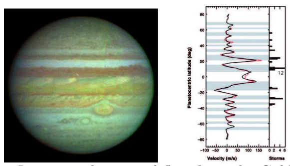
Compressible Convection in the Giant Planets

Kirill Kuzanyan and Chris Jones

School of Mathematics, University of Leeds, UK

Giant Planet Atmospheres

The giant planets Jupiter and Saturn have a 'striped' appearance with dark belts and lighter zones.
The belts and zones are associated with the zonal flow: east-west flows independent of longitude.

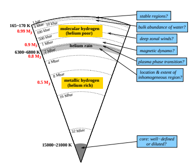


Jupiter and its zonal flow from the Galileo mission (courtesy of NASA)

Long-lived storms, such as the great red spot, are located in the shear zones associated with these zonal flows. The great red spot is located at latitude 23° south in the Galileo mission picture above.
How deep do these zonal flows and storms go into the atmospheres of giant planets? It was believed that they are surface phenomenon, but there is now evidence they may go as deep as 15,000 km below the surface.

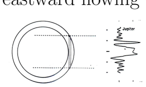
Interior structure of the Giant Planets

Below 15,000 km the pressure inside Jupiter is so high that the material goes into a **metallic hydrogen** phase. The magnetic field in this region locks the fluid together. Zonal flows occur above this magnetic inner core, which in Jupiter occupies the inner 80% by radius.

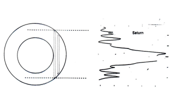


Internal structure of Jupiter, from Guillot et al. 2004

In Saturn, which is smaller than Jupiter, the magnetic core only occupies the innermost 50% by radius. So the zonal flow region is larger. This may be why the zonal flows in Saturn have higher wind speeds and why there is a much broader equatorial eastward flowing current near the equator.



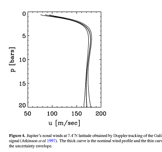
Jupiter: Large radius ratio, narrowly confined bands



Saturn: Smaller radius ratio, less confined bands

The Galileo Probe

To discover how deep the zonal flows go into the atmosphere, the Galileo mission parachuted a probe into Jupiter's atmosphere. It discovered the winds are **not** confined to the surface, they persist into the interior!



At the visible surface, the pressure is about 1 bar. The Galileo probe signalled back until it reached a depth at which the pressure was 20 bar.

Wind velocity as a function of depth from Galileo probe (from Vasavada & Snowman, 2005)

Anelastic polytropic model

Spherical shell $r_i < r < r_o$, $r_i/r_o = \eta$, $r_o - r_i = d$

Hydrostatic equilibrium $\dfrac{dp_0}{dr} = -\dfrac{GM}{r^2}\rho_0$,

the gas law $p_0 = R\rho_0 T_0$,

and the polytropic relation $p_0 = \text{const}\cdot\rho_0^{1+\frac{1}{n}}$,

n the polytropic index, gives $p_0 = p_c\zeta^{n+1}$,
$\rho_0 = \rho_c\zeta^n$, $T_0 = T_c\zeta$, $\zeta = c_0 + c_1 d/r$,
c_0, c_1 are constants defined in terms of η, n and N_ρ

$N_\rho = \log\dfrac{\rho_i}{\rho_o}$: number of density scale heights

$N_\rho \to 0$ is Boussinesq, $N_\rho = 5$ has density about 150 times larger at $r = r_i$ than at $r = r_0$.

Anelastic Continuity equation:

$\nabla\cdot(\rho_0\mathbf{u}) = 0$, as $\dfrac{\partial\rho'}{\partial t}$ is small.

Anelastic Entropy equation:

The eddy diffusion is a source of entropy

$$\rho_0\frac{DS_c}{Dt} = \frac{1}{T_0}\nabla\cdot\ \rho_0 T_0\kappa_T\nabla S_c + \frac{1}{T_0}Q_v\,,$$

$$Q_v = 2\nu\rho_0\left[e_{ij}e_{ij} - \frac{1}{3}(\nabla\cdot\mathbf{u})^2\right], e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

The radiative thermodiffusion is ignored.

Anelastic Momentum equation:

$$\frac{\partial\mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + 2\mathbf{\Omega}\times\mathbf{u} = -\frac{\nabla p}{\rho} - \frac{\mathbf{g}\rho_c}{\rho} + \mathbf{F}_{visc}$$

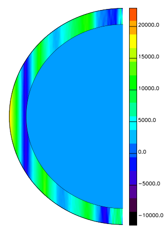
can be written

$$\frac{\partial\mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + 2\mathbf{\Omega}\times\mathbf{u} = -\nabla\left(\frac{p_c}{\rho_0}\right) - \frac{\mathbf{g}\alpha T_0 S_c}{c_p}$$
$$+\frac{\nu}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\frac{\partial u_i}{\partial x_j} + \rho_0\frac{\partial u_j}{\partial x_i}\right) - \frac{2\nu}{3\rho_0}\frac{\partial}{\partial x_i}\left(\rho_0\frac{\partial u_j}{\partial x_j}\right)$$

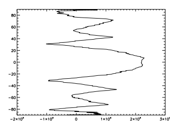
so that the curl eliminates the pressure fluctuation. This is only possible because the equilibrium state is close to adiabatic. Assuming constant ν and κ_T.

Fully Three-Dimensional simulations

After running the simulation for many thousands of rotation periods stable banded structures emerge. However, rather extreme parameter values are needed to see any jets. The flow must have strongly supercritical small scale convection before any significant zonal flows are found. Furthermore, the flow must be rotationally dominated, with the value of Ekman number $E = \nu/\Omega d^2$ lower than 10^{-5}.



Meridional section of the axisymmetric part of u_ϕ (near Boussinesq case).

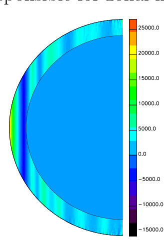


Surface zonal flow as a function of latitude at $E = 6\times10^{-6}$, $Pr = 0.1$, $Ra \approx 40\times Ra_{\text{crit}}$, $\eta = 0.85$, $N_\rho = 0.1$, stress-free conditions at both lower and upper boundaries. The spacing between bands is governed by the **Rhines** scale, at which inertial and Coriolis forces balance.

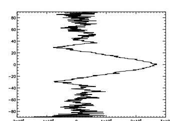
- We found that for lower values of $Pr = \nu/\kappa_T$ more spiralling of convective motions occurs, and so it is easier to obtain larger zonal jets (agreeing with Heimpel et al., 2005).
- High latitude eastward and westward jets occur as in giant planets. Also, at high latitudes the zonal flow is comparable to the magnitude of the asymmetric u_ϕ.

Compressible Models

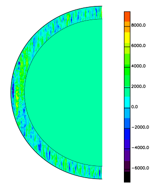
The density varies by many orders of magnitude from the magnetic core to the surface. We have developed a new code to take this into account using the **anelastic** approximation. Entropy replaces temperature, and convective flows are much faster near the surface. Nevertheless, the convection columns responsible for zonal flow generation are still present.



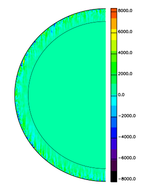
Meridional section of the axisymmetric part of u_ϕ.



Surface zonal flow as a function of latitude at $E = 6\times10^{-6}$, $Pr = 0.1$, $Ra \approx 22\times Ra_{\text{crit}}$, $\eta = 0.85$, $N_\rho = 5$, stress-free conditions at both lower and upper boundaries (strongly compressible case).



Meridional section of u_r in a near Boussinesq case.The convection columns reach right across the shell.



Meridional section of u_r in a strongly compressible case. The velocity is larger near the surface, but columns are still apparent.

- In both Boussinesq and compressible cases, there is a strong eastward (prograde) jet near the equator, as on Jupiter and Saturn (Galileo probe).
- However, in the compressible case the amplitude of jets inside the tangent cylinder is lower and latitudinal extent is significantly thinner.
Heimpel et al. 2005 need $r_i/r_o = 0.9$ to get the equatorial zone the right thickness (case of Jupiter).

Our result suggests this is not necessary:
$r_i/r_o = 0.8 - 0.85$ can fit the observations.

- The parameter range on the horizontal L_{width} and vertical L_{depth} jet scales for formation of large-scale jets:

$$L_{\text{width}} \sim E^{1/3} \ll L_{\text{depth}} \sim N_\rho^{-1} \ll \frac{d}{r_o} = 1 - \eta$$

Boundary conditions

- It is very hard to obtain zonal jets inside the tangent cylinder for no-slip conditions at the inner boundary. In that case no large scale zonal flow in mid-latitudes is established yet, only small scale flows due to individual convective plumes.
- On the contrary, for stress-free conditions at the inner boundary there are coherent zonal jets inside the tangent cylinder.

Questions remaining

- 1. Does the interface region between the magnetic core and the non-magnetic atmosphere act like a no-slip or stress-free boundary?
- 2. Guillot's model of the Jovian interior suggests a smooth transition from molecular H_2/He to metallic H. If so, pumping of atmospheric fluid into the magnetic interior may occur, thus damping out zonal flows. Could this be avoided if there is a sharp transition?
- 3. Possibly only the equatorial prograde jet reaches into the deep interior. The Galileo probe went in close to the equator. The high-latitude zonal jets may not be formed by the global "Taylor-Proudman"-like columnar structure of convection, but may be a surface phenomenon?