Numerical Relativity beyond General Relativity
status, challenges and new directions

Helvi Witek

Department of Physics & Illinois Center for Advanced Studies of the Universe
University of Illinois at Urbana-Champaign

presentation with Luis Lehner

GREFT20: Probing Effective Theories of Gravity in Strong Fields and Cosmology
27 August 2020

1. **Numerical (General) Relativity in a nutshell**
2. **Status: Numerical Relativity beyond the standard model**
3. **Interludium I: new directions and approaches – a discussion**
4. **Part II: Roadblocks and a potential way through (Luis Lehner)**

Numerical (General) Relativity

$$=$$

Solve

$$G_{\mu\nu} = 16\pi T_{\mu\nu}$$

in $3+1$ dimensions

on High-Performance Computing Facilities

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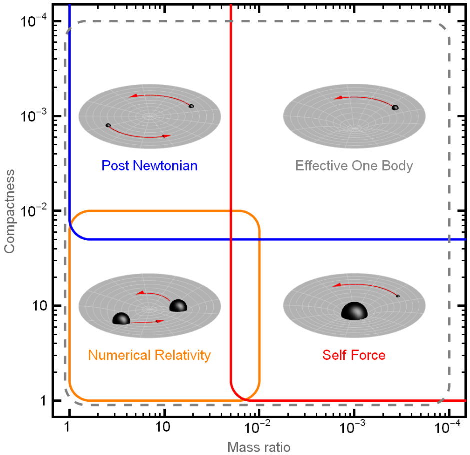
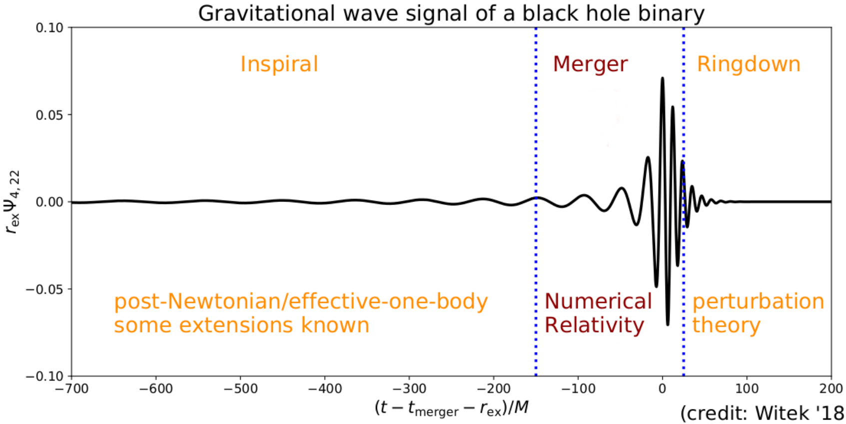
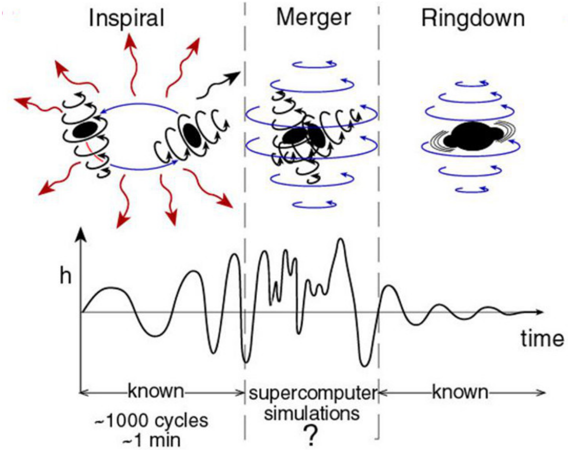
The realms of Numerical Relativity



(https://www.wikiwand.com/en/Two-body_problem_in_general_relativity)



(credit: Witek '18)

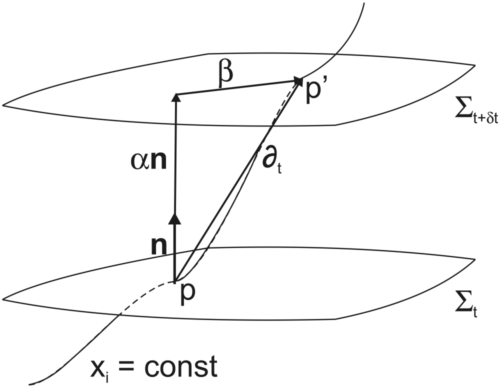


(credit: Thorne; pre-NR breakthrough by Pretorius '05)

Numerical General Relativity in a nutshell

- make space and time dependence explicit: foliation of spacetime $\mathcal{M} = \Sigma_t \times R$
- 4D metric $g_{\mu\nu} \longrightarrow$ 3D metric γ_{ij}, lapse α, shift β^i
- extrinsic curvature $K_{ij} = -\gamma^{\mu}{}_{i}\gamma^{\nu}{}_{j}\nabla_{\mu}n_{\nu} = -\frac{1}{2}\mathcal{L}_n\gamma_{ij}$

  $\Rightarrow$ kinematic evolution equation $\partial_t\gamma_{ij} \simeq -2\alpha K_{ij}$



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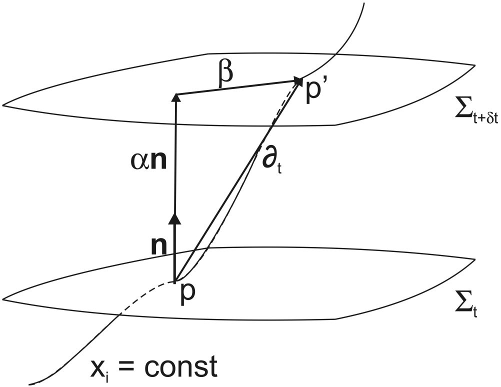

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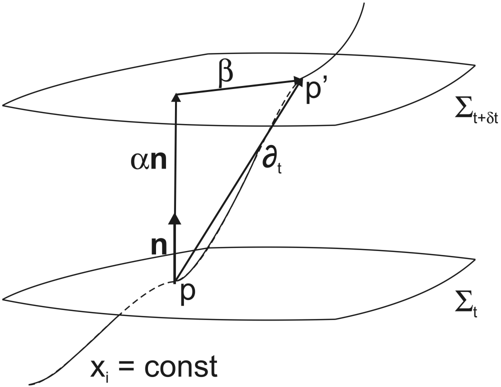

Numerical General Relativity in a nutshell

Dynamics: decompose field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 16\pi T_{\mu\nu}$$

geometry matter

Constraint equations
- elliptic PDEs
- initial conditions

Evolution equations
- hyperbolic (time evolution) PDEs
- well-posed initial value formulation
- supplement with gauge conditions

Diagnostic tools
- Wave extraction
- (apparent) horizon finder

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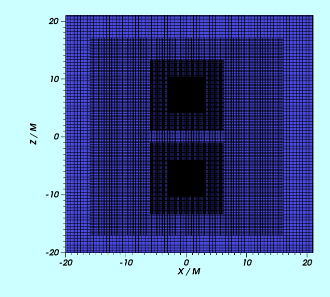
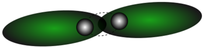


Technical implementation

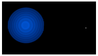
- numerical schemes
  (e.g. method of lines with finite differences and RK time integrator)
- adaptive mesh refinement
- high performance computing

Numerical Relativity
beyond the standard model

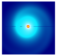
Black holes as cosmic particle detectors



Black hole mimickers boson stars



Testing the nonlinear regime of gravity



Numerical Relativity in modified gravity

$$\begin{aligned}
\mathcal{L} = \quad & f_0(\phi)R - \omega(\phi)(\nabla\phi)^2 - V(\phi) + \mathcal{L}_{\rm M}[\Psi, A^2(\phi)g_{ab}] && \text{scalar-tensor theory} \\
& + f_1(\phi)\left(Riem^2 - 4R_{ab}R^{ab} + R^2\right) + f_2(\phi)\,{}^*R_{abcd}R^{abcd} && \text{quadratic gravity} \\
& + \ldots && \text{Horndeski} \\
& + \ldots && \text{Lorentz violation} \\
& + \ldots &&
\end{aligned}$$

Numerical Relativity in modified gravity

black-hole formation

- Einstein-æther theory (Garfinkle et al '07)
- Gauss–Bonnet gravity (Benkel, Sotiriou, **HW** '16; Ripley & Pretorius '19, '20, Dima et al '20)
- Horndeski gravity (Ripley & Pretorius '19, Bernard et al '19, Figueras & França '20)
- (massive) scalar-tensor theory (Gerosa et al '16, Sperhake et al '17, Rosca-Mead et al '19, '20)

compact binaries

- scalar-tensor theory (Barausse et al '12, Shibata et al '13, Healy et al '11, Berti et al '13)
- Einstein-Maxwell-Dilaton models (Hirschmann et al '17)
- dynamical Chern-Simons gravity (Okounkova et al '17 – '19)
- scalar Gauss–Bonnet gravity (**HW** , L. Gualtieri, P. Pani, T. P. Sotiriou '18, Okounkova '20)

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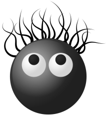
Example: NR in scalar Gauss–Bonnet gravity

Example: scalar Gauss-Bonnet gravity (sGB) (Kanti et al '95)

$$G_{\mu\nu} = \frac{1}{2} T^{\Phi}_{\mu\nu} - \frac{\alpha_{\rm GB}}{4} \mathcal{G}^{\rm GB}_{\mu\nu} \,, \qquad \Box\Phi = -\frac{\alpha_{\rm GB}}{8} f'(\Phi) \mathcal{R}_{\rm GB}$$

with $\mathcal{R}_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} \sim (\partial\partial g_{\mu\nu})^2$

- low-energy limit of quantum gravity candidates
- black holes with scalar hair for $f(\Phi) \sim \Phi$
  (Kanti et al '95, Pani et al '09, '11, Stein et al '11, Sotiriou & Zhou '14, Ayzenberg & Yunes '14, Maselli et al '15, Benkel, Sotiriou, **HW** '16, '17; **HW**, Gualtieri, Pani, Sotiriou '19)
- black hole scalarization for $f(\Phi) \sim \Phi^2$
  (Silva et al '17; Doneva et al '17; Antoniou et al '17; Silva et al '18; Macedo et al '19; Doneva et al '19; Ripley & Pretorius '20; Dima et al '20)



Example: NR in scalar Gauss–Bonnet gravity

- well-posed initial value formulation necessary for numerical stability
  (for higher derivative theories see Choquet-Bruhat '88; Delsate, Hilditch, **HW** '14; Papallo & Reall '17; J. Cayuso et al '17; Allwright & Lehner '18; Kovacs & Reall '20; Kovacs '20; R. Cayuso & Lehner '20)
- full 3+1 formulation? double-valued Hamiltonian (Julié & Berti '20; **HW** , Gualtieri, Pani '20)
- expansion in coupling $\epsilon = \alpha_{\rm GB}/\ell^2 \ll 1$ (see Okounkova et al '17 for dCS)

$$\epsilon^0 : \; G^{(0)}_{ab} = \frac{1}{2} T^{(0)}_{ab} \qquad \Box^{(0)}\Phi^{(0)} = 0 \qquad \Rightarrow \; (g^{(0)}_{ab}, \Phi^{(0)}) = (g^{\rm GR}_{ab}, 0)$$

$$\epsilon^1 : \; G^{(1)}_{ab} = 0 \qquad \Box^{(0)}\Phi^{(1)} = -f' \mathcal{R}^{(0)}_{\rm GB} \qquad \Rightarrow \; (g^{(1)}_{ab}, \Phi^{(1)}) = (0, \Phi^{(1)})$$

$$\epsilon^2 : \; G^{(2)}_{ab} = \frac{1}{2} T_{\rm eff}(g^{(0)}_{ab}, \Phi^{(1)}) \qquad \Box^{(0)}\Phi^{(2)} = -\frac{M^2}{4} f'_{(1)} \mathcal{R}^{(0)}_{\rm GB}$$

Discussion appetizer (1 slide to go)

- $+$ structure applicable to theories with EFT-type expansion
- $+$ pathway towards "parametrized numerical relativity"
- - missing physics? nonlinear effects?
- - validity?

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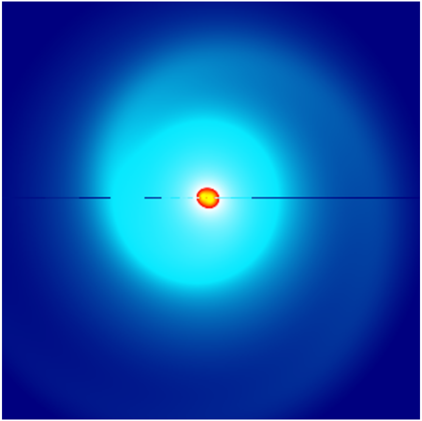
Example: NR in scalar Gauss–Bonnet gravity

Proof-of-principle: evolution of hairy black holes



(**HW**, Gualtieri, Pani, Sotiriou '19)

Interludium I: Discussion

Guiding principle?

- Developing the numerical infrastructure takes 1-2 years per theory
- Covering parameter space takes years and large amount of resources!
  (black hole mass ratio, spins; coupling parameters; . . .)
- Which extensions of the standard model are scientifically most interesting?
  - physical motivation from cosmology / HEP?
  - existence, uniqueness and stability of solutions? Is flat space stable?
- Which extensions of the standard model are practical from NR perspective?
- Which extensions represent classes of theories and capture generic features?

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Methodologies?

- "Parametrized" versus theory-specific numerical relativity?
- effective-field theory approach versus "full theory"?
  - EFT $\rightarrow$ hyperbolic equations with sources
  - What might we miss in EFT?
  - Nonlinear effects, treatment of "secular" effects, validity?
- well-posedness of specific theory?
  - Maths: well-posed initial value formulation
  - Physics: well-defined (theoretically well-posed) model
- reformulations of higher derivative theories inspired by hydrodynamics (Luis' discussion)

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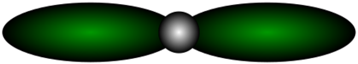
Appendix

Black holes as cosmic particle detectors

(Press & Teukolsky '72; Damour et al '76; Detweiler '80; Zouros & Eardley '79; Cardoso et al '05; Dolan '07; Rosa & Dolan '11; Pani et al '12; **HW** et al '12; Dolan '12; Shlapentokh-Rothman '14; Okawa, **HW** et al '14; Brito et al '15; Zilhao, **HW** et al '15; Moschidis '16; East '17, '18; Frolov et al '18; Dolan '18; Ficarra, Pani, **HW** '19, Baumann et al '18, '19, Herdeiro et al '19, Siemonsen & East '19, Creci, Vandoren, **HW** '20, . . .)

- superradiant instability
  $\rightarrow$ formation of bosonic condensates



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BHs ($M_\odot$):	10^{10}	10^{6}	10^{3}	50	3
μ (eV):	10^{-21}	10^{-17}	10^{-14}	10^{-12}	10^{-11}

- axion-like particles as dark matter candidates, string axiverse
  (Peccei & Quinn '77, Arvanitaki & Dubovsky '10, '11, Kodama & Yoshino '11, Hui et al '16, Baumann et al '18, '19, . . .)
- "hairy" black-holes (Herdeiro et al '14, Hui et al '19, Clough et al 19, . . .)
- Any ultra-light bosonic field coupled to gravity
  $\Rightarrow$ black holes as probe for BSM particles complementary to traditional colliders

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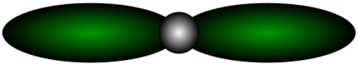

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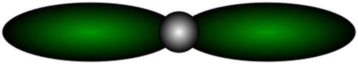

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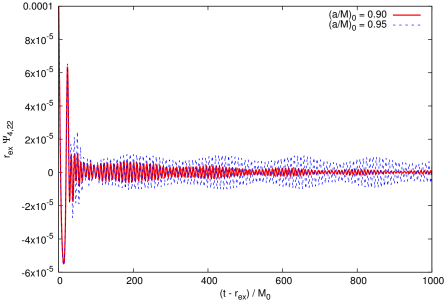
Black holes as cosmic particle detectors

Observable signatures:

- gaps in spin–mass phase space of black hole population
  (Arvanitaki et al '09, '10; Pani et al '12; Brito et al '15-'20; Ficarra et al '18; ...)
- black hole shadow (Herdeiro et al '19; Creci, Vandoren, **HW** '20,...)
- gravitational waves with $f_{22} \sim 20 \left[\frac{M}{M_\odot}\right]^{-1}$ kHz

  (Arvanitaki et al '14; Yoshino et al '13; Okawa, **HW**, Cardoso '14; Zilhão, **HW** '15, East et al '17-'20)

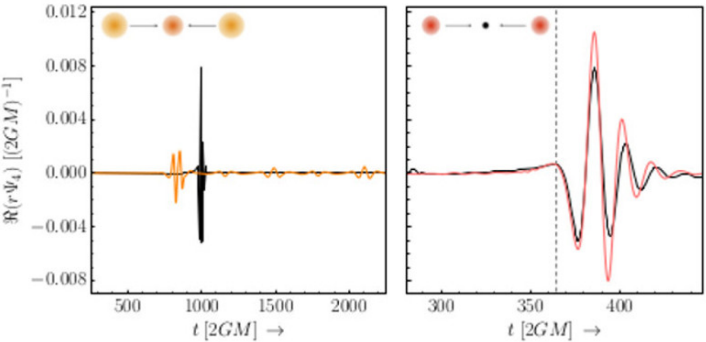


(**HW** & Zilhão; code: Einstein Toolkit & Canuda)

- Ongoing: BH binary evolution
  (Baumann et al '18 – '20, Wong et al '19, Hang & Zhang '19, Berti et al '19...)

Did we really detect the black holes of GR?

- (scalar) boson stars (Liebling & Palenzuela '12, Palenzuela et al '17, Helfer et al '18, Bezares et al '18, Alcubierre et al '19, ...),
- Proca stars (Sanchis-Gual et al '18)
- black holes with near-horizon fluctuations (Liebling et al '17)
- axion stars and black holes or neutron stars (Clough, Dietrich et al '18)



Example: boson stars (Helfer et al '18)