant perturbation

T_{FL}

insulating phase

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irrelevant perturbation

T_FL

insulating phase

quantum phase transition at $T_\mathrm{FL} \sim T_\mathrm{F} \Leftrightarrow \dfrac{v^2}{J} \sim M^{-1} \sim \dfrac{J}{N}$ (cf. Lunkin *et al.* 18)

theoretical formulation

start from extended $G\Sigma$ action

$$S[G,\Sigma] \equiv \sum_a S_0[G^a,\Sigma^a] + \sum_{\langle ab\rangle} S_\mathrm{T}[G^a,G^b]$$

project to mean-field/reparameterization sector

$$S_0[h] = -\,m \sum_a \int d\tau\,\{h^a,\tau\},$$

$$S_\mathrm{T}[h] = -\,w \sum_{\langle ab\rangle} \int d\tau_1 d\tau_2 \left(\frac{h_1'^a h_2'^a}{[h_1^a - h_2^a]^2} \times \frac{h_1'^b h_2'^b}{[h_1^b - h_2^b]^2} \right)^{1/4}$$

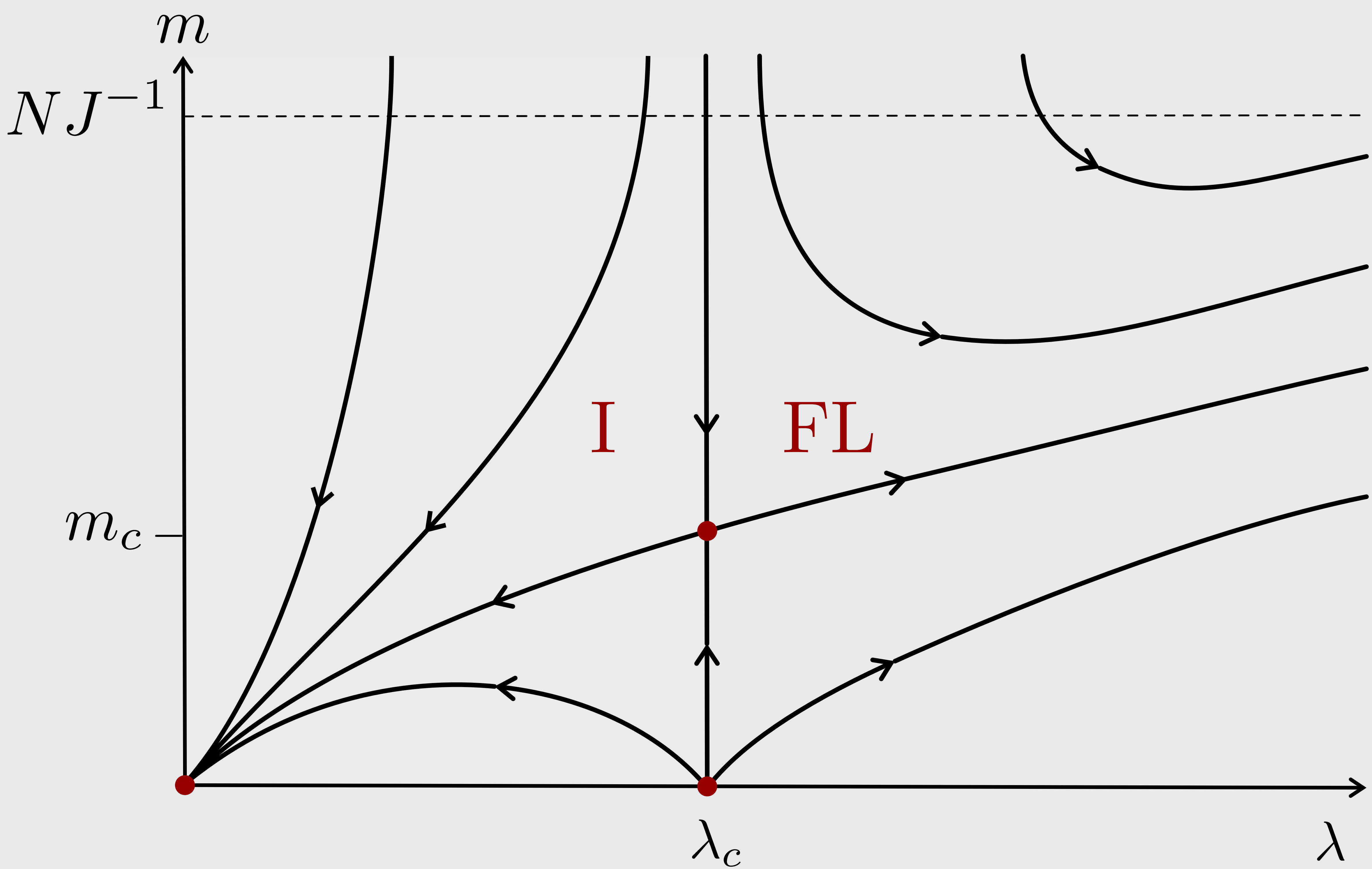
two parameter theory defined through dimensionful constants

$$[m] = \mathrm{time}$$

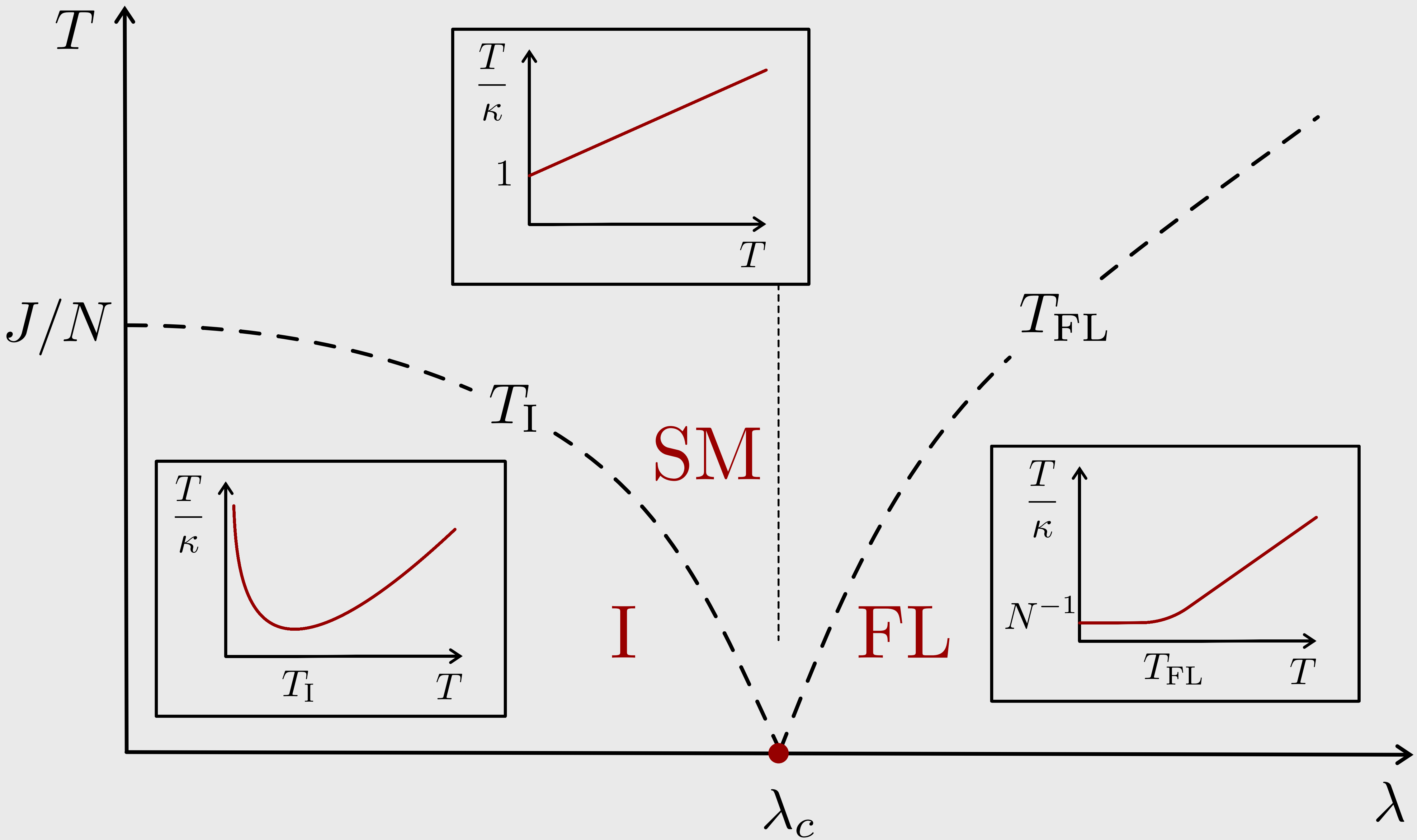
$$[w] = \mathrm{energy}$$

run RG: $'h = h_\mathrm{s} + h_\mathrm{f}'$

RG flow cont'd



… leading to phase diagram



summary

SYK cell a paradigm of interacting quantum matter

analytically approachable

physics governed by universal quantum fluctuations

a building block for the modeling of correlated phases

Holographic interpretation (Maldacena & Stanford, 16; Almheiri & Polchinksi, 16)

Consider 2d Einstein-Hilbert action

also constant

positive cosmological constant

$$S = \frac{\phi_0}{16\pi G} \int \sqrt{g}(R + \Lambda)$$

gravitational constant

action invariant under conformal deformations of 2d space (because it is topological)

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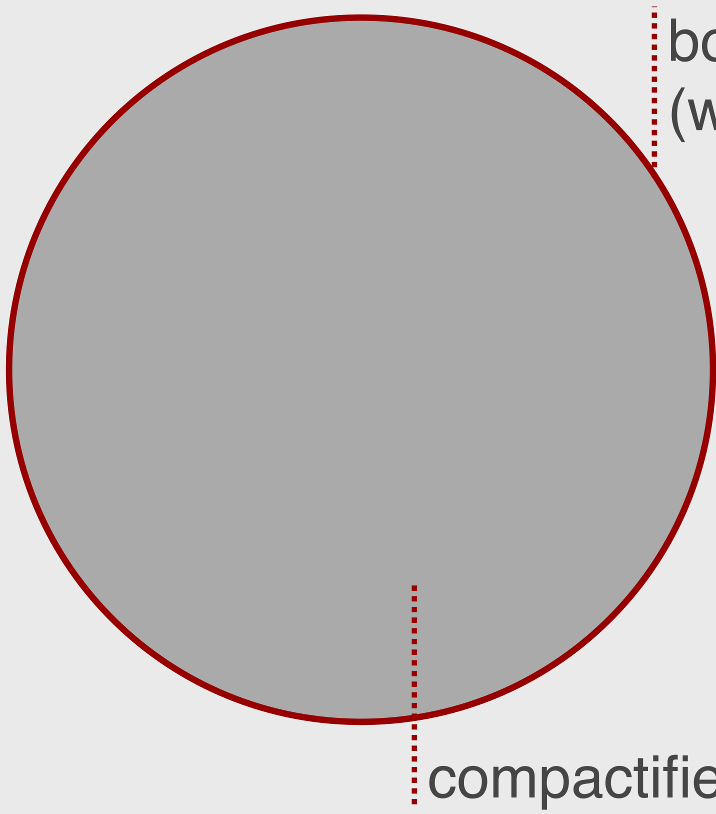

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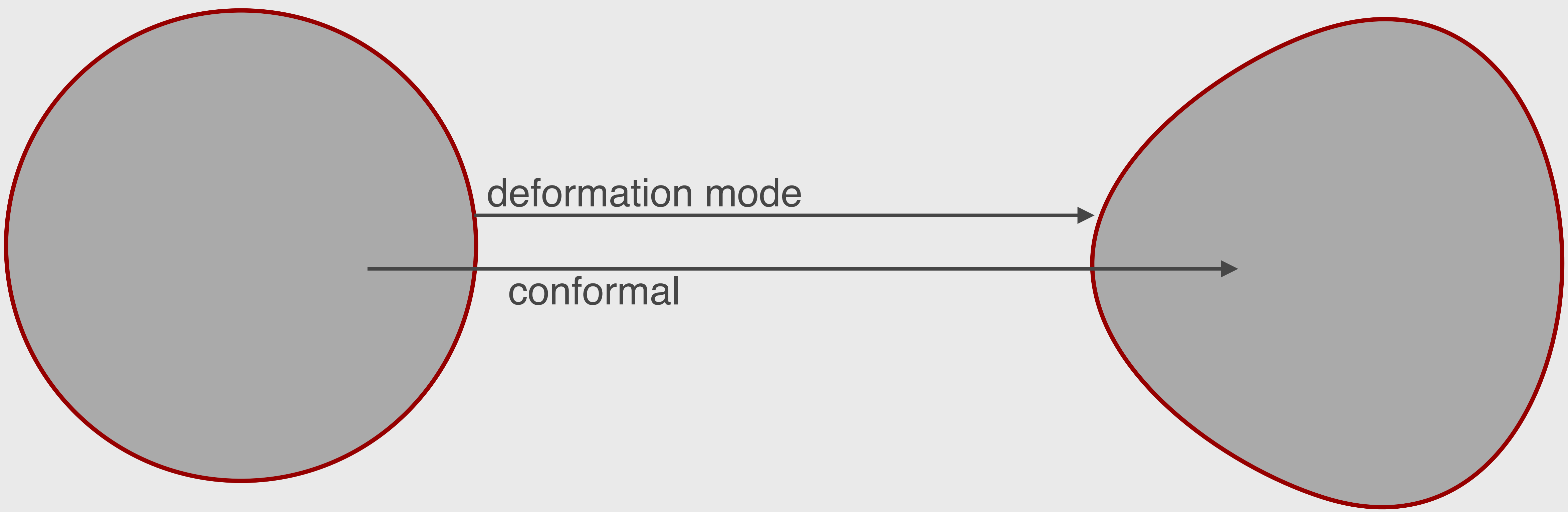

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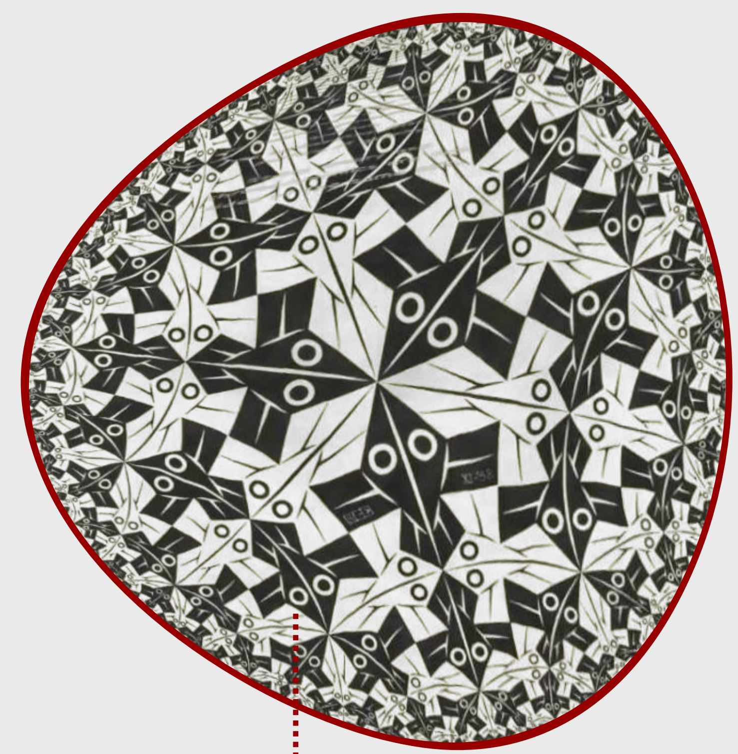

AdS metric

AdS metric (spontaneously) breaks symmetry to *SL(2,R).* Reparameterization Goldstone modes without action.

Holographic interpretation (continued)

Improve situation by upgrading pure gravity action to dilaton action

now a field

$$S = \frac{\phi_0}{16\pi G}\int \sqrt{g}(R+\Lambda) \longrightarrow \frac{1}{16\pi G}\int \sqrt{g}\phi(R+\Lambda)+\ldots$$

Jackiw Teitelboim gravity

This action (i) is non-topological, (ii) fluctuations of the dilaton field weakly break conformal symmetry and (iii) afford physical interpretation if AdS2 action is seen as boundary theory of higher dimensional extremal black hole.

Combination (i-iii) motivates boundary with conformal invariance breaking and signatures of quantum chaos.