Power-law correlated 2D SU(6) quantum paramagnets

Congjun Wu

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Ref: Z. Cai, H. H. Hung, L. Wang, Yi Li, and C. Wu, arxiv:1207.6843.

Z. Cai, H. H. Hung, L. Wang, D. Zheng, and C. Wu, arxiv1202.6323.

C. Wu, Physics 3, 92 (2010).

Related past works:

C. Wu, J. P. Hu, and S. C. Zhang, Phys. Rev. Lett. 91, 186402 (2003).

C. Wu, Mod. Phys. Lett. B 20, 1707 (2006) (brief review).

H. H. Hung, Y. P. Wang, C. Wu, Phys. Rev. B 84, 054406 (2011).

KITP, Sept 18, Frustrated magnet workshop

Collaborators

Zi Cai	(UCSD→Ludwig-Maximilians Univ.)
Hsiang-hsuan Hung	(UCSD→UIUC→ UT Austin)
Yi Li	(UCSD)
Dong Zheng	(Tsinghua/UCSD→ industry)
Lei Wang	(ETH, Zurich)

Collaborators on past related works: S. C. Zhang (Stanford), J. P. Hu (Purdue), S. Chen and Y. P. Wang (IOP, CAS).

Acknowledgments: J. Hirsch (UCSD), Y. Takahashi (Kyoto Univ.) , Kivelson(Stanford), F. Zhou (UBC), T. L. Ho (OSU).

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Outline

- **Introduction: a novel system for quantum magnetism.**

Large hyperfine-spin ultra-cold alklai and alklaine-earth fermions in optical lattices.

Why they are interesting?

Large spin enhances rather than suppresses quantum spin fluctuations due to large symmetries of SU(2N), Sp(2N).

- Brief-review the generic Sp(4) symmetric in spin-3/2 systems – unification of AFM, SC and CDW.

http://online.kitp.ucsb.edu/online/coldatoms07/wu2/

- Evidence of power-law correlated spin-correlations of SU(6) Hubbard model at half-filling – a Quantum Monte-Carlo study.

- Thermodynamic properties of SU(6) Hubbard model: enhancement of Pomeranchuk cooling - QMC

Analytic and numeric efforts for spin-liquids

- Bosonic large N – Neel, dimer ordering.

  Arovas, Auerbach PRB1988, Sachdev and Read, Nucl. Phy. B 1989.

- Fermionic large N -- spinon Fermi surface, Dirac point, etc.

  Affleck and Maston PRB1988, Hermele et al PRB2004, Lee, Nagaosa, Wen, RMP2006

- RVB, quantum dimer model, etc.

  Anderson 1973; Rokhsar, Kivelson PRL1988; Fradkin, Kivelson Mod. Phys. Lett 1990; Moessner and Sondhi PRL 2001.

- Frustration -- ring exchange, J1-J2 square lattice, Kagome, etc.

  Jiang, Fisher, Sheng, Motrunich et al 2008-2012; Jiang, Yao, Balents PRB 2012; Yan, Huse, White Science 2011.

- Weak Mott-insulators – QMC: honeycomb lattice, square lattice with π-flux.

  Meng et al, Nature 2010, Sorella et al arxiv2012, Chang and Scalettar PRL 2012.

Large spin fermions with alkaline-earth and alkali atoms

- High symmetries (e.g. Sp(2N)/SU(2N)) difficult to access in solid state systems.

- Theoretical investigations.

  Wu, Hu, Zhang, Chen, Wang (2003 ---);

  Azaria, Lecheminant (2006 ---);

  V. Gurarie, M. Hermele, A. Rey, P. Zoller, et al. (2010 ---).

- Why are they related to this workshop?

Strong quantum spin fluctuations!

Another system for quantum disordered Mott-insulating states besides solid state systems.

Experiment progress of multi-component fermions

Selected for a Viewpoint in *Physics*
90401 (2010) PHYSICAL REVIEW LETTERS wee 5 NOVE

PRL 105, 190401 (2010)

Realization of a SU(2) × SU(6) System of Fermions in a Cold Atomic Gas

Shintaro Taie,[1,*] Yosuke Takasu,[1] Seiji Sugawa,[1] Rekishu Yamazaki,[1,2] Takuya Tsujimoto,[1] Ryo Murakami,[1] and Yoshiro Takahashi[1,2]

)2 (2010) PHYSICAL REVIEW LETTERS

PRL 105, 030402 (2010)

Degenerate Fermi Gas of ^{87}Sr

B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian

Viewpoint

Physics 3, 92 (2010)

Exotic many-body physics with large-spin Fermi gases

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Published November 1, 2010

The experimental realization of quantum degenerate cold Fermi gases with large hyperfine spins opens up a new opportunity for exotic many-body physics.

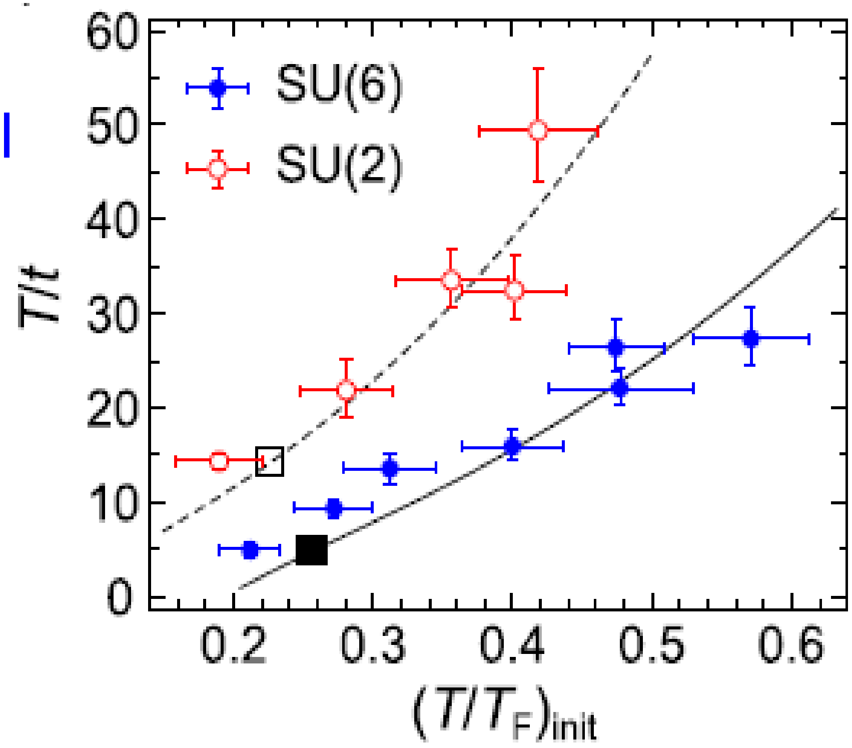
Current experiment progress on alkaline-earth atoms

- ^{132}Yb (F=5/2) and ^{87}Sr (F=9/2) → quantum degeneracy in optical traps.

- ^{132}Yb fermions → 3D cubic optical lattices → Mott-insulating states.

- Temperatures can reach the order of t, but are still higher than the AF exchange energy scale J.

- An interesting T scale difficult to reach in solids – S. Kivelson.



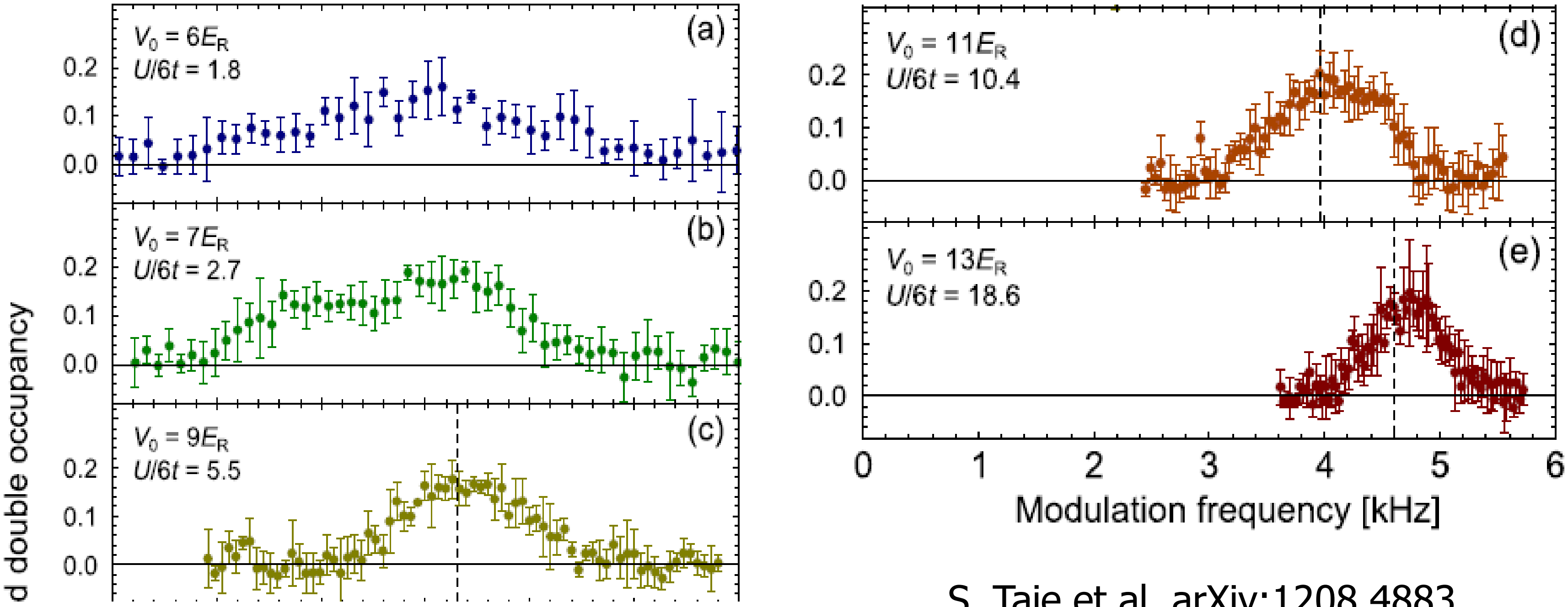
$$N = 1.9 \times 10^4, V = 11 E_R,$$

$$t/h = 63.7\,\text{Hz}, U/h = 4.0\,\text{kHz}$$

S. Taie et al, arXiv:1208.4883.

Observation of Mott gap

- The Mott-state of one fermion per site.
- Create charge excitations (doublons) from periodically modulating lattice potentials.
- The charge gap measured from resonance spectra.



S. Taie et al, arXiv:1208.4883.

Classical (large S): large-spin solid state systems

• Hund's rule coupled electrons → large onsite spin.

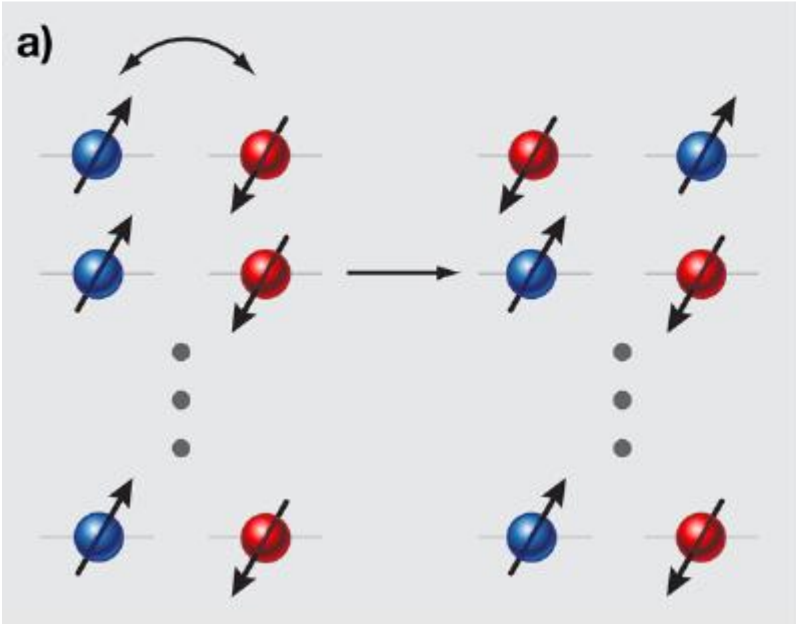
• Inter-site coupling is dominated by exchanging a single pair of electrons.

• ΔS_z only +1 or -1. Quantum spin-fluctuations are suppressed by 1/S.

• In solid state systems, the larger the spin is, the more classical the physics is.

• Bilinear exchange dominates

$$\frac{t^2}{U}\vec{S}_i \cdot \vec{S}_j + \frac{t^4}{U^3}(\vec{S}_i \cdot \vec{S}_j)^2 + \ldots$$



C. Wu, Mod. Phys. Lett. (2006); Physics 3, 92 (2010).

Large-spin cold atoms: **Not classical but quantum!**

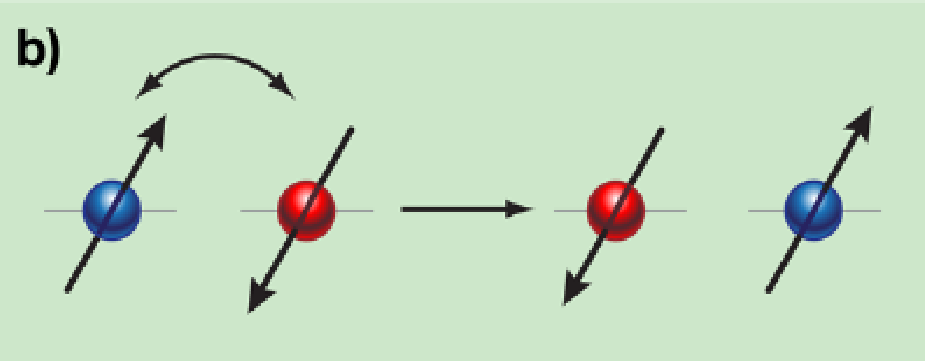
- Large-spin cold fermion moves as a whole object. The exchange of a pair of fermions can completely flip spin-configuration.

$$\Delta S_z = \pm 1, \pm 2, \ldots \pm S$$

- Quantum fluctuations are enhanced by the large number of spin components.

- Bilinear, bi-qudratic, bi-cubic terms, etc., are all at equal importance.

$$\vec{S}_i \cdot \vec{S}_j, (\vec{S}_i \cdot \vec{S}_j)^2, (\vec{S}_i \cdot \vec{S}_j)^3$$



C. Wu, Mod. Phys. Lett. (2006); Physics 3, 92 (2010).

Large N NOT large S! SU(2N), Sp(2N=2S+1)

• Alkaline-earth atoms have fully-filled electron-shells, thus their hyperfine spin is just nuclear spin.

• Interactions are insensitive to nuclear spin components → an obvious SU(2N) symmetry.

• SU(2N) symmetry is not generic for spin-dependent interactions, say, alkali fermions.

• SU(2N) → Sp(2N): SU(2N) generators which are **odd** under time-reversal transformation span the Sp(2N) algebra.

Illustration by Dick Codor.

From Auerbach's book.

C. Wu et al, PRL 2003, C. Wu and S. C. Zhang PRB 2005; C. Wu, Mod. Phys. Lett. (2006); C. Wu Physics 3, 92 (2010).

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Spin-3/2 atoms are special with hidden Sp(4) symmetry!

- Spin 3/2 atoms: ^{132}Cs, ^{9}Be, ^{135}Ba, ^{137}Ba, ^{201}Hg.

- Extend Hubbard model to spin-3/2 fermions. Only two independent interaction parameters U_0 and U_2.

$$H = \sum_{\langle ij\rangle,\alpha} -t\{c^+_{i,\alpha} c_{j,\alpha} + h.\ c.\} - \mu \sum_i c^+_{i,\alpha} c_{i,\alpha}$$

$$+ U_0 \sum_i \eta^+(i)\eta(i) + U_2 \sum_{m=\pm 2,\pm 1,0} \chi^+_m(i)\chi_m(i)$$

$$\eta^+(i) = \sum_{\alpha\beta} \langle 00 | \tfrac{3}{2}\tfrac{3}{2};\alpha\beta\rangle c^+_\alpha(i) c^+_\beta(i)$$

$$\chi^+_m(i) = \sum_{\alpha\beta} \langle 2m | \tfrac{3}{2}\tfrac{3}{2};\alpha\beta\rangle c^+_\alpha(i) c^+_\beta(i)$$

- Exact **Sp(4)** symmetry regardless of dimensionality, external potential, and lattice geometry!

Sp(4) in spin 3/2 systems ←→ SU(2) in spin ½ systems

C. Wu et al, PRL 2003, C. Wu, Mod. Phys. Lett. (2006); Physics 3, 92 (2010).

U(4) → Sp(4) algebra

- Total degrees of freedom: $\psi^{+}_{\alpha} M_{\alpha\beta} \psi_{\beta}$ $4^2=16=1+3+5+7.$

1 density operator and 3 spin operators are far from complete.

	rank:	
	0	$1,$
	1	S_x, S_y, S_z
$M_{\alpha\beta}$	2	$\xi^a_{ij} S_i S_j (a = 1 \sim 5):$
	3	$\xi^a_{ijk} S_i S_j S_k (a = 1 \sim 7)$

↔ $S_x^2 - S_y^2, S_z^2 - \frac{5}{4},$
$\{S_x, S_y\}, \{S_y, S_z\}, \{S_z, S_x\}$

- **Spin-quadrupole matrices**: the same Γ-matrices in Dirac equation.

$$\Gamma^a = \xi^a_{ij} F_i F_j , \quad \{\Gamma^a, \Gamma^b\} = 2\delta_{ab}, \quad (1 \le a, b \le 5)$$

Hidden conserved quantities: **spin-octupoles**

- Both $S_{x,y,z}$ and $\xi^{a}_{ijk} S_i S_j S_k$ are conserved, which span the Sp(4) algebra. Spin-3/2 Hubbard model is Sp(4) invariant!!

3+7=10 $\Gamma^{ab} = \frac{i}{2}[\Gamma^a, \Gamma^b] \quad (1 \le a < b \le 5)$

- **The time-reversal odd kernel of U(4) is Sp(4).**

1 scalar + 5 vectors + 10 generators = 16

		Time Reversal
1 density:	$n = \psi^{+}\psi;$	even
5 spin-quadrupole:	$n_a = \frac{1}{2}\psi^{+}\Gamma^{a}\psi;$	even
3 spins + 7 spin-octupole:	$L_{ab} = \frac{1}{2}\psi^{+}\Gamma^{ab}\psi;$	odd

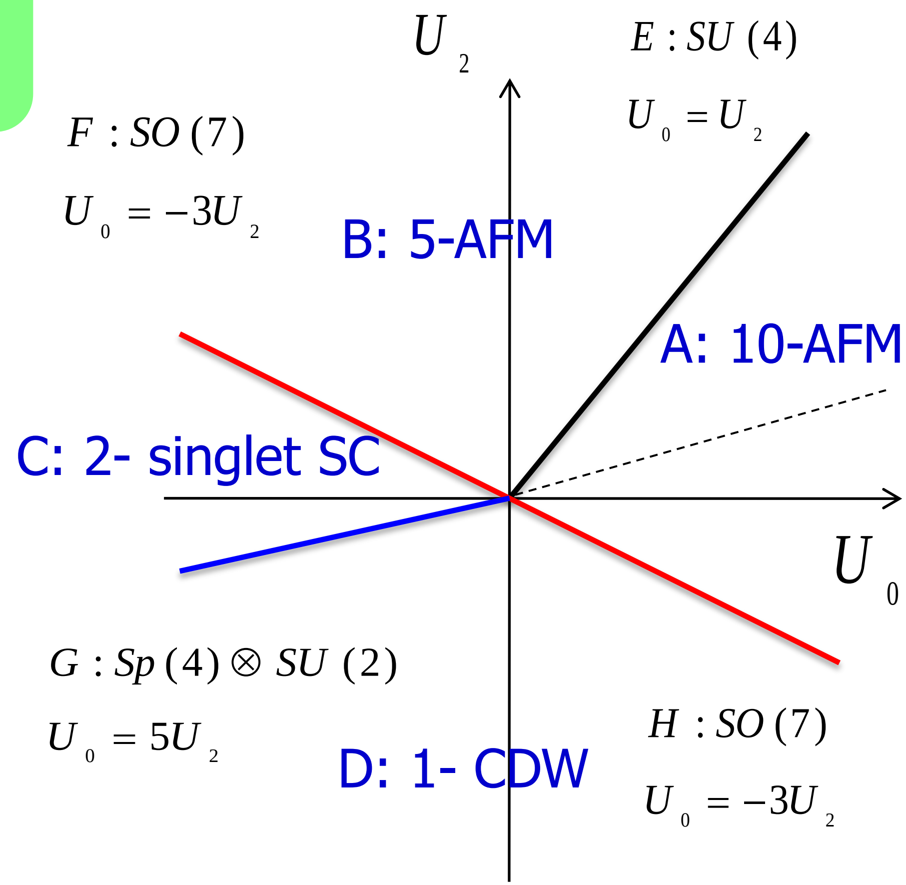
Unify AFM, SC, CDW with **exact** symmetries extended from Sp(4) in bipartite lattice at half-filling

- AFM (5-spin quadrupole) + SC (singlet) by SO(7) symmetry.

c.f. SO(5) theory of high Tc: 3-AF + 2 SC=5.

- CDW + SC (singlet) by pseudo-spin SU(2) symmetry. Generalization of eta-pairing.

- AFM(10-spin+spin octupole) +SC (10-quintet)+ CDW by the adjoint rep. of SO(7).



More technical details

Brief Review

Modern Physics Letters B, Vol. 20, No. 27 (2006) 1707–1738

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HIDDEN SYMMETRY AND QUANTUM PHASES IN SPIN-3/2 COLD ATOMIC SYSTEMS

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Received 31 August 2006

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- Introduction -a novel system to explore quantum magnetism.

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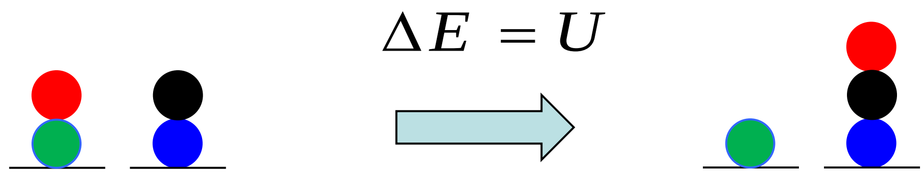
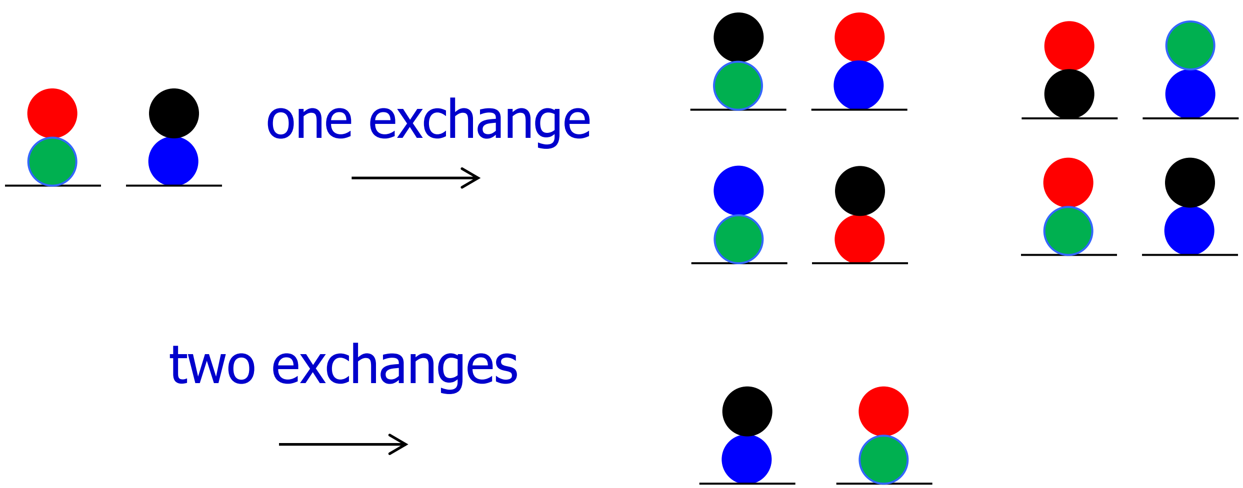
SU(2N) Hubbard model at half-filling

$$H = -t \sum_{\langle ij \rangle, \sigma=1}^{2N} \{ c_{i,\sigma}^{+} c_{j,\sigma} + h.c. \} + \frac{U}{2} \sum_i (n_i - N)^2 \qquad n_i = \sum_{\sigma=1}^{2N} n_{i,\sigma}$$

- In the atomic limit, t=0.



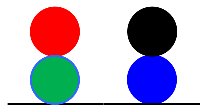
- Turning on t, number of super-exchange processes scales as N^2.



Enhancement of quantum spin fluctuations

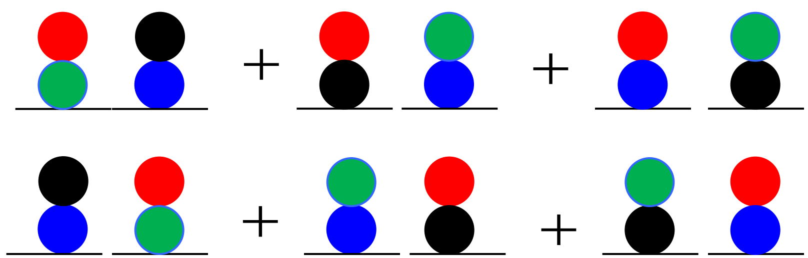
- As increasing 2N, the Neel states become unfavorable.

$$\Delta E = -2N \frac{t^2}{U}$$



classic-Neel

$$\Delta E = -2N(N+1)\frac{t^2}{U}$$



bond SU(2N) singlet

- Bond dimer state consists of $\binom{2N}{N}$ resonating Neel configurations.
- As N goes infinity, bond dimer ordering is realized (Sachdev + Read).

SU(2N) generators and Casimir

- The SU(2N) generators on each site *i*.

$$J_{\alpha\beta}(i) = c^{+}_{\alpha}(i)c_{\beta}(i) - \frac{\delta_{\alpha\beta}}{2N}\sum_{\sigma=1}^{2N} c^{+}_{\sigma}(i)c_{\sigma}(i) \qquad \sum_{\alpha} J_{\alpha\alpha}(i) = 0$$

- If a site is half-filled (N-fermions per site). Its degeneracy is $\binom{2N}{N}$ and the Casimir is

$$C_2(2N,N) = \tfrac{1}{2}\sum_{\alpha\beta} J_{\alpha\beta}(i)J_{\beta\alpha}(i) = \frac{N(2N+1)}{4}$$

- SU(2N) Heisenberg model with self-conjugate Rep.

$$H_{Heisenberg} = \frac{2t^2}{U}\sum_{\alpha\beta}\{J_{\alpha\beta}(i)J_{\beta\alpha}(j) - \frac{1}{2}n(i)n(j)\} + \text{bi} - \text{quadratic} \quad \text{terms}$$

Sign-problem free QMC at half-filling

- Hubbard-Stratonovich transformation in the density-channel involving imaginary numbers.

$$e^{-U\Delta\tau(n_i-N)^2} = \sum_{l=\pm1,\pm2} \gamma_i(l) e^{i\eta_i(l)\sqrt{U\Delta\tau}(n_i-N)} + O(\Delta\tau^4)$$

$$\gamma(\pm1) = 1+\frac{\sqrt{6}}{3}, \quad \gamma(\pm2) = 1-\frac{\sqrt{6}}{3},$$
$$\eta(\pm1) = \pm\sqrt{2(3-\sqrt{6})}, \quad \eta(\pm2) = \pm\sqrt{2(3+\sqrt{6})}.$$

F. F. Assaad, cond-mat/9806307.
F. F. Assaad, and M. Imada, J. Phys. Soc. Jpn 65, 189 (1996)

- Particle-hole transformation for ♦=N+1, …, 2N.

$$c_{i,\sigma} \to (-)^i c^+_{i,\sigma}, \quad c^+_{i,\sigma} \to (-)^i c_{i,\sigma}, \quad n_i - N \to \sum_{\sigma=1}^{N} n_{i,\sigma} - \sum_{\sigma=N+1}^{2N} n_{i,\sigma}$$

- The functional determinant after integrating out fermions is a product of complex-conjugate pairs, thus positive-definite.

QMC at half-filling

• Thermodynamic and ground-state properties can be simulated with a high numeric precision.

• Our simulation results:

AFM long-range order for SU(4) --- weaker ordering than SU(2)

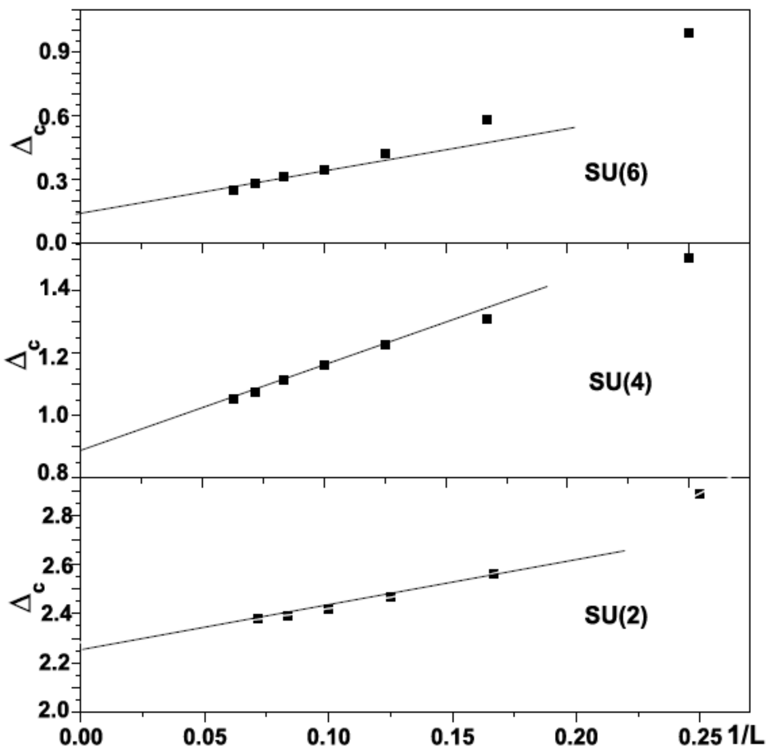
Evidence for algebraic spin correlations for SU(6) – gapless quantum paramagnets, or, spin liquid?.

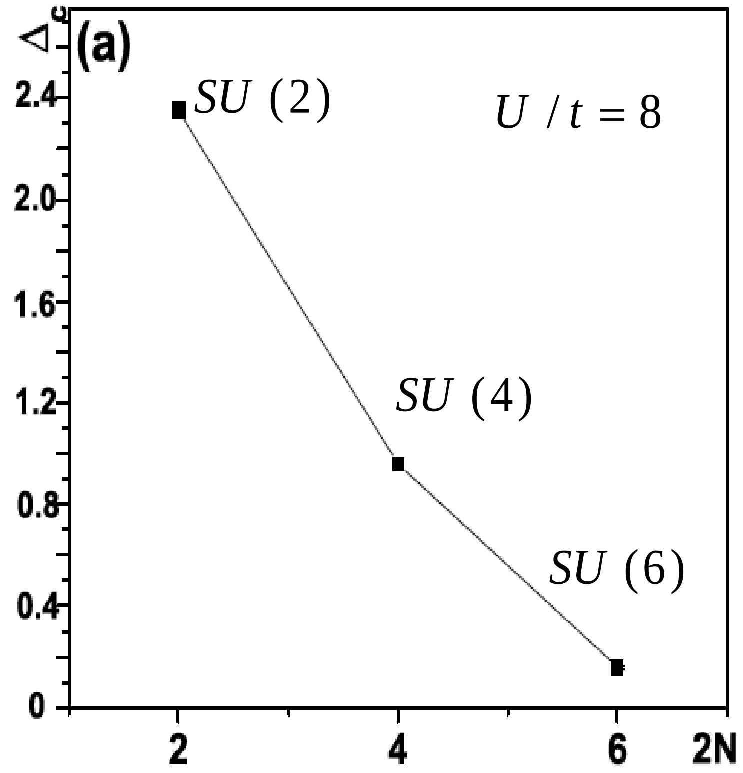
Enhancement of Pomeranchuk cooling at half-filling in the SU(6) case.

Z. Cai, H. H. Hung, L. Wang, Yi Li, and C. Wu, arxiv:1207.6843.
Z. Cai, H. H. Hung, L. Wang, C. Wu, arXiv:1202.632.

Confirm Mott gap: extracting single particle gap from Green's function

$$G(i,i,\tau) = \langle G \,|\, c^{+}_{\alpha}(i,\tau) c_{\alpha}(i,0) \,|\, G \rangle \to e^{-\Delta_{ch}\tau}$$





- Single-particle gap is weakened by increasing 2N.

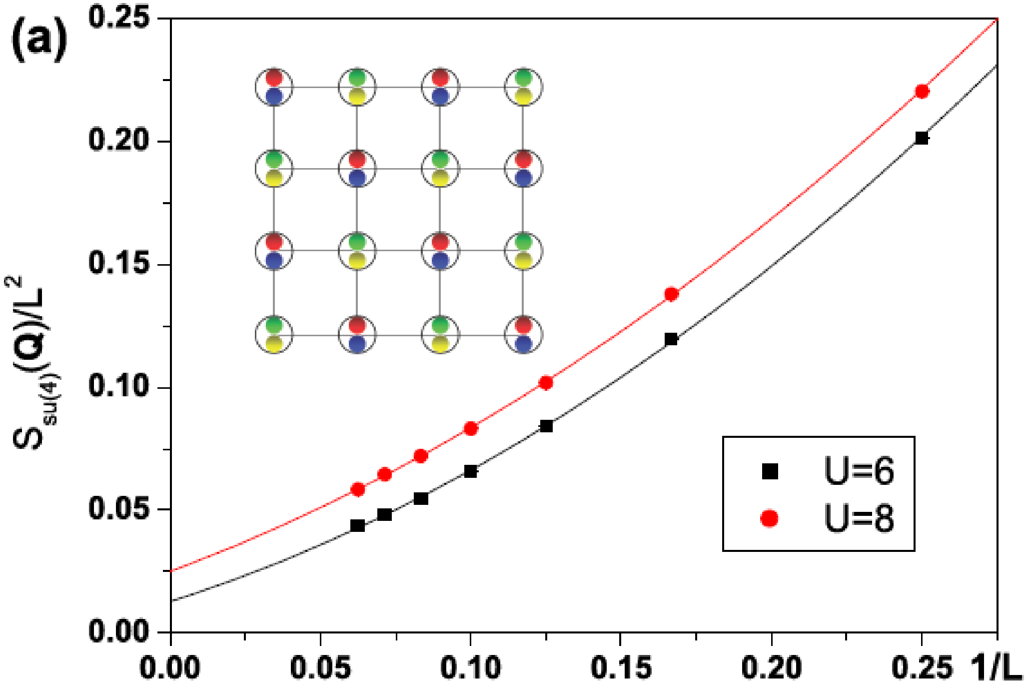
AFM long-range-ordering of SU(4) Hubbard model

- AF spin structure factor: equal time spin-spin correlation.

$$S_{SU(2N)}(\vec{Q}) = \frac{1}{C_2(2N,N)} \sum_{\alpha\beta} \frac{1}{2} \langle J^{\alpha\beta}(\vec{Q}) J^{\beta\alpha}(\vec{Q}) \rangle \qquad \vec{Q} = (\pi, \pi)$$

$$C_2(2N = 4, N) = \frac{N(2N+1)}{4}$$

$$J_{\alpha\beta}(\vec{Q}) = \frac{1}{L} \sum_{i=1}^{L^2} e^{i\vec{Q}\cdot\vec{r}_i} J_{\alpha\beta}(i)$$



- At U=8, $\frac{1}{L^2} S_{SU(4)}(\vec{Q}) \to 0.025$

For the SU(2) case, the Heisenberg model

$$\frac{1}{L^2} S_{SU(2)}(\vec{Q}) \to 0.125$$

SSE on Heisenberg model, Sandvik, PRB,56,11678.

Vanishing of spin gap of the SU(4) Hubbard model

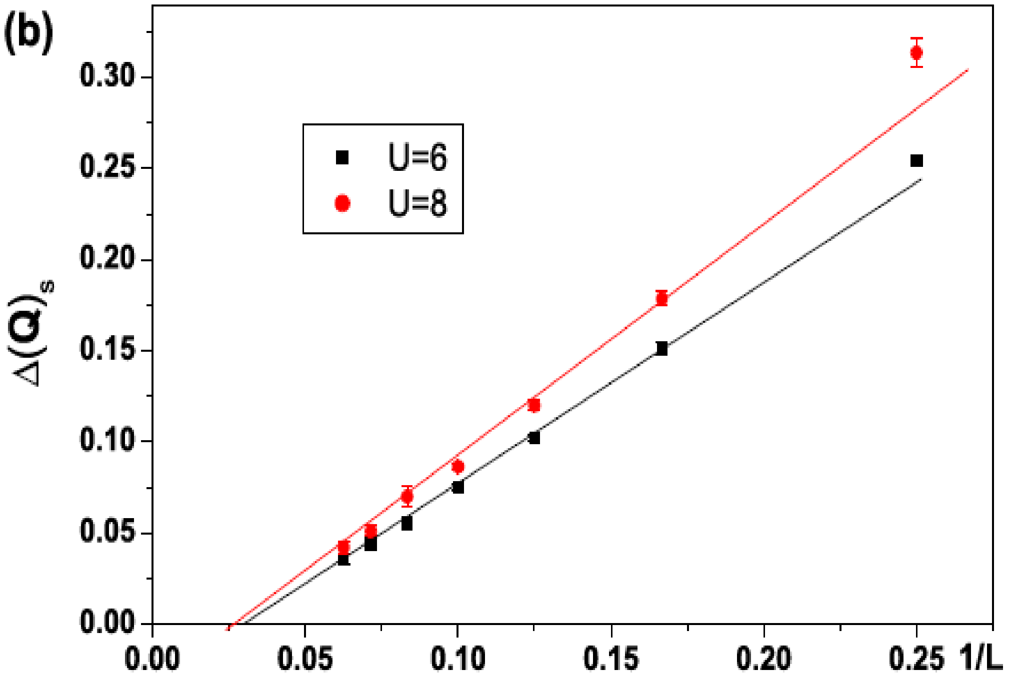
- Spin gap scales to zero: measured from non-equal time decay.

$$S_{structure}(\vec{Q},\tau) = \left\langle G \mid J_{\alpha\beta}(\vec{Q},\tau) J_{\beta\alpha}(-\vec{Q},0) \mid G \right\rangle \rightarrow e^{-\Delta_{sp}\tau}$$

- Consistent with the AFM long-range order.

- 6 dimensional Goldstone manifold U(4)/[U(2)*U(2)].

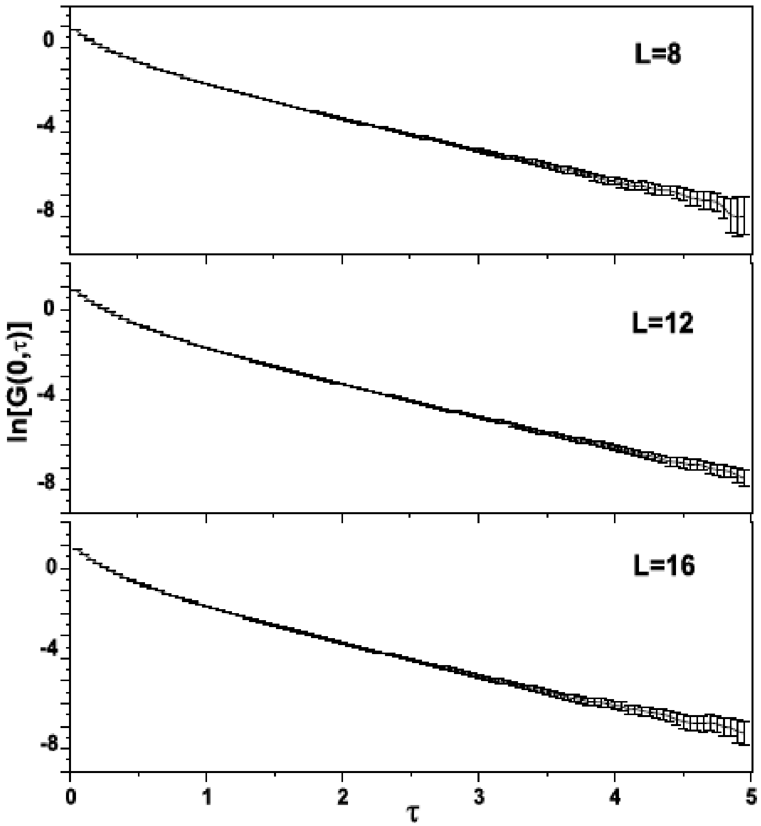
- Our result is consistent with the variational MC study.

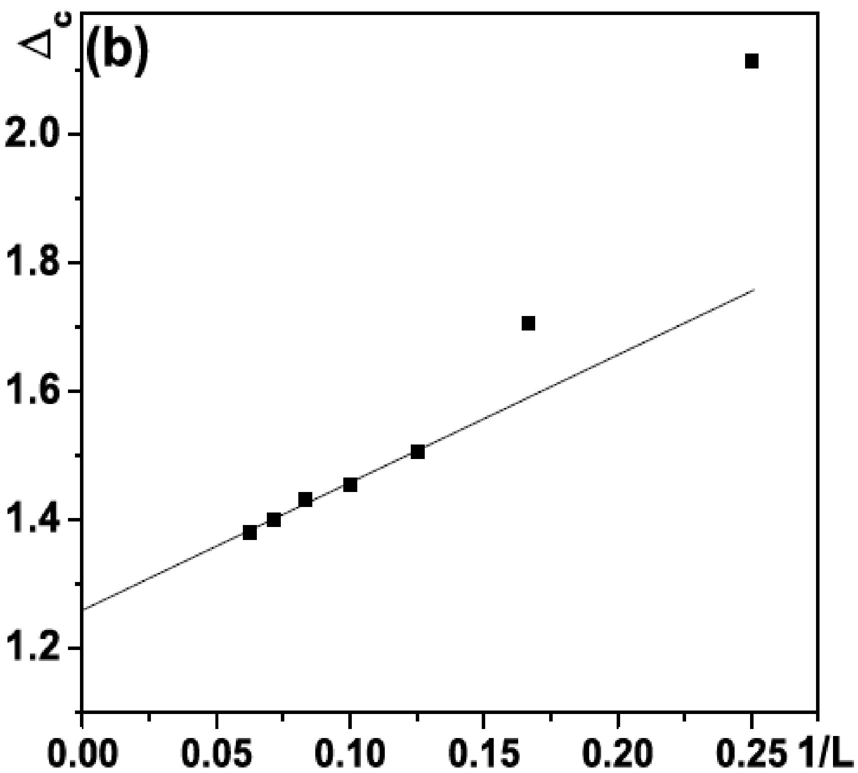


A. Paramekanti and J. B. Marston, J. Phys. Cond. Matt. 19, 125215 (2007).

SU(6) at half-filling: single-particle gap (U/t=12)

- Finite scaling of the charge gap ~ 1.26 → Mott insulator
- Green's function: $G(i,i,\tau) = \langle G \mid c_\alpha^+(i,\tau) c_\alpha(i,0) \mid G \rangle \rightarrow e^{-\Delta_{ch}\tau}$

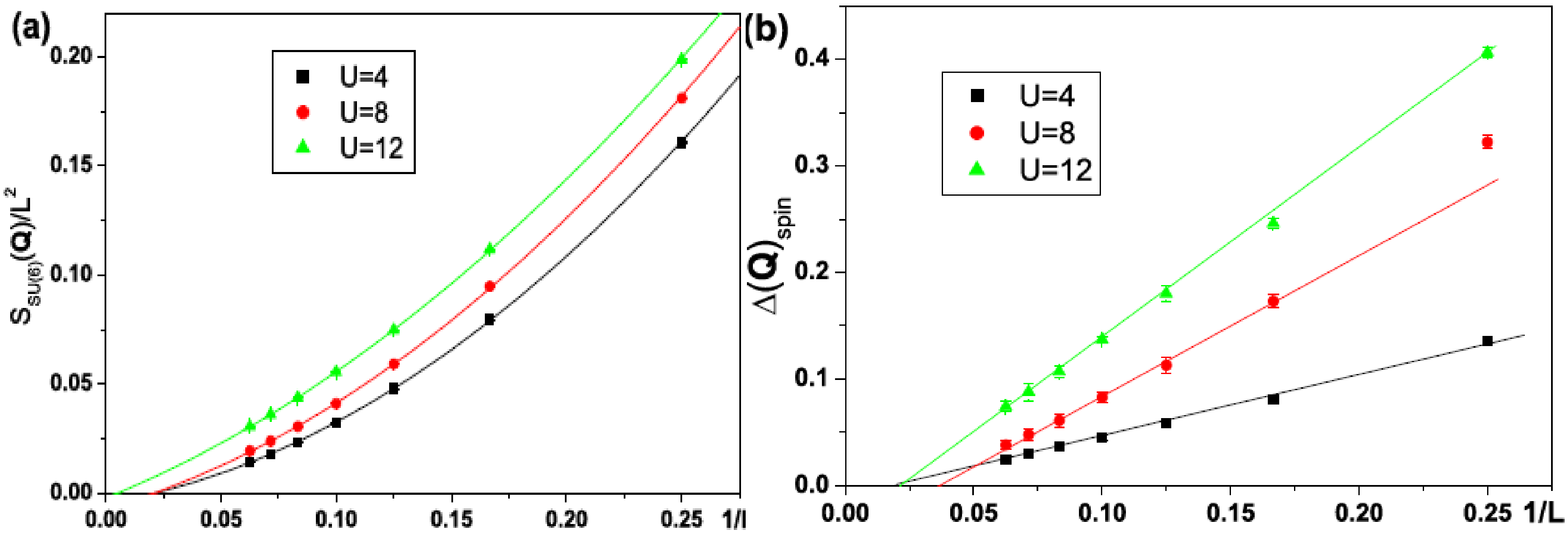




SU(6) Hubbard model: vanishing of Neel ordering and spin gap

spin structure factor

spin gap scaling



SU(6) Hubbard mode at U=12: farthest points spin correlation

- Power-law spin correlation. $\eta \approx 1.17$ for U=12.

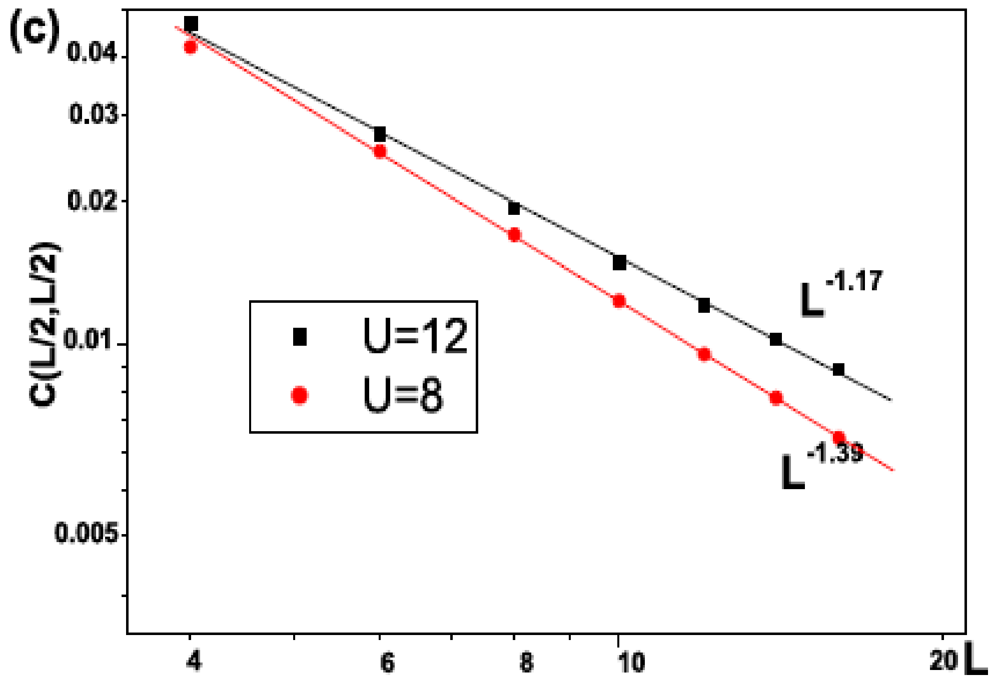
$$C_{SU(6)}(L/2, L/2) = \frac{1}{C_2(6,3)} \sum_{\alpha\beta} \frac{1}{2} \left\langle G \,|\, J_{\alpha\beta}(0,0) J_{\beta\alpha}(\frac{L}{2}, \frac{L}{2}) \,|\, G \right\rangle \sim L^{-\eta}$$

- $\eta \approx 1.11$ for U= 12 fitted from the scaling of structure factor.

$$S_{SU(6)}(\vec{Q}) = AL^{-2} + BL^{-\eta}$$

short-range

quasi-long-range

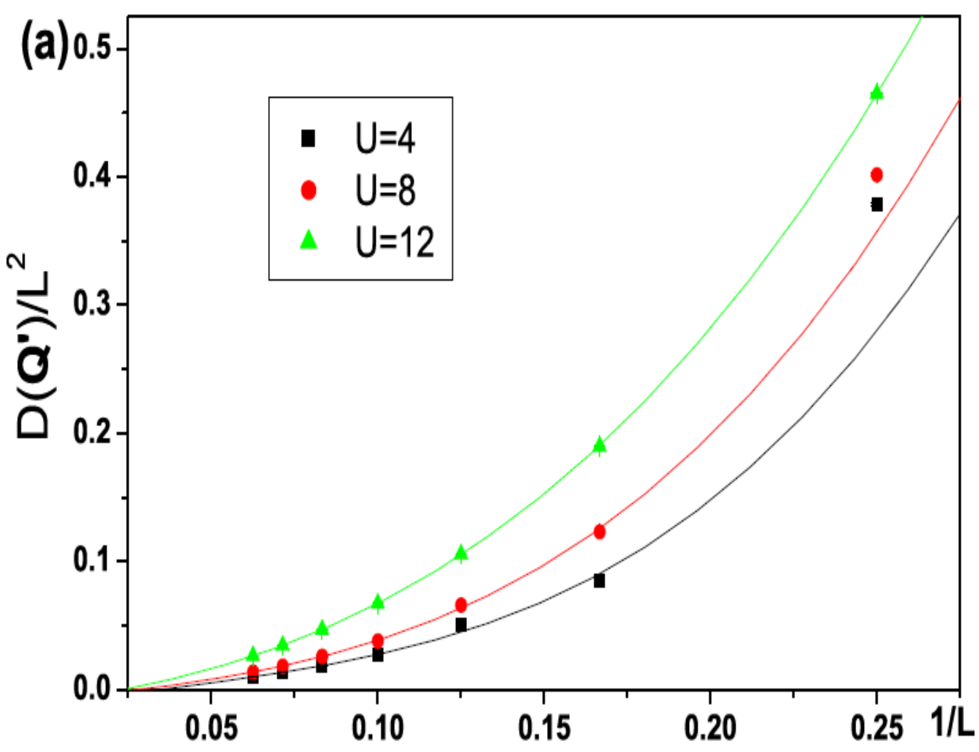


SU(6) Hubbard mode U=12: vanishing of dimer ordering

- Structure-factor of bond kinetic energy operators at Q=(□, 0)

$$D_{ij} = \sum_{\alpha} c^{\dagger}_{i,\alpha} c_{j,\alpha} + h.c.$$

- Fitted with AL^{-2} → short-range correlation.

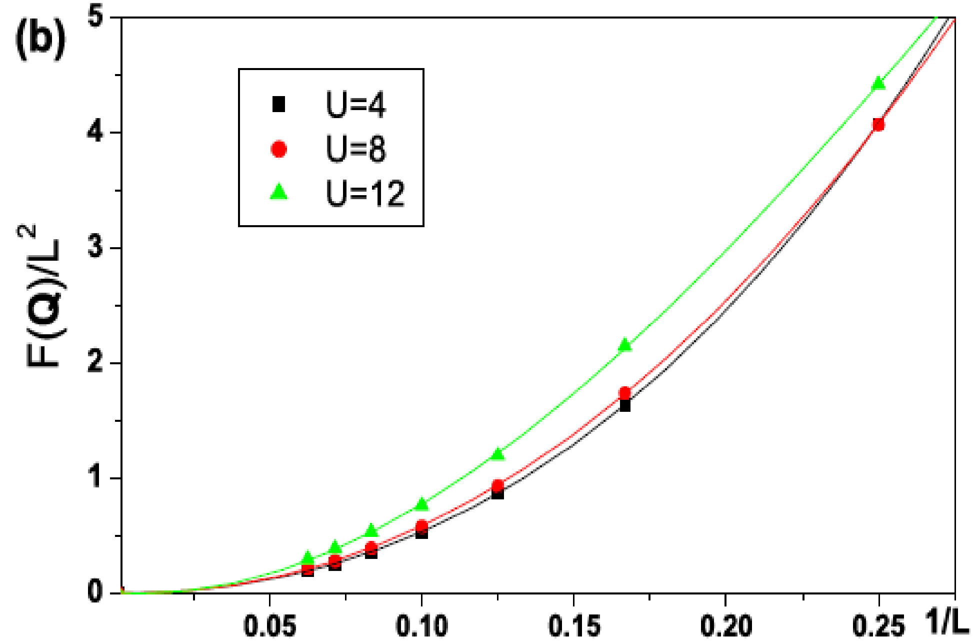


SU(6) Hubbard mode U=12: vanishing of DDW ordering

- Structure-factor of the bond current operators at Q=(□, □)

$$F_{ij} = \sum_{\alpha} i(c^{\dagger}_{i,\alpha} c_{j,\alpha} - h.c.),$$

- Fitted with AL^{-2} → short-range correlation.



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Pomeranchuk cooling

- Fermions in Mott-insulating states can hold more entropy than in the Fermi liquid states.

- In Mott-insulators, all the sites contribute to entropy through spin-configurations, while in Fermi liquids, only fermions close to Fermi surfaces contributes.

- Driving a spinful Fermi liquid to Mott-insulating states, or, crystalline solids, leads to cooling --- Pomeranchuk cooling proposed in He-3 system.

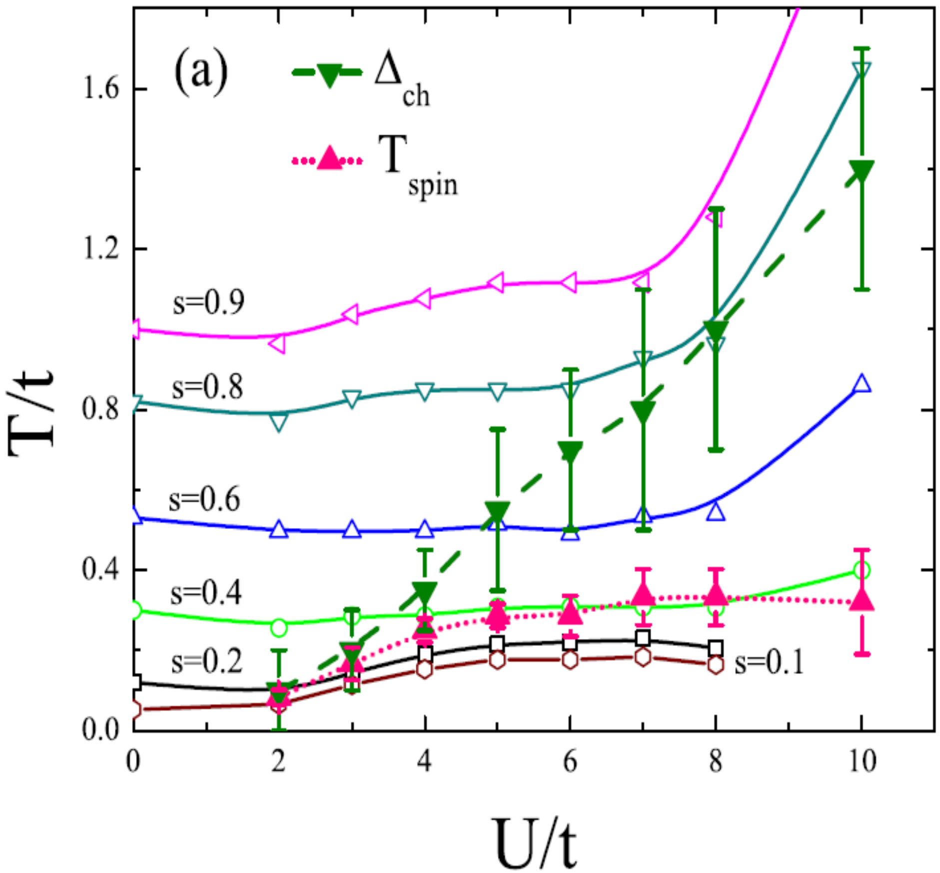
- Pomeranchuk cooling is more efficient for large spin systems due to the enhanced entropy capability.

S. Taie, arXiv 1208.4883; K. R. Hazzard, et al PRA 2012, Z. Cai et al, arXiv:1207.6843.

Inefficiency of Pomeranchuk cooling of SU(2) fermions

- The iso-entropy curve for spin-1/2 Hubbard model at half-filling – QMC by T. Paiva et al, PRL 2010.

- The ordering tendency of the SU(2) AFM suppresses the spin entropy.



T. Paiva, et al, PRL 104, 066406 (2010).

Entropy capability per particle for half-filled SU(2N) Hubbard model

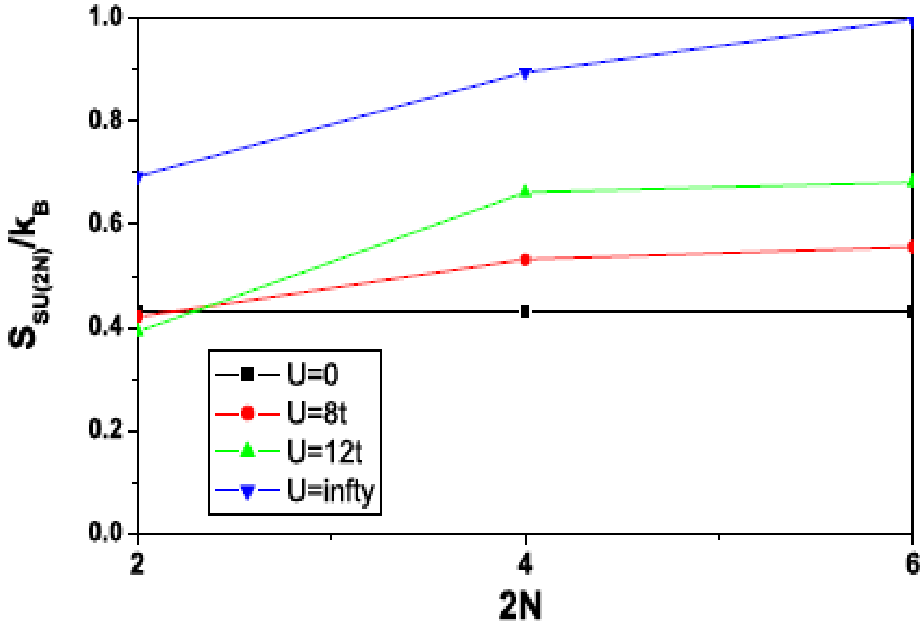


FIG. 1: Entropy per particle $S_{su(2N)}$ for the SU$(2N)$ Hubbard model at half-filling v.s. $2N$ in a 10×10 square lattice. The temperature is fixed at $T/t = \frac{1}{3}$. The line of $U/t = \infty$ is from the results of Eq. 3.

- Entropy per particle at U→ infinity and N→ infinity.

$$\frac{S_{Su(2N)}}{k_B} = \frac{1}{N} \ln \frac{(2N)!}{N!N!} \underset{N \to \infty}{\to} \ln 4$$

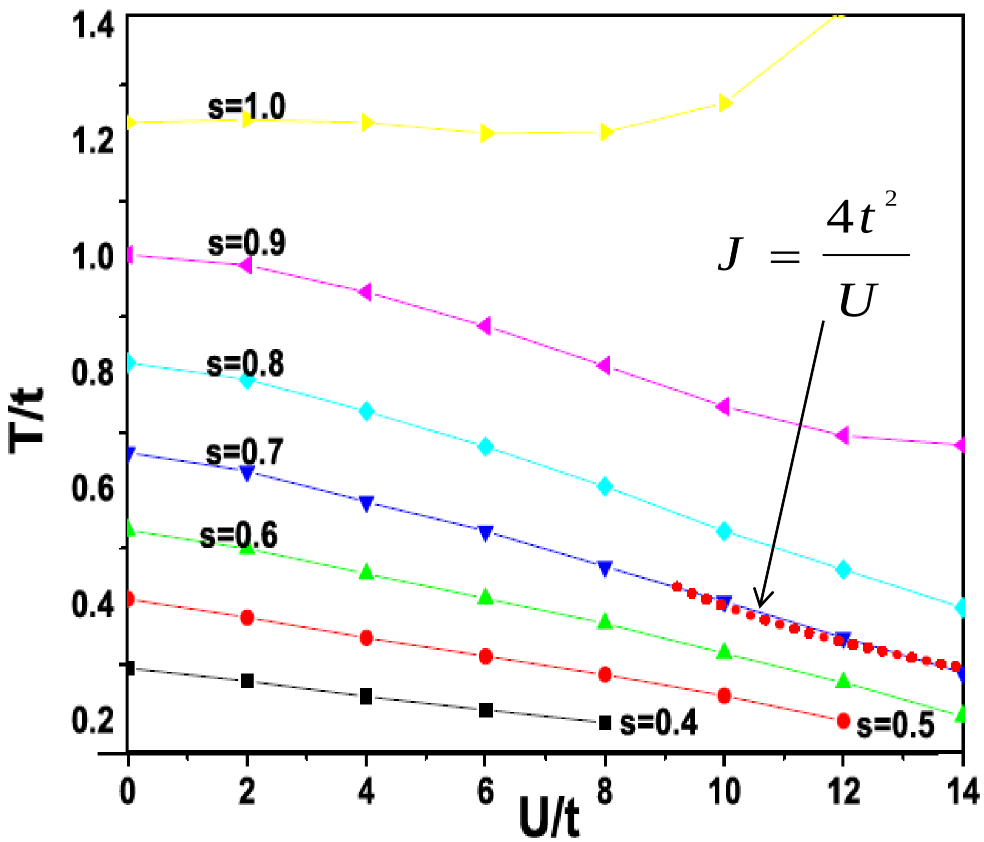
Pomeranchuk cooling for SU(6) fermions at half-filling

- Iso-entropy curve at half-filling (three-particle per site).

$$S_{su(2N)} = S/(NL^2)$$

$$\frac{S_{su(2N)}(T)}{k_B} = \ln 4 + \frac{E(T)}{T} - \int_T^{\infty} dT' \frac{E(T')}{T'^2},$$

- As entropy per particle s<0.7, increasing U can cool the system below the anti-ferro energy scale J.



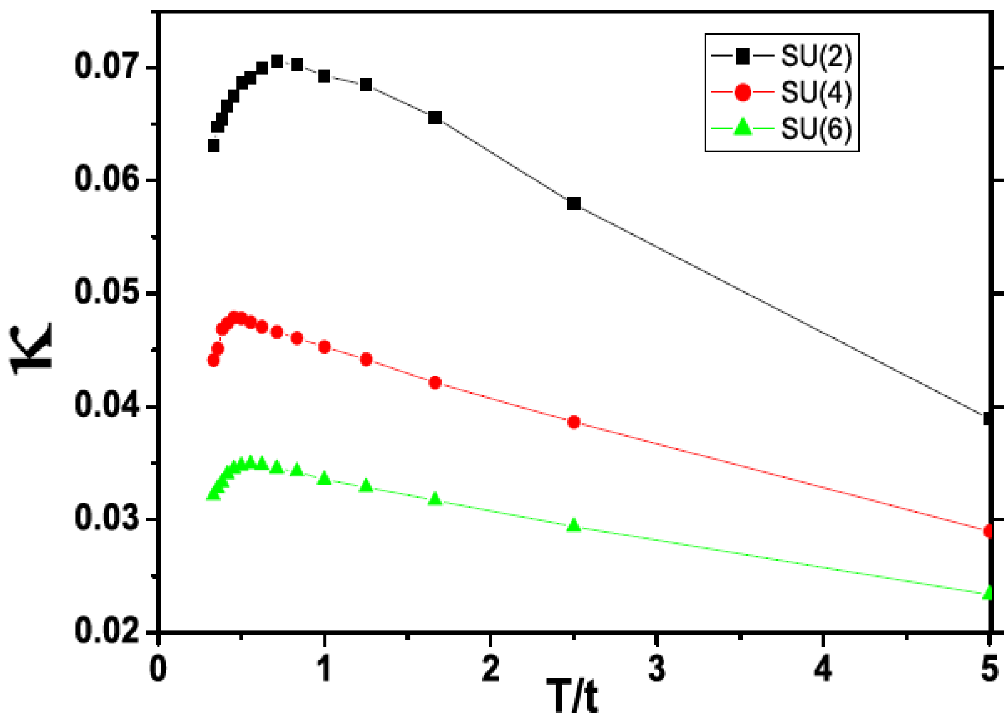
Sample size 10×10

Z. Cai, H. H. Hung, L. Wang, D. Zheng, and C. Wu, arxiv1202.6323.

Compressibility

- Charge fluctuation energy scale.

$$\kappa_{SU(2N)} = \frac{1}{L^2}\frac{\partial N_f}{\partial \mu} = \frac{1}{TL^2}(\langle \hat{N}_f^2 \rangle - \langle \hat{N}_f \rangle^2)$$



The normalized compressibility $\kappa_{su(2N)}/(2N)$ v.s. T

Sample size 10×10

$U/t = 4$

Z. Cai, H. H. Hung, L. Wang, D. Zheng, and C. Wu, arxiv1202.6323.

Magnetic susceptibility v.s. T

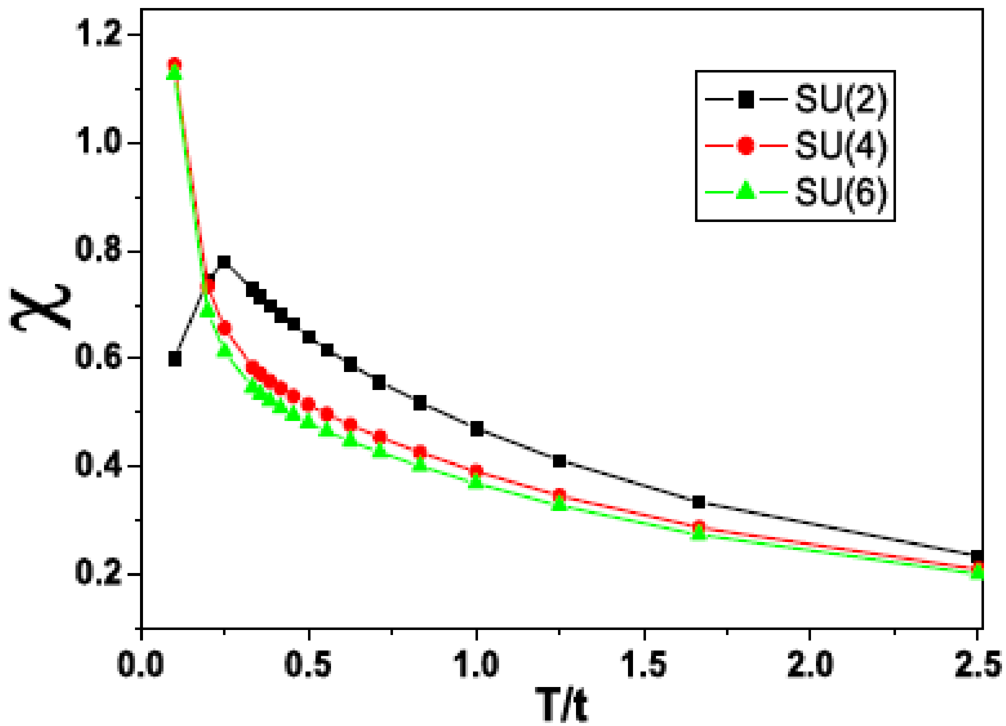


FIG. 4: The normalized SU($2N$) susceptibilities $\chi_{su(2N)}$ v.s. T with fixed $U/t = 4$ for $2N = 2, 4, 6$

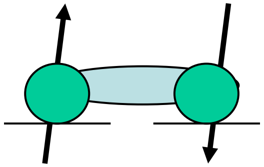
Sample size 10×10

Z. Cai, H. H. Hung, L. Wang, D. Zheng, and C. Wu, arxiv1202.6323.

¼-filling (one particle per site) -- "color magnetism"

C. Wu, Phys. Rev. Lett. 95, 266404 (2005); Hung, Wang, and Wu, PRB 05446, (2011)

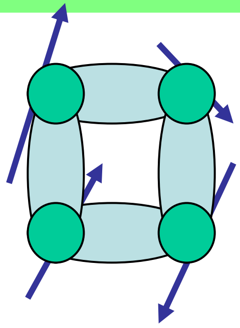
- Strong spin fluctuations: N=4.
- When the onsite Neel ordering is suppressed, multi-site correlations develop.
- spin-1/2: 2 sites to form an SU(2) singlet.



- 4 sites to form an SU(4) singlet. Each site belongs to the fundamental Rep.

baryon-like $$\frac{\varepsilon_{\alpha\beta\gamma\delta}}{4!}\psi_{\alpha}^{+}(1)\psi_{\beta}^{+}(2)\psi_{\gamma}^{+}(3)\psi_{\delta}^{+}(4)\left|0\right\rangle$$

Bossche et. al., Eur. Phys. J. B 17, 367 (2000).



- c. f. QCD. At least three quarks form an SU(3) color singlet: baryons; multi-particle color/magnetic correlations.

Conclusion

- **Large-spin cold fermions are quantum-like NOT classical**!

- Spin-3/2 Hubbard model unifies AFM, SC and CDW phases with exact symmetries extended from Sp(4).

- Power-law spin correlations in the half-filled SU(6) Hubbard model.

- Pomeranchuk cooling of the SU(6) Hubbard model.

- Exotic "color magnetism" exhibits dominant multi-particle correlations.