Optimal Transport Reconstruction and Peaks Theory for Energy

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with
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Reconstruction of density and displacements
Excursion set peaks in energy

Optimal Transport:

Assume initial density field uniform (same for all cosmologies); solve for displacements

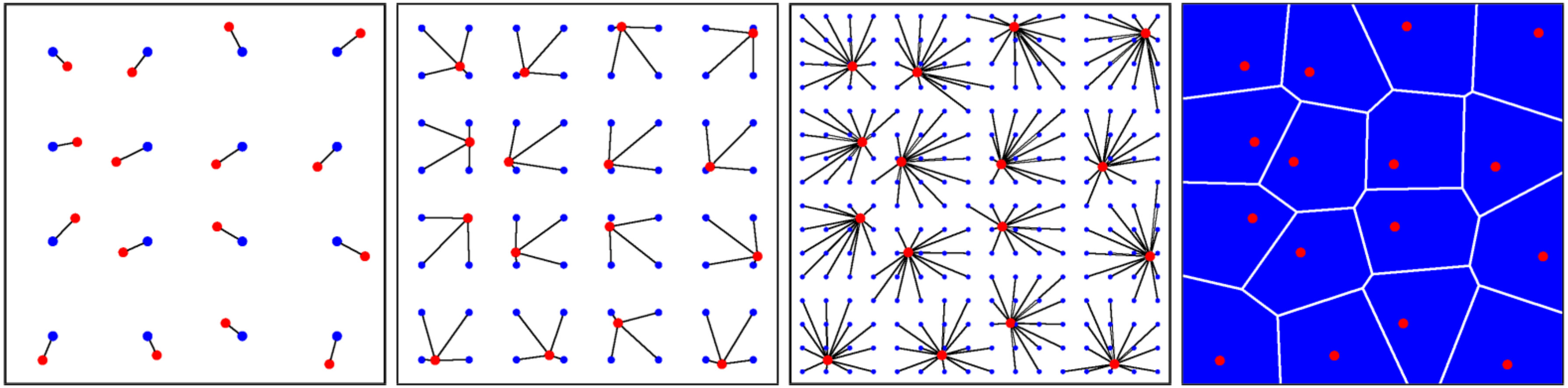
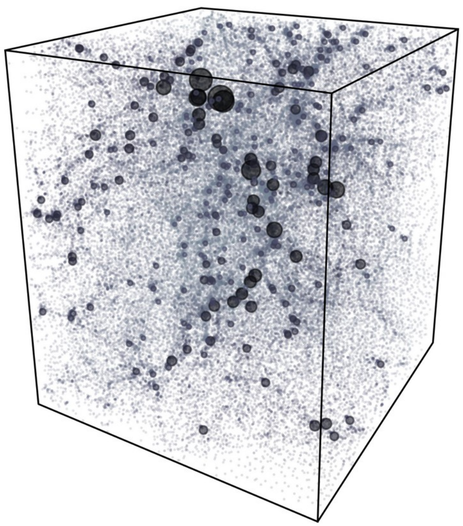


Figure 52: From discrete-discrete to semi-discrete optimal transport problem. The red points show the distribution of matter at current time and the blue points represent the initial condition by a regular grid. From left to right, we increase the precision by using a finer grid for the initial positions (Lévy et al., 2020).

Optimal transport

(Nikakhtar et al. 2022)

$z = 0$

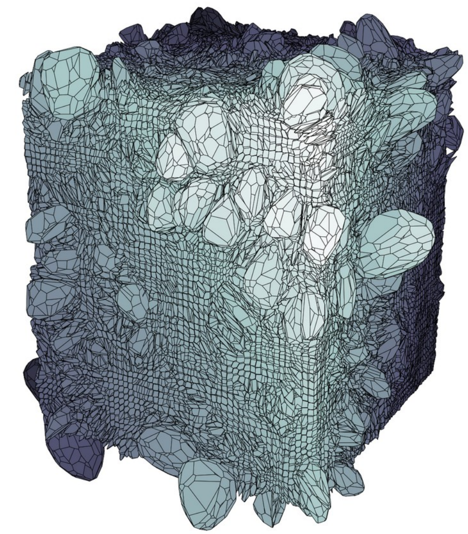


Weighted Semi-discrete OT Reconstruction:

Computing Laguerre cells V_i^ψ

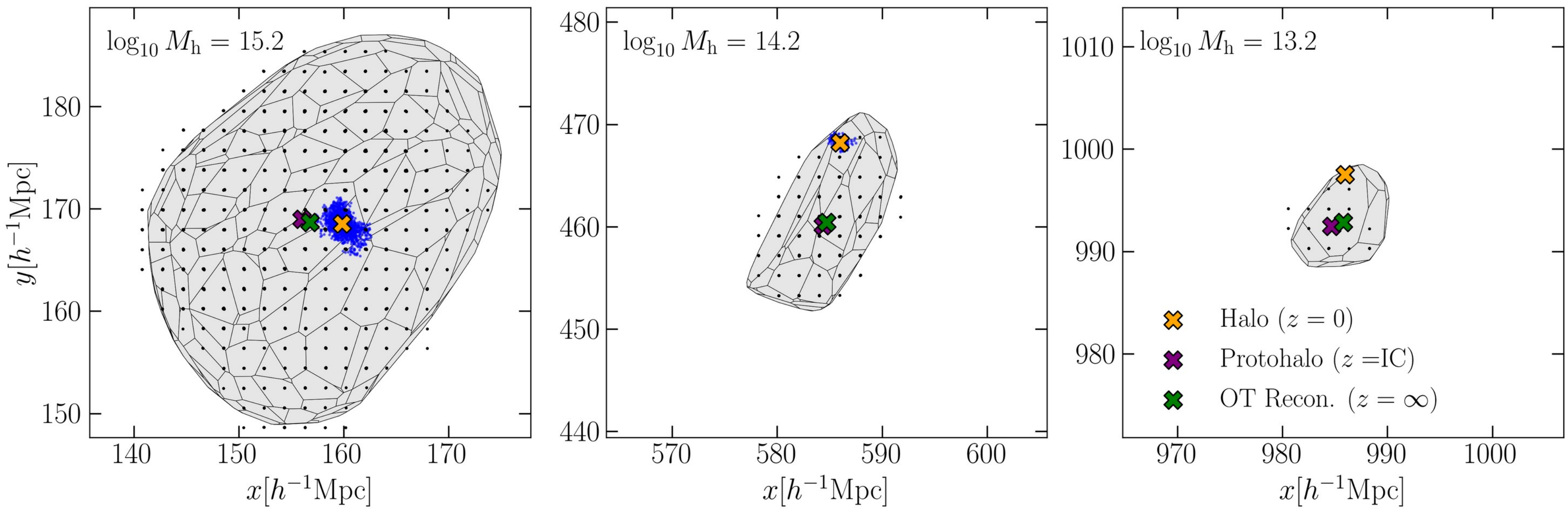
$$V_i^\psi = \left\{ \mathbf{q} \;\middle|\; \tfrac{1}{2}|\mathbf{x}_i - \mathbf{q}|^2 - \psi_i < \tfrac{1}{2}|\mathbf{x}_j - \mathbf{q}|^2 - \psi_j,\ \forall j \neq i \right\}$$

$z = \infty$



Reconstructs displacements, hence protohalo positions, shapes

Optimal transport



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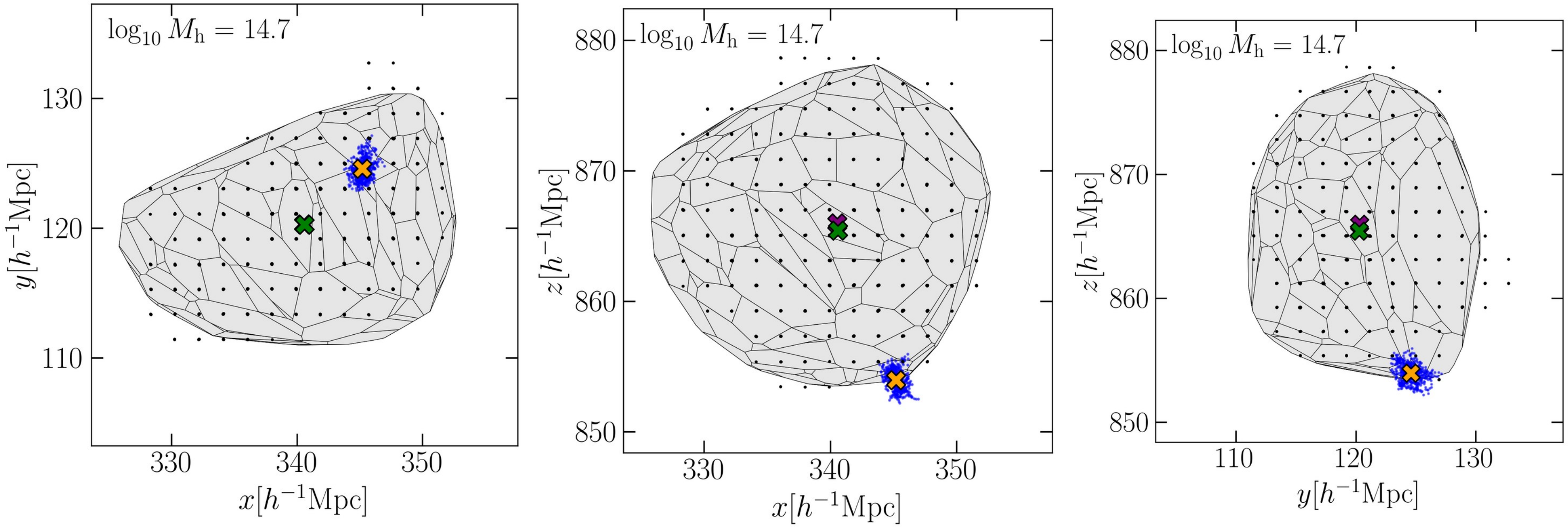
Optimal transport



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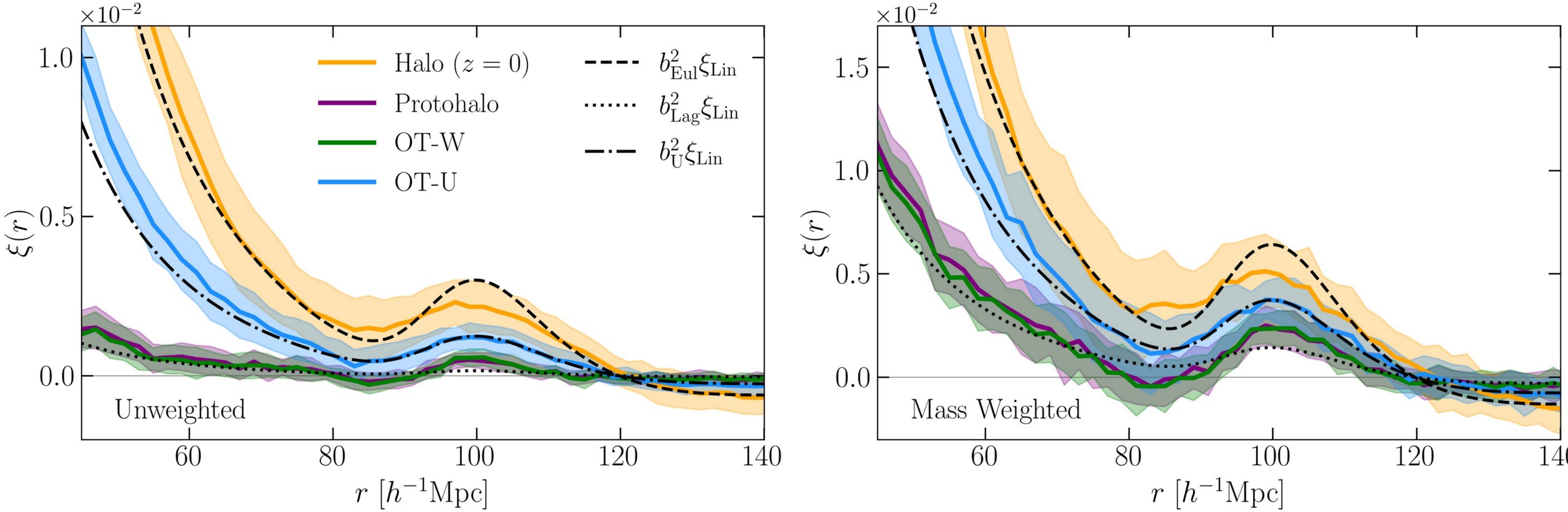
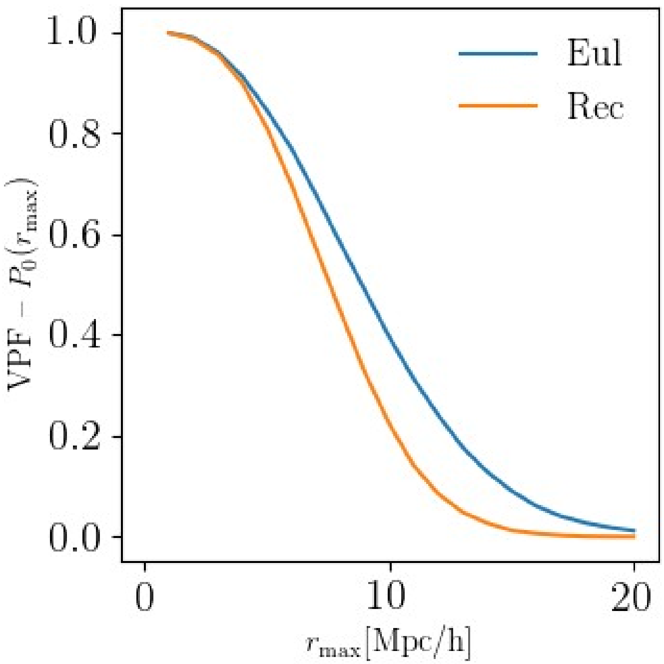
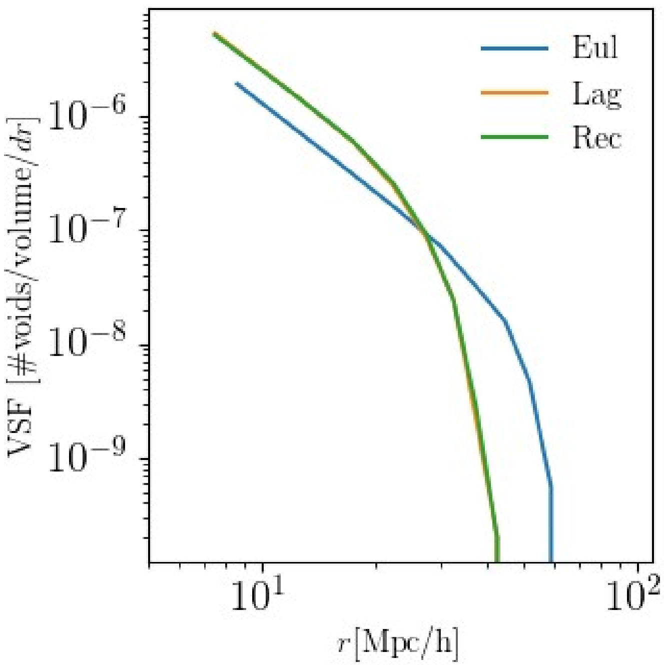
Optimal transport



Reconstructs two-point statistics (mass weighted ~ HOD galaxies)

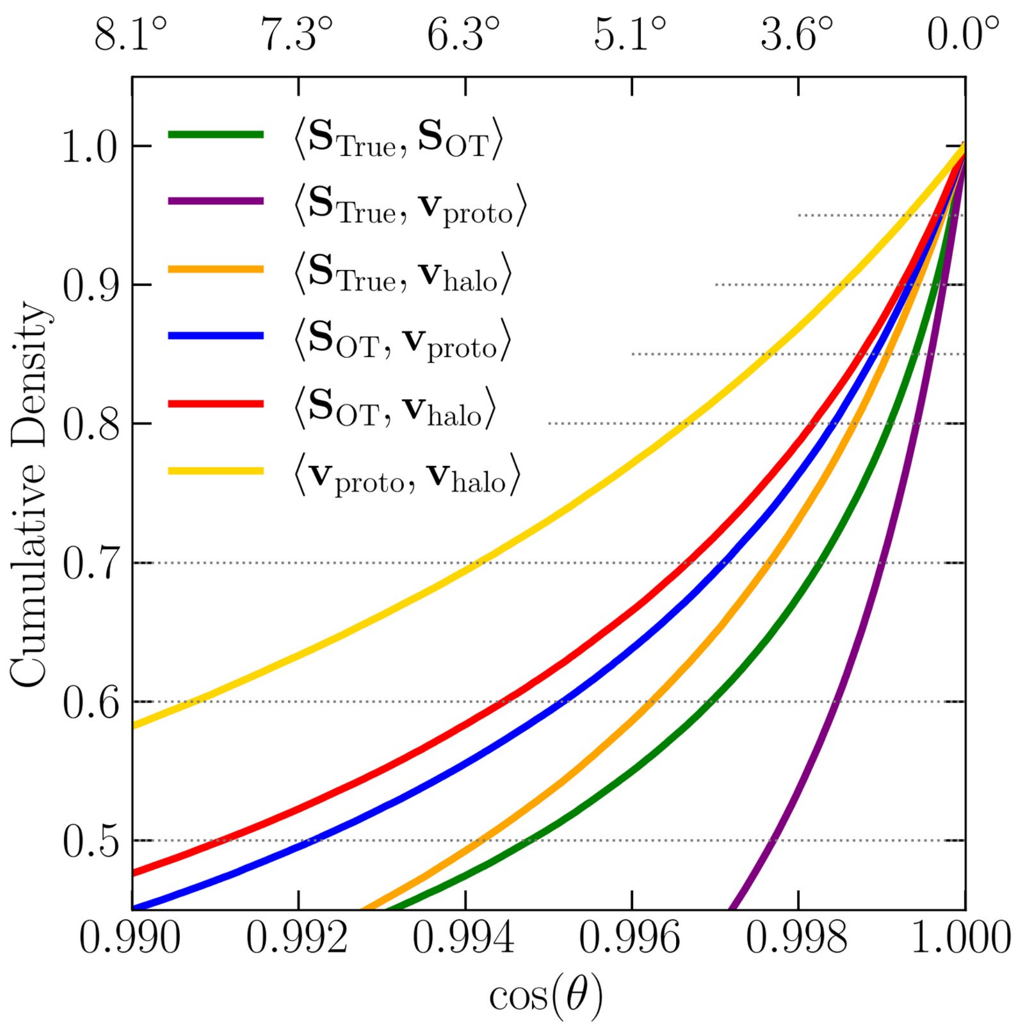
Optimal transport





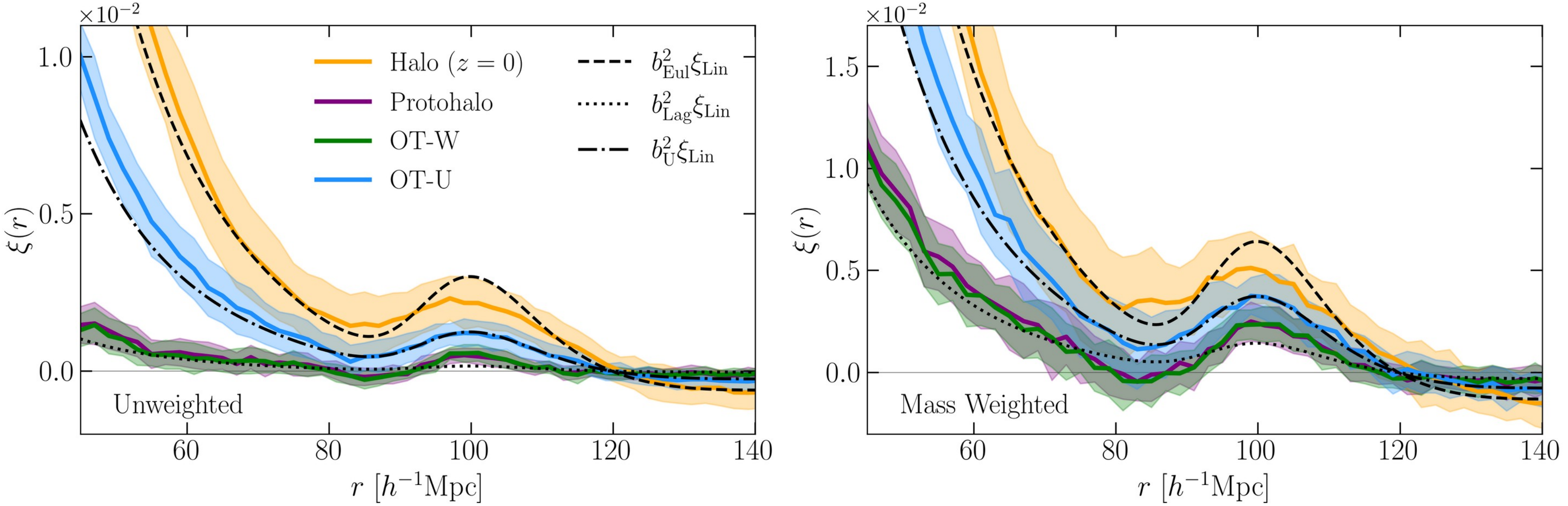
Reconstructs n-point statistics:
Void PDF (hence kNN), Void sizes

Optimal transport



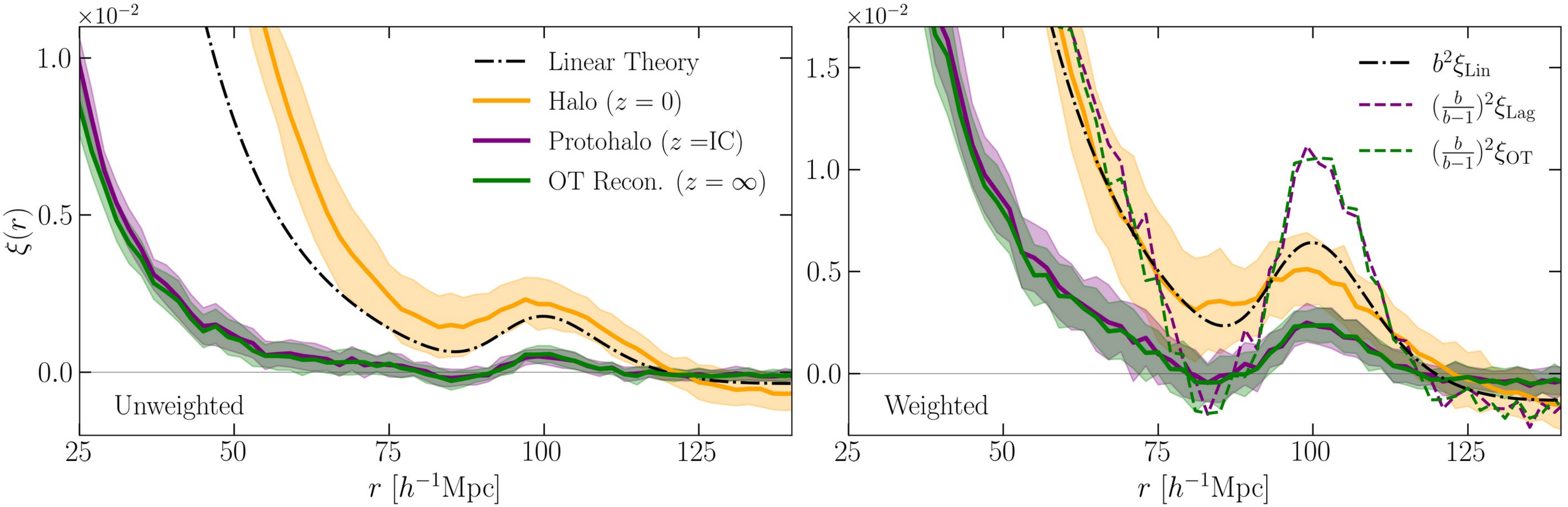
Reconstructs displacements; BAO-kSZ synergy?

Optimal transport



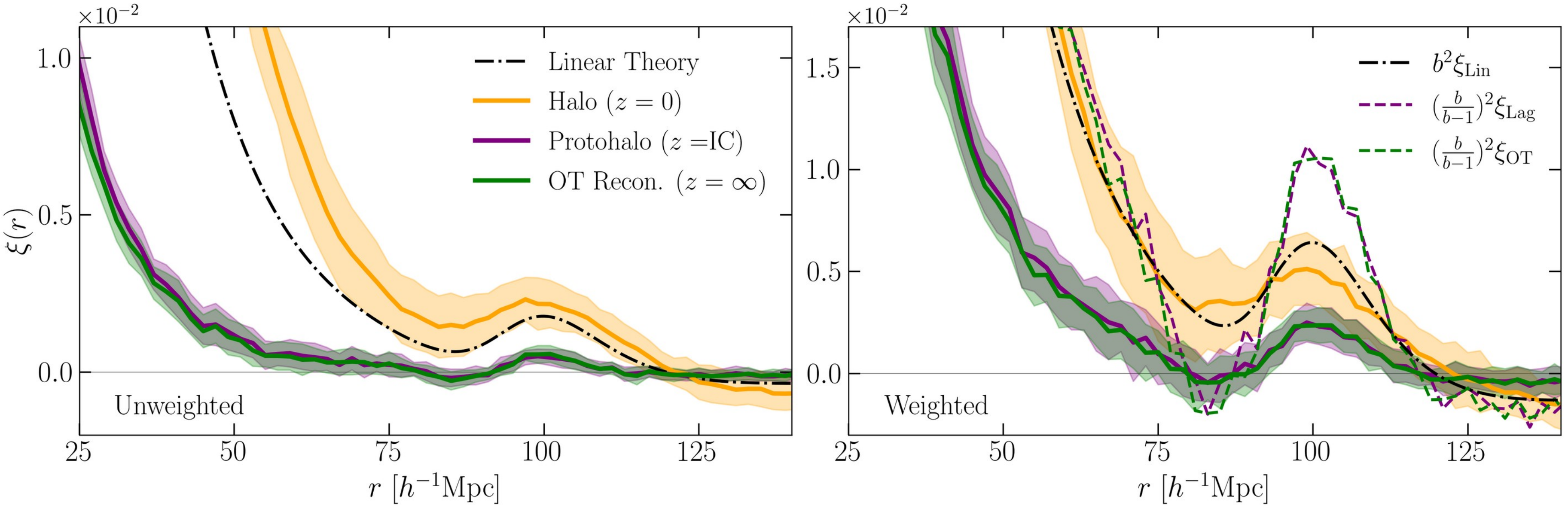
Reconstructs 2-point statistics (mass weighted ~ HOD)

Optimal transport



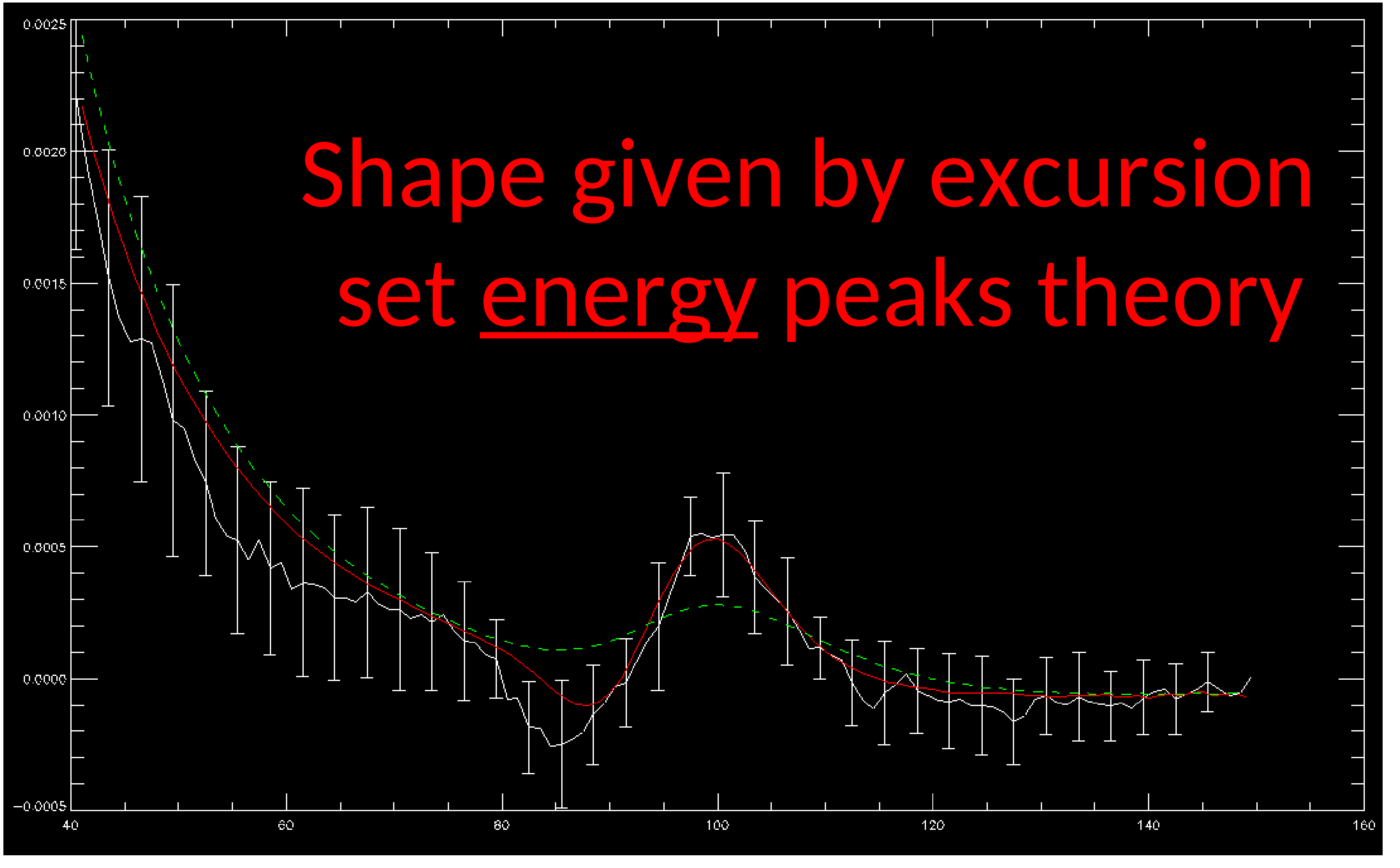
Reconstructed 2-point statistics scale as expected – enables determination of bias factor b

Optimal transport



Reconstructed 2-point statistics scale as expected – but shape different from pure linear theory!

Optimal transport



Excursion set peaks

Spherical evolution model suggests initial overdensity determines subsequent evolution

Peaks in smoothed density field tractable but

- What smoothing filter? What smoothing scale?

Set scale by additional 'multiscale' requirement that density is smaller on larger smoothing:

$$dn/d\ln R \sim p(\delta_R=\delta_c,\ \partial_i\delta_R=0,\ \partial_{ij}\delta_R<0,\ y_R>0)$$

where $y_R = -\,d\delta_R/d\ln R$. Each extra constraint shifts mean and variance of Gaussian pdf.

In principle:

$M \sim R^3$ so $dn/d\ln M = (dn/d\ln R)/3$

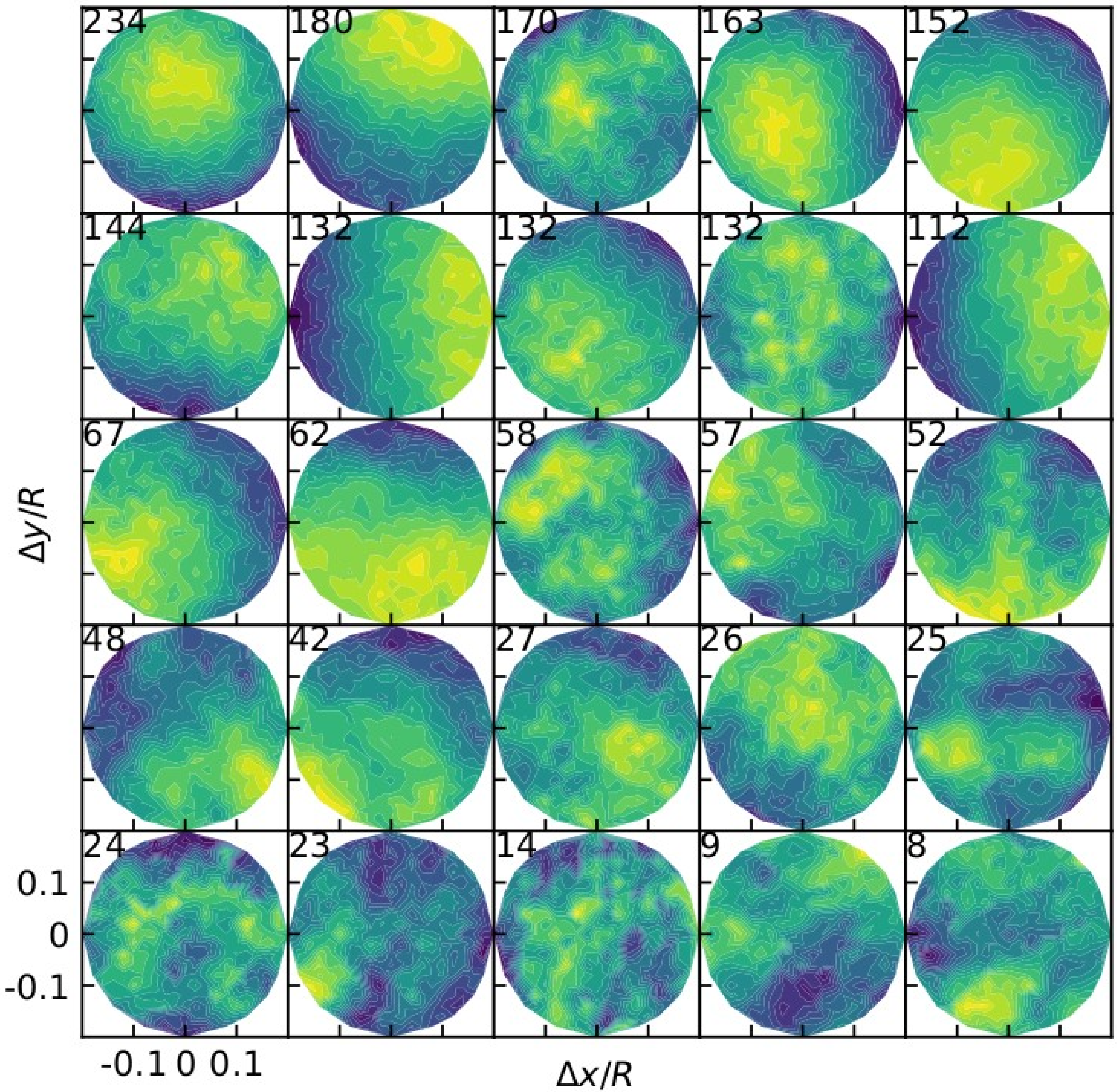
Cosmology constraints because pdf depends on P(k)

Problem in practice:

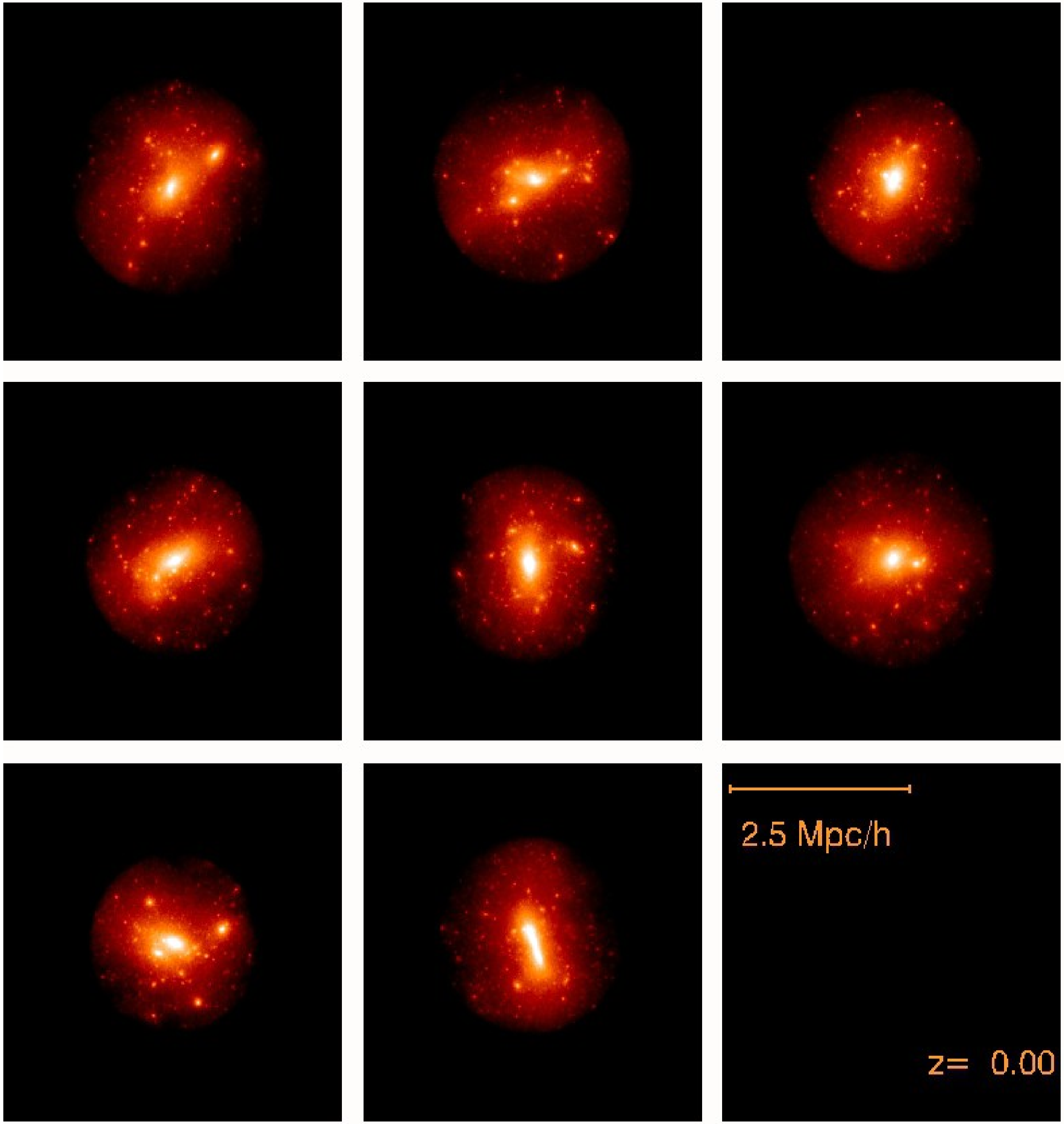
For a tophat smoothing filter, some integrals which define the relevant variances diverge.

So one plays games with the tophat smoothing filter (typically 'smooth' its edges).

Is there a more principled way out?

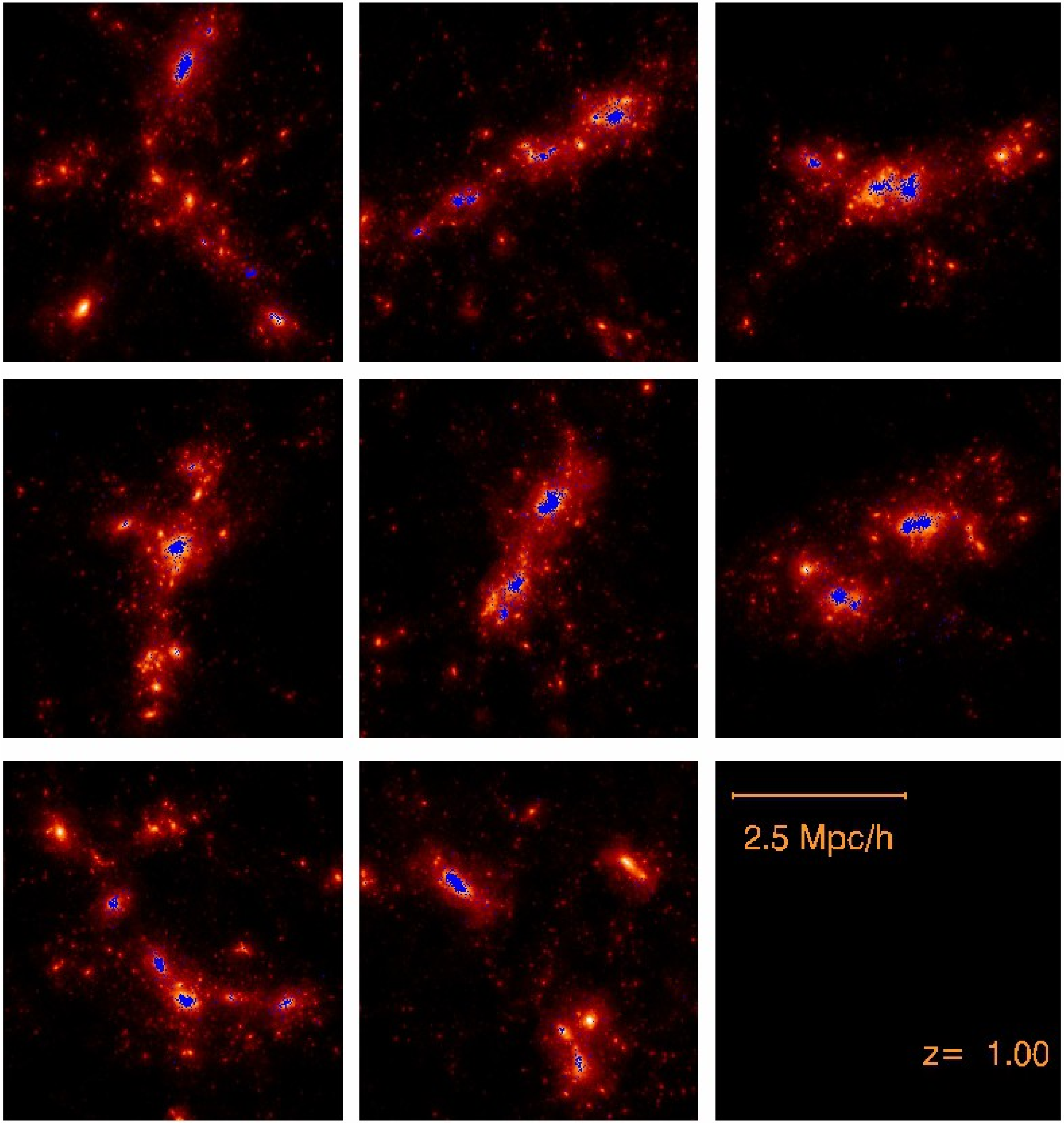


Density smoothed within patch centered on protohalo is noisy, miscentered, especially at lower mass



8 halos, $10^{15}M_{sun}$ at z=0 in ΛCDM

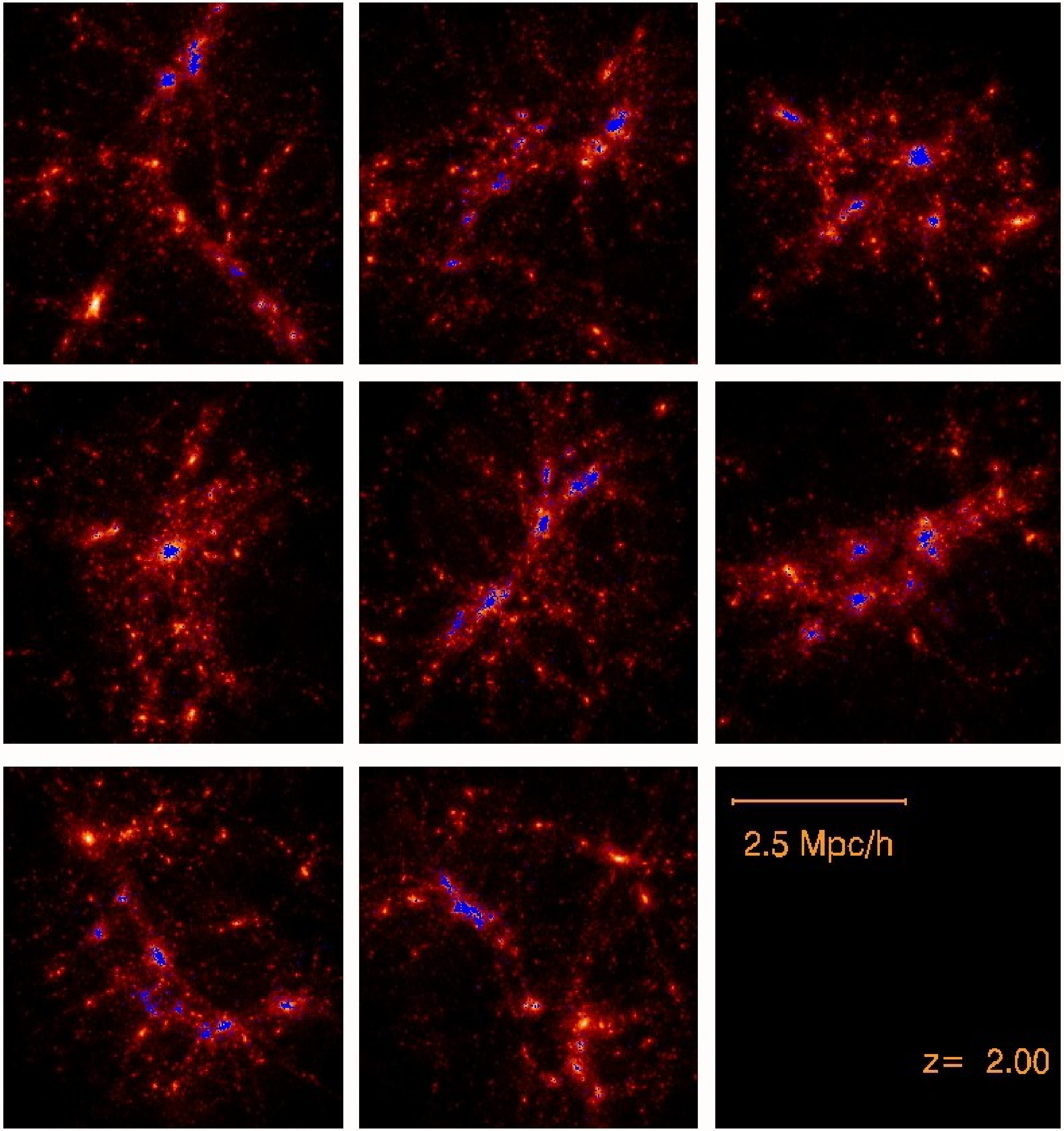
Only dark matter particles within R_{200} shown



Same objects at z=1

Blue shows dark matter within 20kpc at z=0

(Springel et al.)



Same objects at z=2

Blue shows dark matter within 20kpc at z=0

(Springel et al.)

Why does this work at all?

- Collapse is lumpy, not smooth
  - Centers of virialized subclumps at early time end up in center of virialized halo at later time
  - Spherical collapse has rank ordering in binding energy 'built-in'
- Collapse is anisotropic, not spherical
  - Monopole of full anisotropic solution is given by SC at all orders

A solution?

Consider energy conservation instead:

$dn/d\ln R \sim p(\delta_R=\delta_c,\ \partial_i\delta_R=0,\ \partial_{ij}\delta_R<0,\ d\delta_R/d\ln R<0)$

$\rightarrow p(\varepsilon_R= \varepsilon_c,\ \partial_i\varepsilon_R=0,\ \partial_{ij}\varepsilon_R<0,\ d\varepsilon_R/d\ln R<0)$

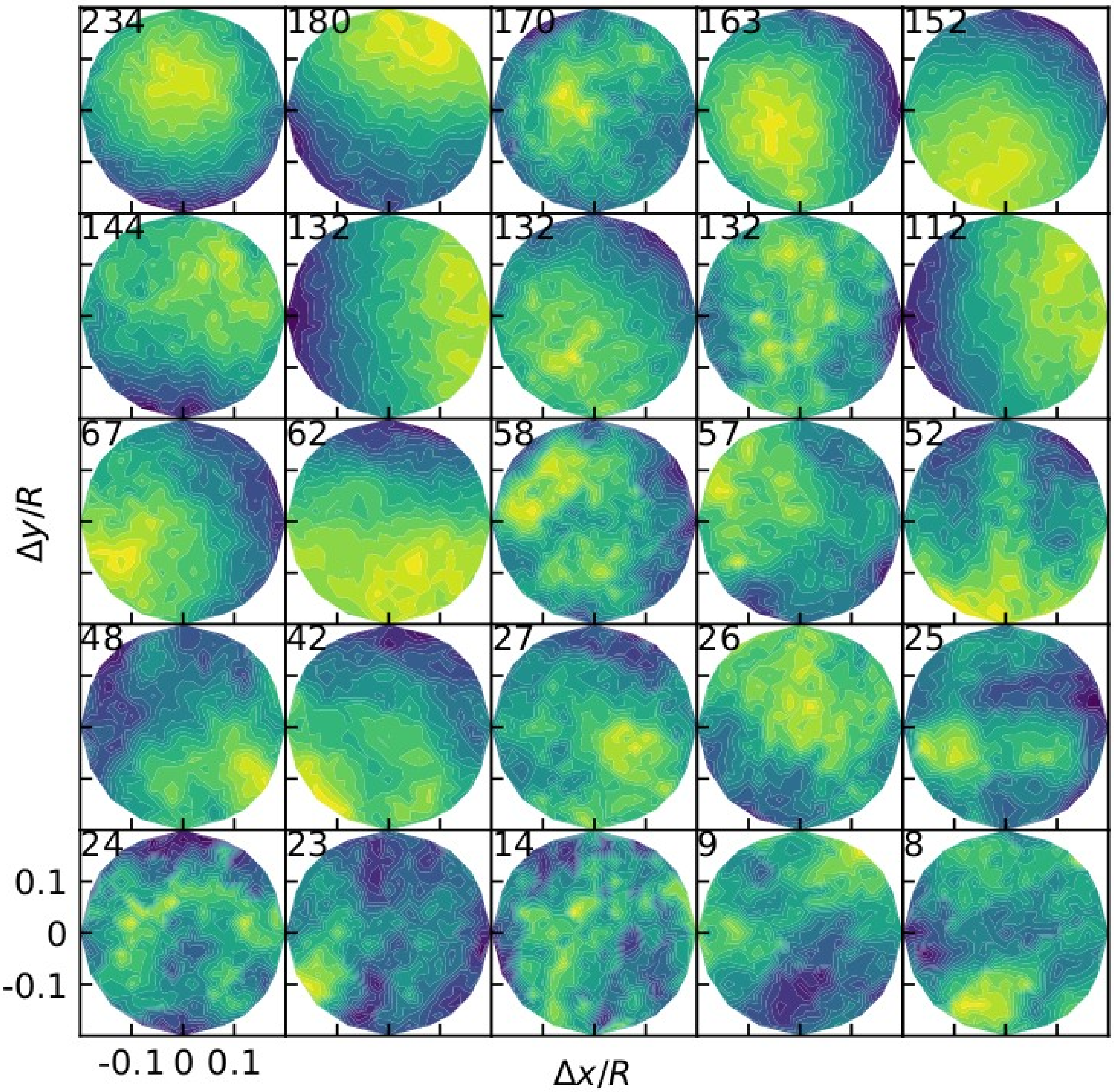
No extra complication. Moreover,

$3j_1(x)/x \rightarrow 15j_2(x)/x^2$ (like 'damped tophat')

And $\partial_i\varepsilon_R=0$ means dipole=0;

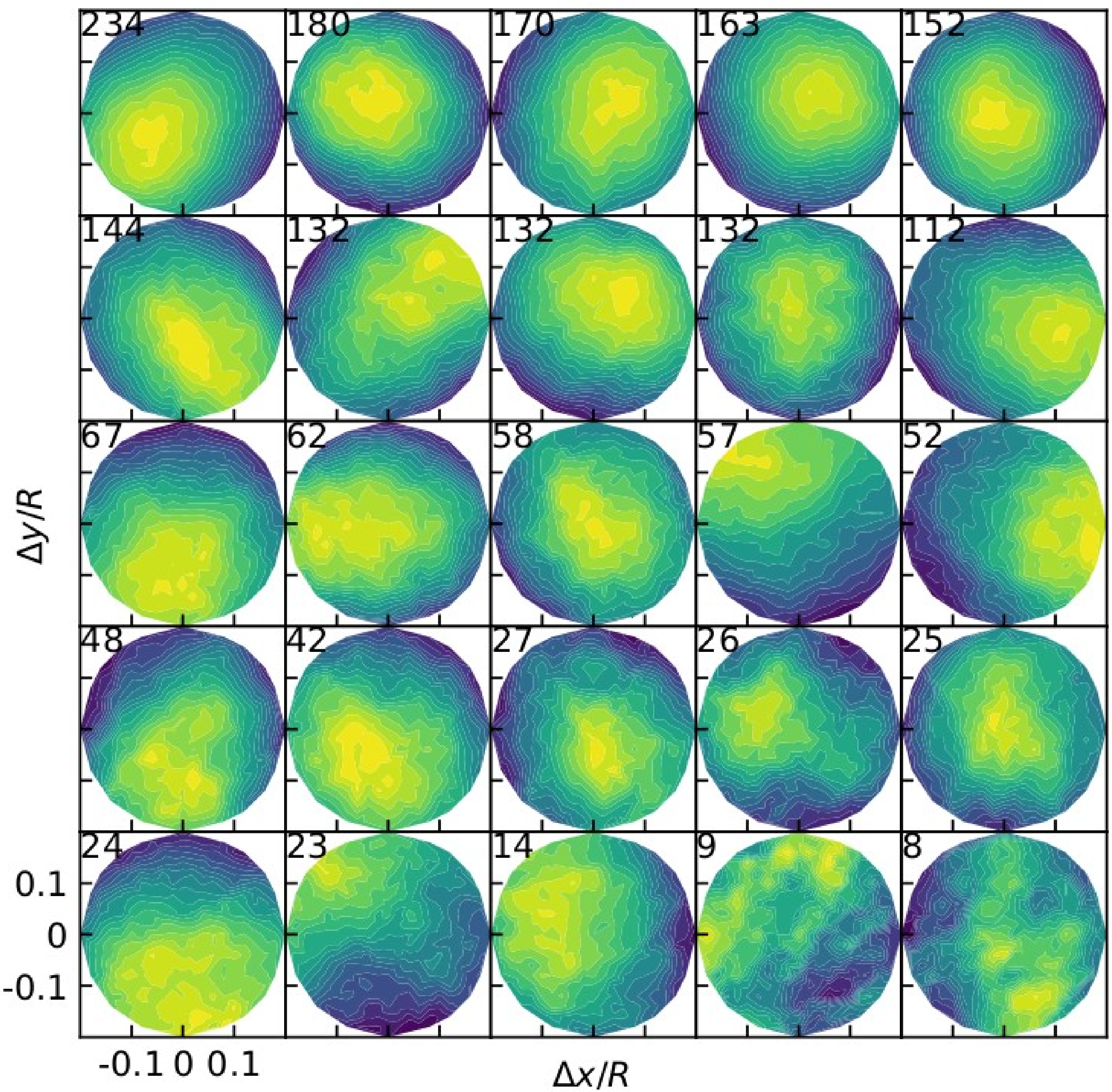
$\partial_{ij}\varepsilon_R<0$ means $d\delta_R/dR<0$;

$d\varepsilon_R/d\ln R$ at fixed $\partial_{ij}\varepsilon_R$ is just δ_R.



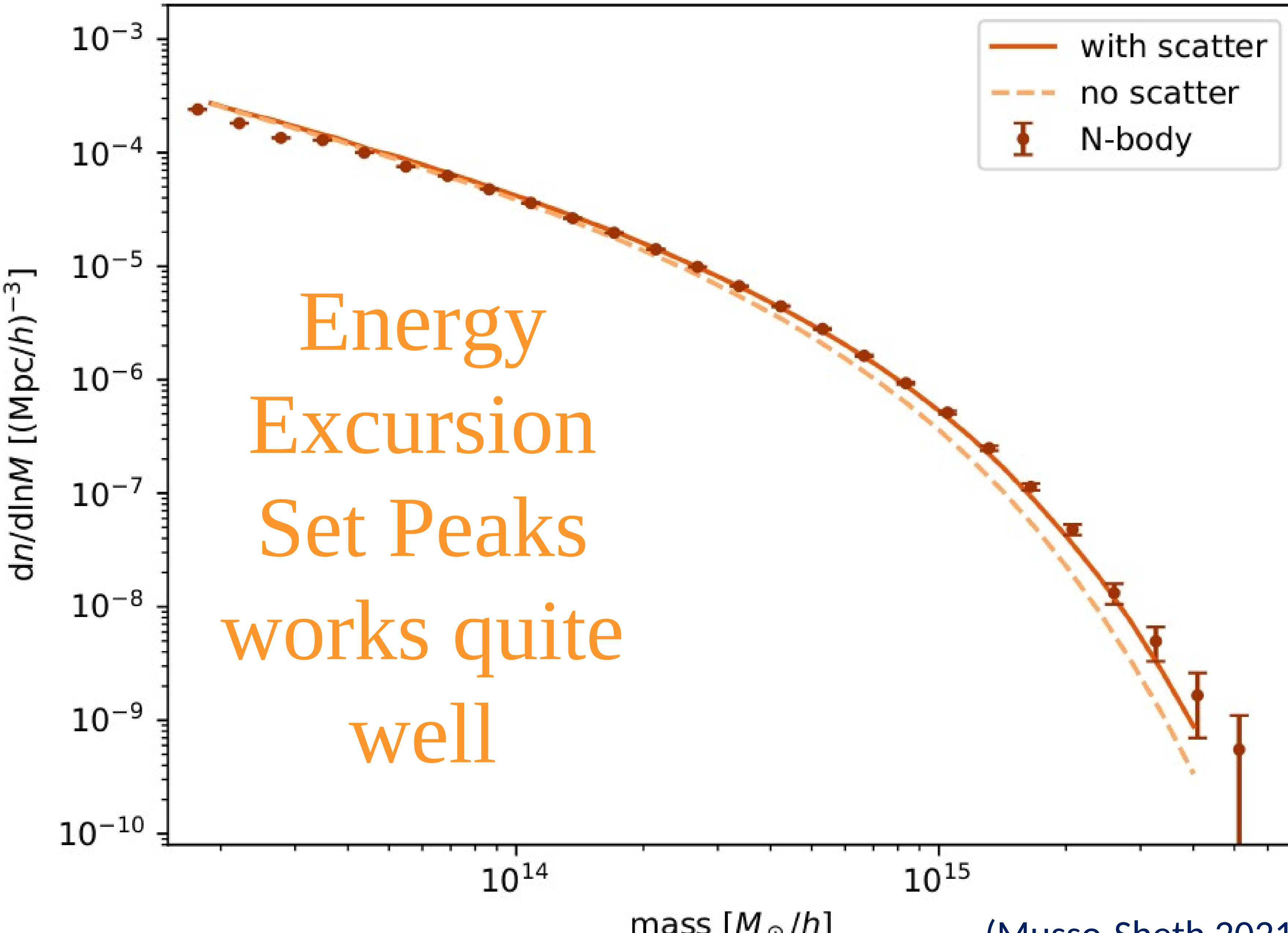
Density smoothed within patch centered on protohalo is noisy, miscentered, especially at lower mass

(Musso-Sheth 2021)



Energy smoothed within patch centered on protohalo is less noisy, ~centered, also at lower mass

(Musso-Sheth 2021)

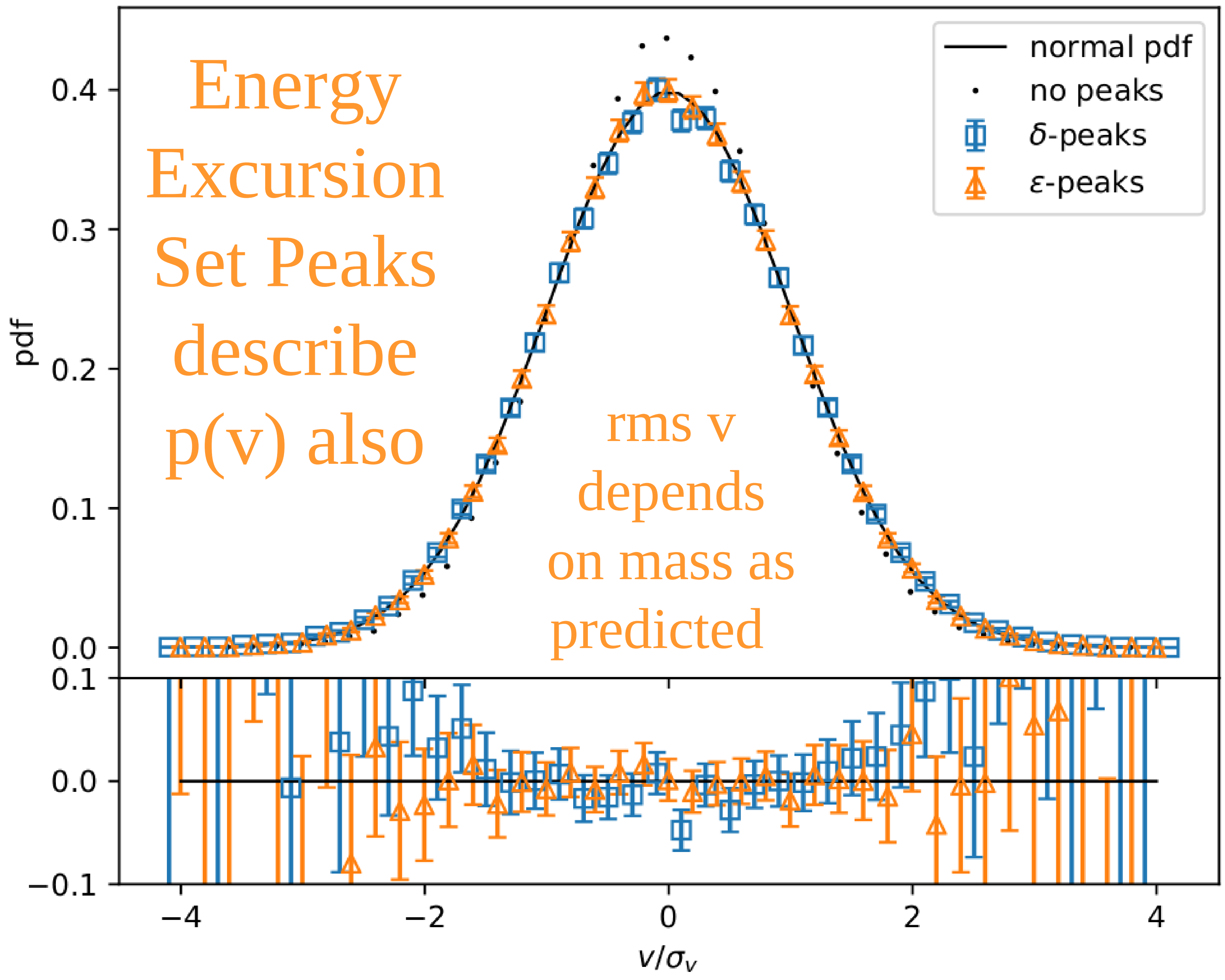


(Musso-Sheth 2021)

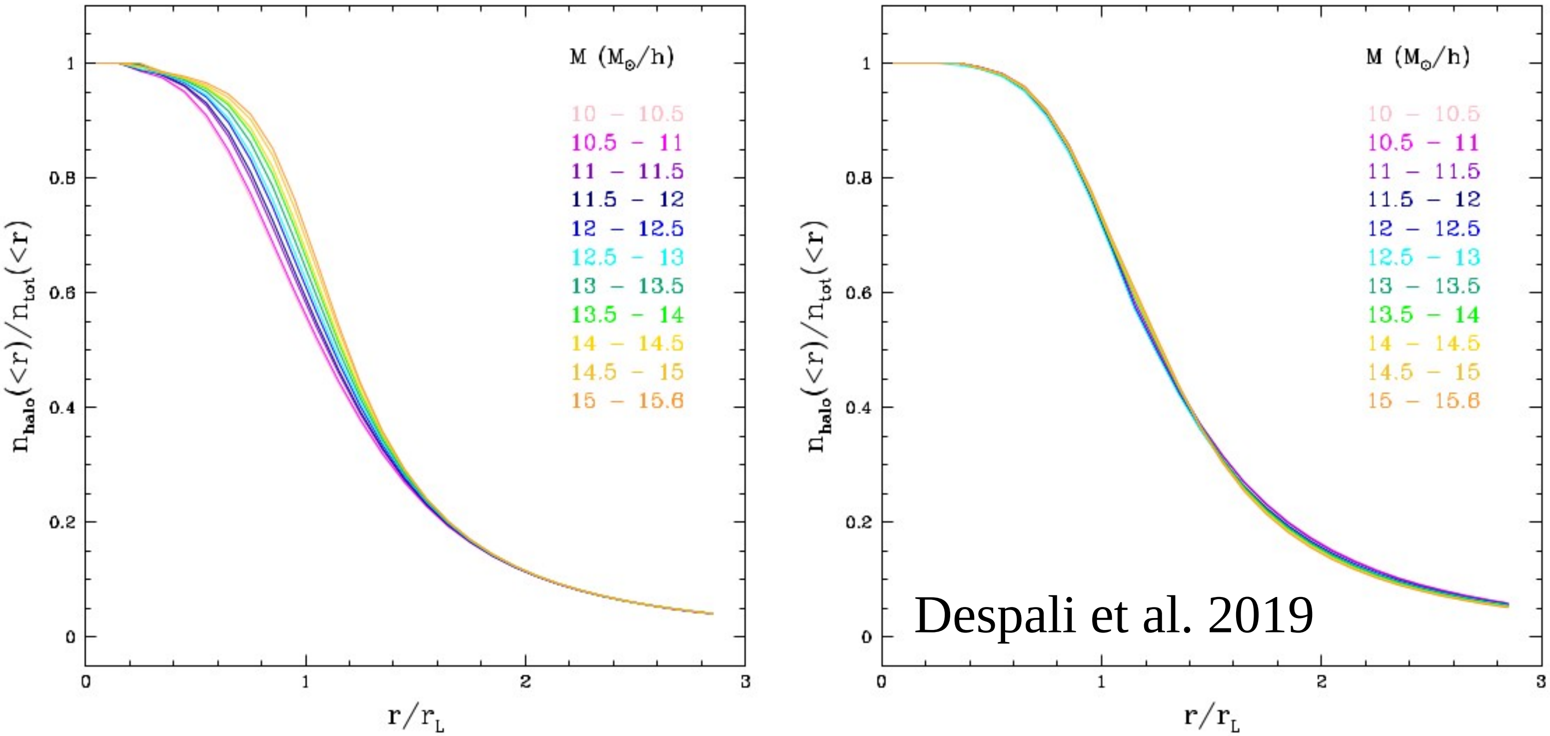
Energy Excursion Set Peaks works quite well

Each constraint modifies (biases) mean and shifts variance

Results in both spatial bias and velocity bias

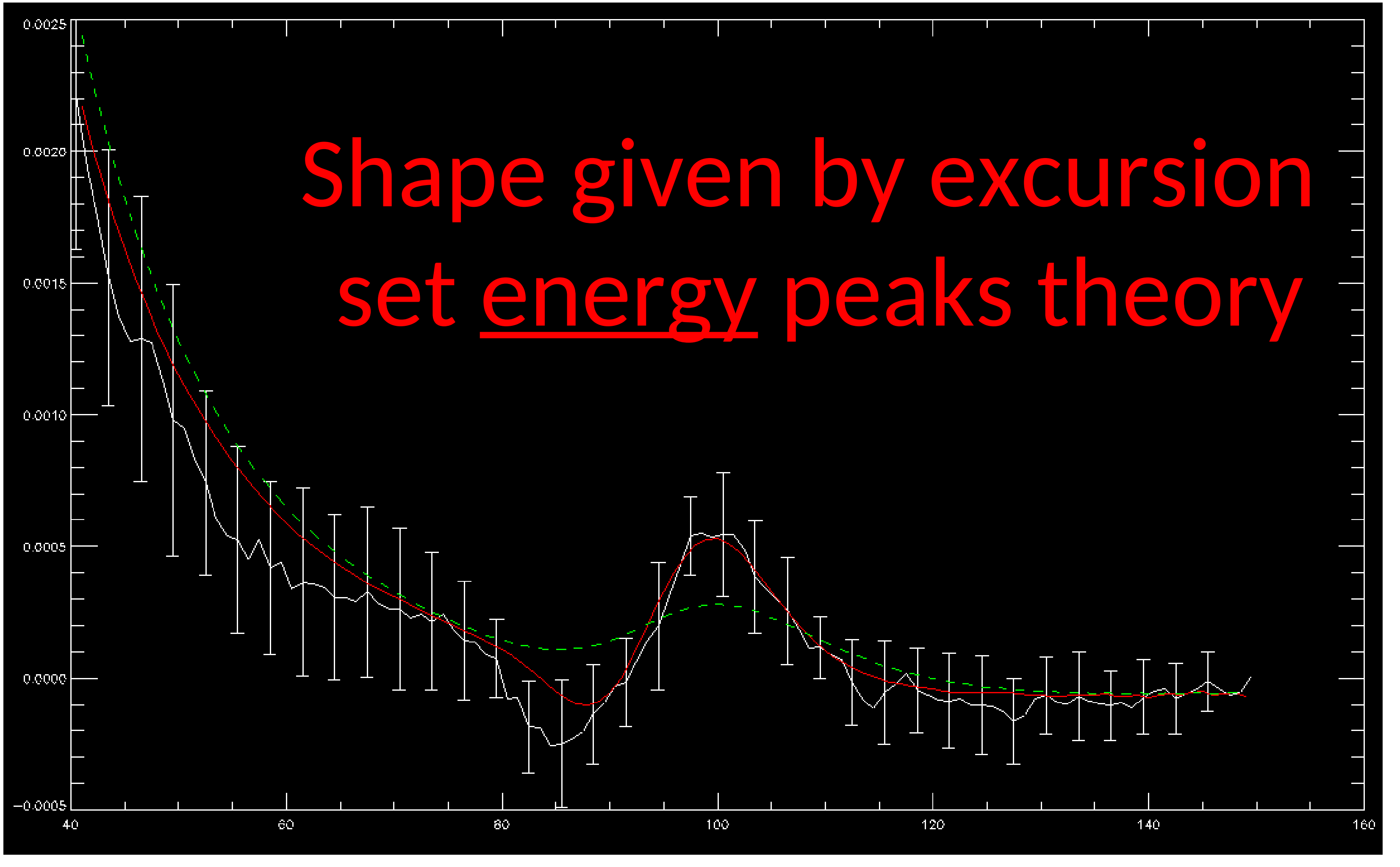

Energy Excursion Set Peaks describe p(v) also
rms v depends on mass as predicted
normal pdf
no peaks
δ-peaks
ε-peaks
pdf
0.4
0.3
0.2
0.1
0.0
0.1
0.0
−0.1
−4
−2
0
2
4
v/σv

Why displacements better (see Marcello Musso)



Anisotropic (Lagrangian) smoothing is better (more painful!)

Optimal transport



Summary

Nice convergence of BAO reconstruction efforts
and
understanding of cosmic web