Knot Revolutions

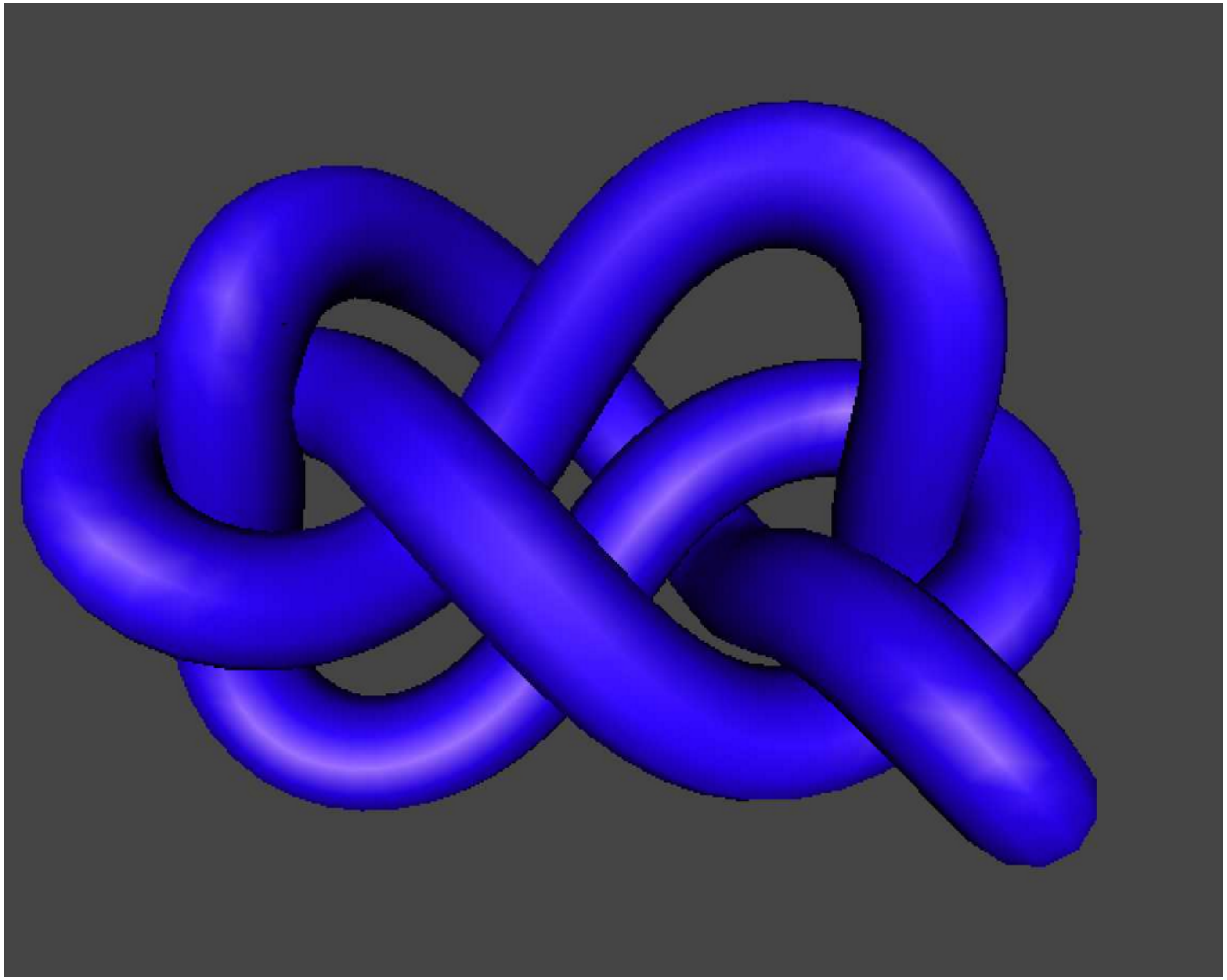
Louis H. Kauffman, UIC
www.math.uic.edu/~kauffman

<kauffman@uic.edu>

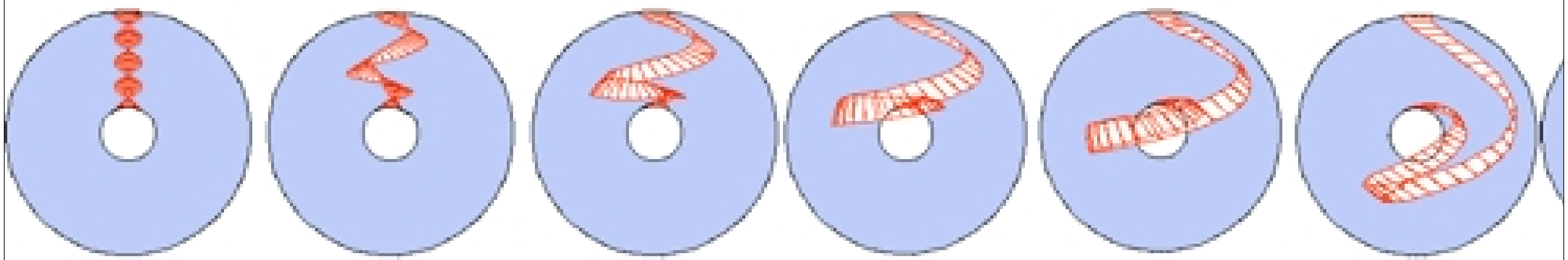
ChalkTalk

UC SANTA BARBARA
Kavli Institute for
Theoretical Physics

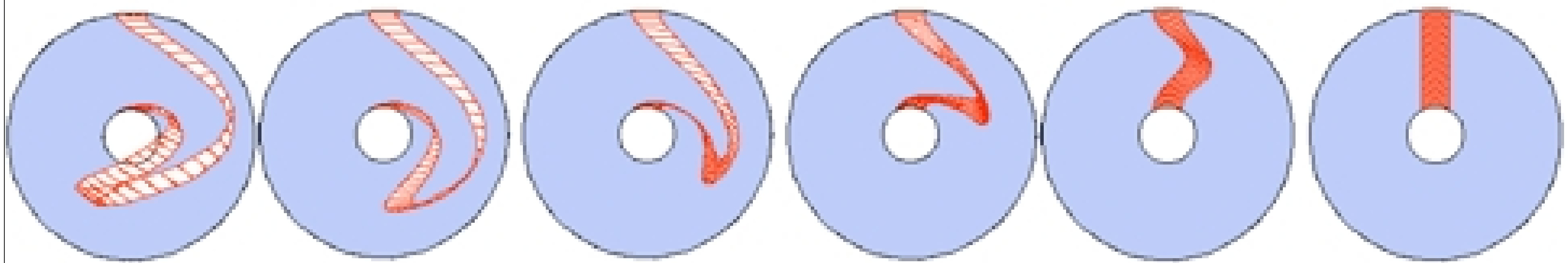
November 28, 2018.



Is it Knotted?



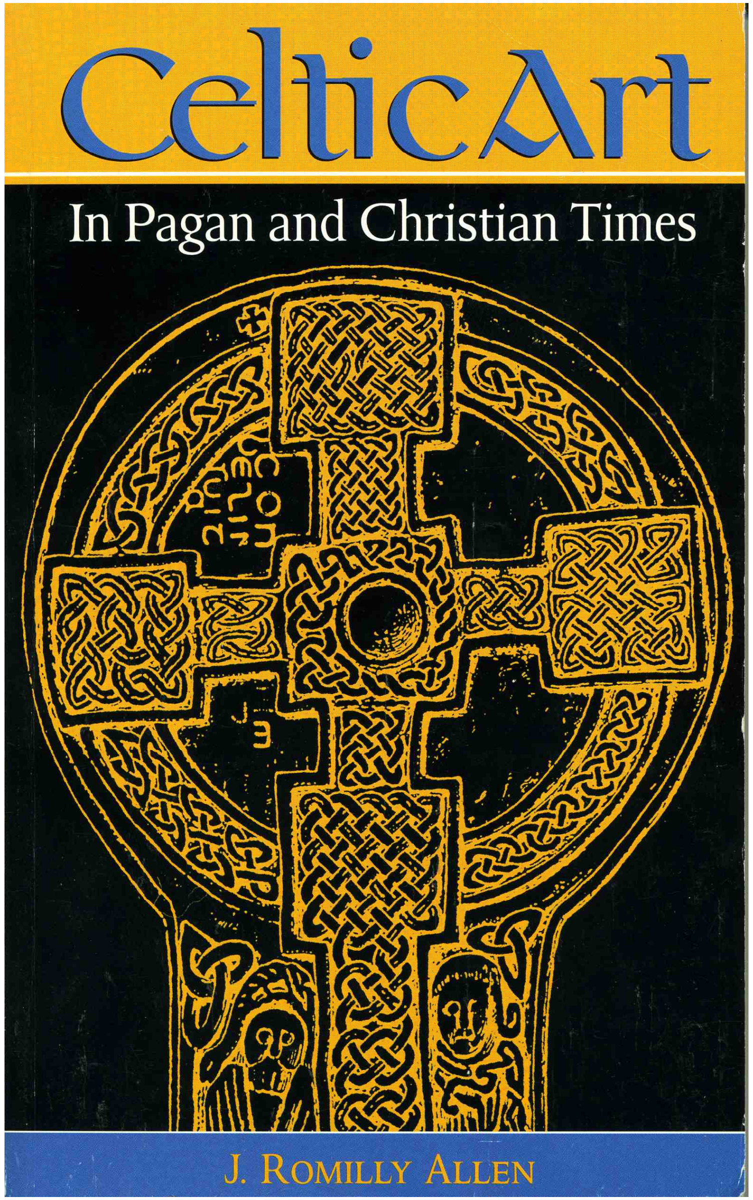
Dirac String Trick Storyboard, courtesy of Louis Kauf



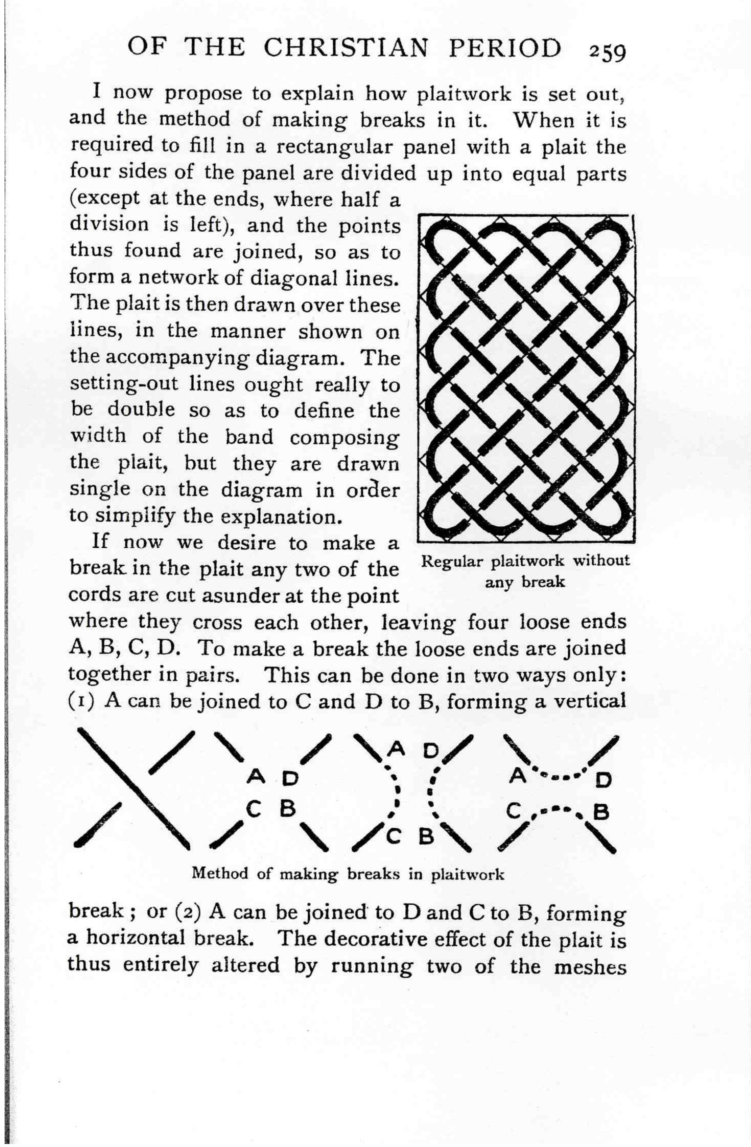
y of Louis Kauffman, University of Illinois, Chicago


Air on the
Dirac Strings

Published 1900.



A Theory for construction of Celtic Weaves.



I now propose to explain how plaitwork is set out, and the method of making breaks in it. When it is required to fill in a rectangular panel with a plait the four sides of the panel are divided up into equal parts (except at the ends, where half a division is left), and the points thus found are joined, so as to form a network of diagonal lines. The plait is then drawn over these lines, in the manner shown on the accompanying diagram. The setting-out lines ought really to be double so as to define the width of the band composing the plait, but they are drawn single on the diagram in order to simplify the explanation.

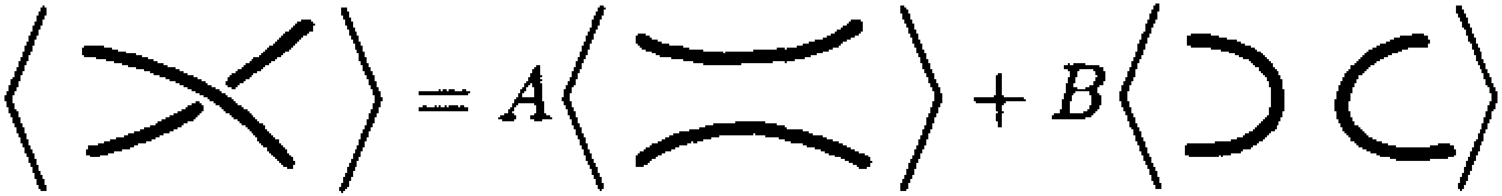
Regular plaitwork without any break

If now we desire to make a break in the plait any two of the cords are cut asunder at the point where they cross each other, leaving four loose ends A, B, C, D. To make a break the loose ends are joined together in pairs. This can be done in two ways only: (1) A can be joined to C and D to B, forming a vertical

Method of making breaks in plaitwork

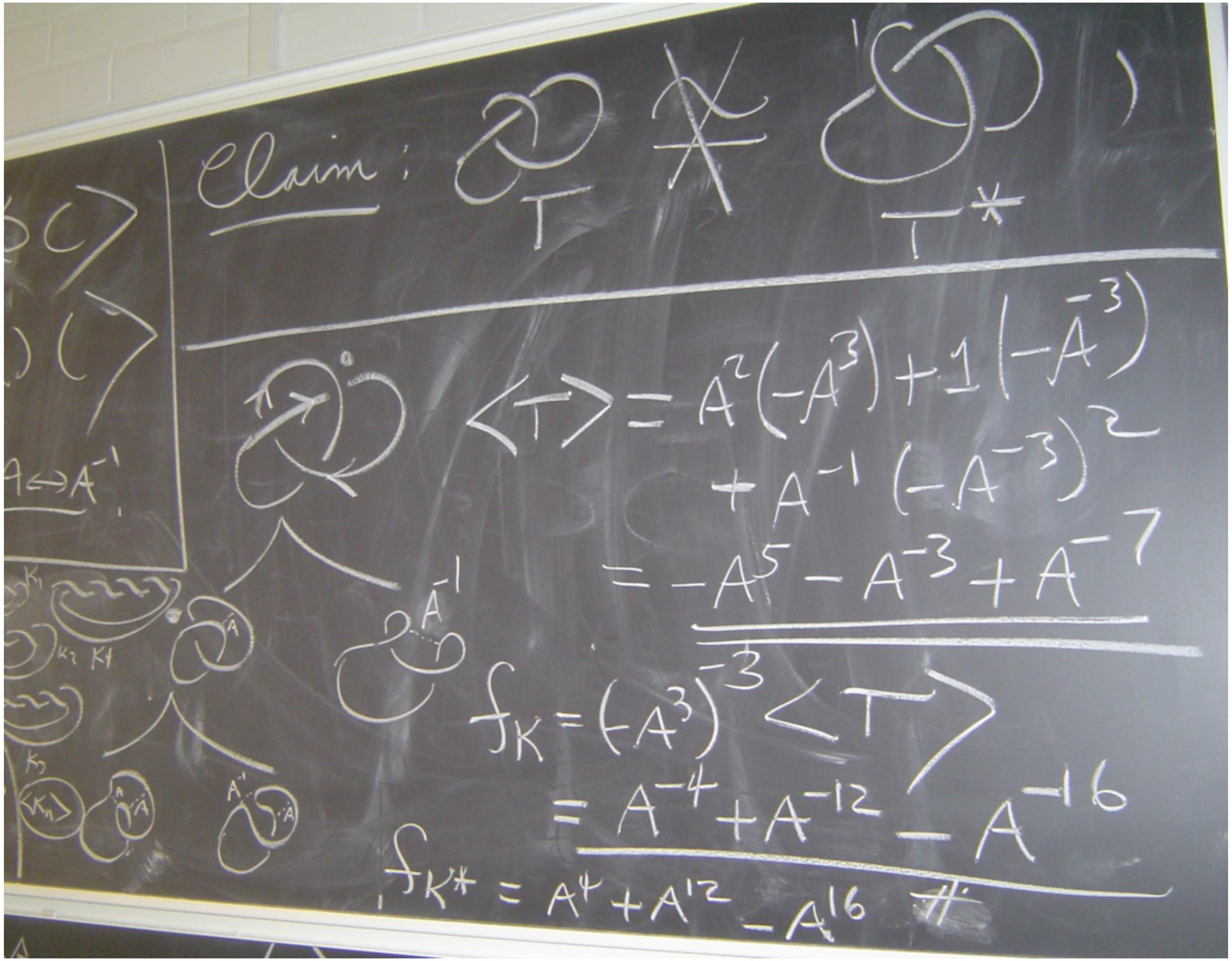
break; or (2) A can be joined to D and C to B, forming a horizontal break. The decorative effect of the plait is thus entirely altered by running two of the meshes

83 Years Later ...

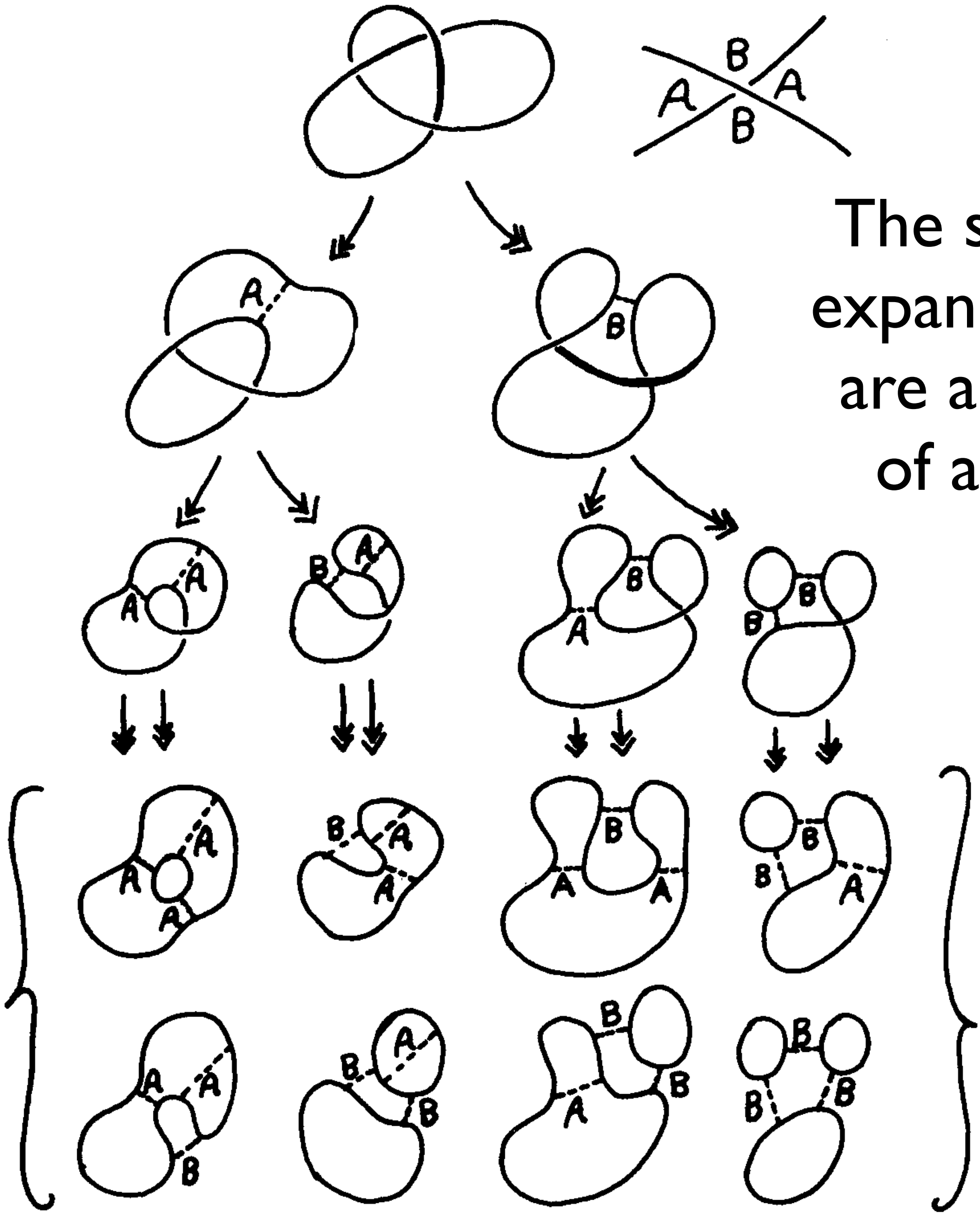


Your lecturer wrote down the equation above (not having read Romily Allen, who did not make his theory into an equation) and this began, with the help of the previously discovered Jones polynomial, a long story of developing relationships among topology, combinatorics, statistical mechanics, quantum theory and more.

We will not detail this part of the story, but a hint or two is worthwhile!

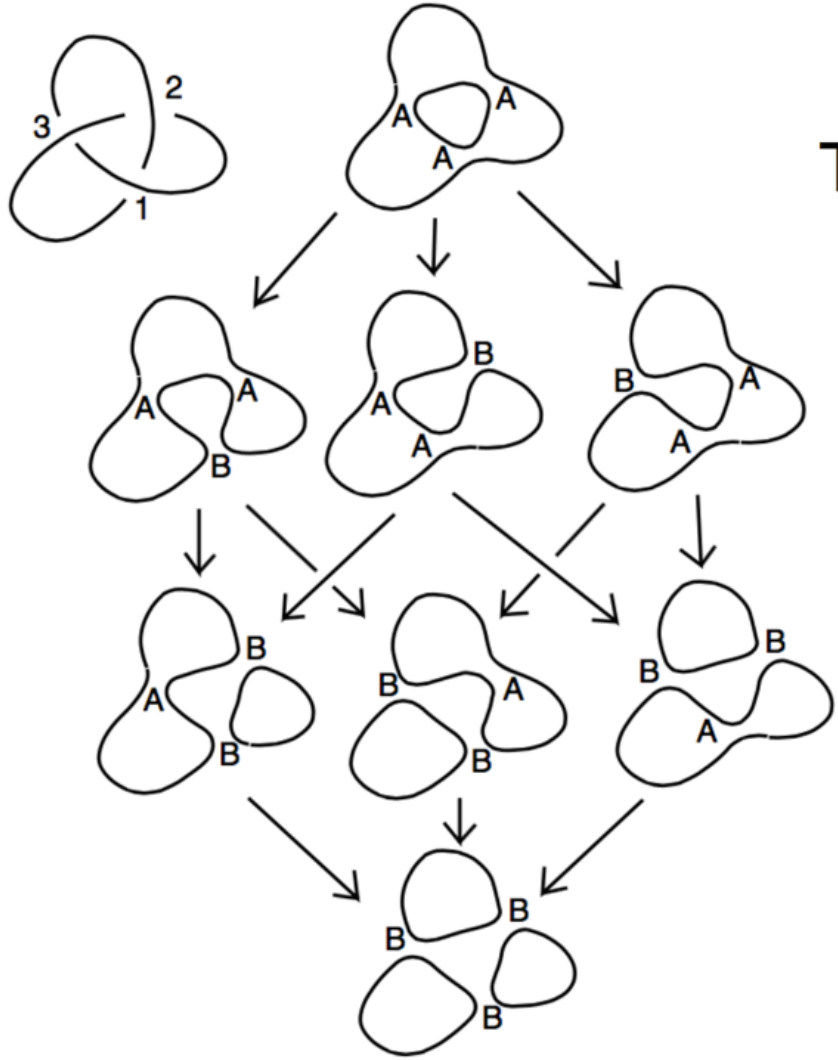


One can calculate information about knots and their mirror images.



The set of states in the expansion of the bracket are analagous to states of a physical system.

Around 1998, Mikhail Khovanov viewed the states as a category and found remarkable answers to the question below.



Cubism

The bracket states form a category. How can we obtain topological information from this category?

Before the Cateogorification came Quantum Field Theory.

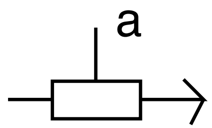
1988 Witten's Integral

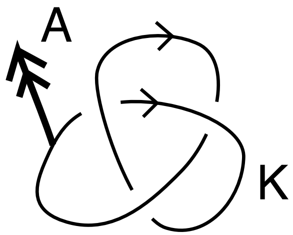
In [49] Edward Witten proposed a formulation of a class of 3-manifold invariants as generalized Feynman integrals taking the form $Z(M)$ where

$$Z(M) = \int DAe^{(ik/4\pi)S(M,A)}.$$

Here M denotes a 3-manifold without boundary and A is a gauge field (also called a gauge potential or gauge connection) defined on M. The gauge field is a one-form on a trivial G-bundle over M with values in a representation of the Lie algebra of G. The group G corresponding to this Lie algebra is said to be the gauge group. In this integral the action $S(M, A)$ is taken to be the integral over M of the trace of the Chern-Simons three-form $A \wedge dA + (2/3)A \wedge A \wedge A$. (The product is the wedge product of differential forms.)

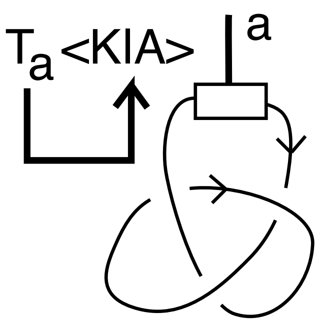
$V \xrightarrow{T^a} V \quad \Longleftrightarrow$

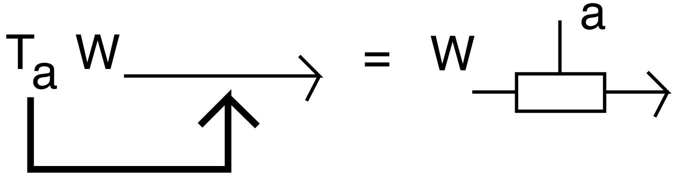




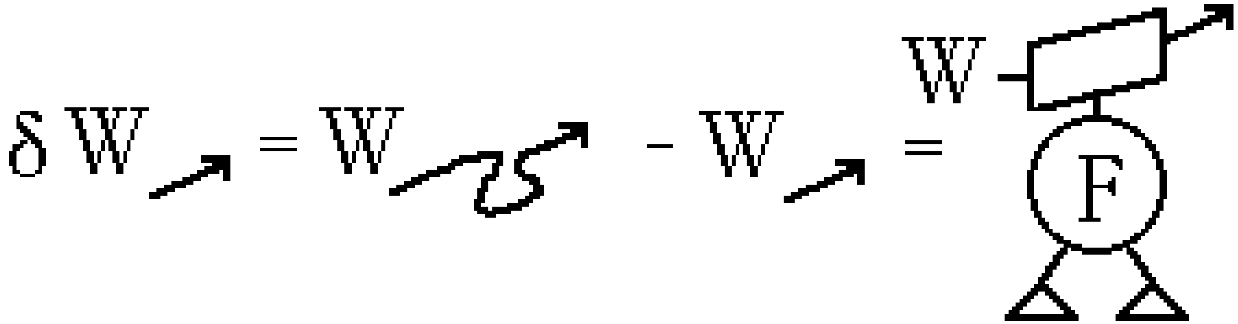
$$W_K(A) = \langle K|A\rangle = \mathrm{tr}\left(P e^{\oint_K A}\right)$$

$$= \prod_{x \,\varepsilon\, K} \left(1 + A^i_a(x) T^a dx_i\right)$$



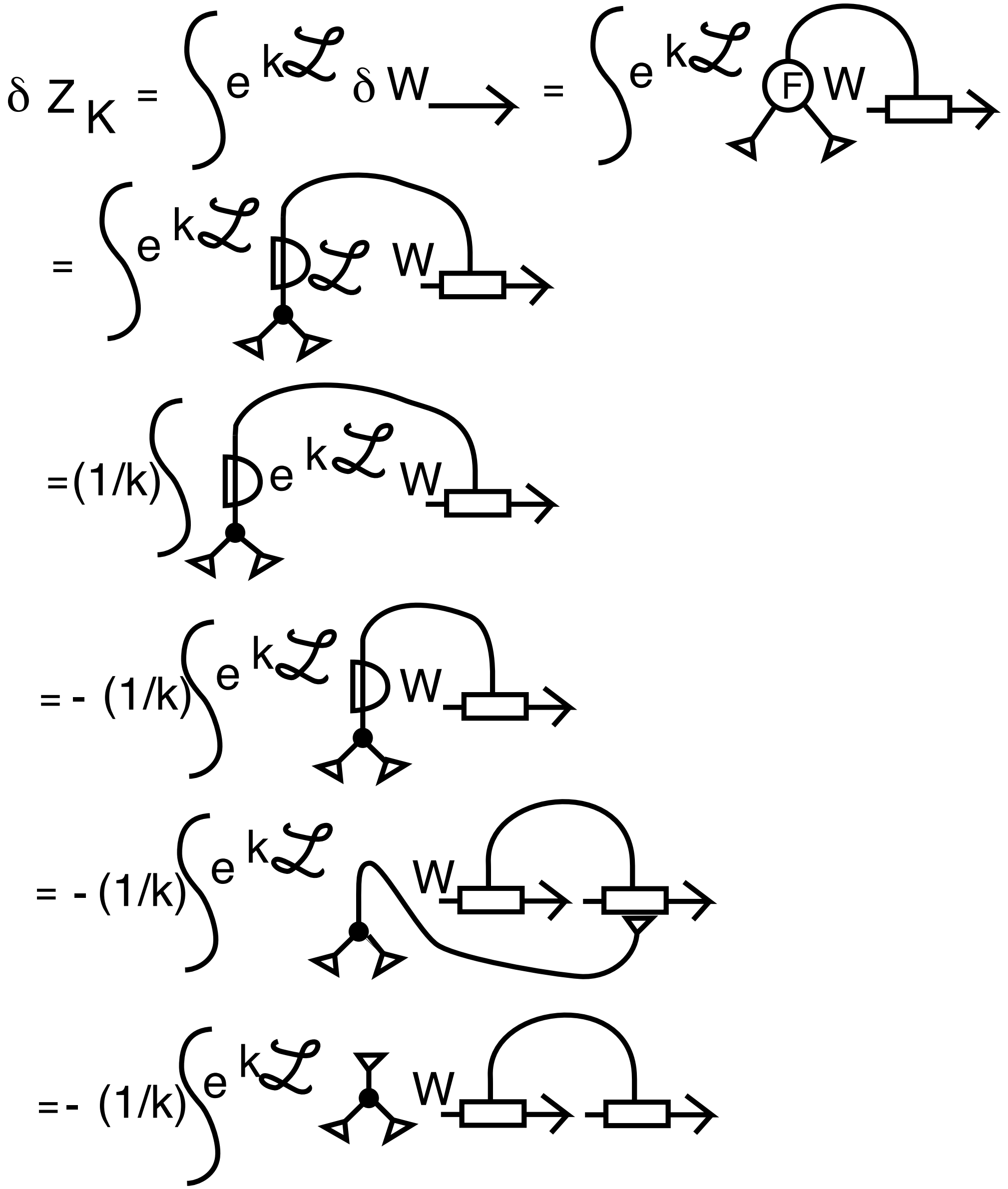


Think of a vector on the knot. As the base of the vector moves by dx the vector changes to (1 + A)v. This is the analog of parallel translation. The gauge field is a connection!

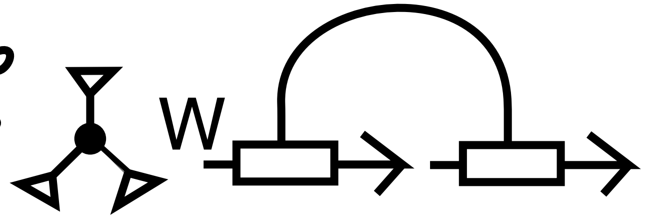


By an interesting calculation,
one finds that if you change the loop by a small amount,
then the Wilson loop changes by an insertion of
Lie algebra coupled with the curvature tensor.

This is just like classical differential geometry
where parallel translation around a small loop
measures curvature.

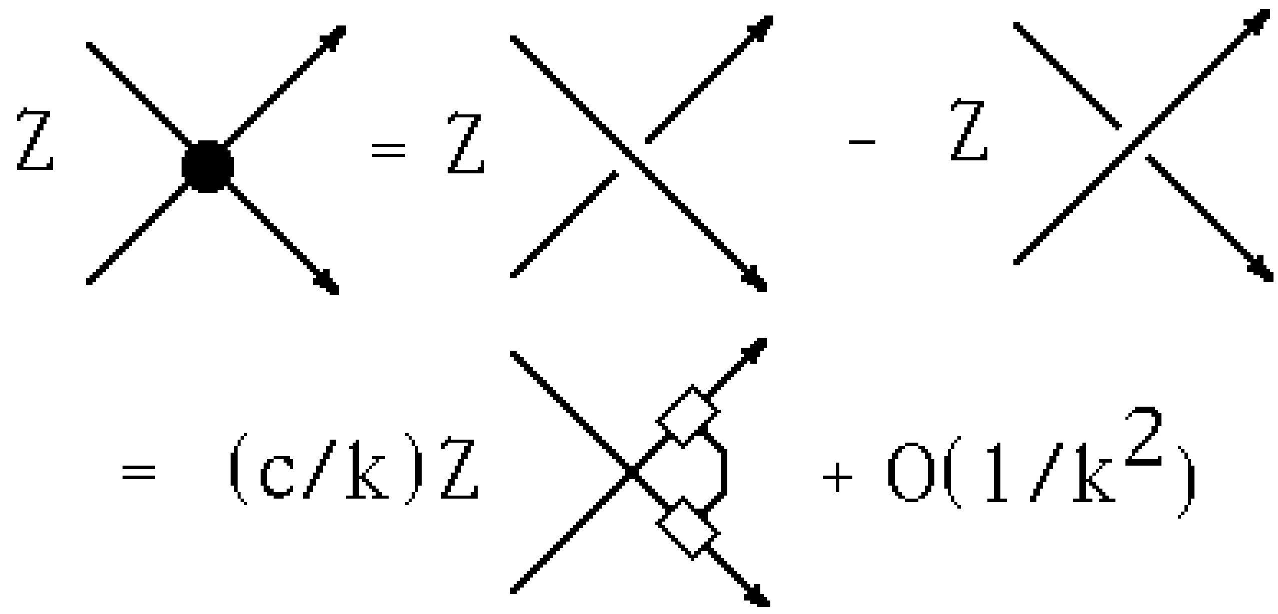

δ Z K = e kℒ δ W = e kℒ F W
= e kℒ ℒ W
=(1/k) e kℒ W
= - (1/k) e kℒ W
= - (1/k) e kℒ W
=- (1/k) e kℒ W

$$\delta Z_K = -(1/k)\int e^{k\mathcal{L}}$$



When you vary the loop,
Witten's integral changes by
the appearance of the volume form

and a double Lie algebra insertion.



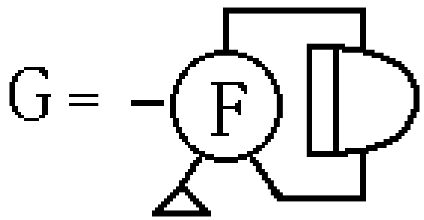
This is what happens when you switch crossings.
You get a "skein relation" involving Lie algebra insertions.

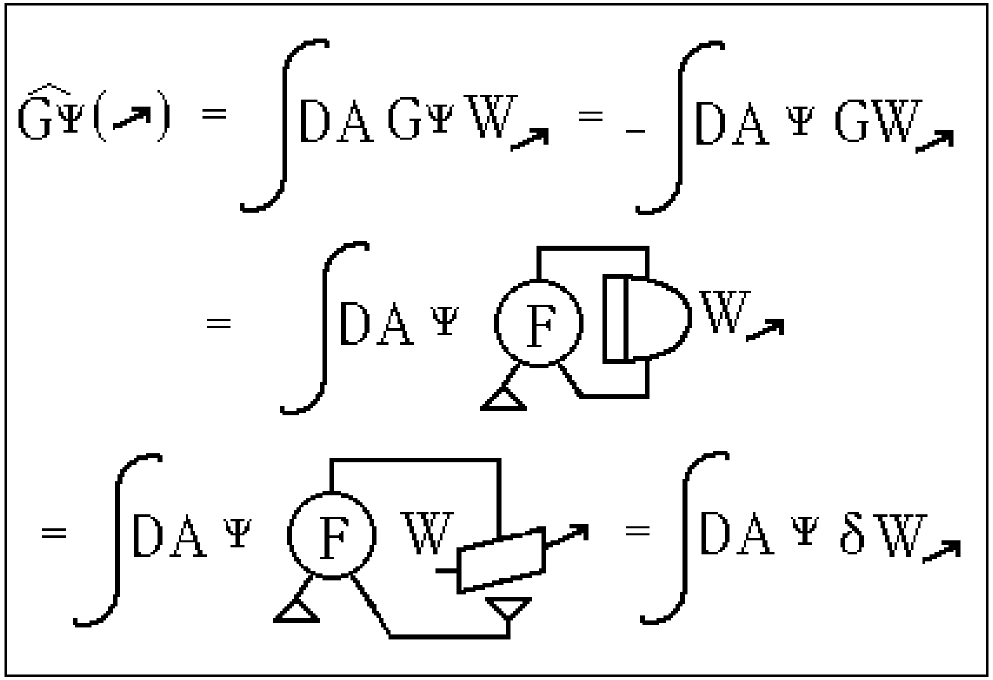
This formula leads directly to the subject of Vassiliev invariants, but we will not discuss that in this talk.

$$\hat{\Psi}(K) = \int DA\, \Psi(A)\, W_K$$

$$\widehat{\Delta\Psi}(K) = \int DA\, \Delta\Psi(A)\, W_K$$
$$= -\int DA\, \Psi(A)\, \Delta W_K$$

The Loop Transform: Start with a function defined on gauge fields. Integrate it against a Wilson loop and get a function defined on knots.
Transform differential operations from the category of functions on gauge fields to the category of functions on knots.



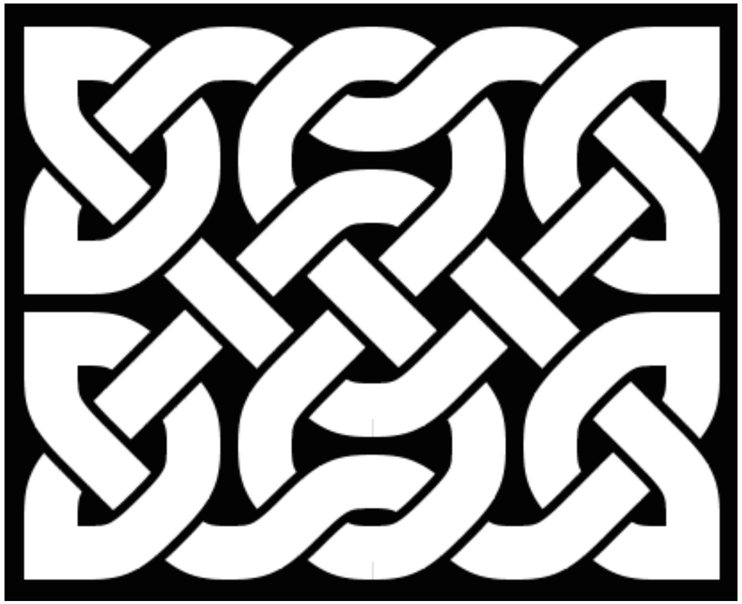


This differential operator occurs in the loop quantum gravity theory of Ashtekar, Rovelli and Smolin.

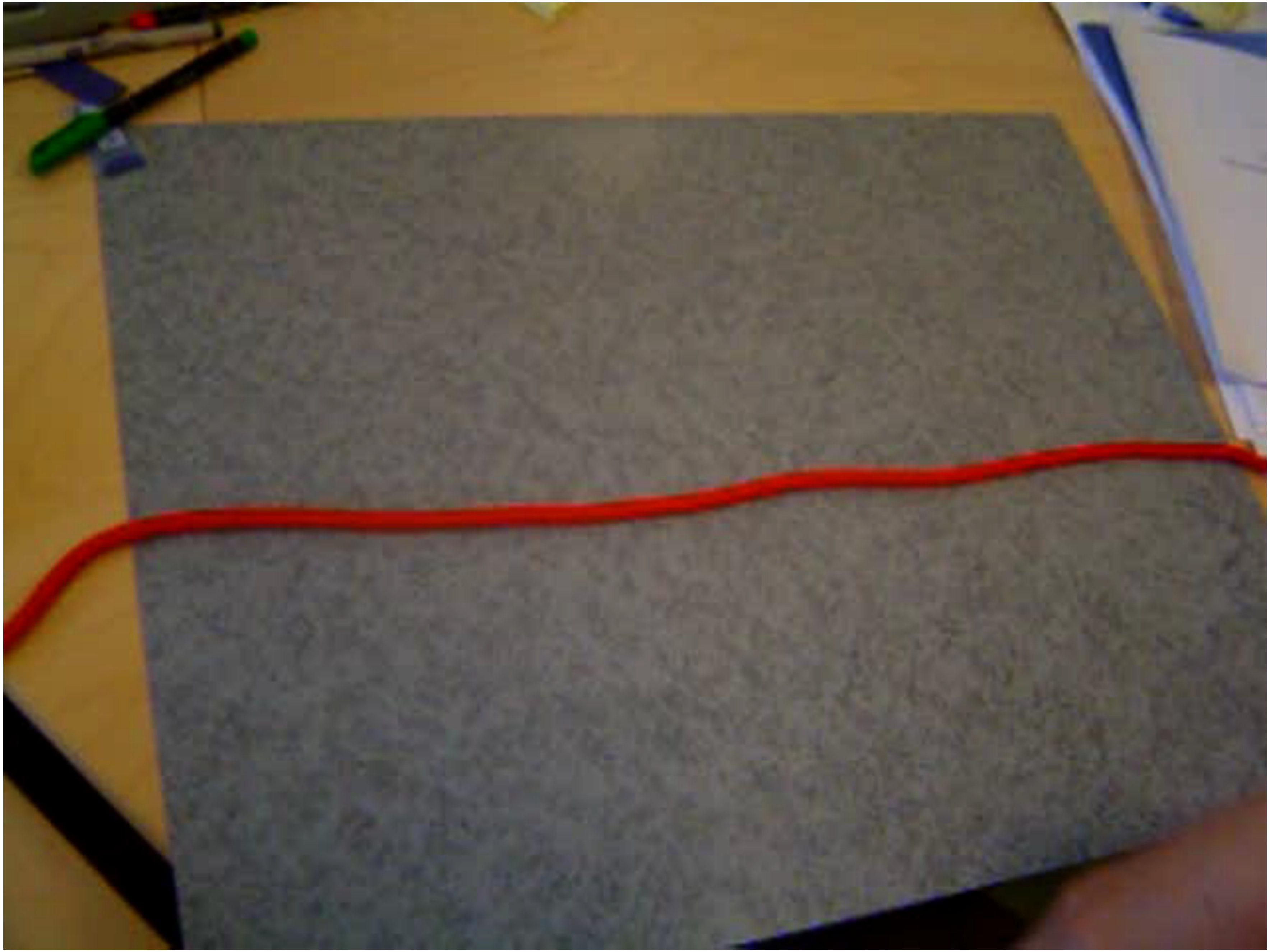
Its transform is the geometric variation of the loop!

We stop here in the discussion of the development of knots and quantum field theory, and other algebraic and physically related methods in knot theory.

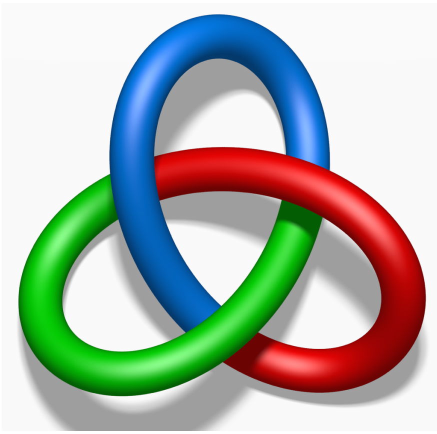
The rest of this talk is about how knots are related to subjects magical, biological and physical.

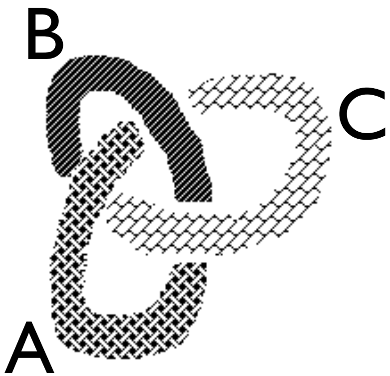



Is it Knotted?



Three-Coloring a Knot



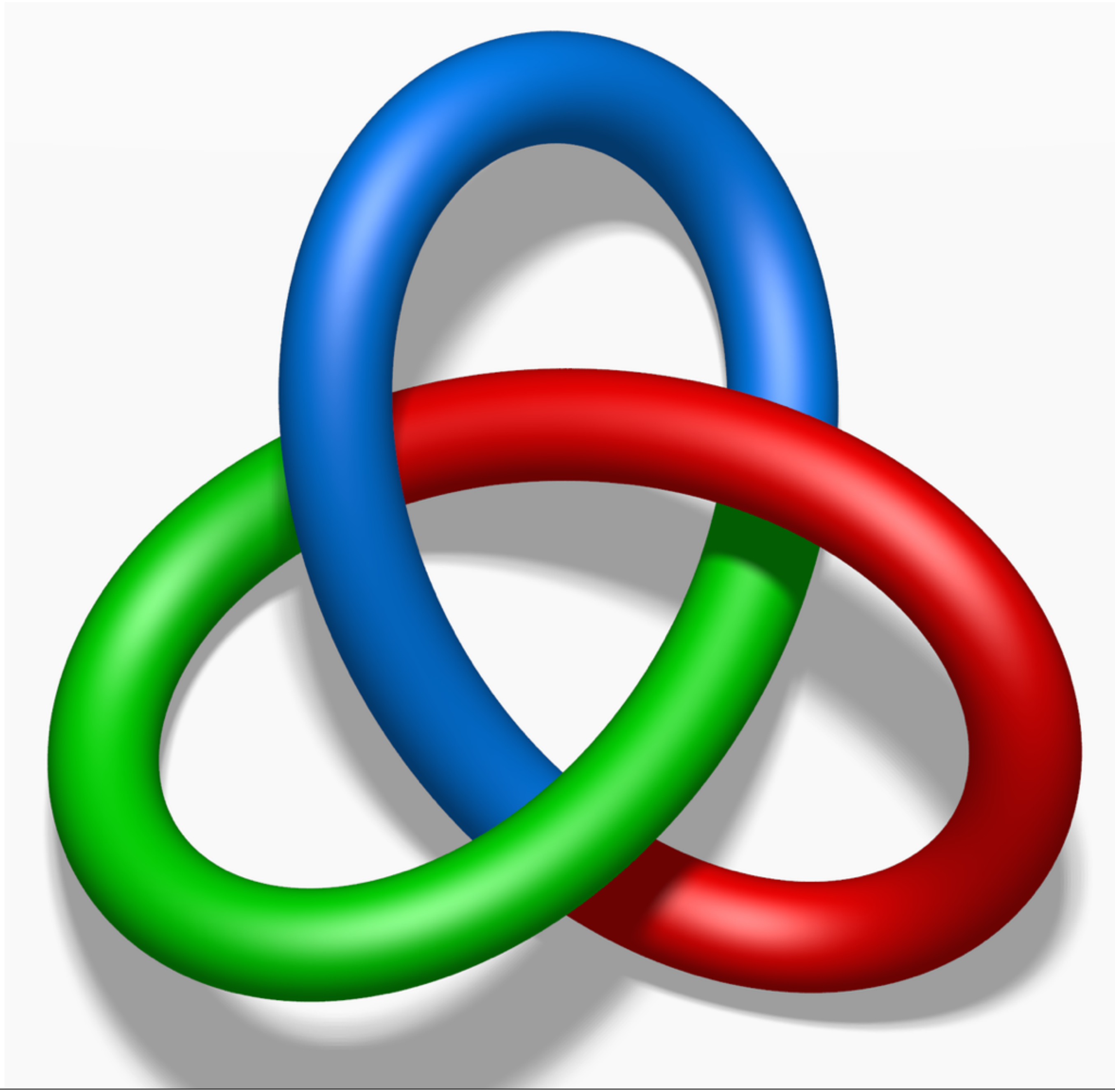


The Rules:

Either three colors at a crossing,

OR

one color at a crossing.



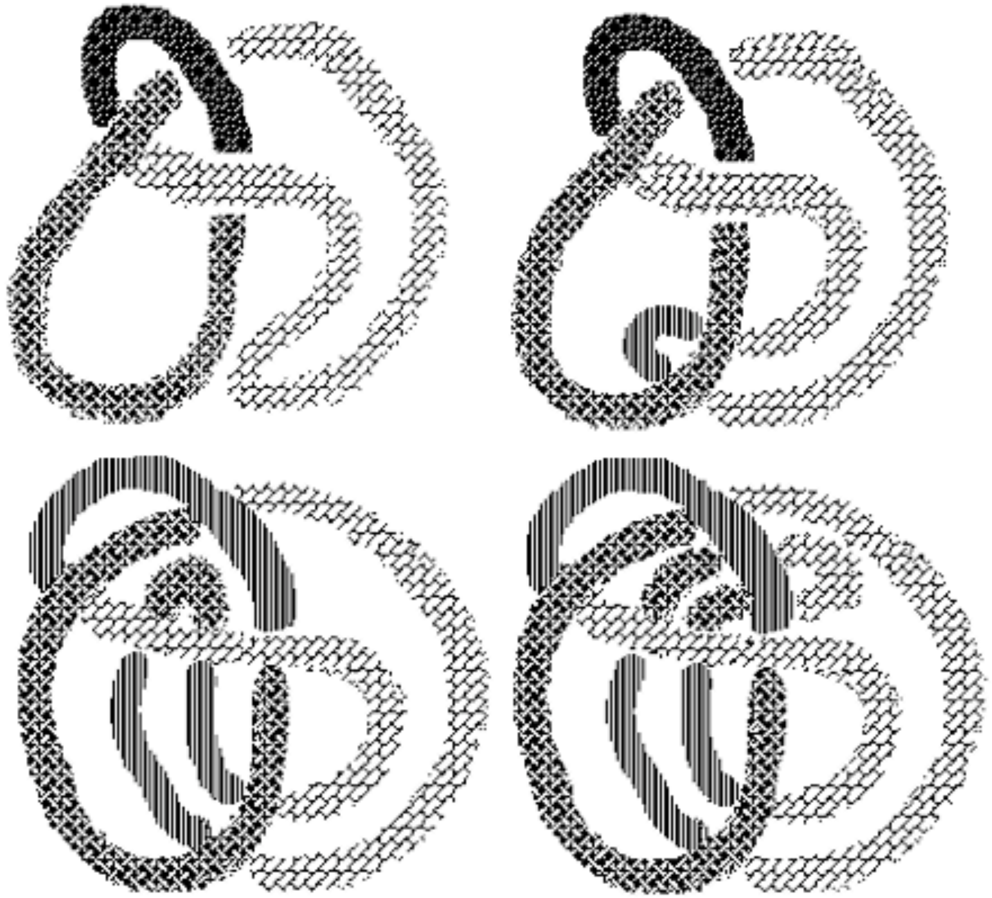


Figure 13 - Inheriting Coloring Under the Type Two Move

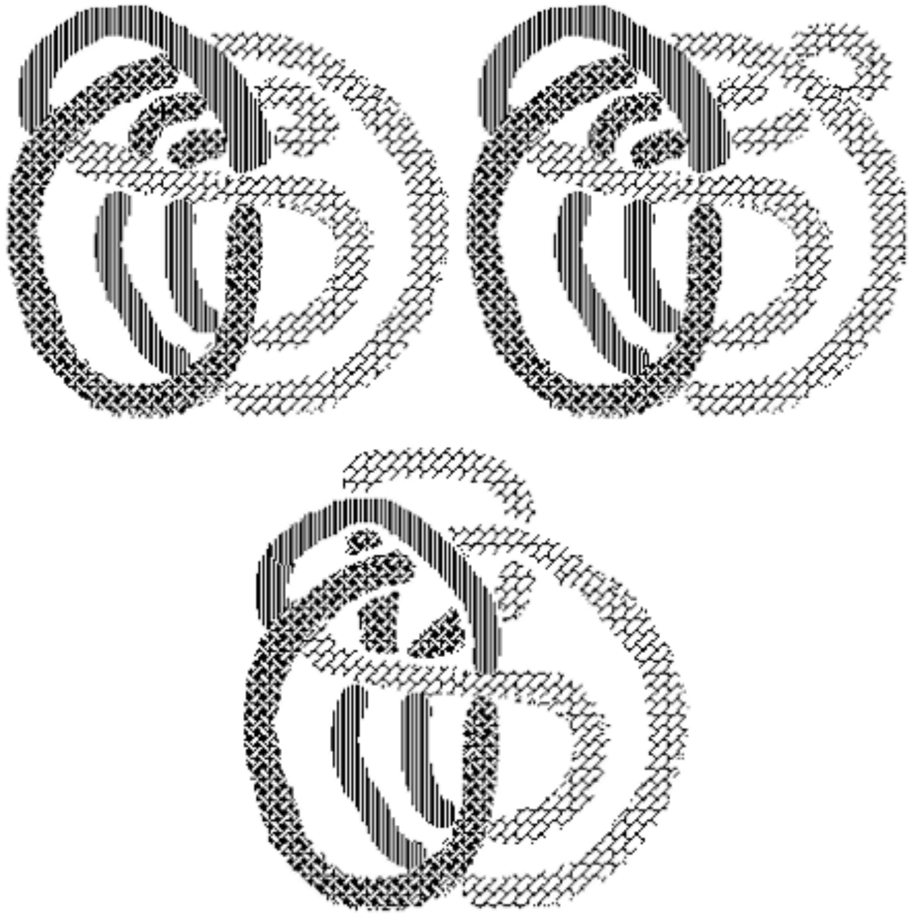


Figure 14 - Coloring Under Type Two and Three Moves

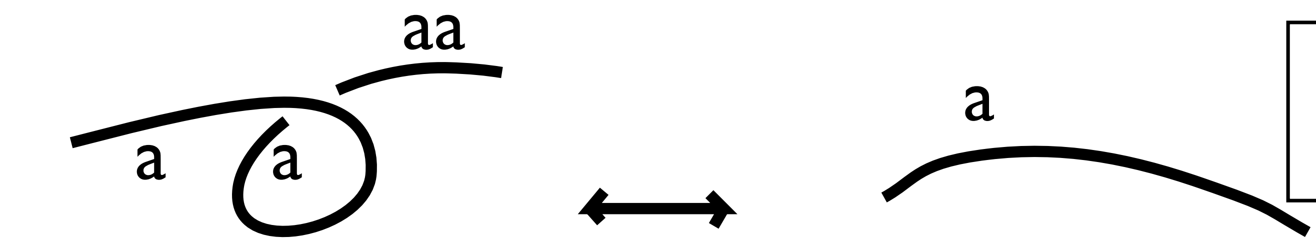
Theorem. The Trefoil Diagram is Knotted.

Proof: Every diagram obtained
from the standard trefoil
by topological changes
uniquely inherits a three-coloring.

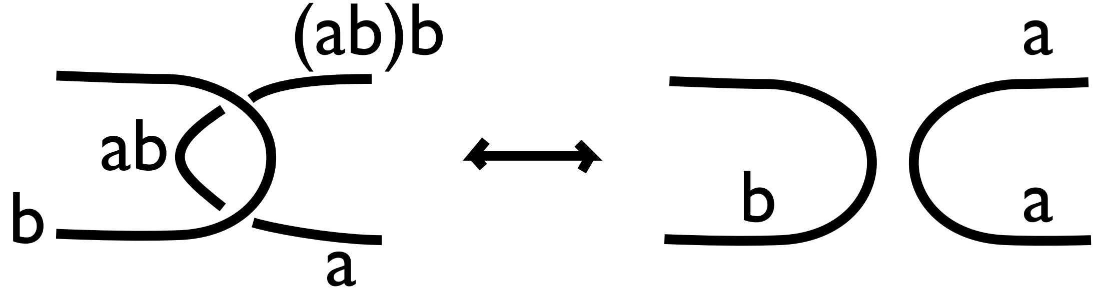
Since an unknot diagram can
have only one color, it follows
that the trefoil is a knot.

Q. E. D.

Exercise: All diagrams topologically related to
the trefoil inherit three colors. No colors are
ever lost.

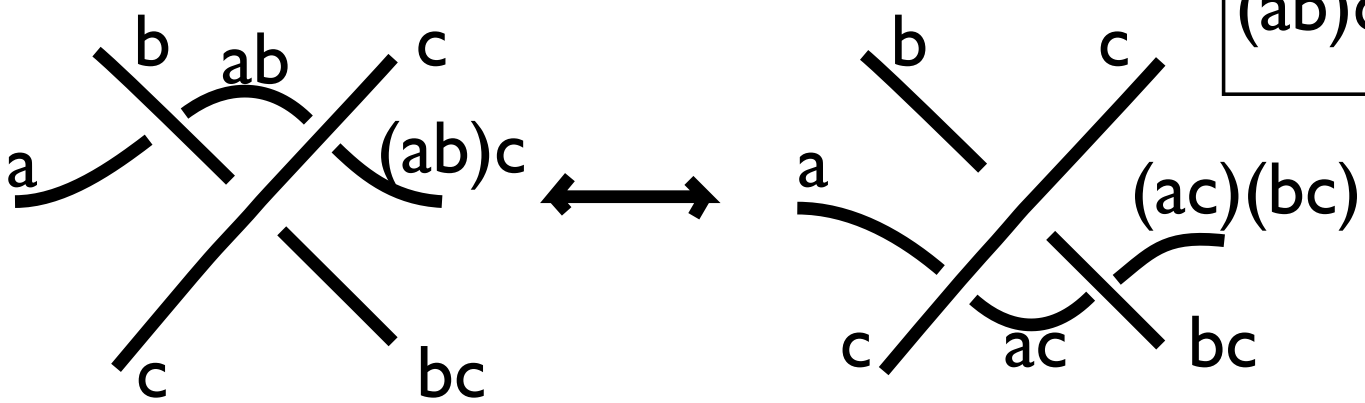


aa= a

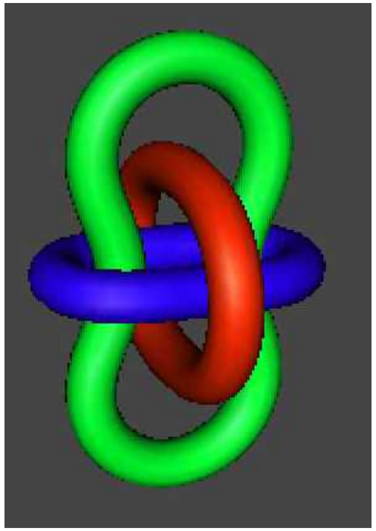


(ab)b = a

(ab)c = (ac)(bc)



Graphs, Diagrams and Reidemeister Moves



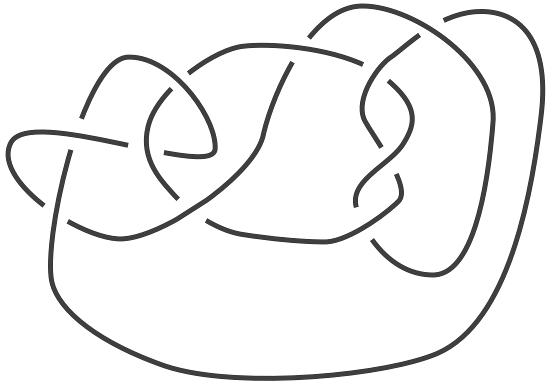
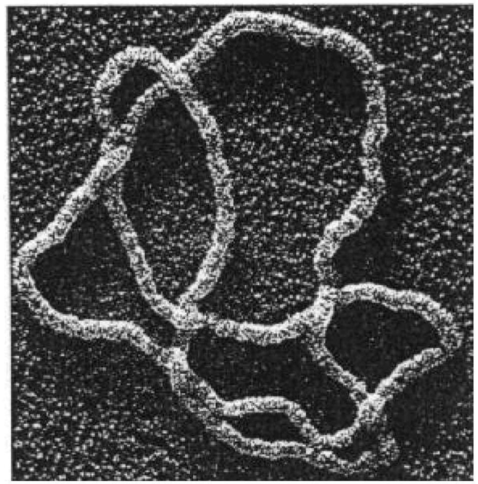


Figure 1 - A knot diagram.



Reidemeister, Alexander and Briggs proved in the 1920's that the three moves suffice for topological equivalence of knots and links.

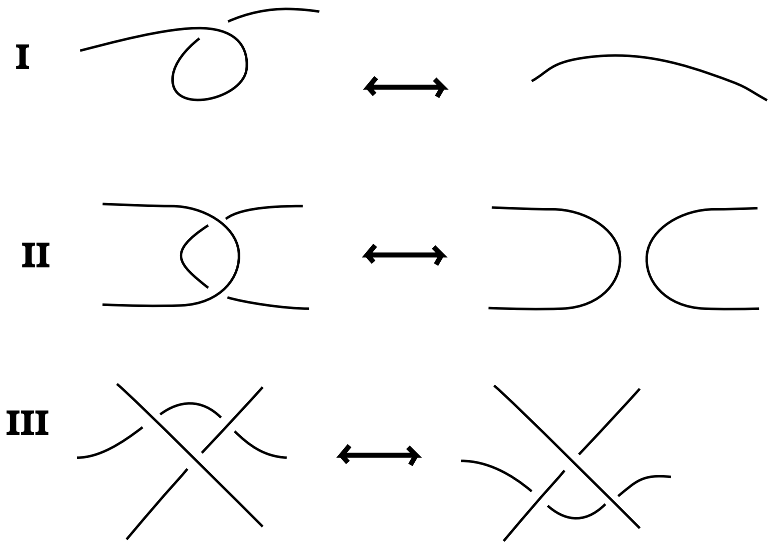
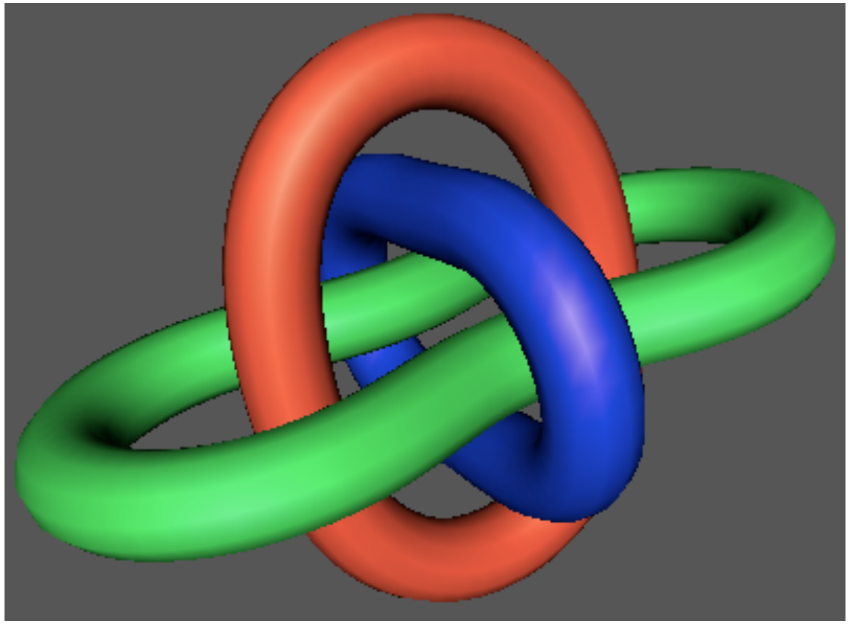


Figure 2 - The Reidemeister Moves.

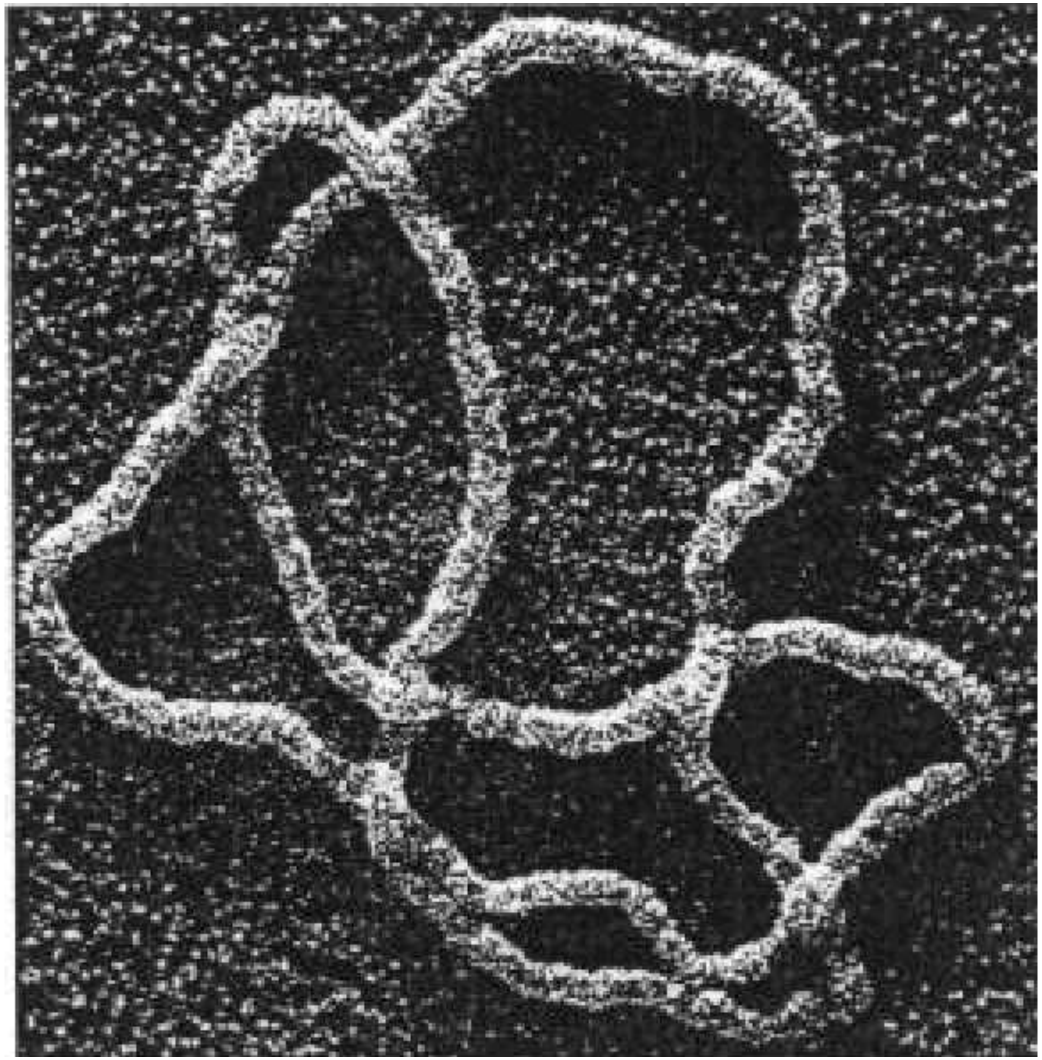
Borromean Rings

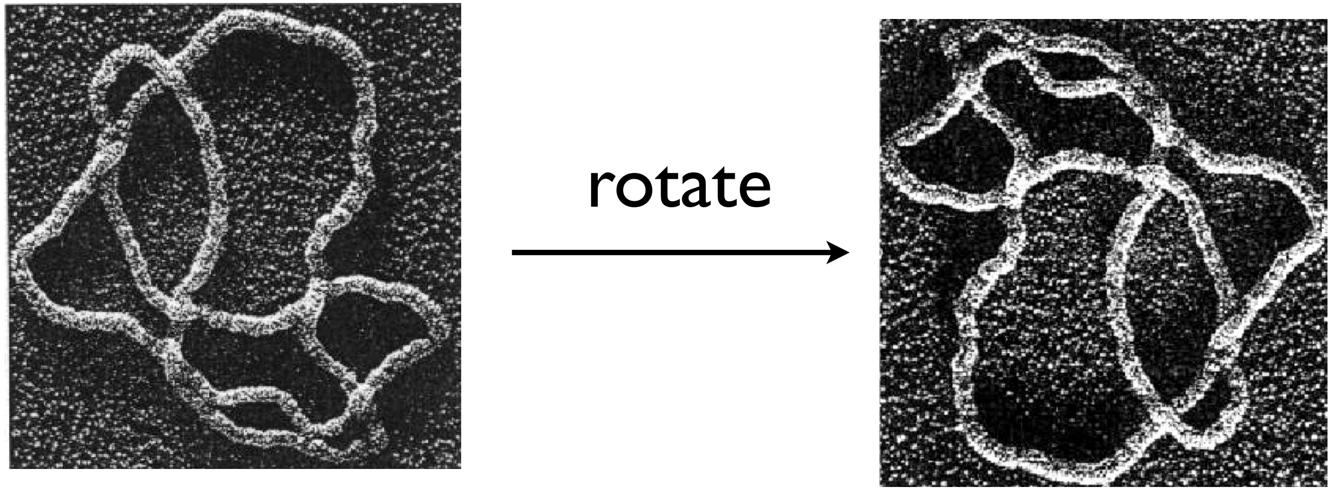


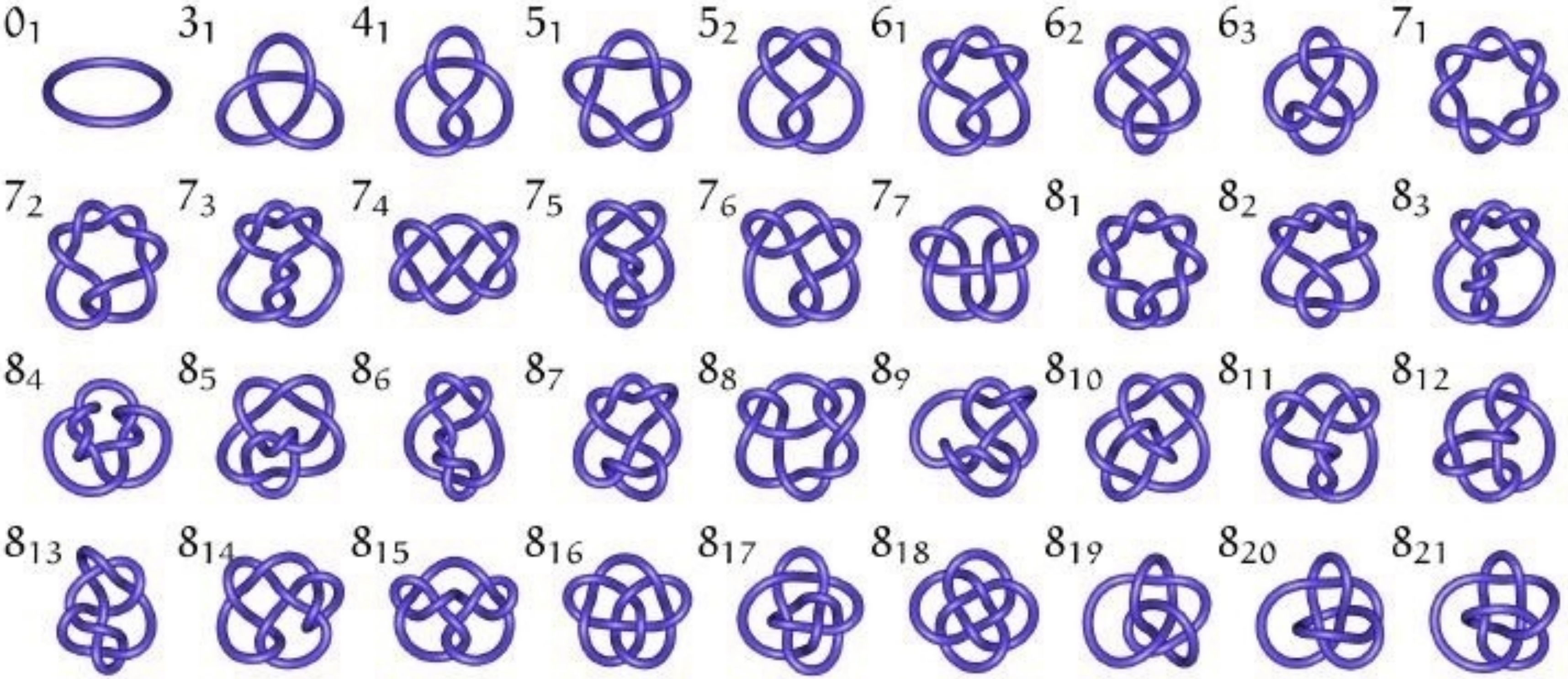
Green surrounds Red.
Red surrounds Blue.
Blue surrounds Green.

This coloring does not obey our rules.
Prove that there is no three coloring
of a diagram of the Rings by our rules.
This implies that the rings are linked!
Why?

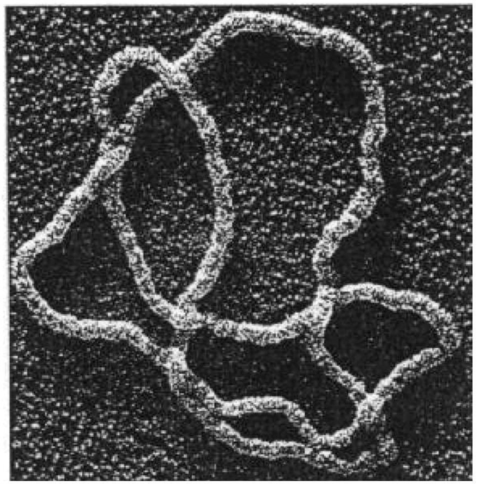
Knotted DNA - Electron Micrograph, Protein Coated DNA Molecule




rotate


0_1 3_1 4_1 5_1 5_2 6_1 6_2 6_3 7_1
7_2 7_3 7_4 7_5 7_6 7_7 8_1 8_2 8_3
8_4 8_5 8_6 8_7 8_8 8_9 8_{10} 8_{11} 8_{12}
8_{13} 8_{14} 8_{15} 8_{16} 8_{17} 8_{18} 8_{19} 8_{20} 8_{21}

DNA Knotting and Recombination



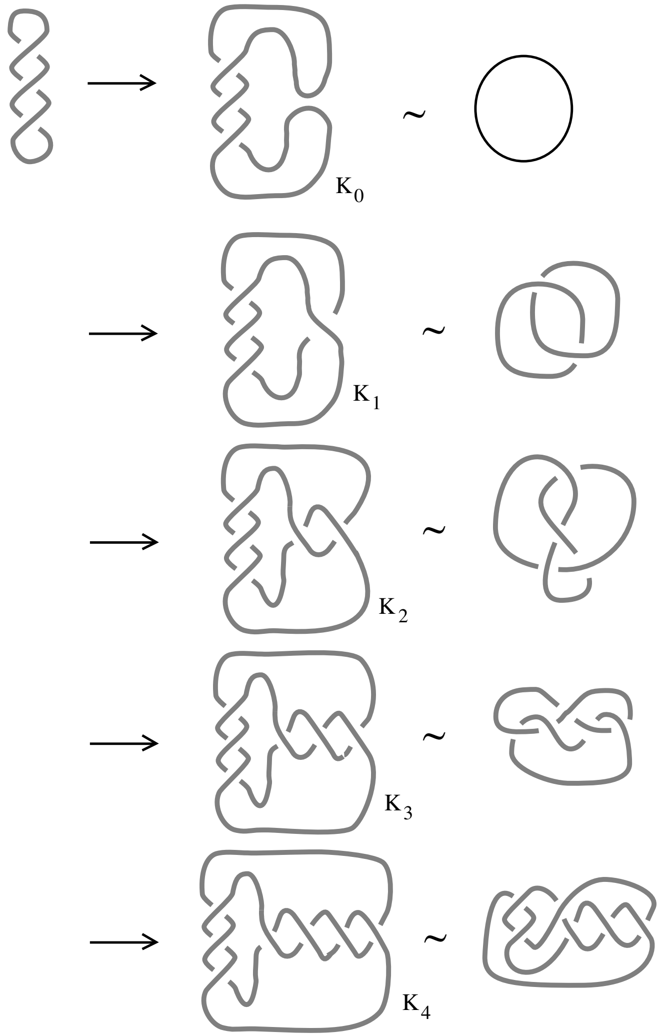
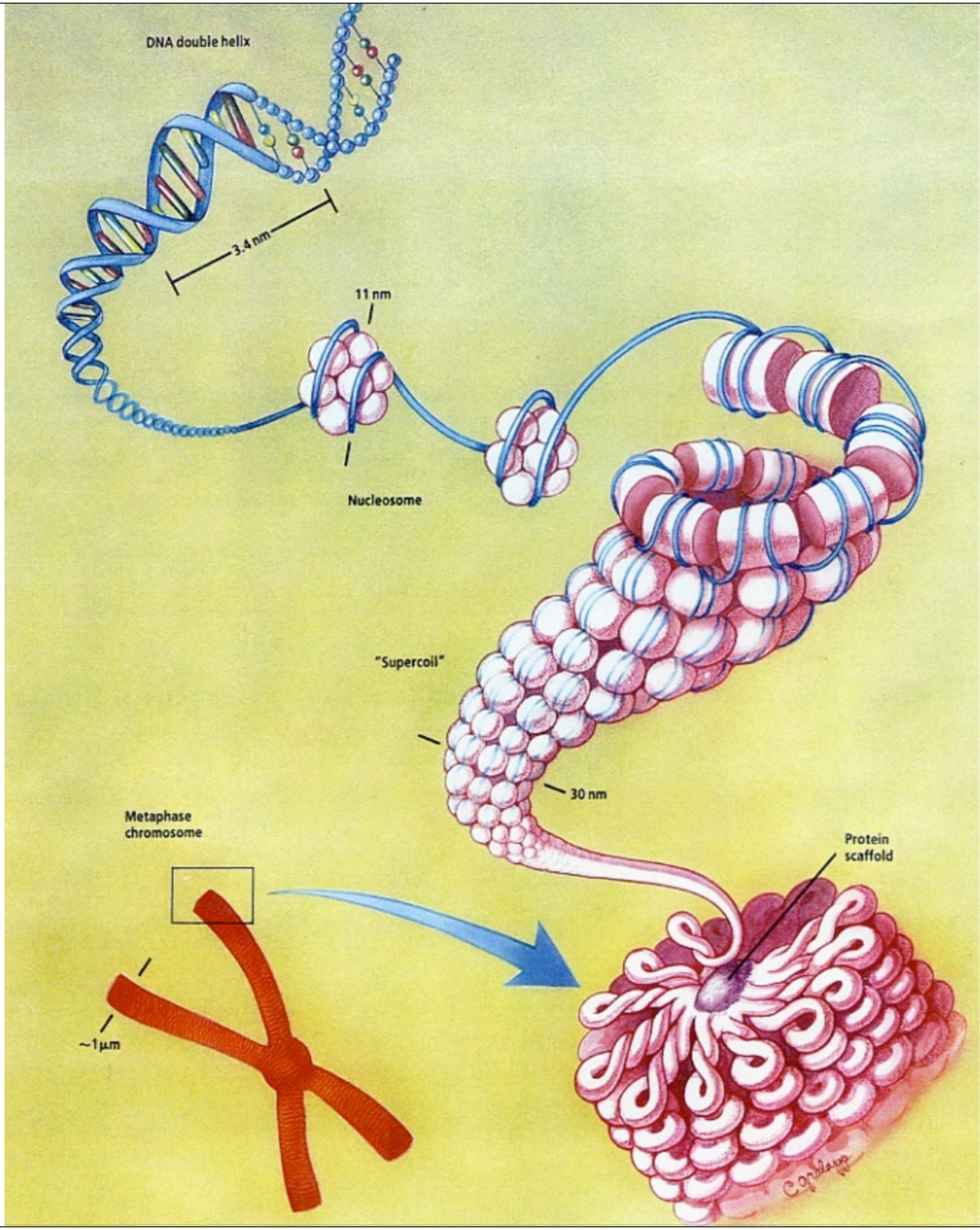
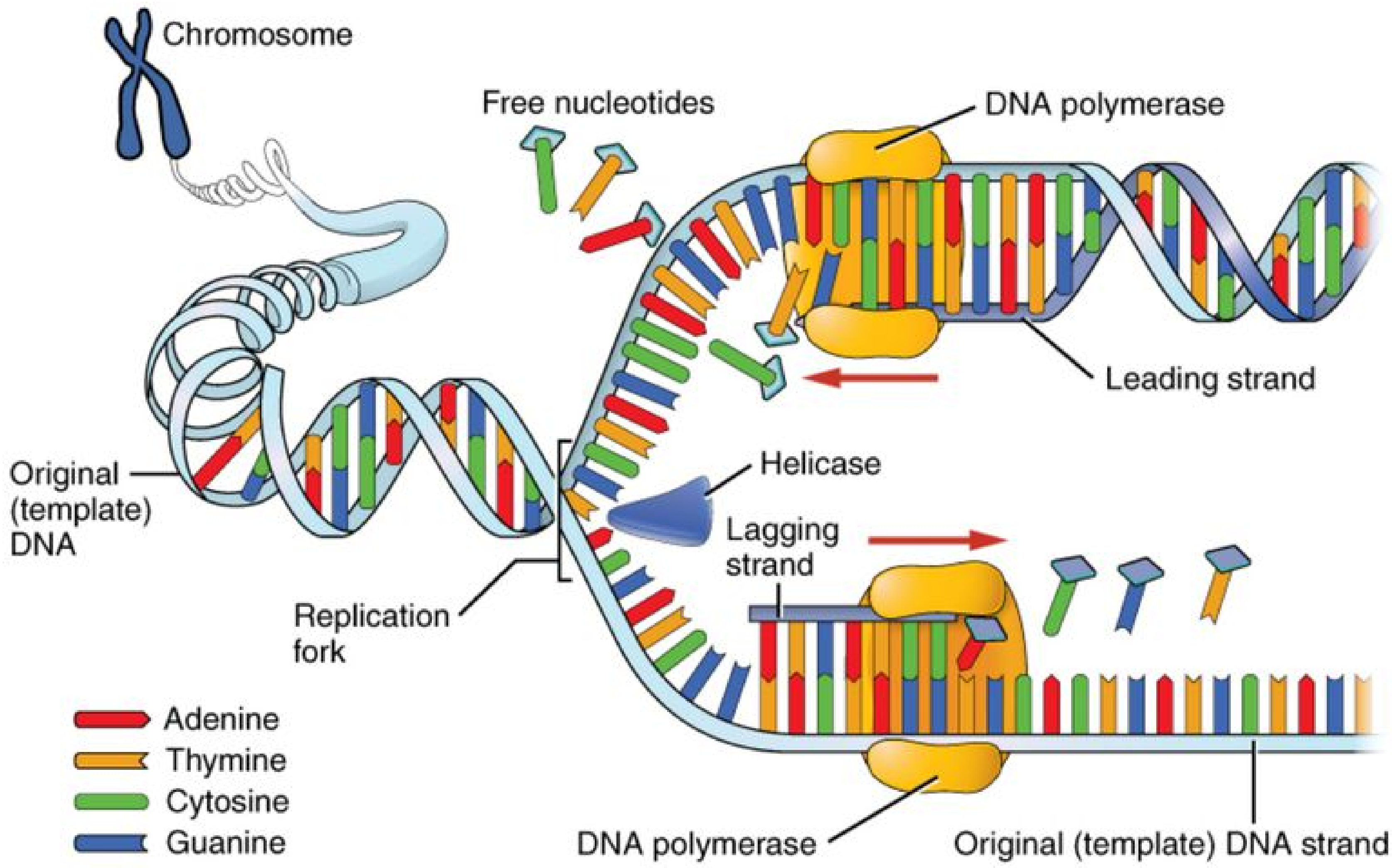
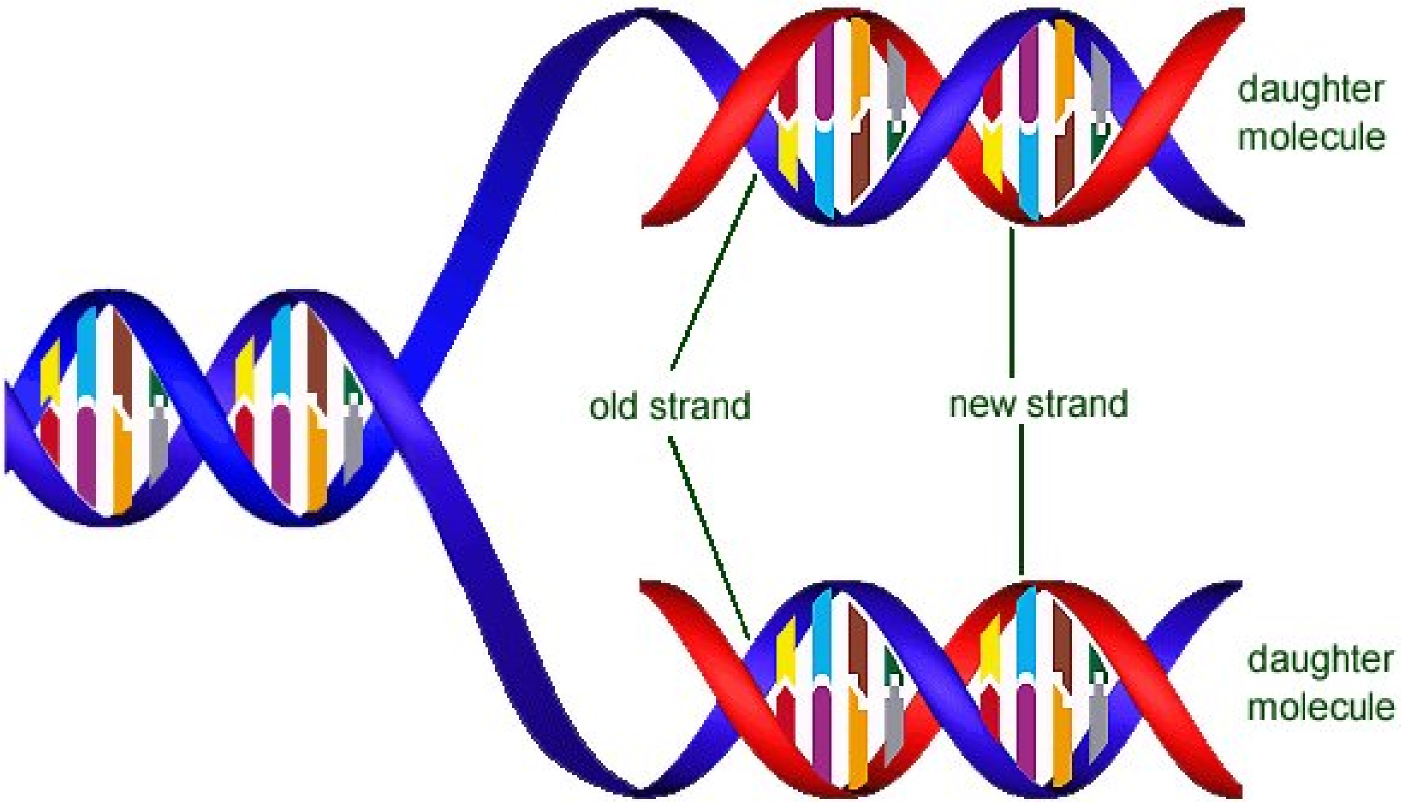


Figure 28 - Processive Recombination with $S = [-1/3]$.


DNA double helix
3.4 nm
11 nm
Nucleosome
"Supercoil"
30 nm
Metaphase chromosome
Protein scaffold
~1µm

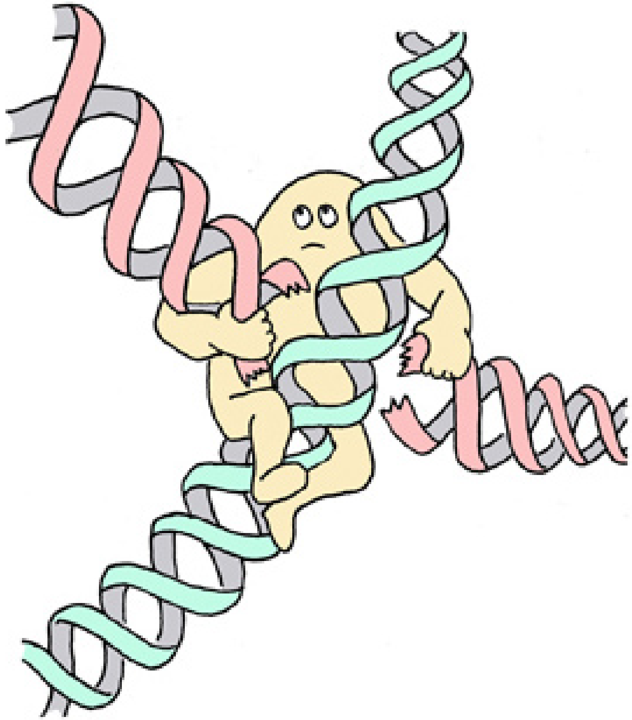


Chromosome
Free nucleotides
DNA polymerase
Leading strand
Helicase
Original
(template)
DNA
Lagging
strand
Replication
fork
Adenine
Thymine
Cytosine
Guanine
DNA polymerase
Original (template) DNA strand



This description of DNA replication ignores all the topological difficulties.

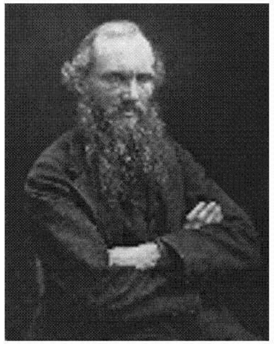
Nature does not ignore the topological problems.
She solves them with Topoisomerase Enzymes that
cut strands to allow passage of strands and the control of linking.



Lord Kelvin's Vortex Atoms

`Idea of knotted strings as fundamental constituents of matter is old`

Lord Kelvin and the
1867 string revolution:

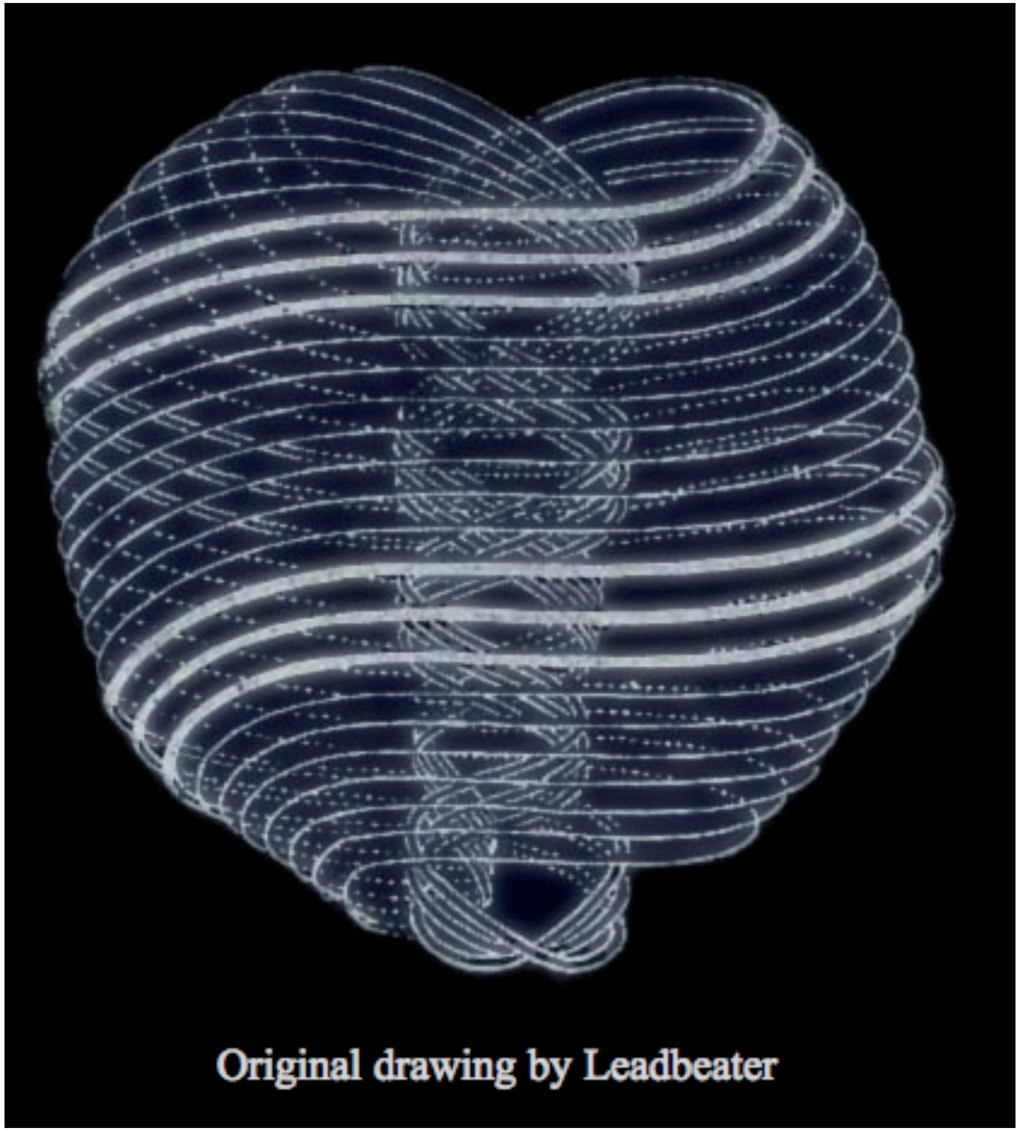


atoms are knotted tubes of aether

- topological stability of knots = stability of matter
- variety of knots = variety of chemical elements

For decades considered as *the* theory of fundamental Matter

`Maxwell:` *Kelvin's theory satisfies more of the conditions than any atom hitherto considered*



From the same period as Kelvin, the “vortex atom” of the visionaries Besant and Leadbeater.

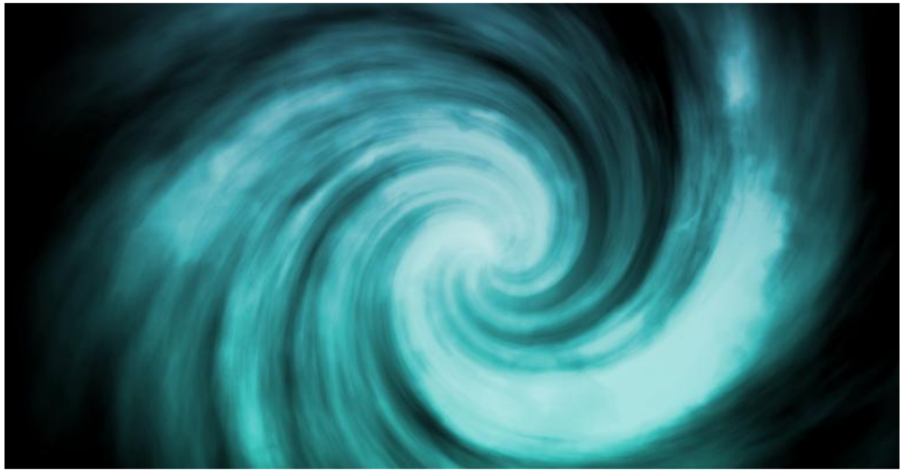
https://en.wikipedia.org/wiki/History_of_knot_theory

Knots were studied from a mathematical viewpoint by Carl Friedrich Gauss, who in 1833 developed the Gauss linking integral for computing the linking number of two knots. His student Johann Benedict Listing, after whom Listing's knot is named, furthered their study.
In 1867 after observing Scottish physicist Peter Tait's experiments involving smoke rings, Thomson came to the idea that atoms were knots of swirling vortices in the æther. Chemical elements would thus correspond to knots and links. Tait's experiments were inspired by a paper of Helmholtz's on vortex-rings in incompressible fluids. Thomson and Tait believed that an understanding and classification of all possible knots would explain why atoms absorb and emit light at only the discrete wavelengths that they do. For example, Thomson thought that sodium could be the Hopf link due to its two lines of spectra.[1]
Tait subsequently began listing unique knots in the belief that he was creating a table of elements. He formulated what are now known as the Tait conjectures on alternating knots. (The conjectures were proved in the 1990s.) Tait's knot tables were subsequently improved upon by C. N. Little and Thomas Kirkman.[1]:6
James Clerk Maxwell, a colleague and friend of Thomson's and Tait's, also developed a strong interest in knots. Maxwell studied Listing's work on knots. He re-interpreted Gauss' linking integral in terms of electromagnetic theory. In his formulation, the integral represented the work done by a charged particle moving along one component of the link under the influence of the magnetic field generated by an electric current along the other component. Maxwell also continued the study of smoke rings by considering three interacting rings.
When the *luminiferous æther* was not detected in the Michelson–Morley experiment, vortex theory became completely obsolete, and

<-: [[knot theory ceased to be of great scientific interest]]. :->

Modern physics demonstrates that the discrete wavelengths depend on quantum energy levels.

Knotted Vortices



Creation and Dynamics of Knotted Vortices

Dustin Kleckner[1] & William T. M. Irvine[1]

[1] *James Franck Institute, Department of Physics, The University of Chicago, Chicago, Illinois 60637, USA*

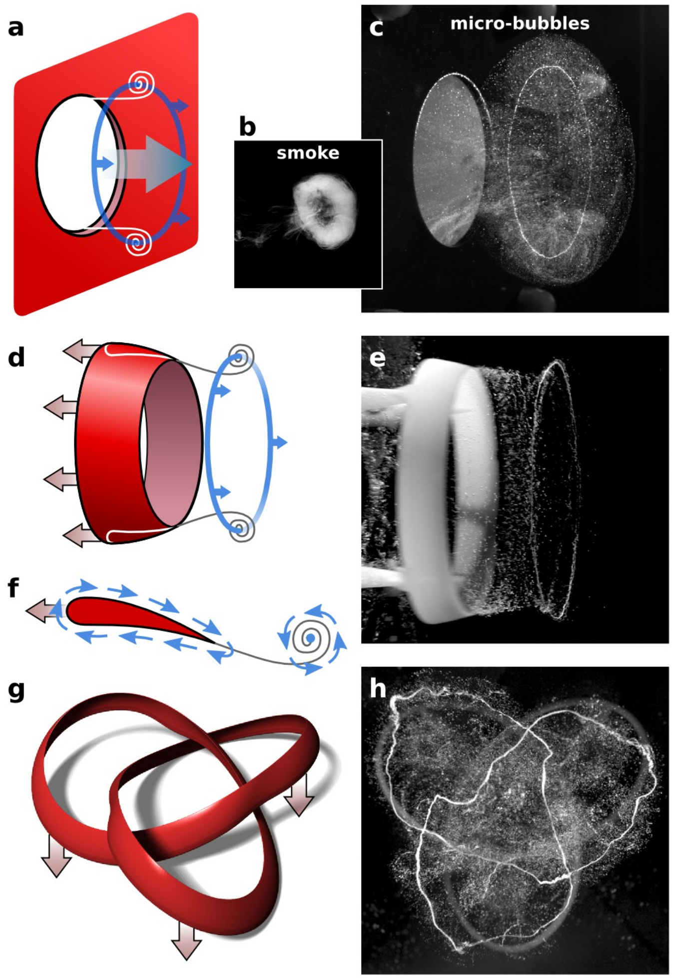
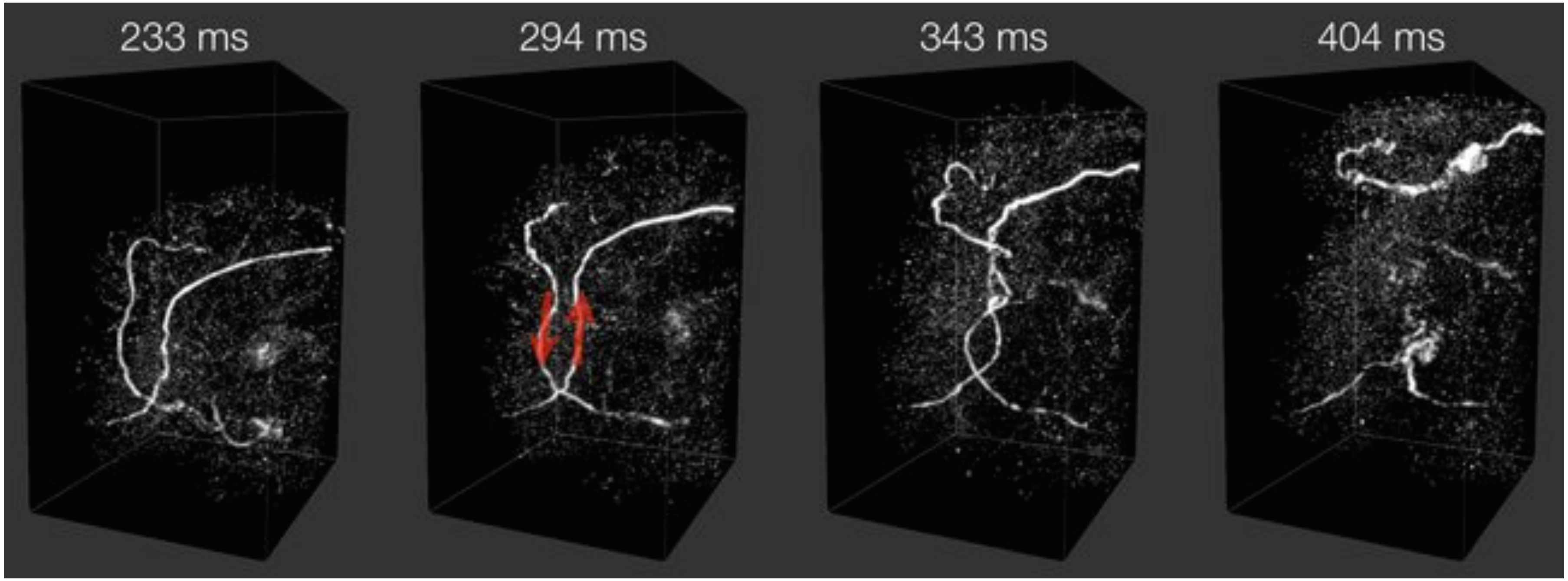
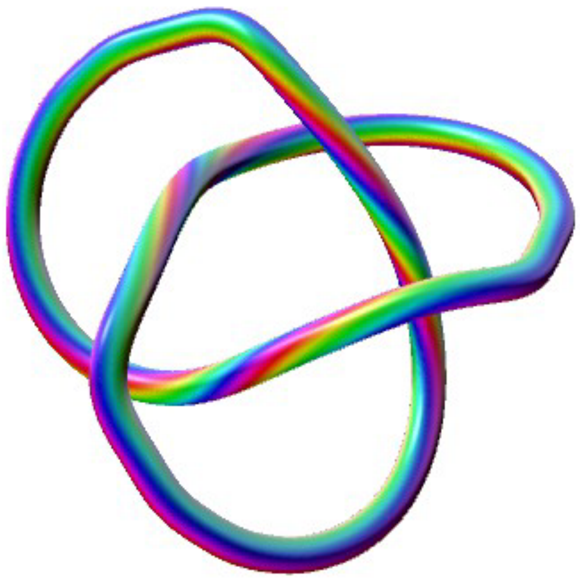


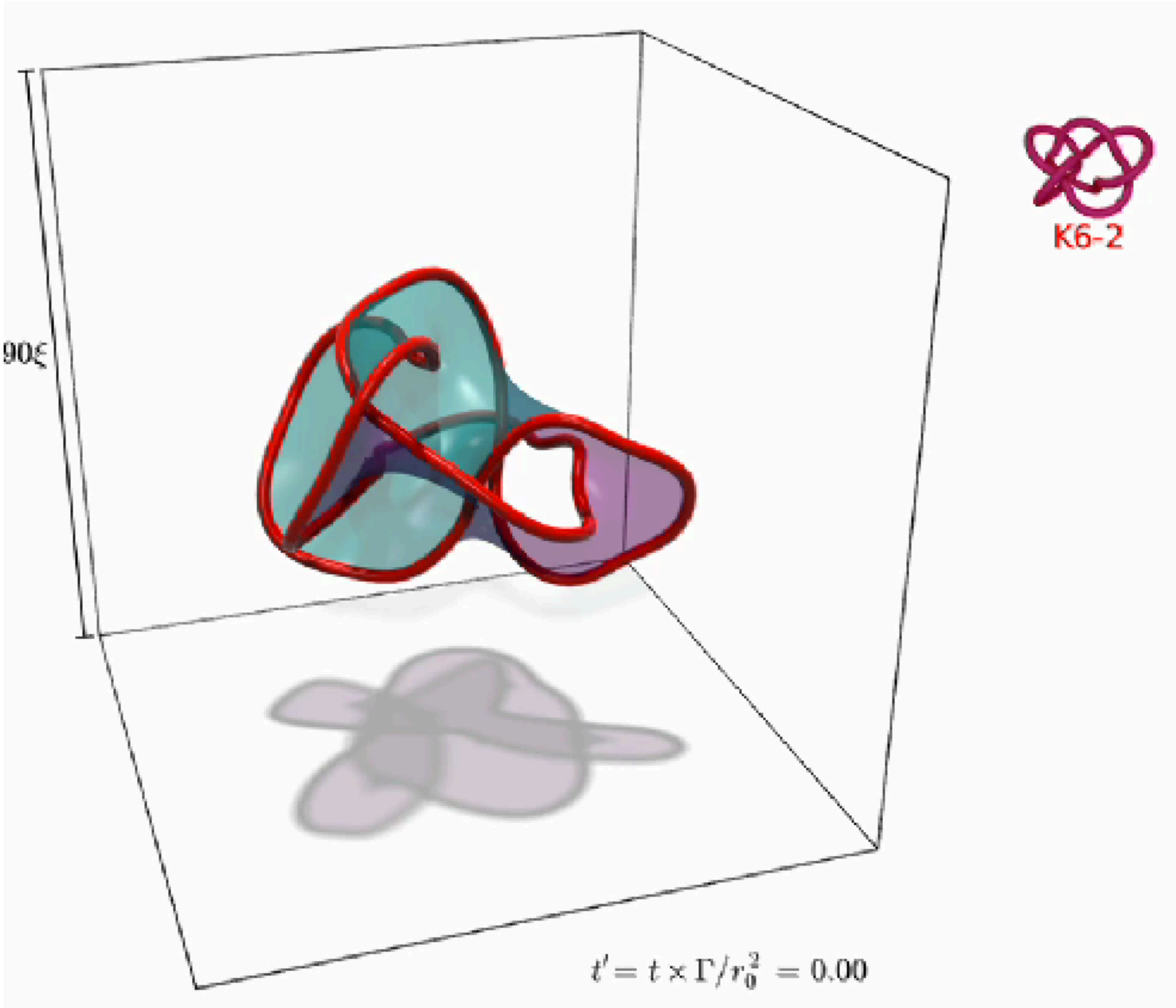
FIG. 1. The creation of vortices with designed shape and topology. **a,** The conventional method for generating a vortex ring, in which a burst of fluid is forced through an orifice. **b,** A vortex ring in air visualized with smoke. **c,** A vortex ring in water traced by a line of ultra-fine gas bubbles, which show finer core details than smoke or dye. **d-e,** A vortex ring can alternatively be generated as the starting vortex of a suddenly accelerated, specially designed wing. For a wing with the trailing edge angled inward, the starting vortex moves in the *opposite* of the direction of wing motion **f,** The starting vortex is a result of conservation of circulation – the bound circulation around a wing is balanced by the counter-rotating starting vortex. **g,** A rendering of a wing tied into a knot, used to generate a knotted vortex, shown in **h**.

Vortex Reconnection

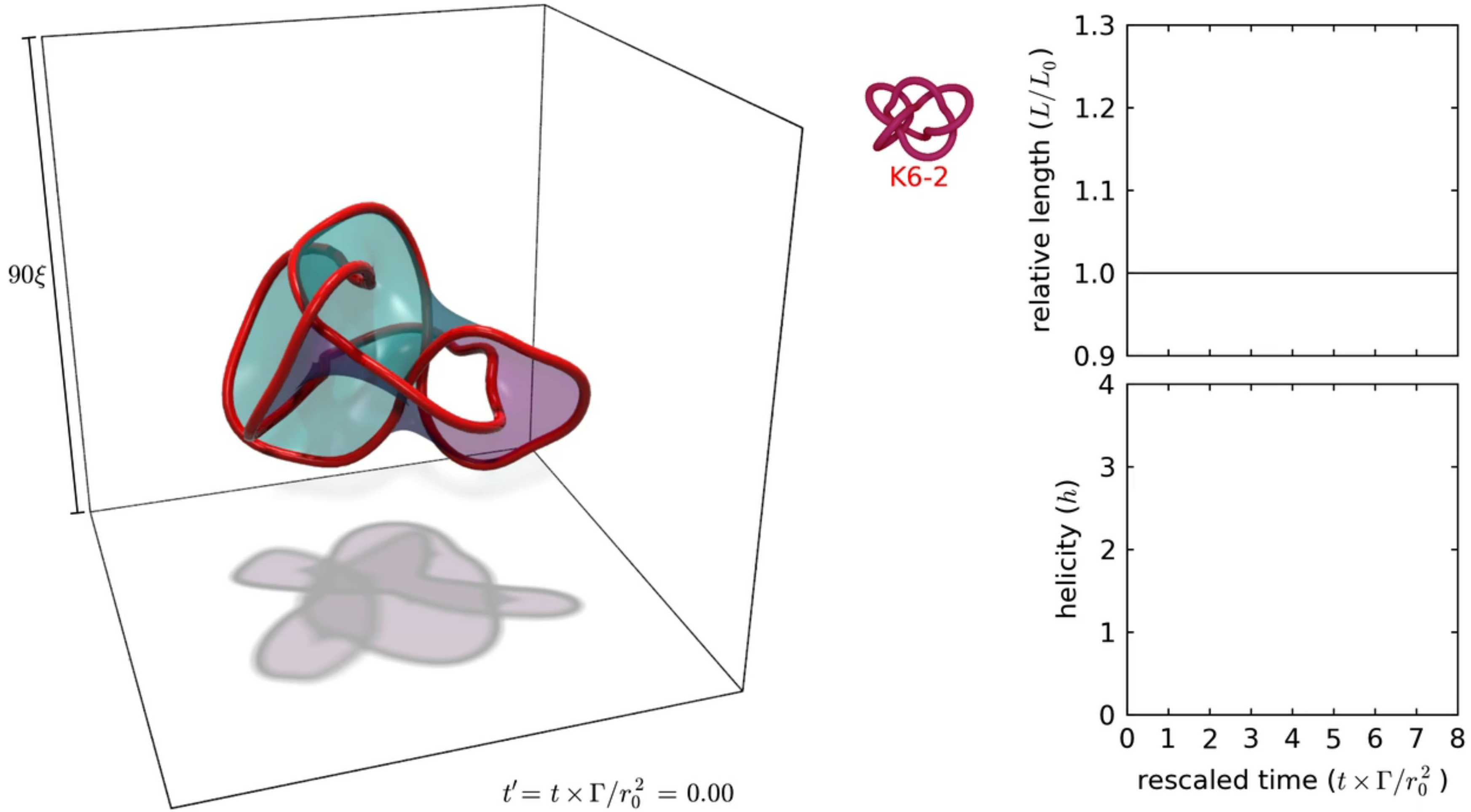




Gross–Pitaevskii evolution by Irvine and Kleckner



90ξ
K6-2
$t' = t \times \Gamma / r_0^2 = 0.00$



90ξ
$t' = t \times \Gamma / r_0^2 = 0.00$
K6-2
relative length (L/L_0)
1.3
1.2
1.1
1.0
0.9
helicity (h)
4
3
2
1
0
0 1 2 3 4 5 6 7 8
rescaled time ($t \times \Gamma / r_0^2$)

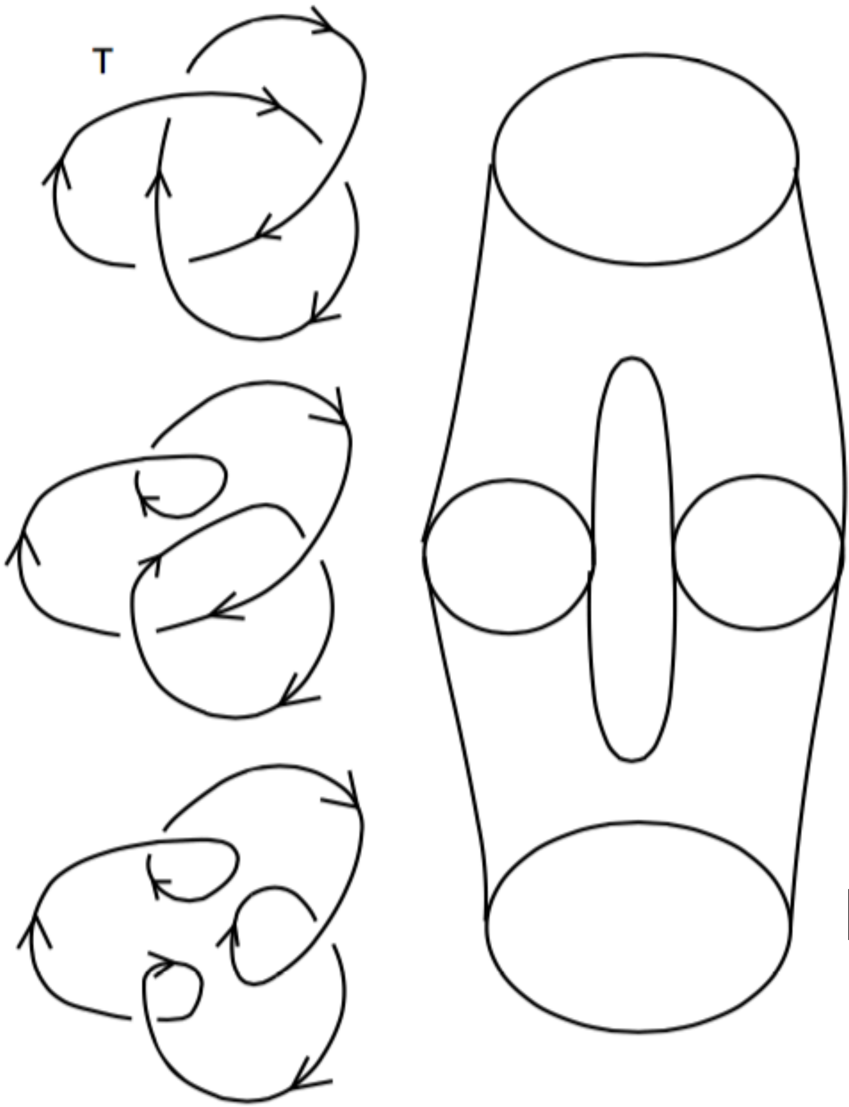


Figure 4: **A Genus One Recombination Sequence**

The WorldLine
of a reconnecting
knot is a surface
in 4-Space.

We can examine the
genus of the surface
(the number of holes).

Each hole corresponds to
two reconnections.

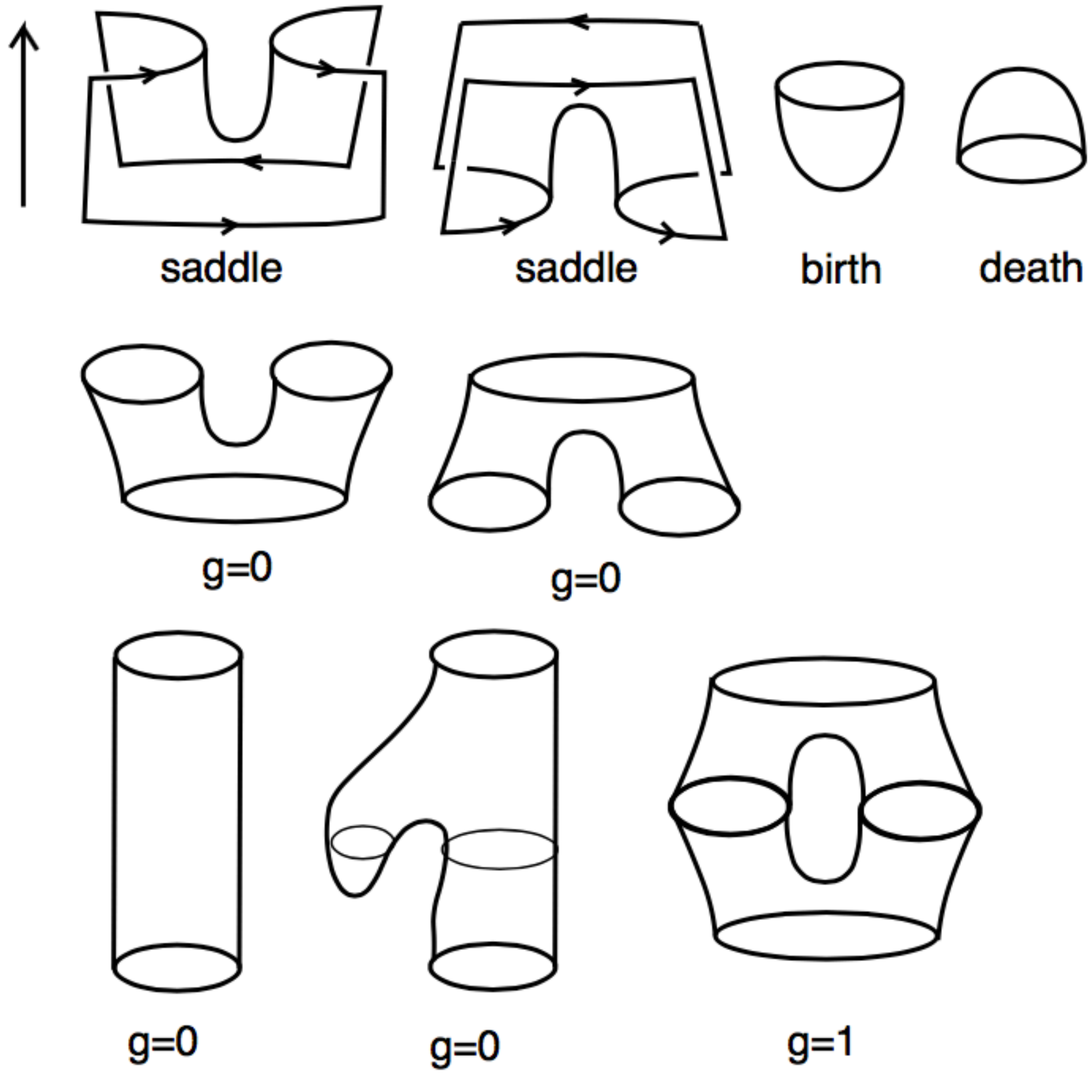


Figure 3: **Saddles, Births and Deaths**

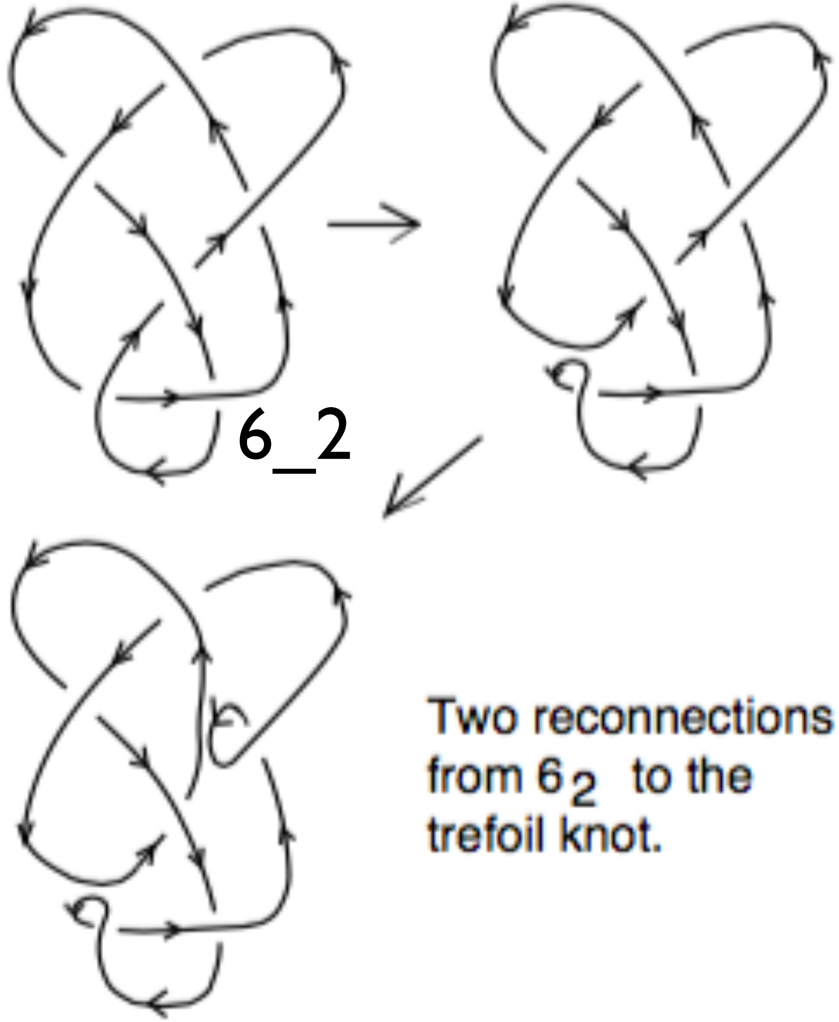


Figure 6: 6_2 has two reconnections to the Trefoil.

Two reconnections from 6_2 to the trefoil and two more to the unknot.

This is a physical sequence as in the simulation.

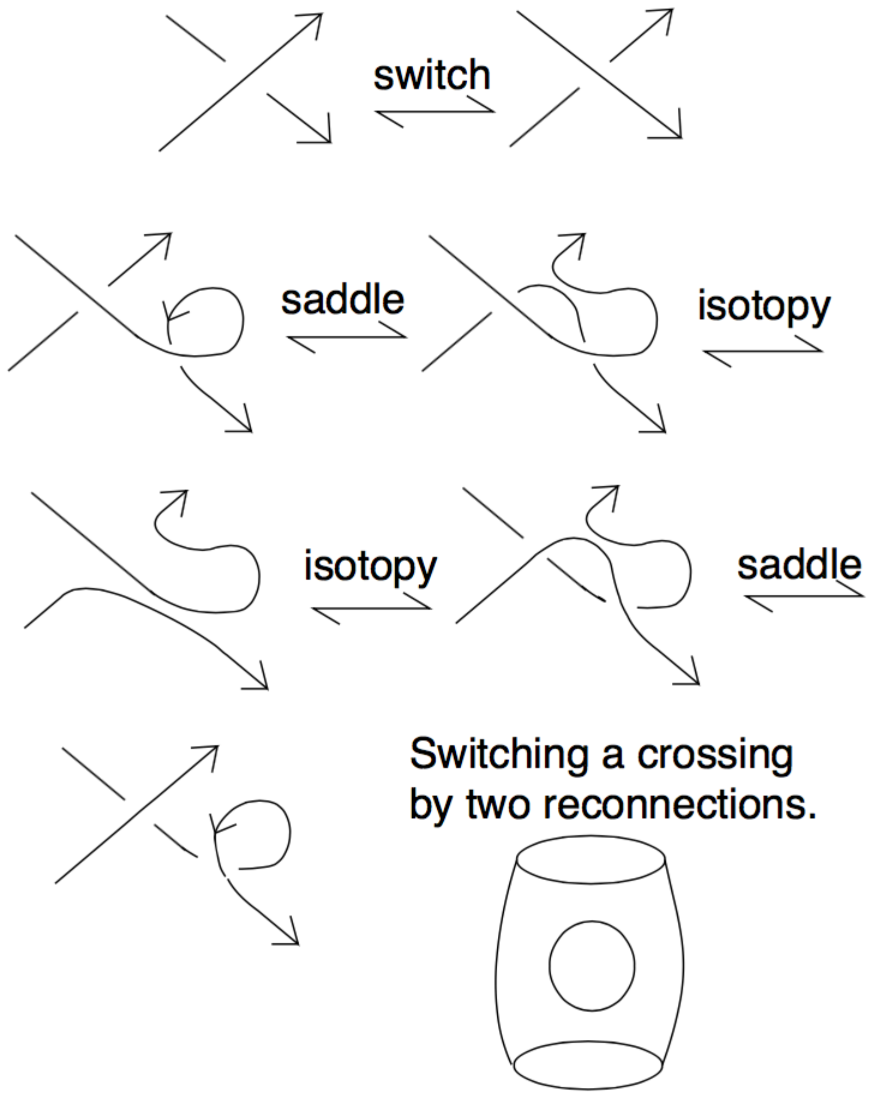
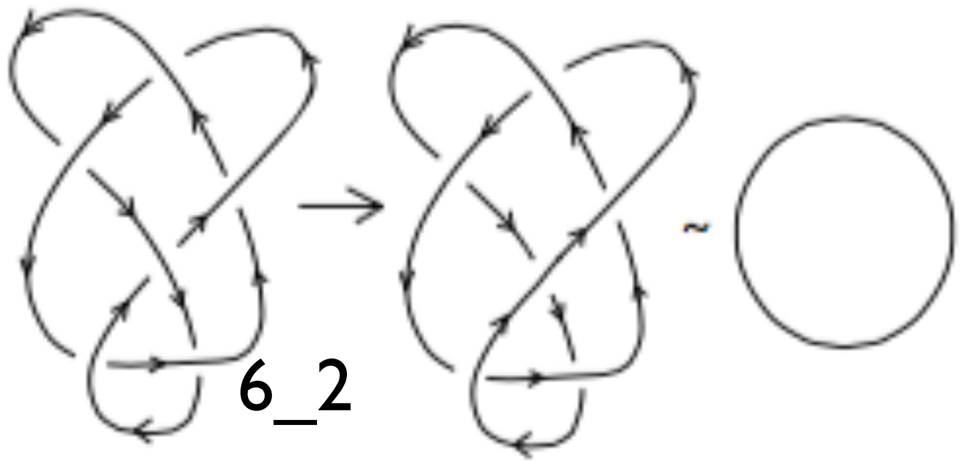


Figure 5: **Crossing Switch in Two Reconnections**

A crossing switch can be accomplished with two reconnections.



One switch
from 6_2 to the unknot.
Hence two reconnections
from 6_2 to the
unknot.

Figure 7: **By Switch 6_2 has two reconnections to the Unknot.**

One crossing
switch takes
6_2 to the unknot.

We have seen that a physical sequence of reconnections takes 6_2 to the unknot in four steps. But in principle this can be done in two steps. We expect this sort of difference between physical pathways of reconnection and available topological pathways.

This phenomenon is under investigation! (LK and William Irvine)

Lower Bounds for the Number of Needed Reconnections for a Knotted Vortex.
(LK and William Irvine)

Let R(K) be the least number of reconnections needed to transform the knot K to a collection of unlinked circles.

There is a classical invariant of knots and links called the Signature(K).

e.g. Signature(Trefoil) = -2 and Signature(6_2) = -2 also.

Theorem. |Signature(K)| <= R(K).

Proof.

2(4-genus (K)) <= R(K)

(each hole is at least two reconnections)

|Signature(K)| <= 2(4-genus(K))

(a fact of classical knot theory)

Therefore |Signature(K)| <= R(K).

Q.E.D

About the Signature and Seifert Pairing

Let F be a spanning surface in three-space for a knot link K. We define a linking number measure of the embedding of F via the *Seifert paring* defined as an assymetric bilinear form

$$\Theta : H_1(F) \times H_1(F) \longrightarrow \mathcal{Z},$$

given by the formula

$$\Theta(x, y) = Lk(x^*, y)$$

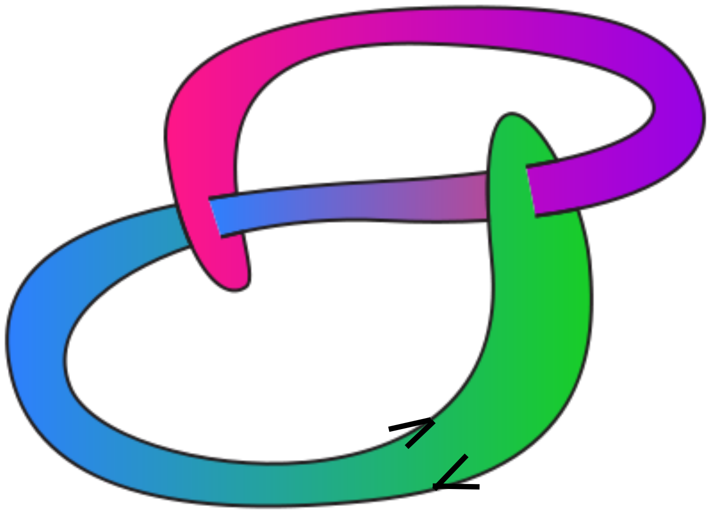
θ	a	b
a	−1	1
b	0	−1

Figure 8: **Seifert Pairing for Surface Bounding Trefoil Knot**

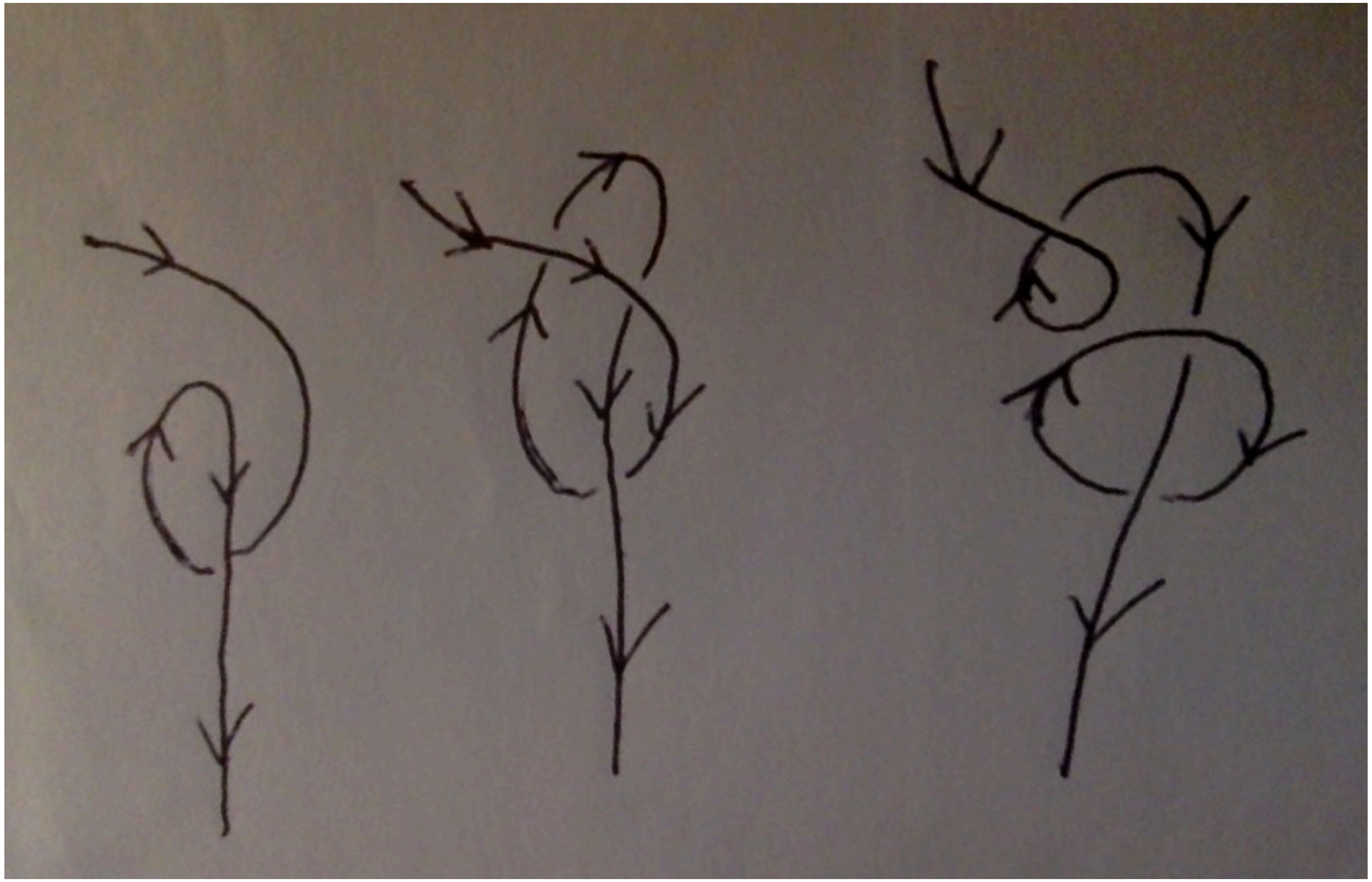
Signature is computed from the (symmetrized) Seifert pairing.

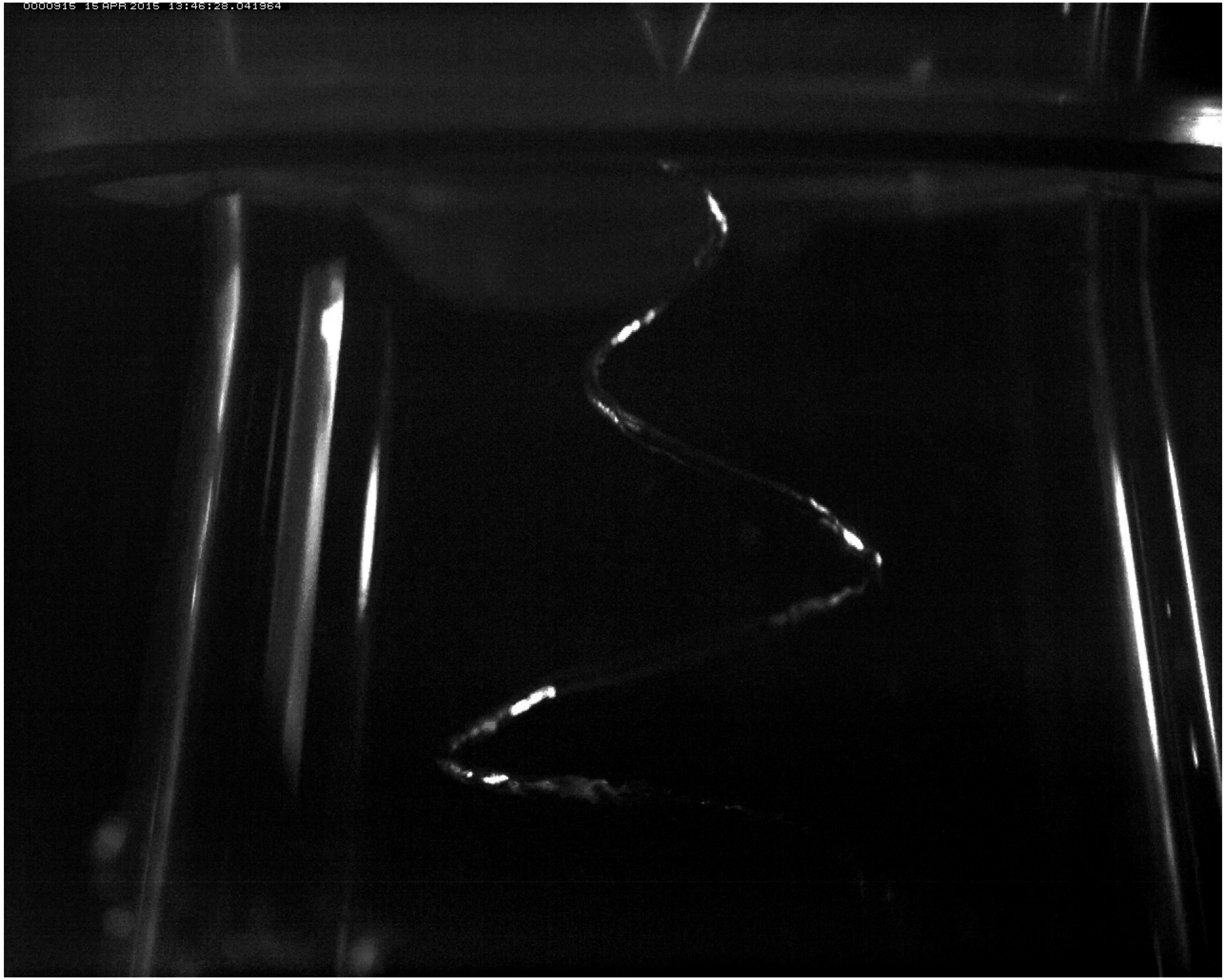
Not all reconnections lead to production of genus.

Consider a slice knot like the one below.
One reconnection is needed. No genus is produced.

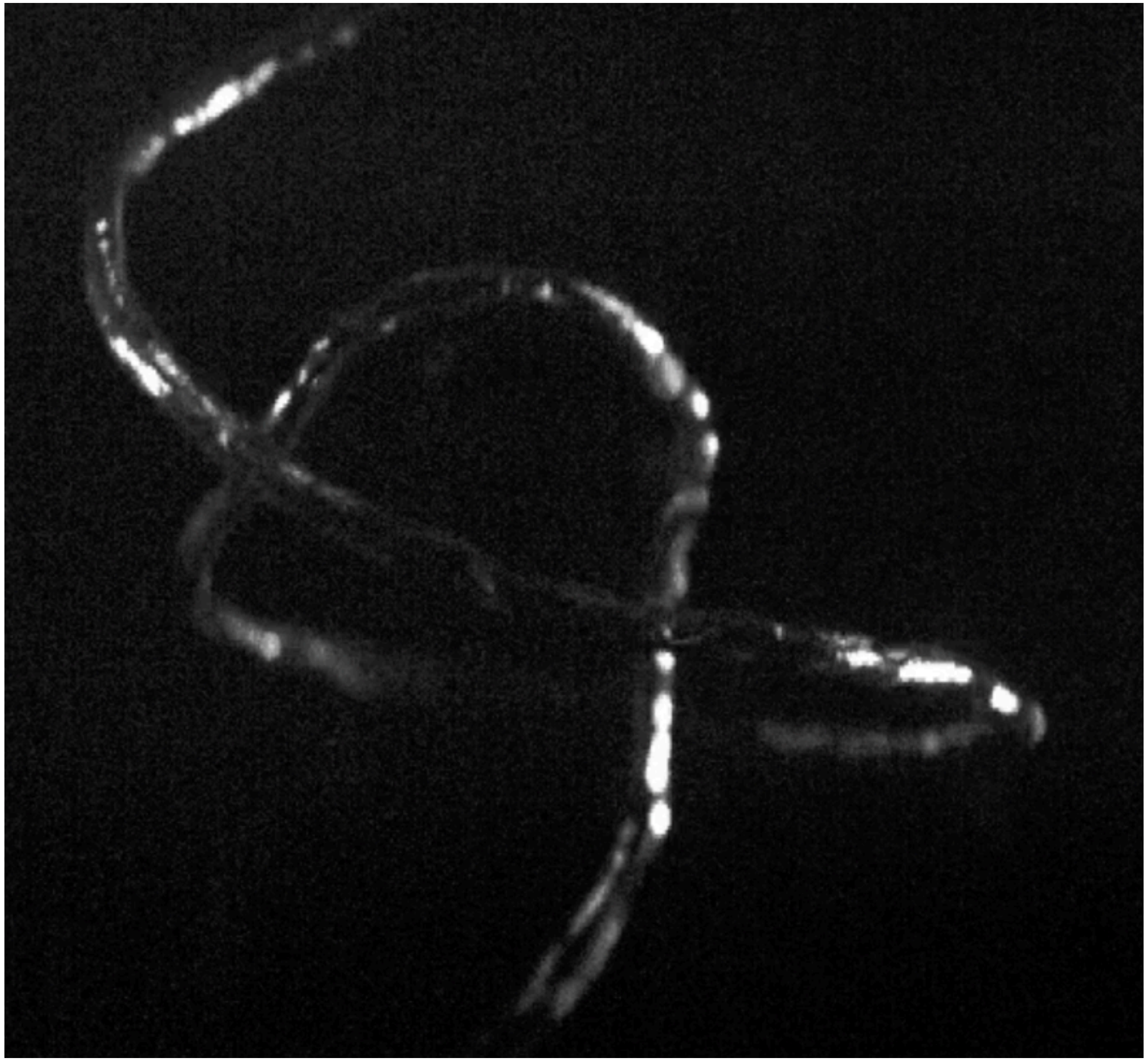


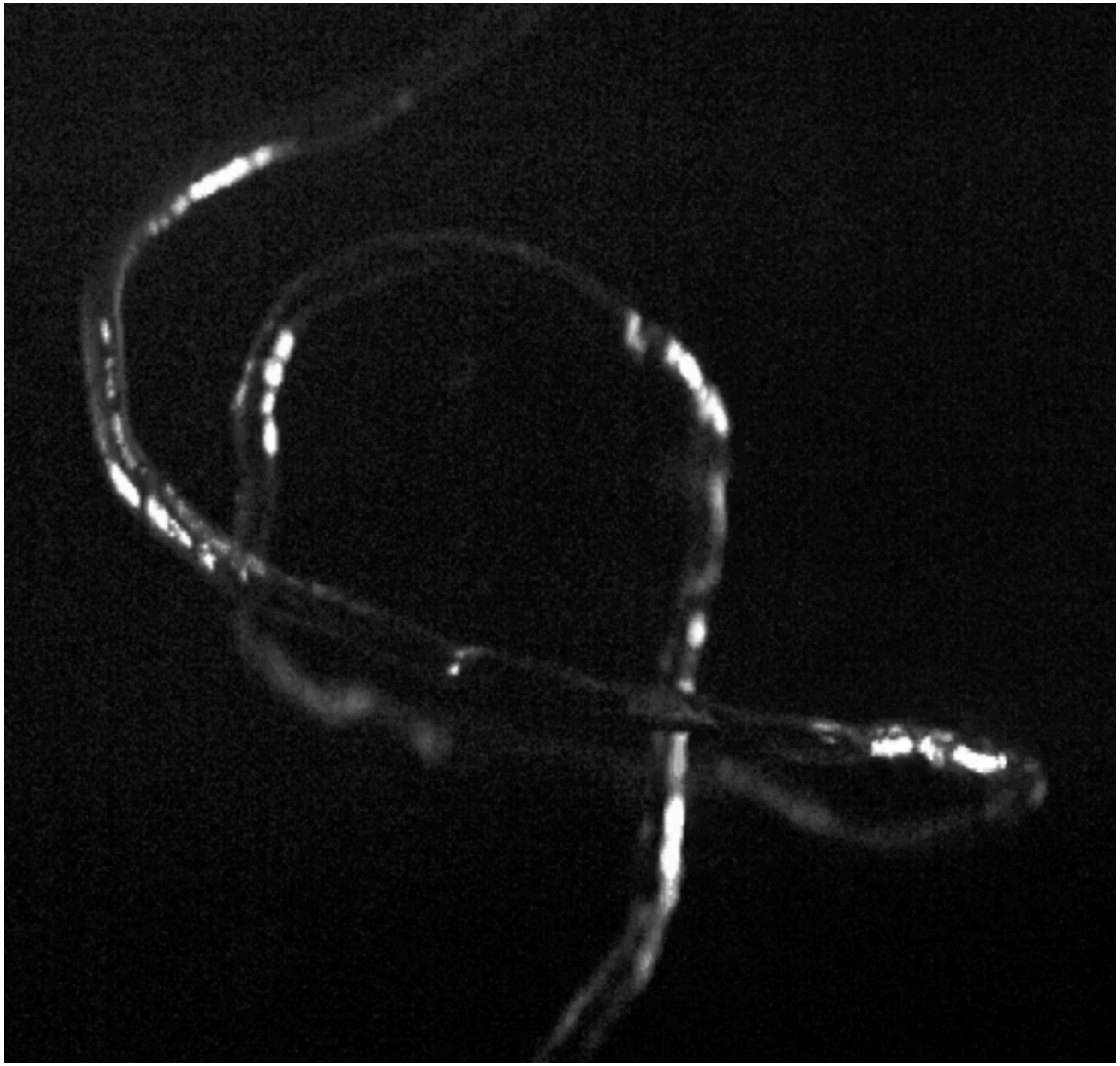
This experiment by Aleeksenko (2016)
shows that it is not so unlikely to switch a crossing after all!

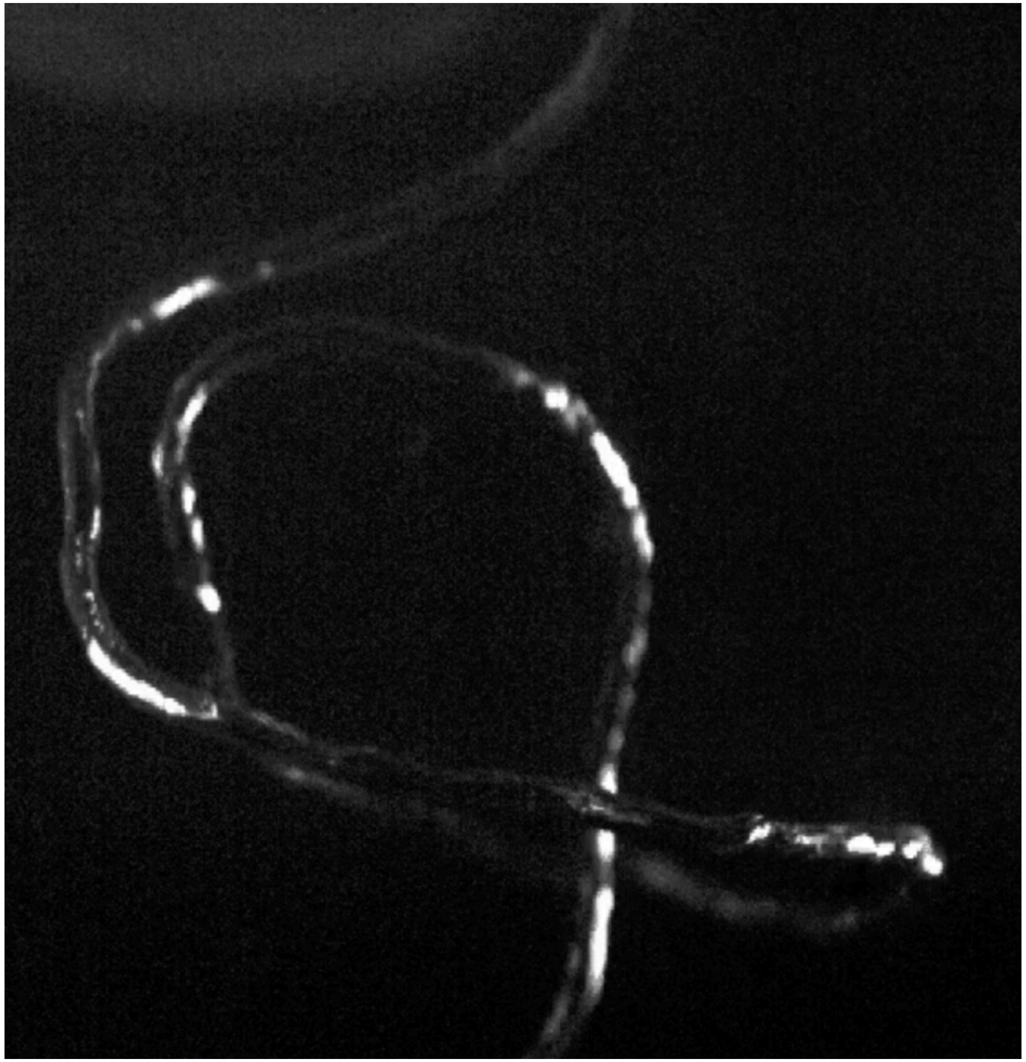


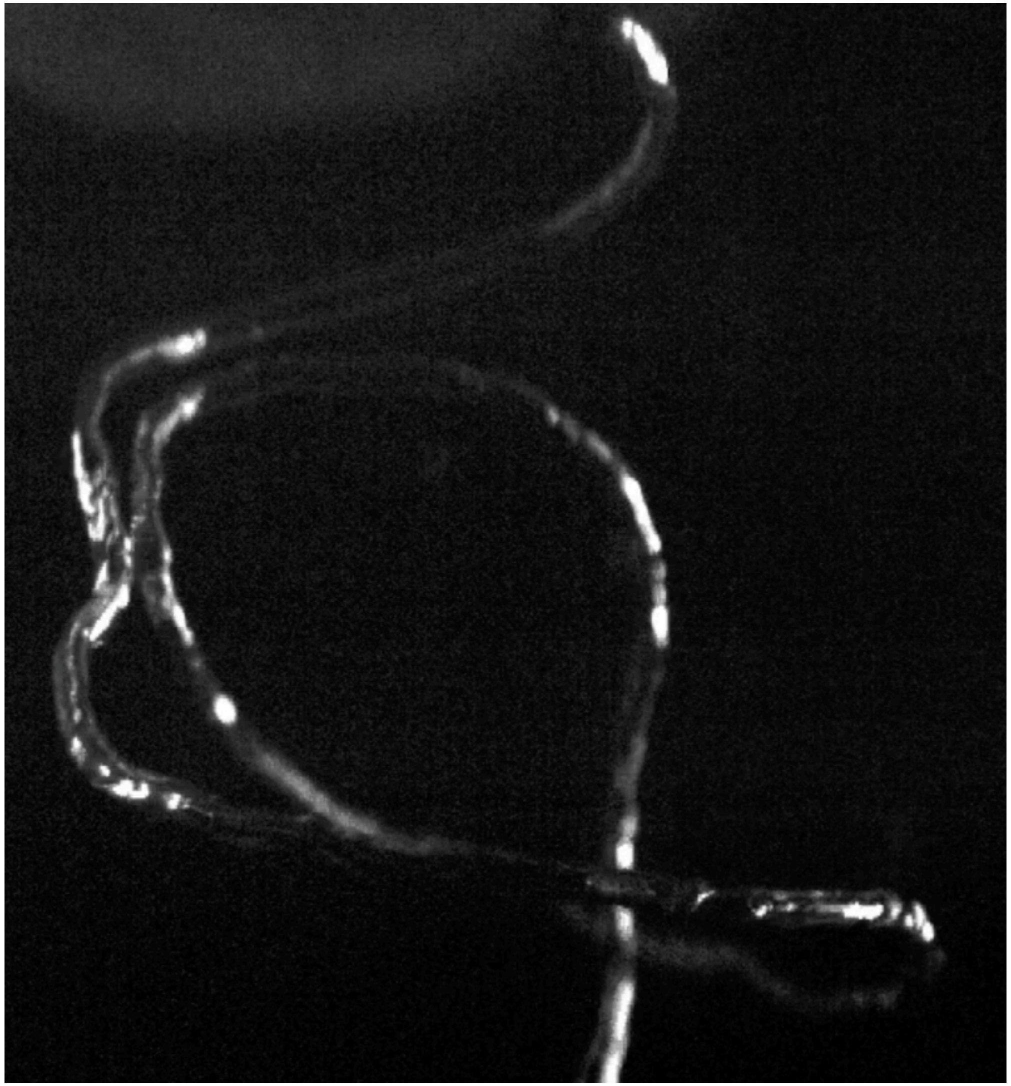


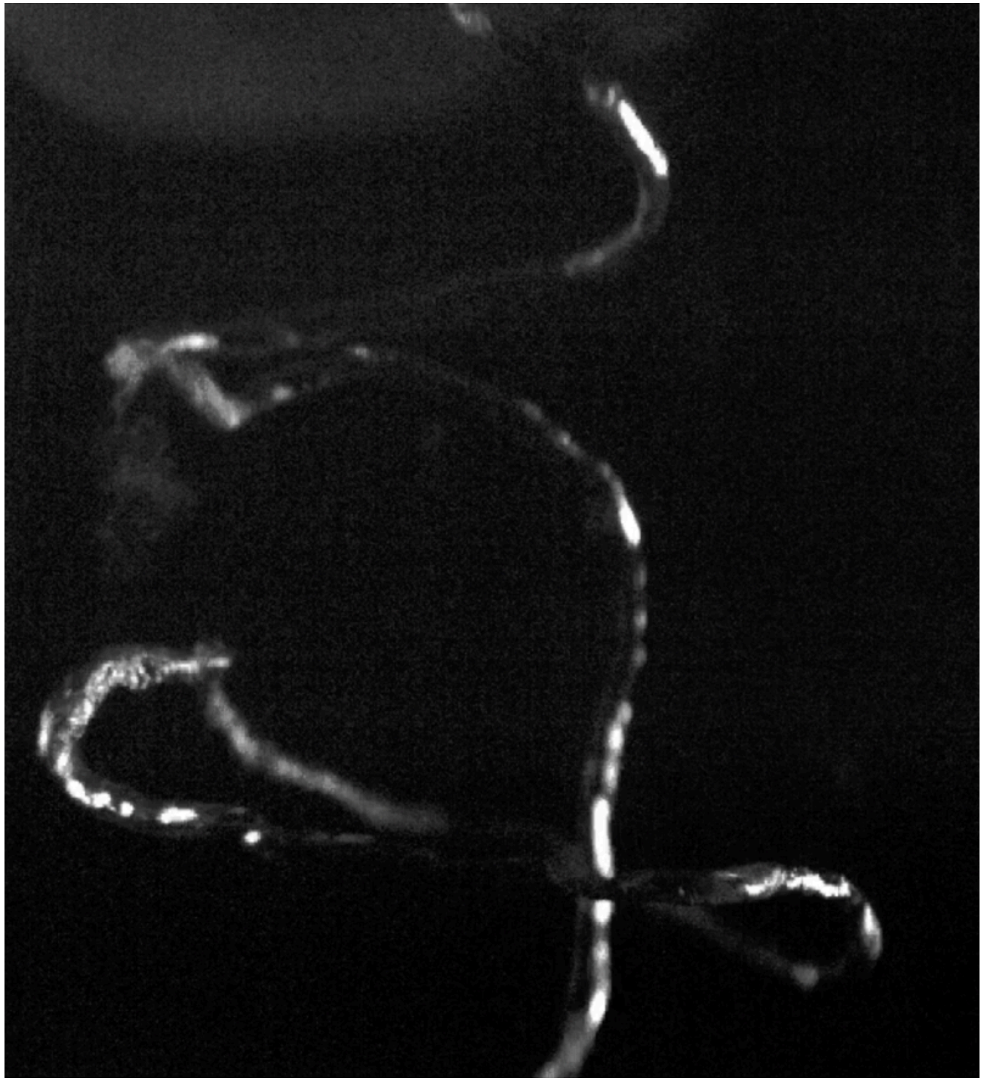
Aleeksenko’s Experiment

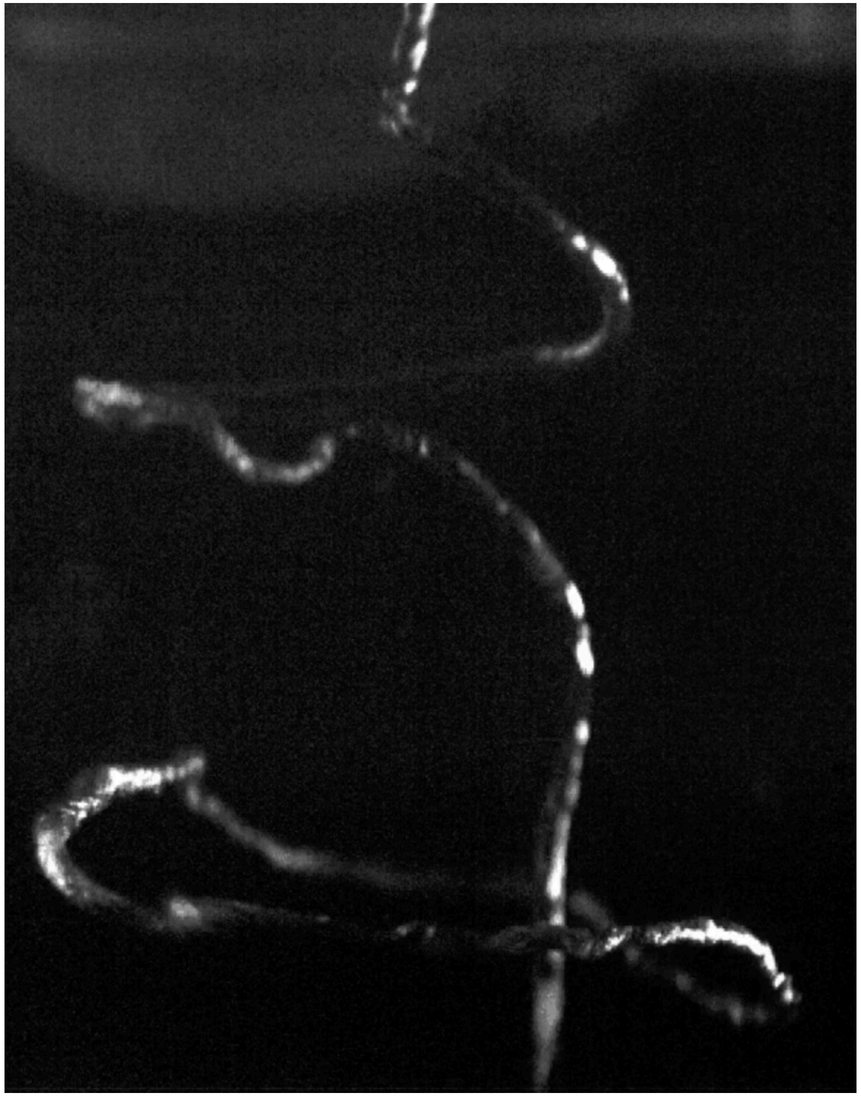


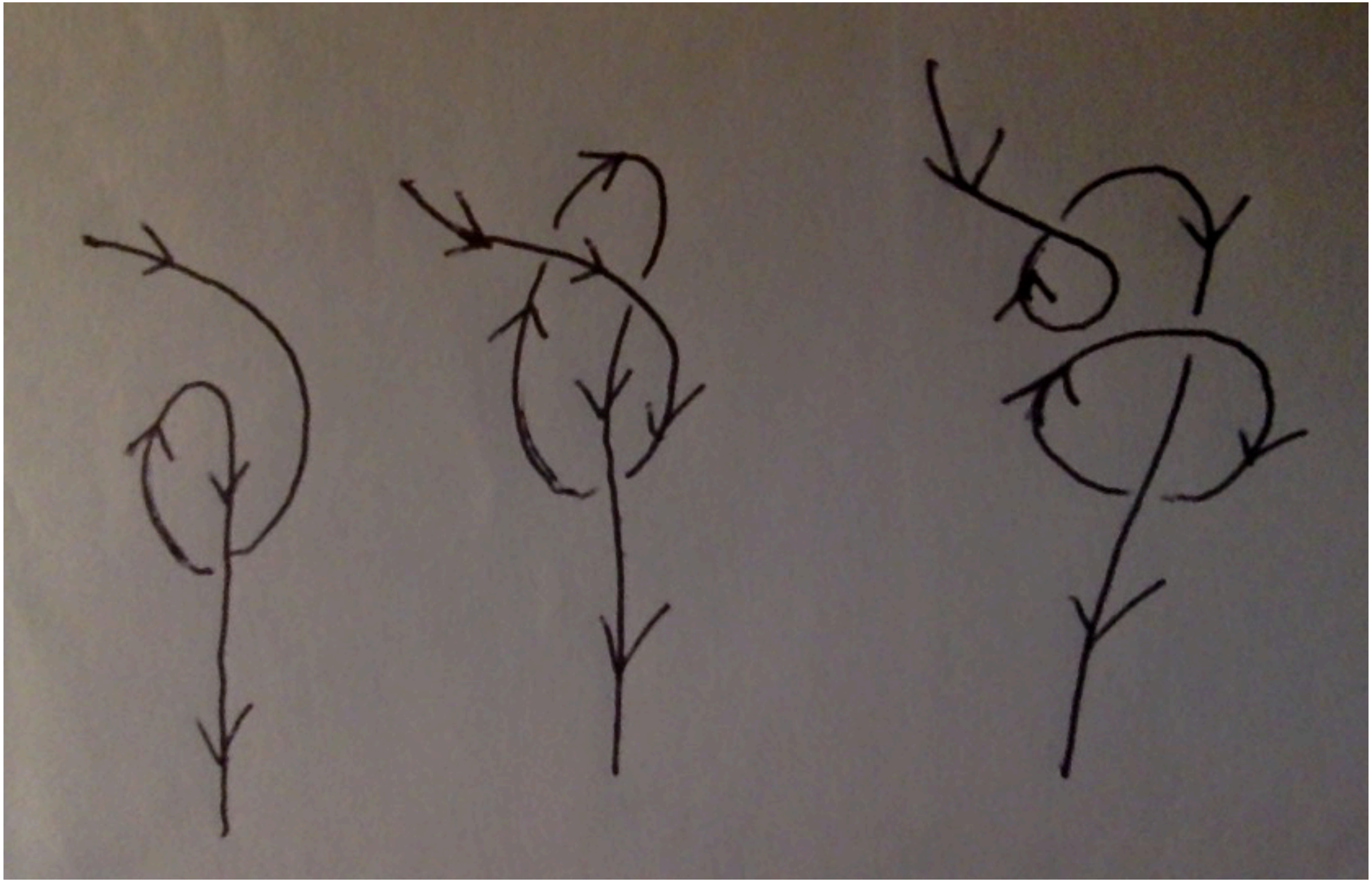












Are elementary particles knotted quantized flux?

PHYSICAL REVIEW D VOLUME 6, NUMBER 2 15 JULY 1972

Flux Quantization and Particle Physics

Herbert Jehle
*Physics Department, George Washington University, Washington, D. C. 20006**
(Received 27 September 1971; revised manuscript received 27 December 1971)

Quantized flux has provided an interesting model for muons and for electrons: One closed flux loop of the form of a magnetic dipole field line is assumed to adopt alternative forms which are superposed with complex probability amplitudes to define the magnetic field of a source lepton. The spinning of that loop with an angular velocity equal to the *Zitterbewegung* frequency $2mc^2/\hbar$ implies an electric Coulomb field, (negative) positive, depending on (anti) parallelism of magnetic moment and spin. The model implies CP invariance. A quark may be represented by a quantized flux loop if interlinked with another loop in the case of a meson, with two other loops in the case of a baryon. Because of the link, their spinning is very different from that of a single loop (lepton). The concept of a single quark does not exist accordingly, and it is seen that a baryon with a symmetric spin-isospin function in the SU(2) $\times$ SU(3) quark representation might not violate the Pauli principle because the wave function representing the relative position of linked loops may be chosen antisymmetric. Weak interactions may be understood to occur when the flux loops involved in the interaction have to cross over themselves or over each other. Strangeness is readily interpreted in terms of the trefoil character of a λ quark: Strangeness-violating interactions imply crossing of flux lines and are thus weak and parity-nonconserving. $\Delta S = \Delta Q$ is favored in such interactions. Intrinsic symmetries may be interpreted in terms of topology of linked loops. Sections I and II give a short résumé of the 1971 paper.

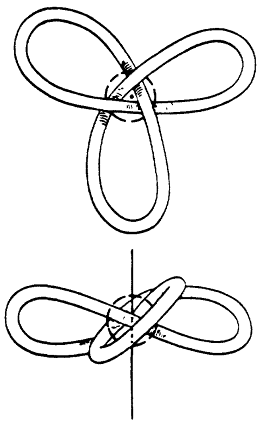


FIG. 2. A trefoil representing a neutrino loop which, like a coasting three-bladed propeller, moves in a helical spinning motion in the direction of the spin axis. In this and in subsequent figures, flux loops are drawn as double lines merely to better visualize the form of the loops. The loops are singular lines, the alternative forms of which define fibration of space. The question of orientation of the magnetic flux is still open; a neutrino might even be a superposition, not only of different loopforms, but also of both signatures of magnetic flux orientation. The difference between electron and muon neutrino is discussed in Sec. IV and in Appendix II of Ref. 1; the distinction is in regard to phase-related versus random-phased probability amplitudes superposition of the contributions of loopform bundles. A *single* loop of this form never represents anything else but a neutrino.

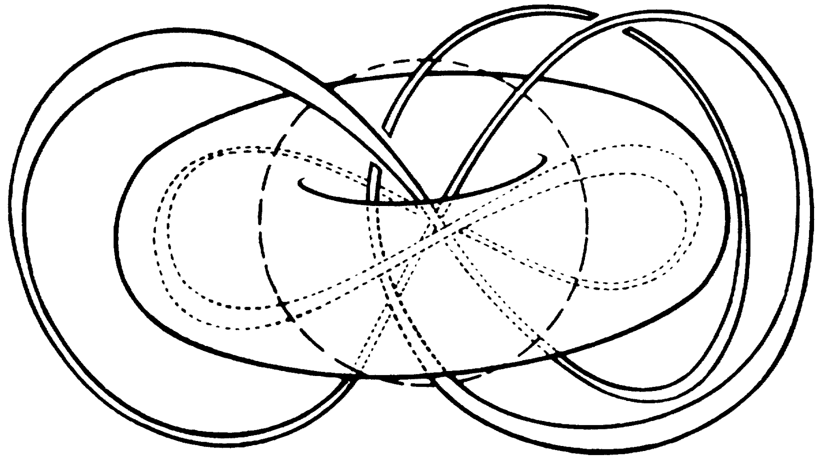


FIG. 4. Spinning-top model. λ and $\bar{\mathfrak{N}}$ quark interlinked, contributing to a meson. To illustrate the topological (knot-theoretical) relationships of the two loops, space is here subdivided by a toroidal surface [dashed lines in Fig. 4(a) which show a doughnut cut in half]. The λ is located entirely outside this doughnut shaped surface, the $\bar{\mathfrak{N}}$ entirely inside. This surface is dividing the fibrated space of λ loopforms from that of $\bar{\mathfrak{N}}$ loopforms; this toroidal interface may arbitrarily shrink or extend itself. Both loops pass through the spherical core region which is indicated by the dashed circle; the two loops may spin independently in a rolling-spinning motion about both the circular and the straight axes.

Jumping forward many years:

Protons are made of quarks.
Quarks are bound by gluon field.
Glueballs are closed loops of
gluon field.

Can glueballs be knotted?!

Are Glueballs Knotted Closed Strings?

Antti J. Niemi*

Department of Theoretical Physics, Uppsala University,
Box 803, S-75 108 Uppsala, Sweden

May 29, 2006

Abstract

Glueballs have a natural interpretation as closed strings in Yang-Mills theory. Their stability requires that the string carries a nontrivial twist, or then it is knotted. Since a twist can be either left-handed or right-handed, this implies that the glueball spectrum must be degenerate. This degeneracy becomes consistent with experimental observations, when we identify the $\eta_L(1410)$ component of the $\eta(1440)$ pseudoscalar as a 0^{-+} glueball, degenerate in mass with the widely accepted 0^{++} glueball $f_0(1500)$. In addition of qualitative similarities, we find that these two states also share quantitative similarity in terms of equal production ratios, which we view as further evidence that their structures must be very similar. We explain how our string picture of glueballs can be obtained from Yang-Mills theory, by employing a decomposed gauge field. We also consider various experimental consequences of our proposal, including the interactions between glueballs and quarks and the possibility to employ glueballs as probes for extra dimensions: The coupling of strong interactions to higher dimensions seems to imply that absolute color confinement becomes lost.

Universal energy spectrum of tight knots and links in physics*

Roman V. Buniy† and Thomas W. Kephart‡
Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235, USA

We argue that a systems of tightly knotted, linked, or braided flux tubes will have a universal mass-energy spectrum, since the length of fixed radius flux tubes depend only on the topology of the configuration. We motivate the discussion with plasma physics examples, then concentrate on the model of glueballs as knotted QCD flux tubes. Other applications will also be discussed.

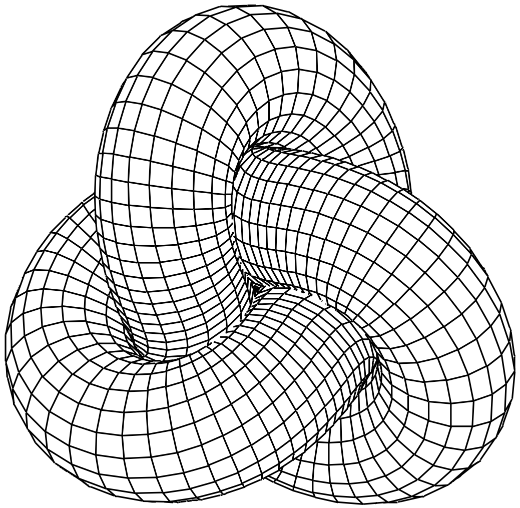


Figure 2: The second shortest solitonic flux configuration is the trefoil knot 3_1 corresponding to the second lightest glueball candidate $f_0(980)$.

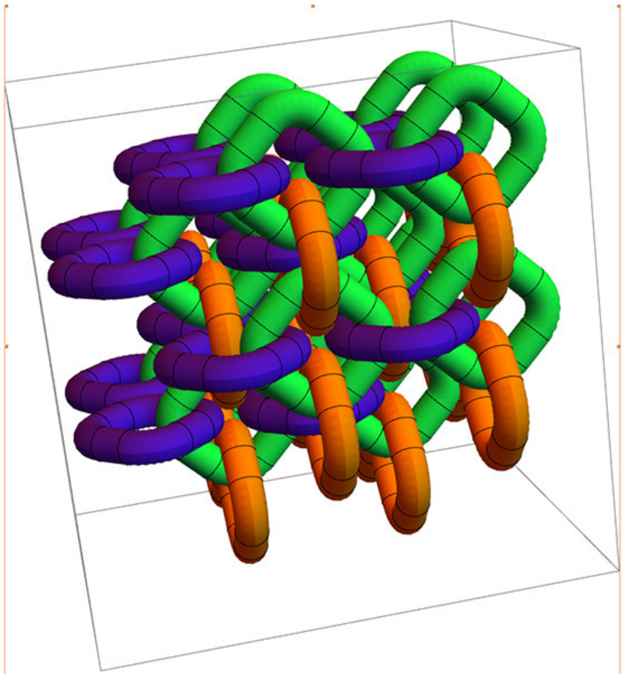
Knotty inflation and the dimensionality of spacetime

Arjun Berera,[1,*] Roman V. Buniy,[2,†] Thomas W. Kephart,[3,‡] Heinrich Päs,[4,§] and João G. Rosa[5,¶]

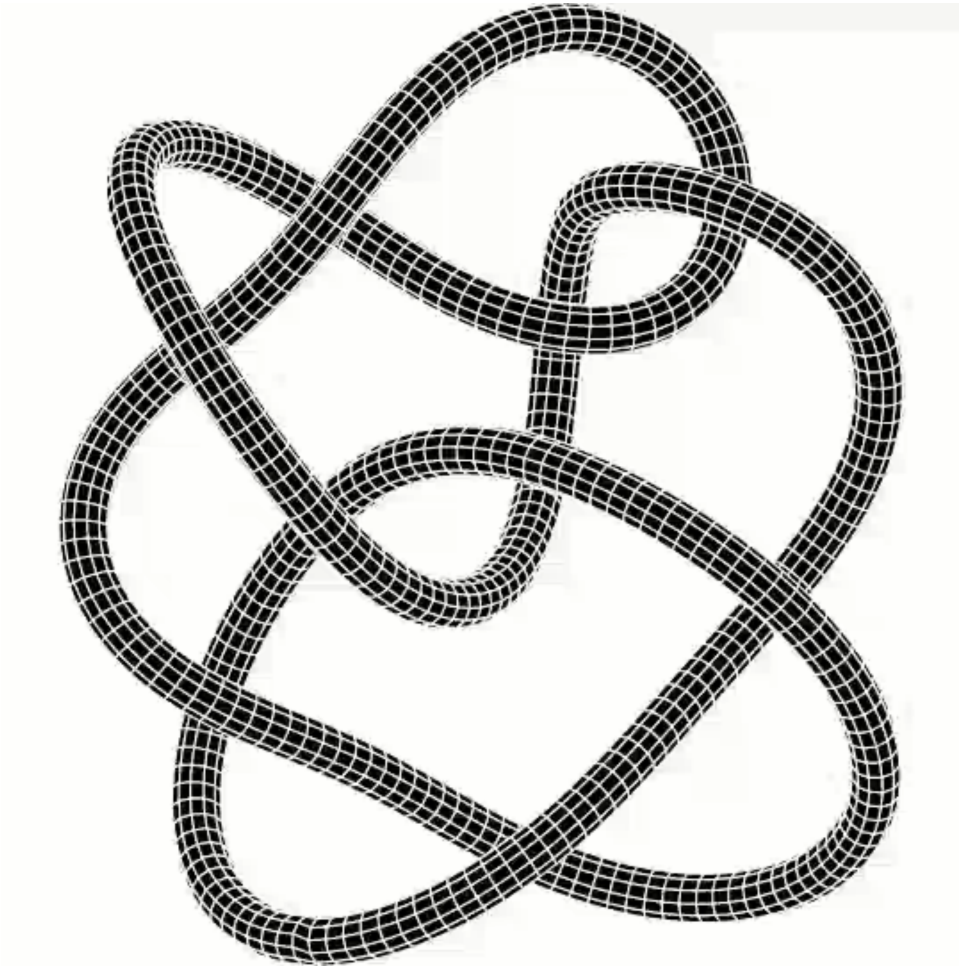
[1] *Tait Institute, School of Physics and Astronomy, University of Edinburgh, Edinburgh, EH9 3JZ, United Kingdom*
[2] *Schmid College of Science, Chapman University, Orange, CA 92866, USA*
[3] *Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235, USA*
[4] *Fakultät für Physik, Technische Universität Dortmund, 44221 Dortmund, Germany*
[5] *Departamento de Física da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal*

(Dated: August 7, 2015)

We suggest a structure for the vacuum comprised of a network of tightly knotted/linked flux tubes formed in a QCD-like cosmological phase transition and show that such a network can drive cosmological inflation. As the network can be topologically stable only in three space dimensions, this scenario provides a dynamical explanation for the existence of exactly three large spatial dimensions in our Universe.



Kephart and Buiny compared the ropelength of knots to observed energy levels of glueballs and found good correlations.



The previous demonstration as made by Jason Cantarella, using his program “ridgerunner”.

http://www.math.uga.edu/~cantarel/

Foundations of Physics, Vol. 31, No. 4, 2001

Knotted Zeros in the Quantum States of Hydrogen

Michael Berry[1]

Received January 8, 2001

Complex superpositions of degenerate hydrogen wavefunctions for the n*th energy level can possess zero lines (phase singularities) in the form of knots and links. A recipe is given for constructing any torus knot. The simplest cases are constructed explicitly: the elementary link, requiring* $n \geqslant 6$*, and the trefoil knot, requiring* $n \geqslant 7$*. The knots are threaded by multistranded twisted chains of zeros. Some speculations about knots in general complex quantum energy eigenfunctions are presented.*

These speculations can be extended to topologies of phase singularity that are not included in the class of torus knots. For example, one can ask whether quantum states can contain zero lines in the form of Borromean rings,[(21)] where three unknotted loops are connected even though no two are linked.

Mobius Strip Particles

A Visualizable Representation of the Elementary Particles

J.S. Avrin*

Abstract

Rudimentary knots are invoked to generate a representation of the elementary particles, a model that endows the particles with visualizable structure. The model correlates with the basic tenets, taxonomy, and interactions of the Standard Model, but goes beyond it in a number of important ways, the most significant being that all particles (hadrons and leptons, fermions and bosons) and interactions share a common topology. Among other consequences of the modeling are the topological basis for isospin invariance and its connection to electric charge, the necessary identity of electron and proton charge magnitudes, and the existence of precisely three generations on the particle family tree. The salient feature of the model is that the elementary particles are viewed not as discrete, point-like objects in a vacuum but rather as sustainable, membrane-like distortions embodying curvature and torsion in and of an otherwise featureless continuum and that their manifest physical attributes correlate with the distortion. There are additional connections to the theories of fiber bundles, superstrings and instantons and, historically, to the work of Kelvin in the mid-nineteenth century and Cartan in the 1920s among others.

(published in Journal of Knot Theory and Its Ramifications)

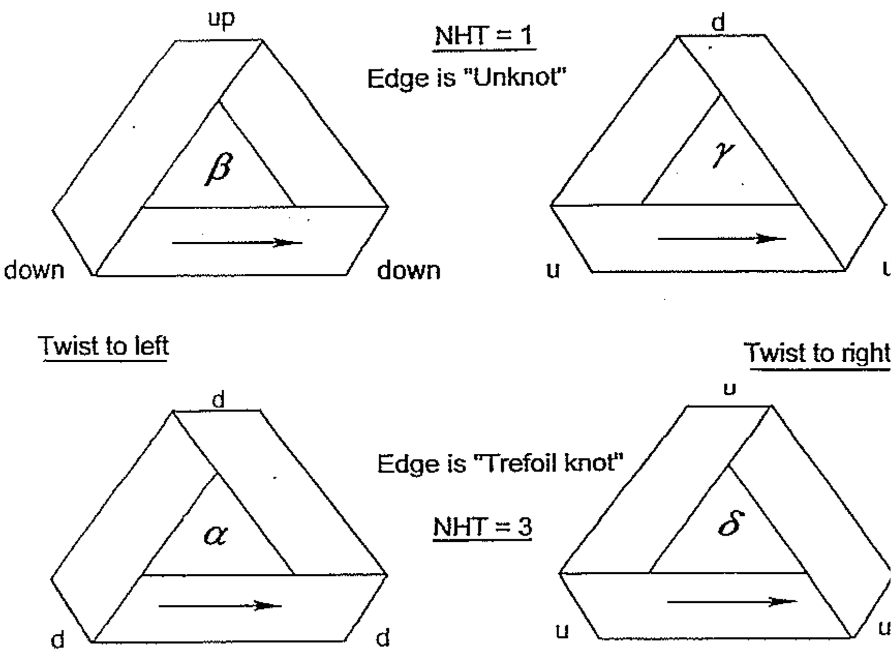


Figure 3a: Basic Set; Spin 1/2 Fermion Diagrams

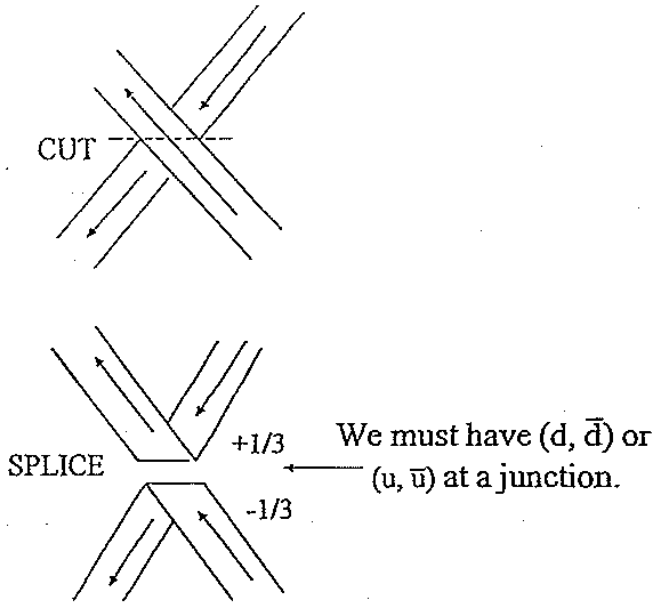


Figure 6: Fission (Cut and Splice)

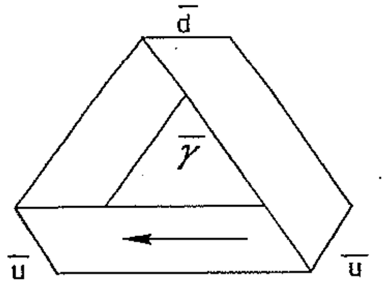
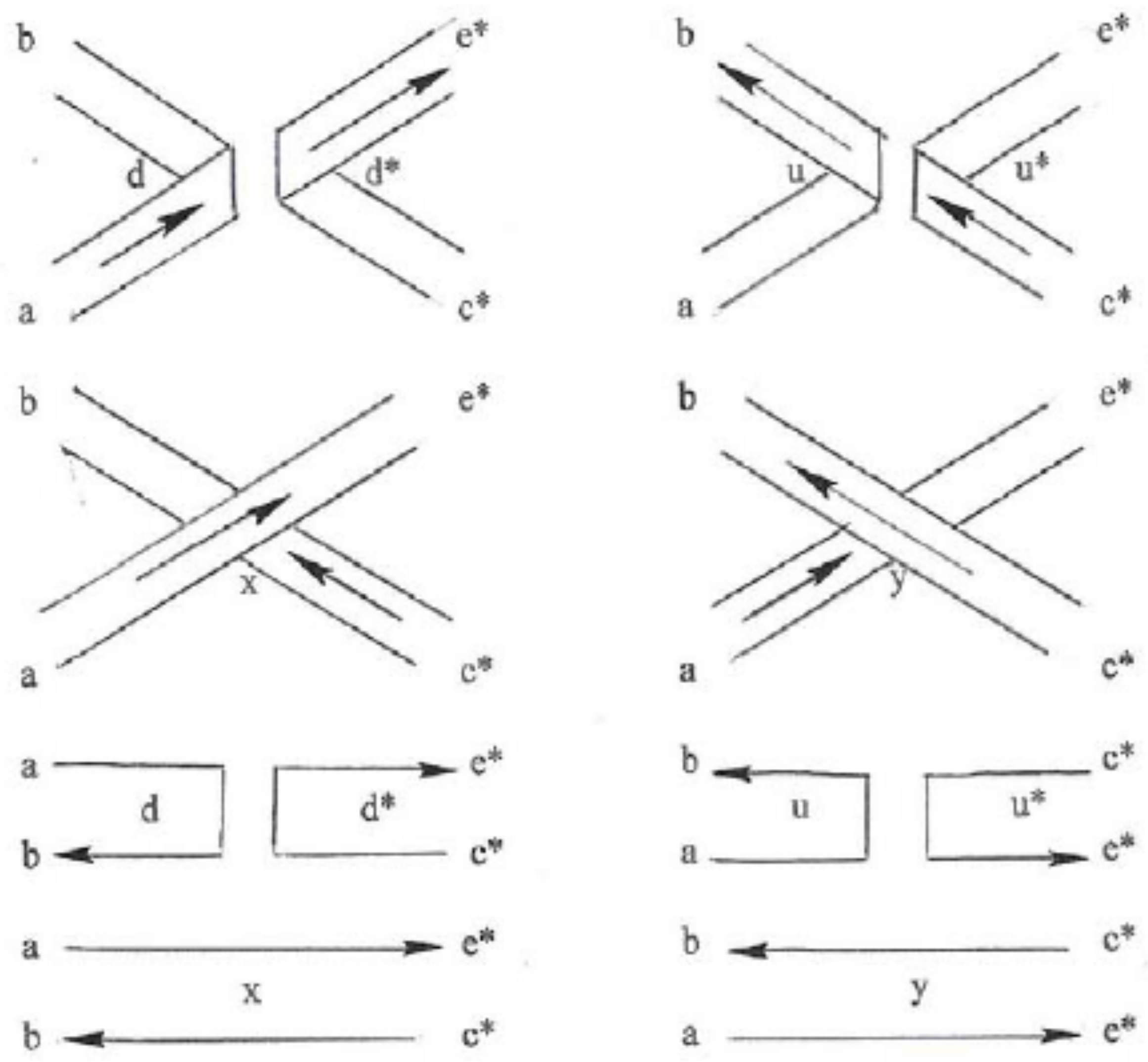
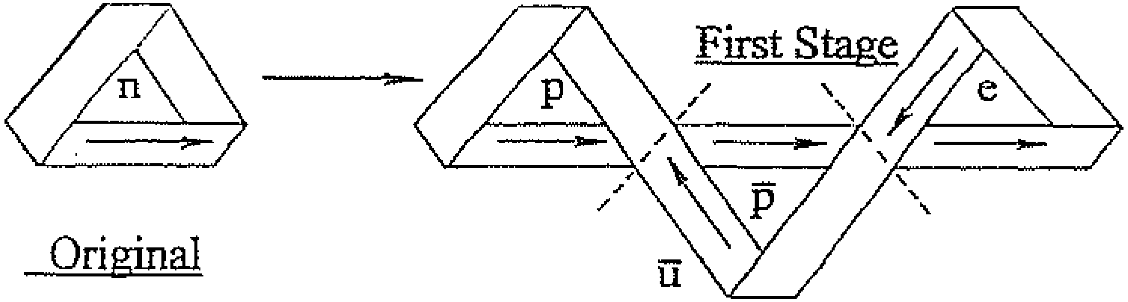


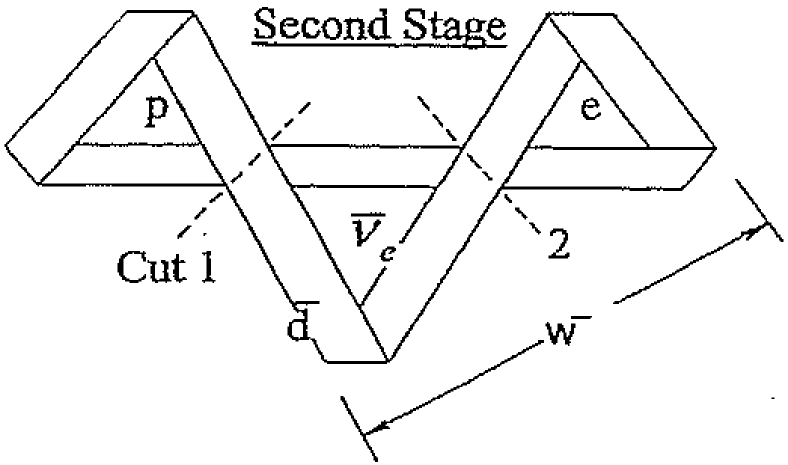
Figure 3b: Antiparticle Diagram for "γ Particle"



b
e*
d
d*
a
c*
b
u
u*
e*
a
c*
b
e*
x
a
c*
b
e*
y
a
c*
a
d
d*
e*
b
c*
b
u
u*
c*
a
e*
a
x
e*
b
c*
b
y
c*
a
e*



- First stage conserves twist and spin but not charge.
- Note the change from $\overline{u}$ to $\overline{d}$ analogous to Standard model process.



$$n \rightarrow p + e + \overline{\nu_e}$$

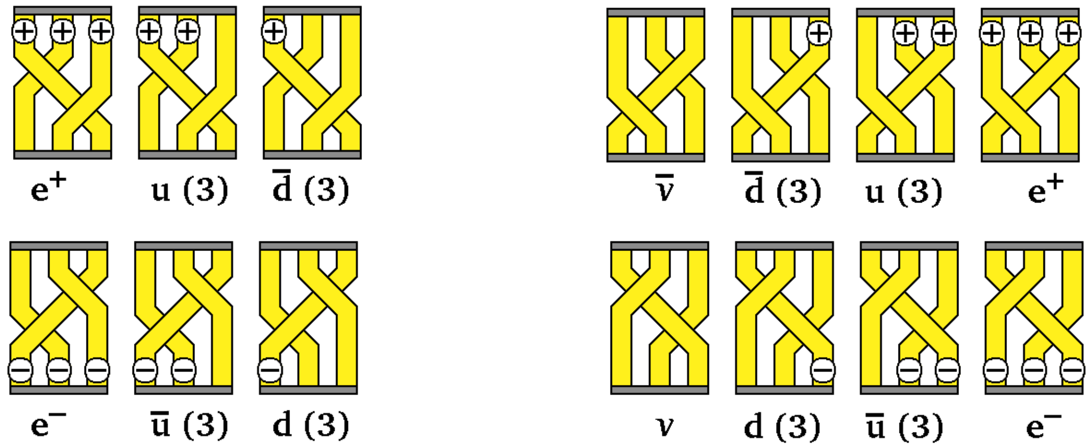
Figure 7: Neutron Decay Model

A topological model of composite preons

Sundance O. Bilson-Thompson*
Centre for the Subatomic Structure of Matter, Department of Physics,
University of Adelaide, Adelaide SA 5005, Australia

(Dated: October 27, 2006)

We describe a simple model, based on the preon model of Shupe and Harari, in which the binding of preons is represented topologically. We then demonstrate a direct correspondence between this model and much of the known phenomenology of the Standard Model. In particular we identify the substructure of quarks, leptons and gauge bosons with elements of the braid group B_3. Importantly, the preonic objects of this model require fewer assumed properties than in the Shupe/Harari model, yet more emergent quantities, such as helicity, hypercharge, and so on, are found. Simple topological processes are identified with electroweak interactions and conservation laws. The objects which play the role of preons in this model may occur as topological structures in a more comprehensive theory, and may themselves be viewed as composite, being formed of truly fundamental sub-components, representing exactly two levels of substructure within quarks and leptons.



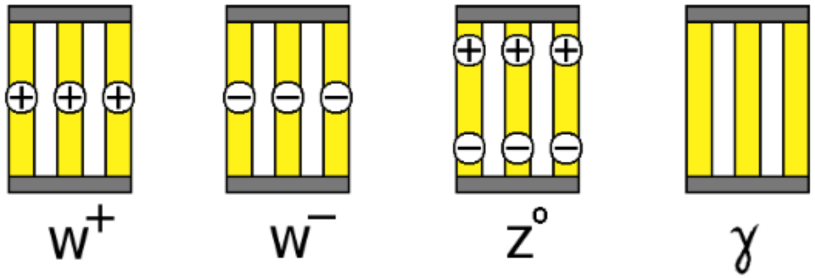


Figure 5. Bosons

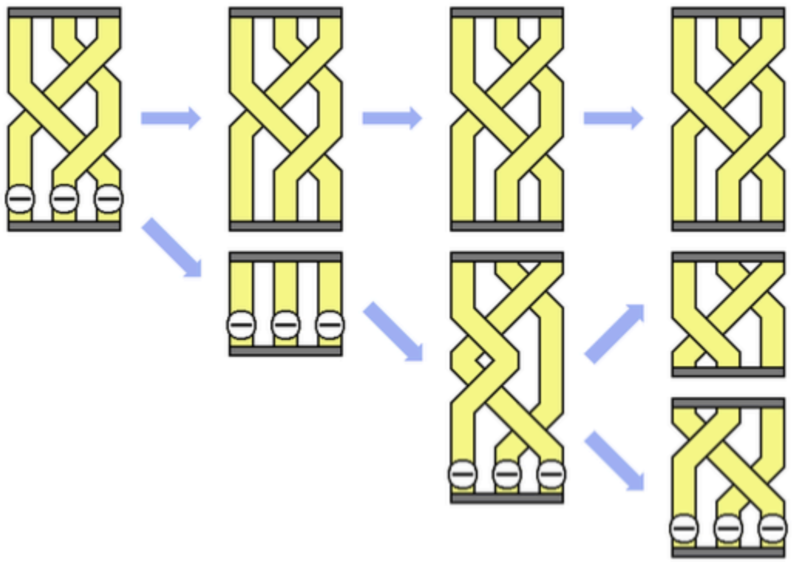


Figure 6. Representation of $\mu \rightarrow \nu_\mu + W_- \rightarrow \nu_\mu + \bar{\nu_e} + e^-$.

Positron

Electron

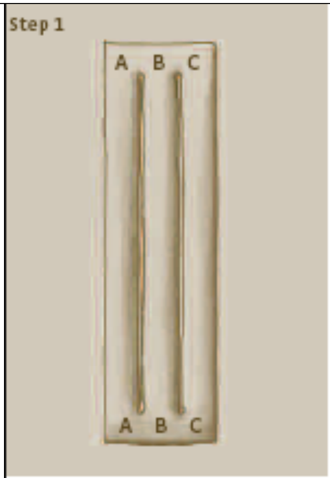
Down quark

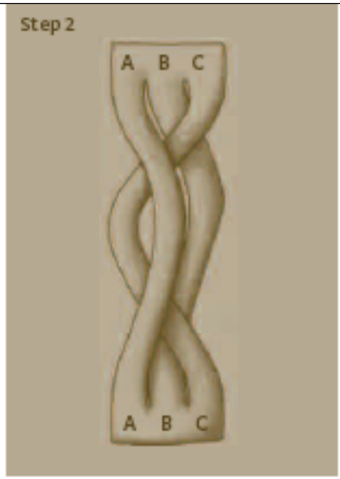
Up quark

The Braided Belt Trick

The mathematics of Sundance Bilson's approach to elementary particles based on the 'braided belt trick" shown in the next slide.

This trick is also the basis for making braided leather belts.



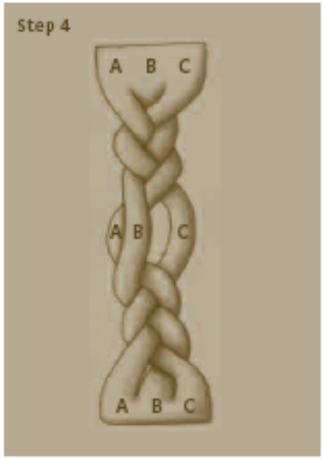


Begin by cutting two slits into a strip of leather.

Be careful not to cut all the way to the ends.

Holding the top flat, pull string C over string B, and pull string A over string C.

Next, pull string B over string A, and pull string C over string B.

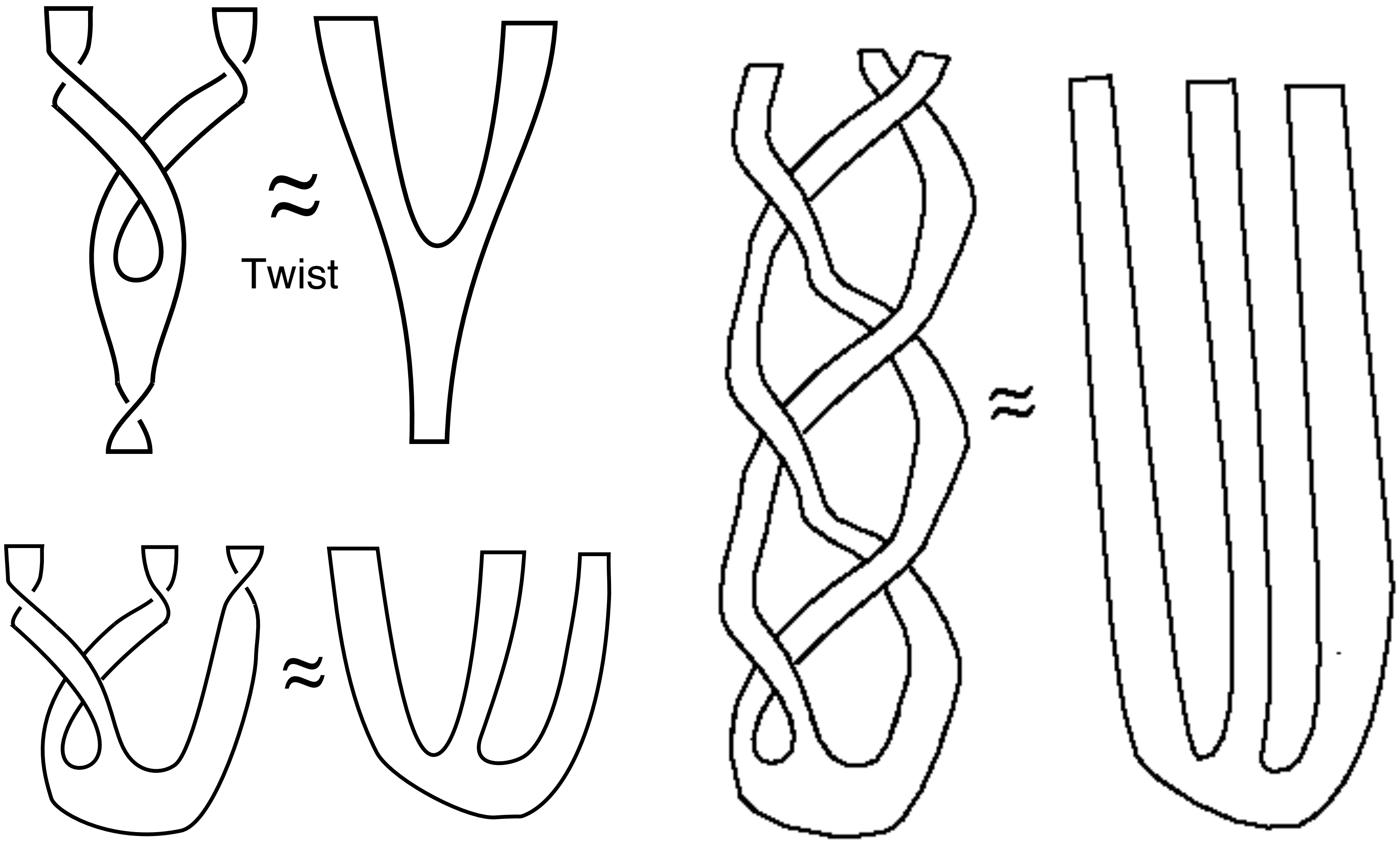


Now pull string A over string C, and pull string B over string A.

Untangle the bottom portion by sliding the bottom end through the open slits.

Continue this pattern until the braid reaches the bottom of the strip.

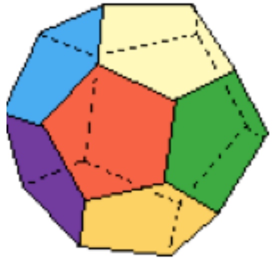
The Braided Belt Trick

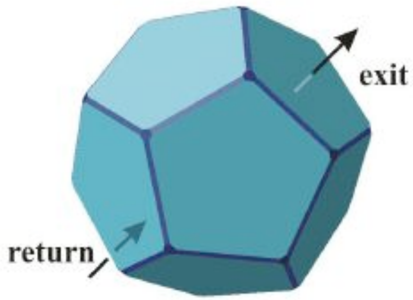


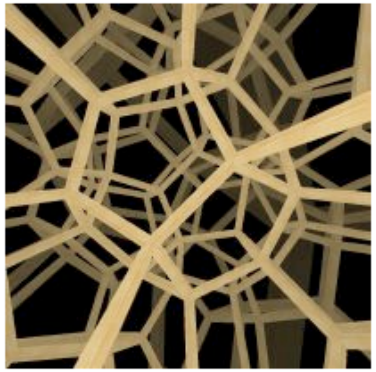
This approach to elementary particle physics is just beginning. We will have to wait and see if elementary particles are braids and if knotted glueballs are real.

After all,
Why Knot?

Is the Geometric Universe a Poincare Dodecahedral Space?

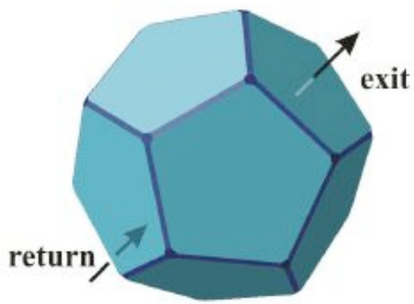


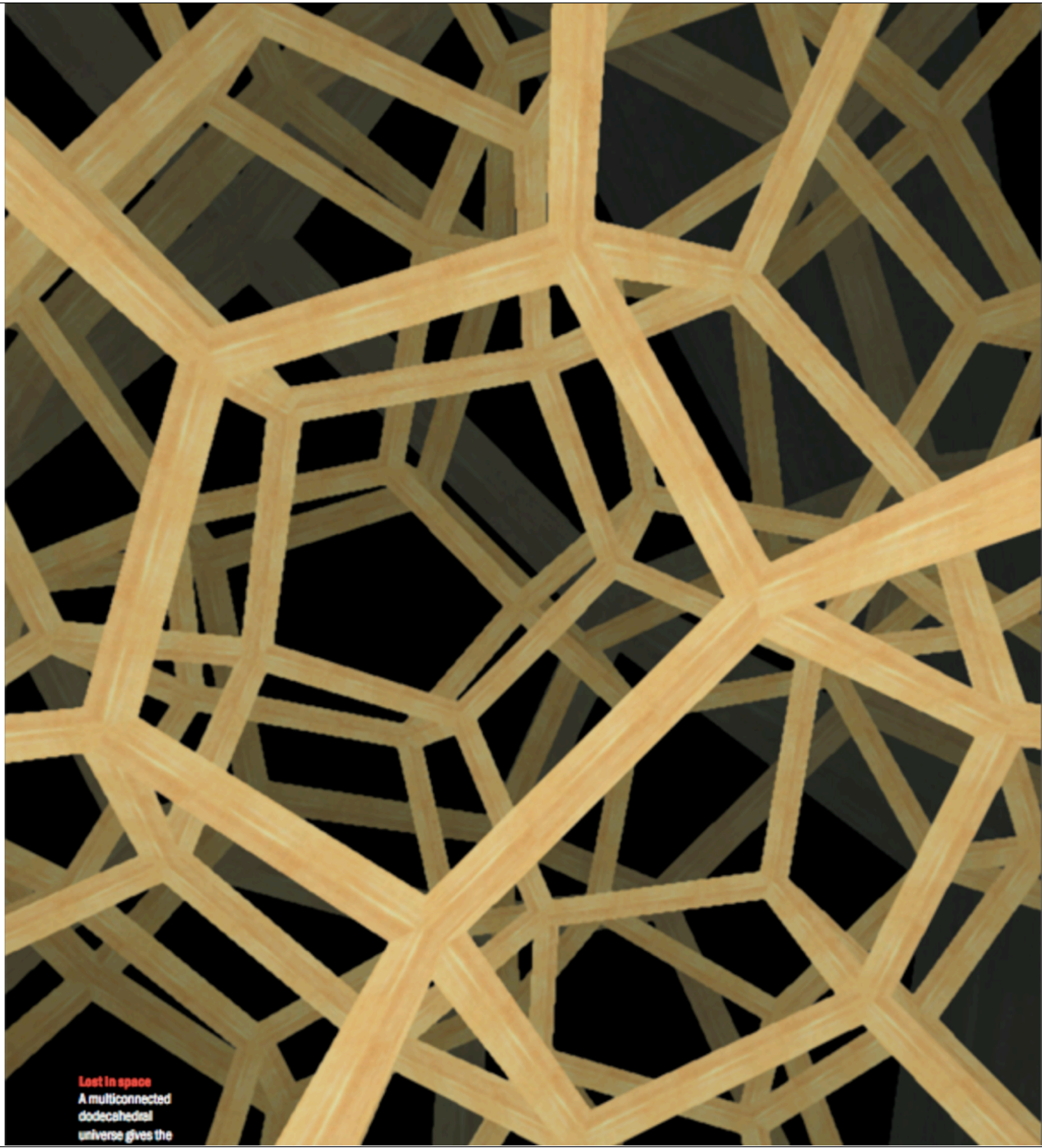




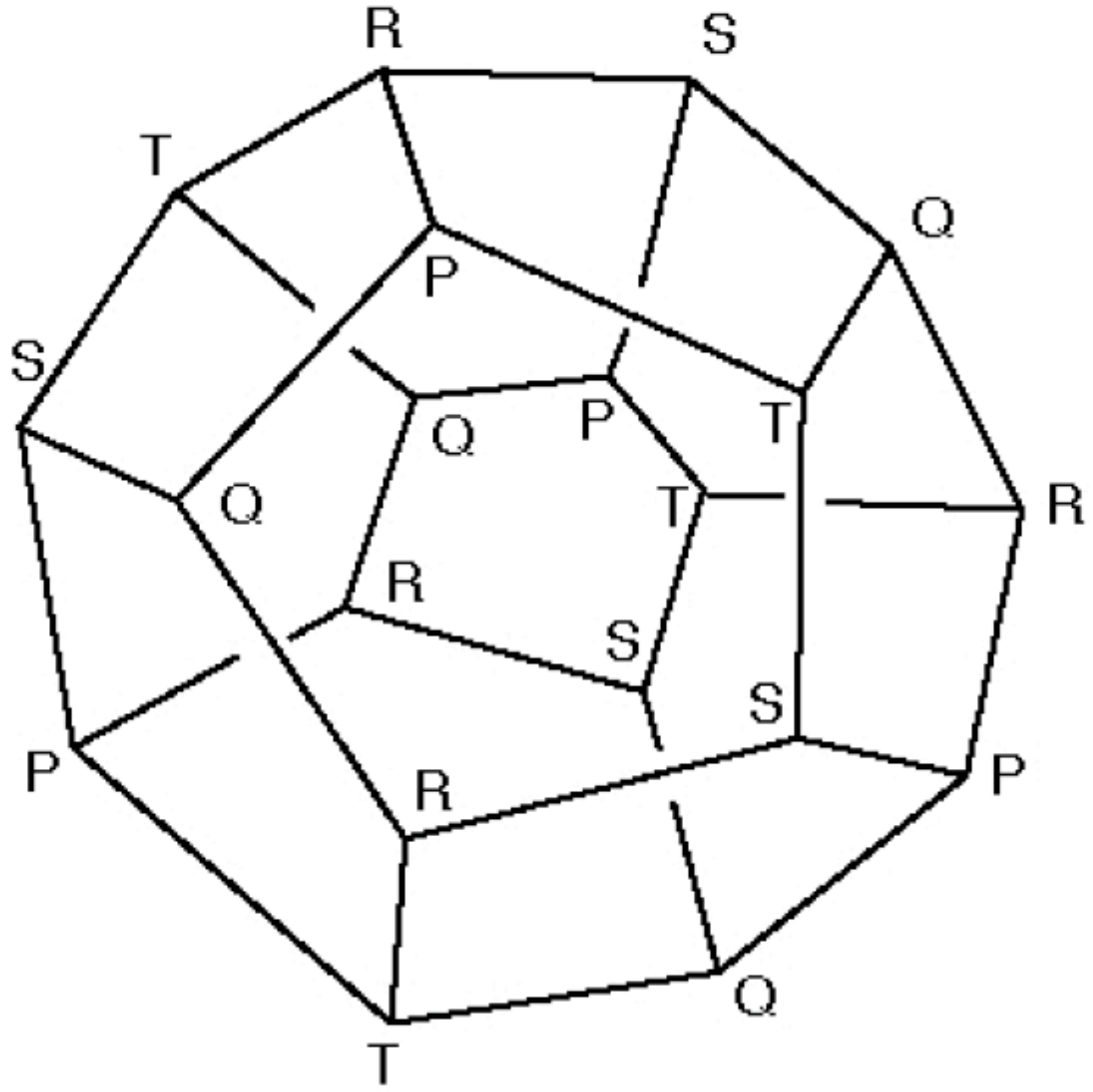
A franco-american team of cosmologists [1] led by J.-P. Luminet, of the Laboratoire Univers et Théories (LUTH) at the Paris Observatory, has proposed an explanation for a surprising detail observed in the Cosmic Microwave Background (CMB) recently mapped by the NASA satellite WMAP. According to the team, who published their study in the 9 October 2003 issue of Nature, an intriguing discrepancy in the temperature fluctuations in the afterglow of the big bang can be explained by a very specific global shape of space (a "topology"). The universe could be wrapped around, a little bit like a "soccer ball", the volume of which would represent only 80% of the observable universe! (figure 1) According to the leading cosmologist George Ellis, from Cape Town University (South Africa), who comments on this work in the "News & Views" section of the same issue: "If confirmed, it is a major discovery about the nature of the universe".

The Poincare Dodecahedral space is
obtained by identifying opposite
sides of a dodedahedron with
a twist.
The resulting space, if you were inside it,
would be something like the next slide.
Whenever you crossed a pentagonal face,
you would find yourself back in
the Dodecahedron.





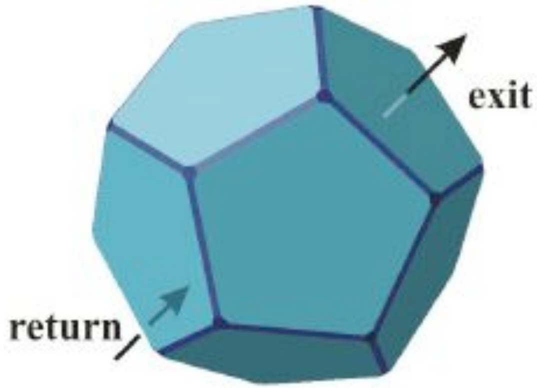
Lost in space
A multiconnected
dodecahedral
universe gives the



R
S
T
Q
P
S
Q
P
T
T
R
Q
R
S
S
P
R
P
Q
T

What Does This Have to do with Knot Theory?

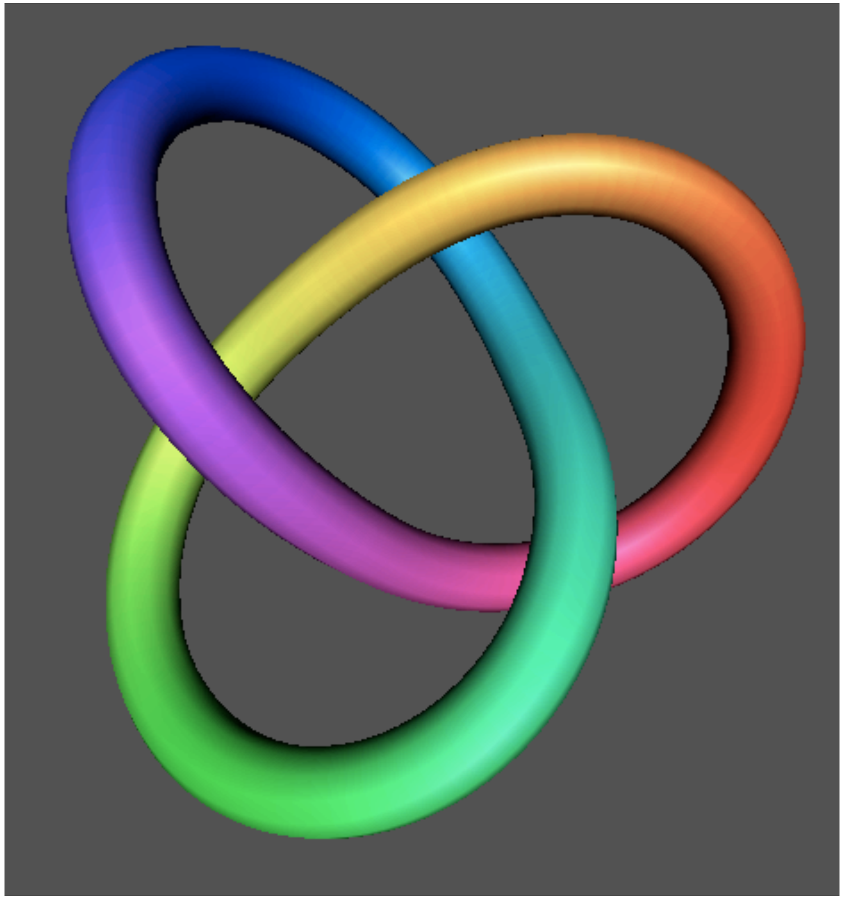
The dodecahedral Space M has Axes of Symmetry: five-fold, three-fold and two-fold.



The dodecahedral space M is the 5-fold cyclic branched covering of the three-sphere, branched along the trefoil knot.

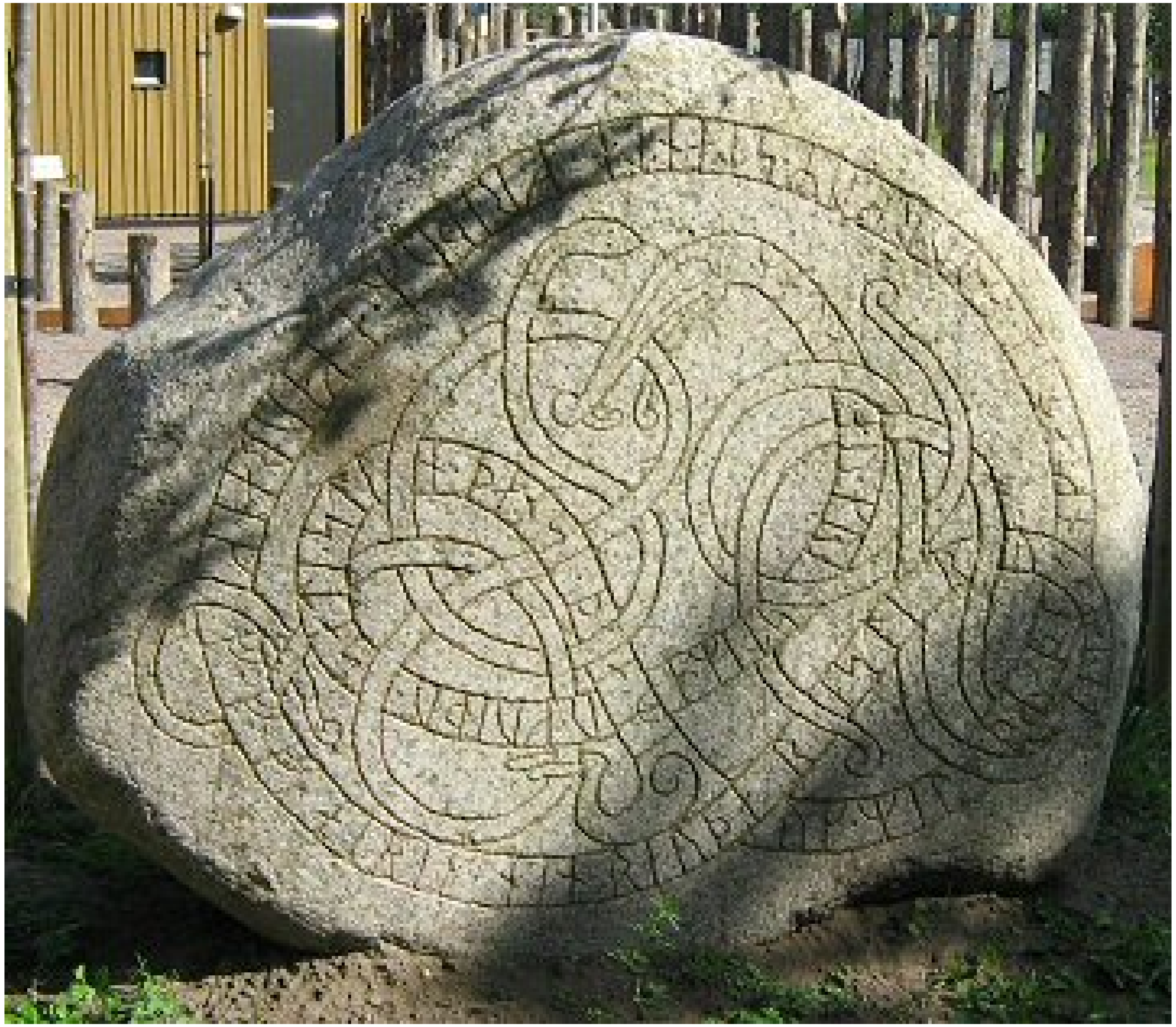
M = Variety(x^2 + y^3 + z^5) Intersected with S^5 in C^3.

CK



So perhaps the trefoil knot is the key to the universe.

Thank you for your attention!



Knots and Quantum Field Theory

From Feynman's Nobel Lecture

The character of quantum mechanics of the day was to write things in the famous Hamiltonian way - in the form of a differential equation, which described how the wave function changes from instant to instant, and in terms of an operator, *H*. If the classical physics could be reduced to a Hamiltonian form, everything was all right. Now, least action does not imply a Hamiltonian form if the action is a function of anything more than positions and velocities at the same moment. If the action is of the form of the integral of a function, (usually called the Lagrangian) of the velocities and positions at the same time

$$S = \int L(\dot{x}, x)\, dt$$

then you can start with the Lagrangian and then create a Hamiltonian and work out the quantum mechanics, more or less uniquely. But this thing (1) involves the key variables, positions, at two different times and therefore, it was not obvious what to do to make the quantum-mechanical analogue.

L = Kinetic Energy - Potential Energy

Classical Mechanics: Extremize Integral of L over the paths from A to B.

So that didn't help me very much, but when I was struggling with this problem, I went to a beer party in the Nassau Tavern in Princeton. There was a gentleman, newly arrived from Europe (Herbert Jehle) who came and sat next to me. Europeans are much more serious than we are in America because they think that a good place to discuss intellectual matters is a beer party. So, he sat by me and asked, "what are you doing" and so on, and I said, "I'm drinking beer." Then I realized that he wanted to know what work I was doing and I told him I was struggling with this problem, and I simply turned to him and said, "listen, do you know any way of doing quantum mechanics, starting with action - where the action integral comes into the quantum mechanics?" "No", he said, "but Dirac has a paper in which the Lagrangian, at least, comes into quantum mechanics. I will show it to you tomorrow."

Next day we went to the Princeton Library, they have little rooms on the side to discuss things, and he showed me this paper. What Dirac said was the following: There is in quantum mechanics a very important quantity which carries the wave function from one time to another, besides the differential equation but equivalent to it, a kind of a kernal, which we might call $K(x', x)$, which carries the wave function j(x) known at time t, to the wave function j(x') at time, $t+e$ Dirac points out that this function K was *analogous* to the quantity in classical mechanics that you would calculate if you took the exponential of ie, multiplied by the Lagrangian $L(\dot{x}, x)$ imagining that these two positions x, x' corresponded t and t+e. In other words,

$$K(x', x) \text{ is analogous to } e^{i\varepsilon L\left(\frac{x'-x}{\varepsilon}, x\right)/\hbar}$$

Professor Jehle showed me this, I read it, he explained it to me, and I said, "what does he mean, they are analogous; what does that mean, *analogous*? What is the use of that?" He said, "you Americans! You always want to find a use for everything!" I said, that I thought that Dirac must mean that they were equal. "No", he explained, "he doesn't mean they are equal." "Well", I said, "let's see what happens if we make them equal."

So I simply put them equal, taking the simplest example where the Lagrangian is ½Mx^2 - $V(x)$ but soon found I had to put a constant of proportionality A in, suitably adjusted. When I substituted $Ae^{i\epsilon L/\hbar}$ for K to get

$$\psi(x', t+\varepsilon) = \int A \exp\left[\frac{i\varepsilon}{\hbar} L\left(\frac{x'-x}{\varepsilon}, x\right)\right] \psi(x, t)\, dx$$

and just calculated things out by Taylor series expansion, out came the Schrödinger equation. So, I turned to Professor Jehle, not really understanding, and said, "well, you see Professor Dirac meant that they were proportional." Professor Jehle's eyes were bugging out - he had taken out a little notebook and was rapidly copying it down from the blackboard, and said, "no, no, this is an important discovery. You Americans are always trying to find out how something can be used. That's a good way to discover things!" So, I thought I was finding out what Dirac meant, but, as a matter of fact, had made the discovery that what Dirac thought was analogous, was, in fact, equal. I had then, at least, the connection between the Lagrangian and quantum mechanics, but still with wave functions and infinitesimal times.

The Taylor expansion is

$$\psi(x,t+\epsilon) = \frac{e^{\frac{-i\epsilon V(x)}{\hbar}}}{A} \int e^{\frac{im\eta^2}{\hbar 2\epsilon}} [\psi(x,t) + \eta \frac{\partial \psi(x,t)}{\partial x} + \frac{\eta^2}{2} \frac{\partial^2 \psi(x,t)}{\partial x^2} + \cdots] d\eta.$$

Now use the Gaussian integrals

$$\int_{-\infty}^{\infty} e^{\frac{im\eta^2}{\hbar 2\epsilon}} d\eta = \sqrt{\frac{2\pi\hbar\epsilon i}{m}},$$

and

$$\int_{-\infty}^{\infty} \eta^2 e^{\frac{im\eta^2}{\hbar 2\epsilon}} d\eta = \sqrt{\frac{2\pi\hbar\epsilon i}{m}} \frac{\hbar\epsilon i}{m}.$$

This rewrites the Taylor series as follows.

$$\psi(x,t+\epsilon) = \frac{\sqrt{\frac{2\pi\hbar\epsilon i}{m}}}{A} e^{\frac{-i\epsilon V(x)}{\hbar}} [\psi(x,t) + \frac{\hbar\epsilon i}{2m} \frac{\partial^2 \psi}{\partial x^2} + O(x^2)].$$

Taking

$$A(\epsilon) = \sqrt{\frac{2\pi\hbar\epsilon i}{m}},$$

we get

$$\psi(x,t) + \epsilon \partial\psi(x,t)/\partial t = \psi(x,t) - \frac{i\epsilon}{\hbar} V(x)\psi(x,t) + \frac{\hbar i\epsilon}{2m} \partial^2\psi/\partial x^2.$$

Hence $\psi(x,t)$ satisfies the Schrödinger equation.

Witten's Integral

In [49] Edward Witten proposed a formulation of a class of 3-manifold invariants as generalized Feynman integrals taking the form $Z(M)$ where

$$Z(M) = \int DAe^{(ik/4\pi)S(M,A)}.$$

Here M denotes a 3-manifold without boundary and A is a gauge field (also called a gauge potential or gauge connection) defined on M. The gauge field is a one-form on a trivial G-bundle over M with values in a representation of the Lie algebra of G. The group G corresponding to this Lie algebra is said to be the gauge group. In this integral the action $S(M, A)$ is taken to be the integral over M of the trace of the Chern-Simons three-form $A \wedge dA + (2/3)A \wedge A \wedge A$. (The product is the wedge product of differential forms.)

With the help of the Wilson loop functional on knots and links, Witten writes down a functional integral for link invariants in a 3-manifold M:

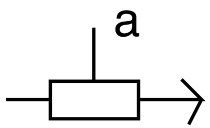
$$Z(M,K) = \int DAe^{(ik/4\pi)S(M,A)} tr(Pe^{\oint_K A})$$

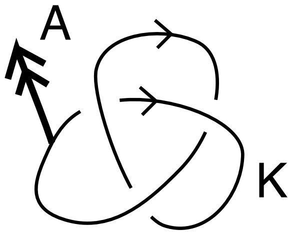
$$= \int DAe^{(ik/4\pi)S} < K|A > .$$

$$A(x) = A_a^k(x) T^a dx_k$$

The gauge field is a Lie-algebra valued one-form on 3-space.

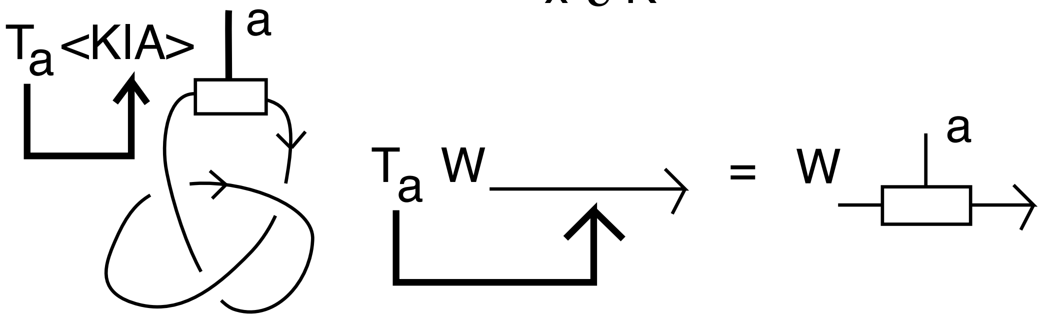
The next slide discusses the nature of the Wilson Loop.

$$V \xrightarrow{T^a} V \iff$$ 



$$W_K(A) = \langle K|A\rangle = \mathrm{tr}\left(P e^{\oint_K A}\right)$$

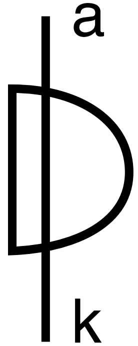
$$= \prod_{x\,\varepsilon\,K} \left(1 + A_a^i(x) T^a dx_i\right)$$



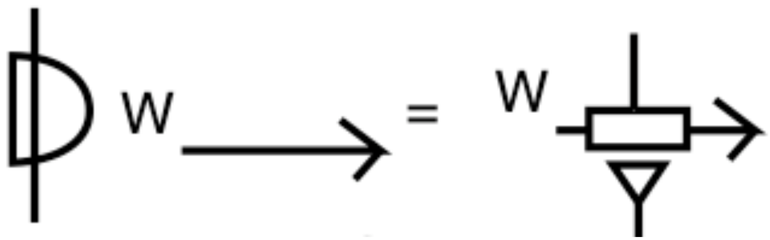
Think of a vector on the knot. As the base of the vector moves by dx the vector changes to (I + A)v. This is the analog of parallel translation. The gauge field is a connection!

$= dx_k$

$$W_K(A) = \langle K|A\rangle = \mathrm{tr}(P e^{\oint_K A}) = \prod_{x \,\varepsilon\, K} (1 + A_a^i(x) T^a dx_i)$$

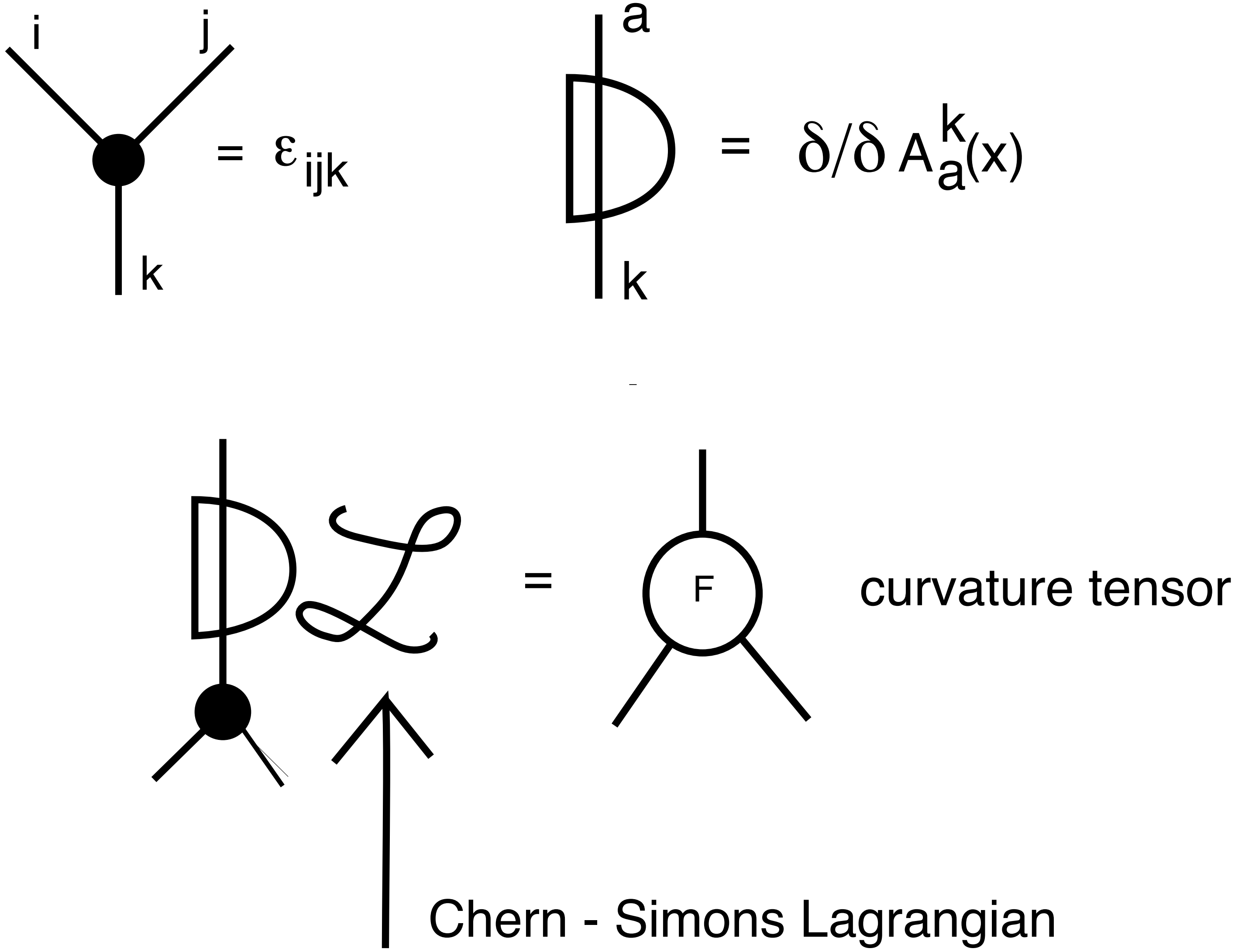


$= \delta/\delta A_a^k(x)$



This diagram defines a symbol for dx_k.

It shows the formula for differentiating a Wilson loop.

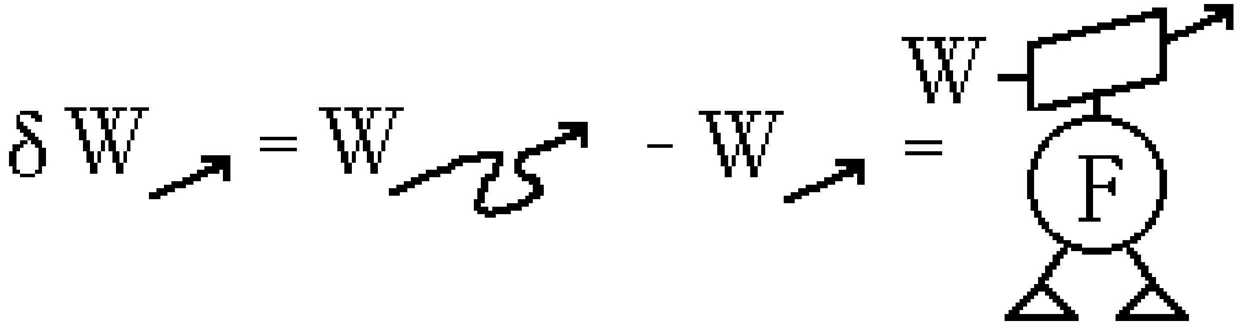


i
j
k
= ε ijk
a
k
= δ/δ A k a (x)
F
=
curvature tensor
Chern - Simons Lagrangian

Curvature is
$dA + A \wedge A$.

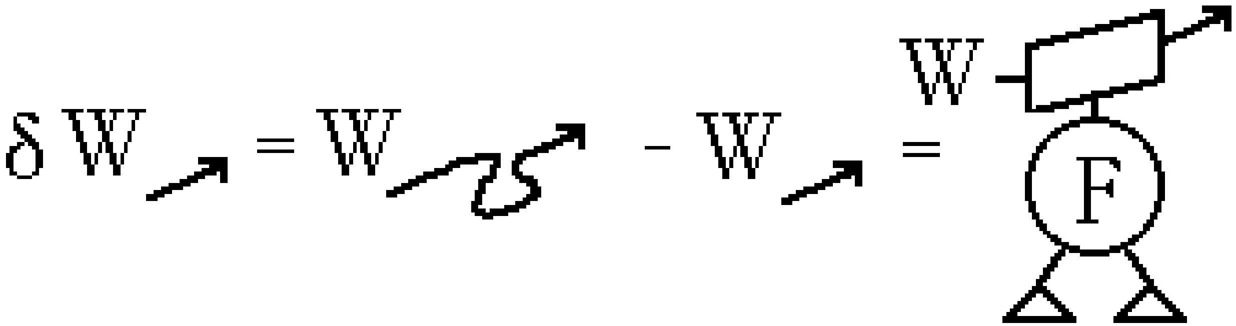
The Chern-Simons Lagrangian is
$L = A \wedge dA + (2/3) A \wedge A \wedge A$.

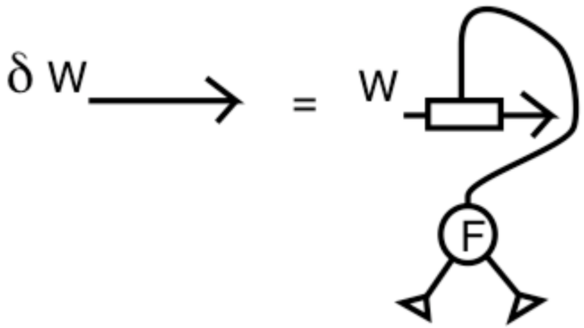
Differentiating L with respect to A
yields curvature.
(But you have to do it in detail to really see this.)



By an interesting calculation,
one finds that if you change the loop by a small amount,
then the Wilson loop changes by an insertion of
Lie algebra coupled with the curvature tensor.

This is just like classical differential geometry
where parallel translation around a small loop
measures curvature.

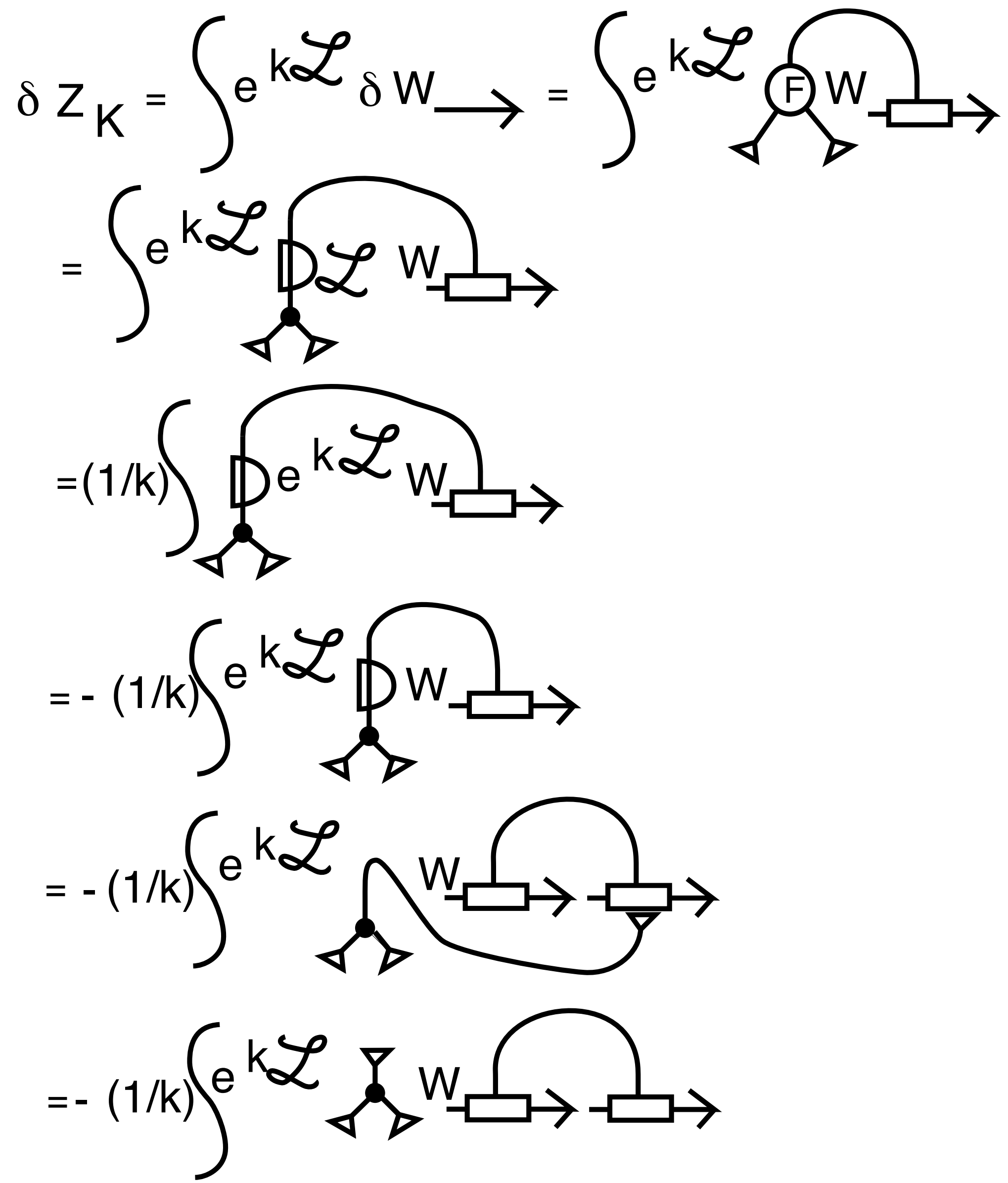




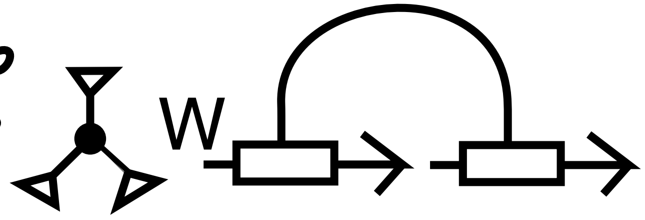
Curvature enters in when one evaluates the varying Wilson loop.

We can put all these facts together and find out how Witten's Integral behaves when we vary the loop.

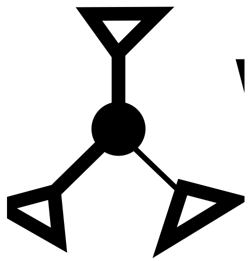
The next slide tells this story in Diagrams.


δ Z
K
=
e
kℒ
δ W
=
e
kℒ
F
W
=
e
kℒ
ℒ
W
=(1/k)
e
kℒ
W
= - (1/k)
e
kℒ
W
= - (1/k)
e
kℒ
W
=- (1/k)
e
kℒ
W

$$\delta Z_K = -(1/k) \int e^{k\mathcal{L}}$$



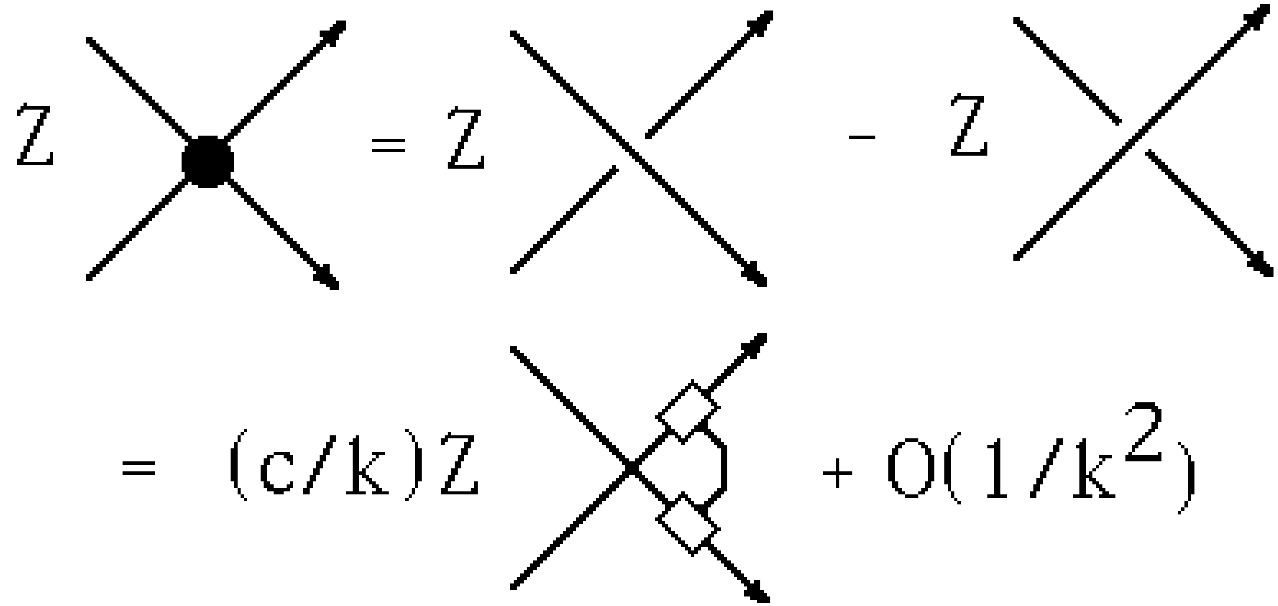
When you vary the loop,
Witten's integral changes by
the appearance of the volume form



and a double Lie algebra insertion.

There will be no change if the the
volume form is zero.
This can happen if the loop deformation
does not create volume.
That is the case for the
second and third Reidemeister moves
since they are "planar".

Hence we have shown (heuristically) that
Z_K is an invariant of "regular isotopy"
just like the bracket polynomial.



This is what happens when you switch crossings.
You get a "skein relation" involving Lie algebra insertions.

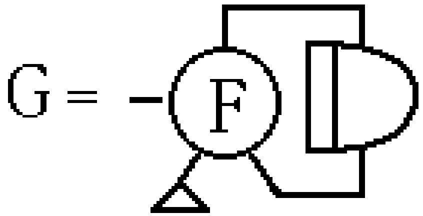
This formula leads directly to the subject of Vassiliev invariants, but we will not discuss that in this talk.

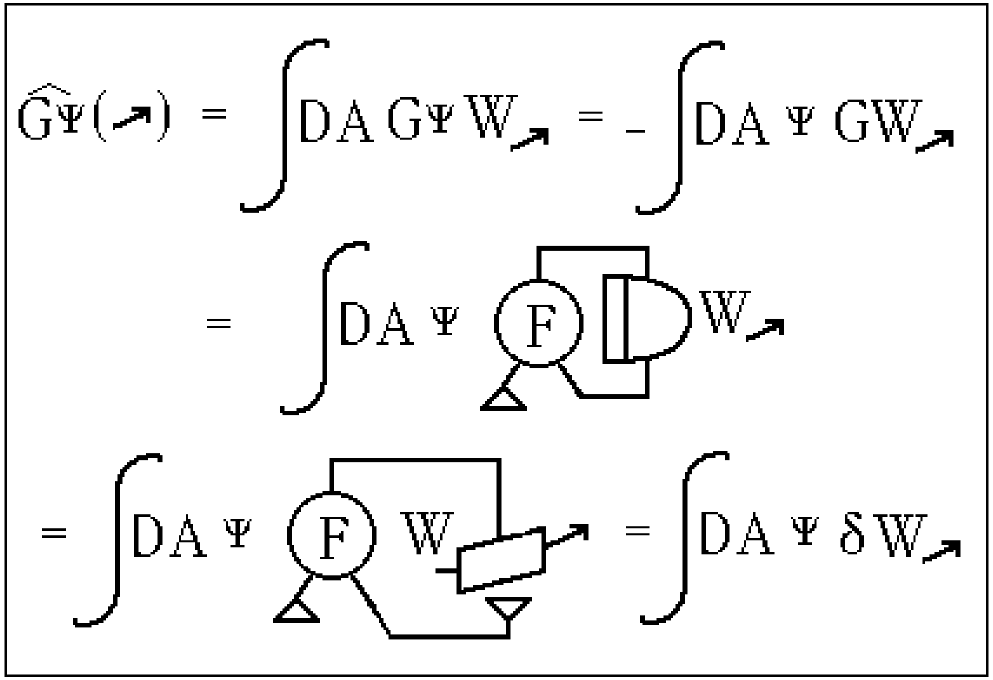
$$\hat{\Psi}(K) = \int DA\, \Psi(A)\, W_K$$

$$\widehat{\Delta\Psi}(K) = \int DA\, \Delta\Psi(A)\, W_K$$

$$= -\int DA\, \Psi(A)\, \Delta W_K$$

The Loop Transform: Start with a function defined on gauge fields. Integrate it against a Wilson loop and get a function defined on knots.
Transform differential operations from the category of functions on gauge fields to the category of functions on knots.



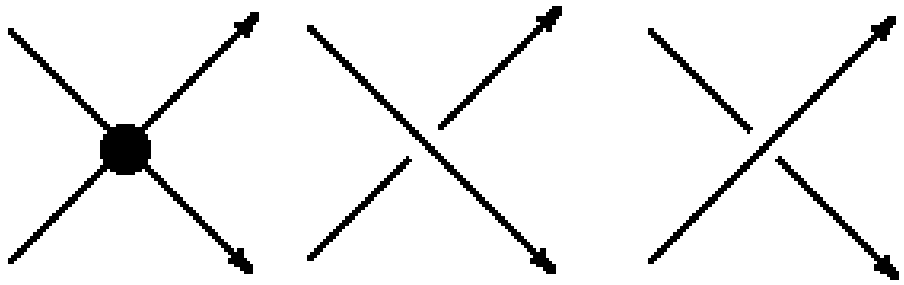


This differential operator occurs in the loop quantum gravity theory of Ashtekar, Rovelli and Smolin.

Its transform is the geometric variation of the loop!

The loop transform enabled
Ashtekar, Rovelli and Smolin to
see that the exponentiated
Chern-Simons Lagrangian could be seen
as a state of quantum gravity
and
that knots are fundamental
to this approach to a theory of
quantum gravity.

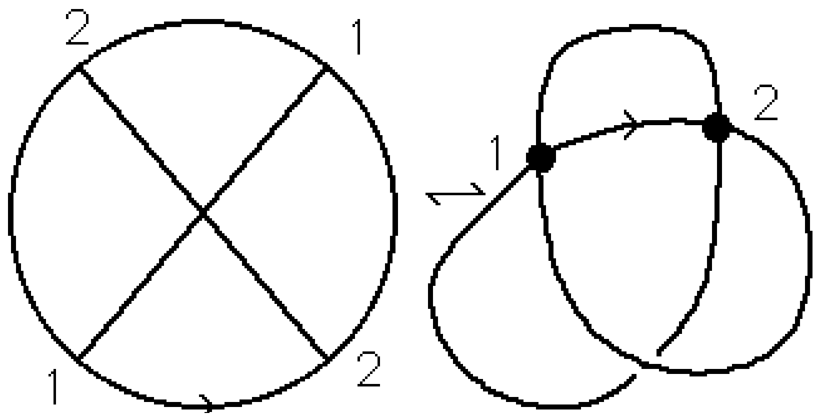
Knots, Links and Lie Algebras
Vassiliev Invariants



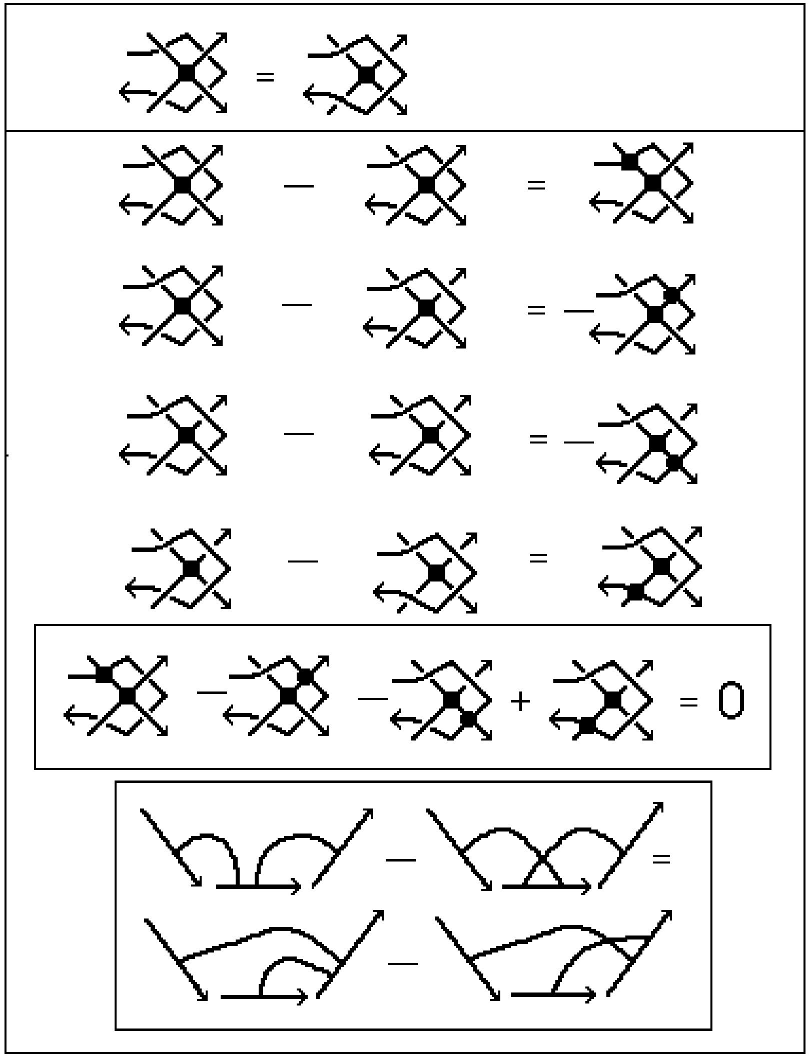
$v(K|*) = v(K|+) - v(K|-)$

Skein Identity

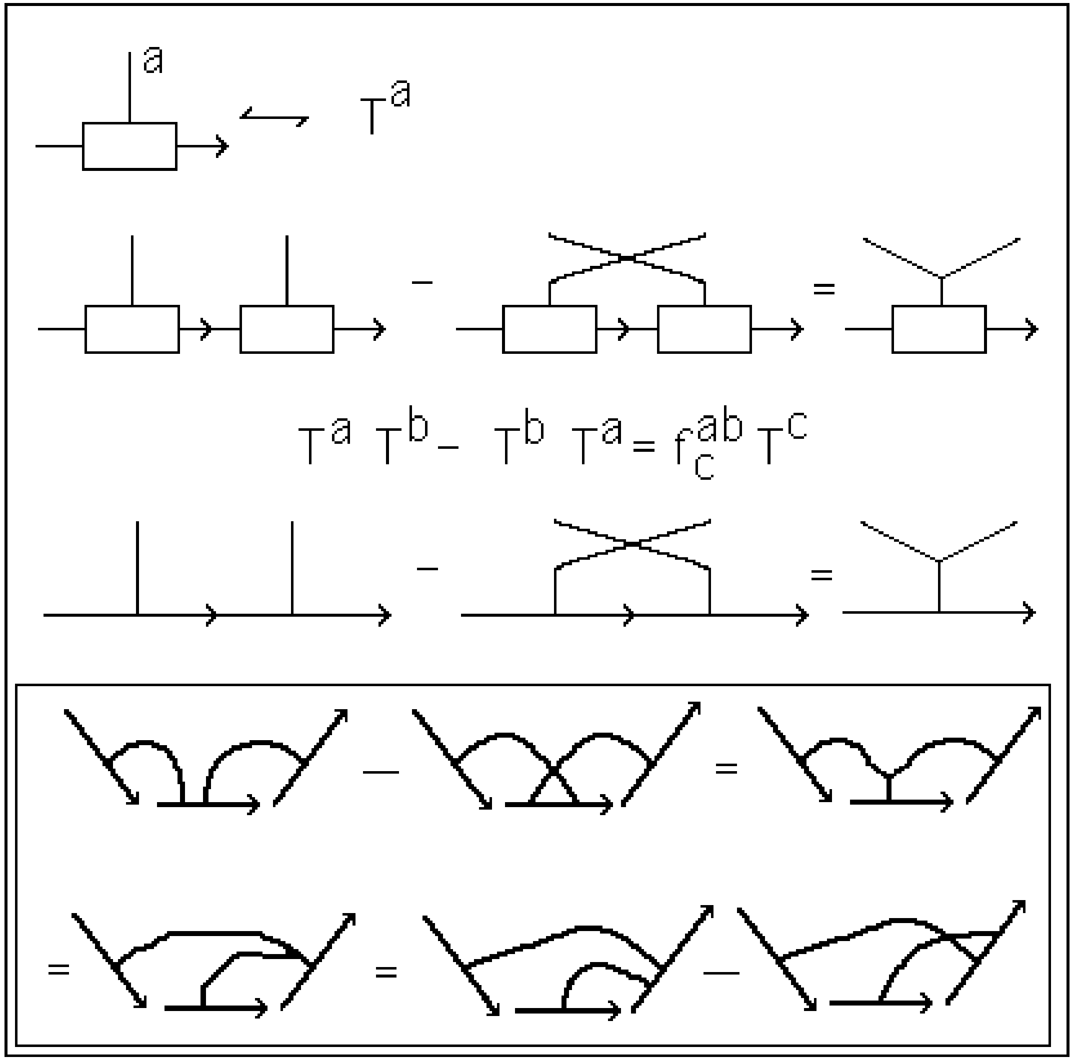
Chord Diagram



Four-Term Relation From Topology



Four Term Relation from Lie Algebra



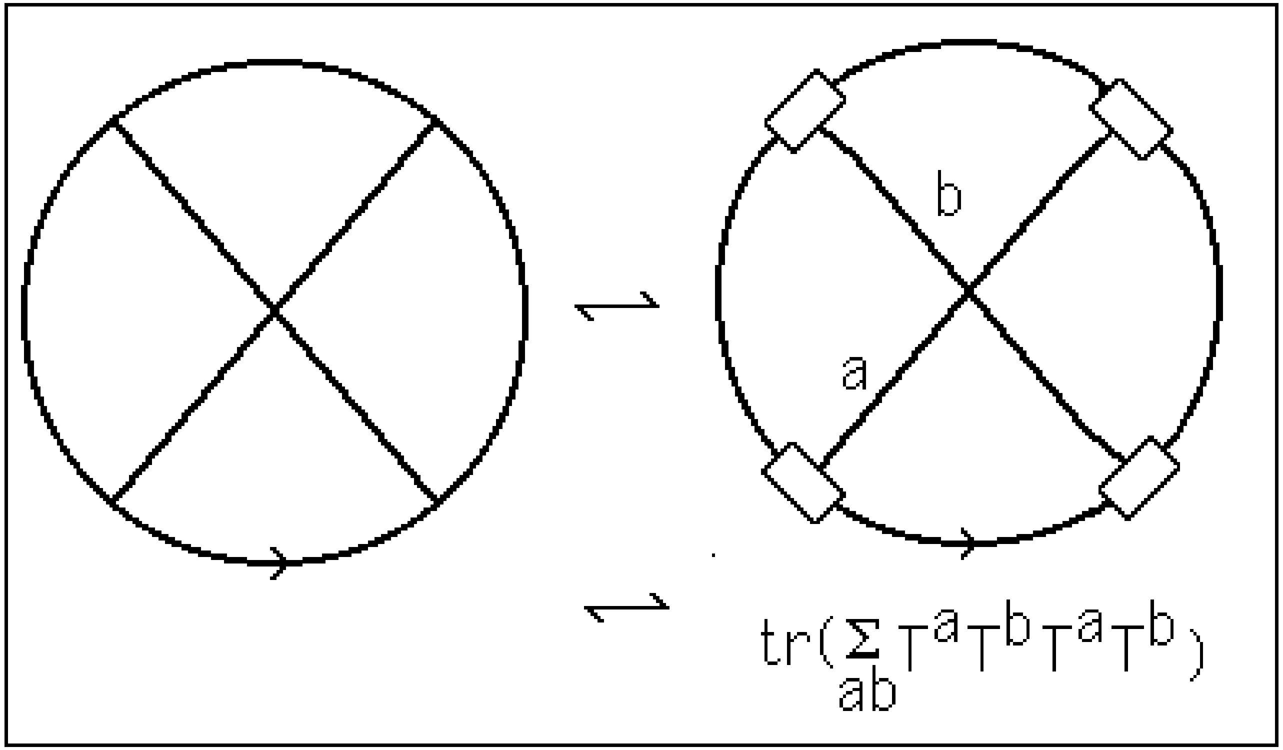
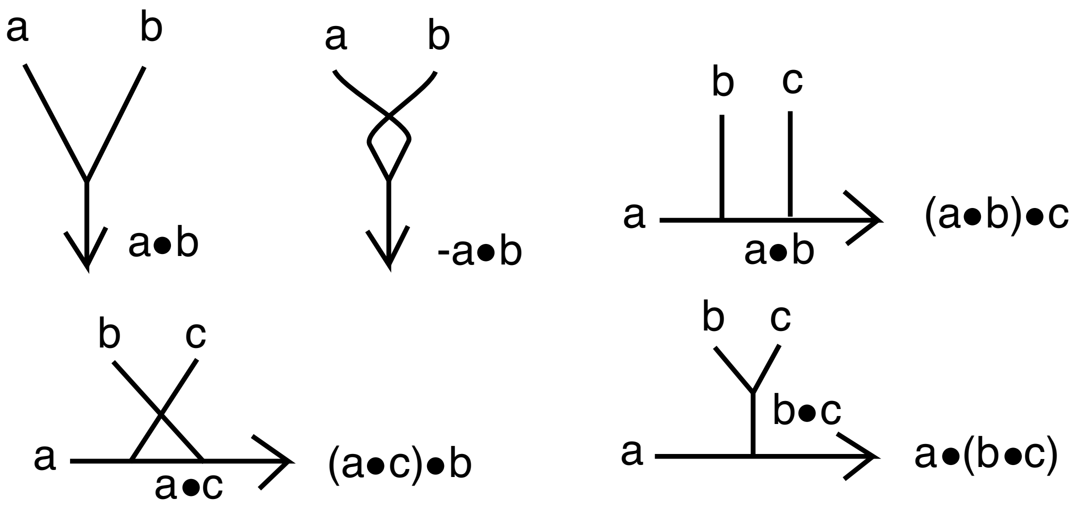
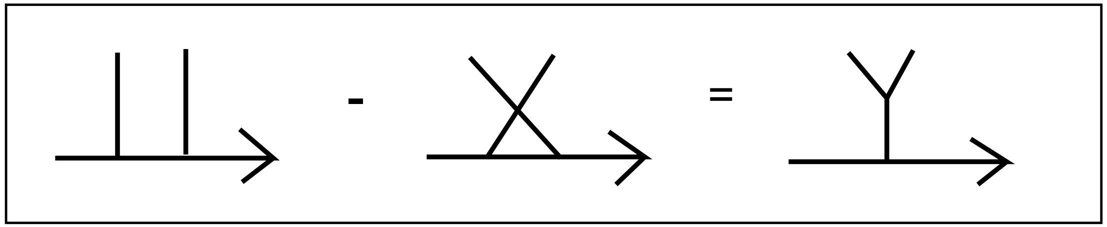


FIGURE 12. **Calculating Lie Algebra Weights.**

The Jacobi Identity





(a•b)•c   - (a•c)•b = a•(b•c)
Hence
(a•b)•c + b•(a•c) = a•(b•c).

Lie algebras and Knots are linked
through the Jacobi Identity.

This is part of a mysterious
connection
whose roots we do not yet fully
understand.