Black holes, fuzzballs and foam

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P. Berglund, E.G. Gimon and TSL, hep-th/0505167

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Motivation

- The fuzzball conjecture offers a promising approach to the black hole information paradox

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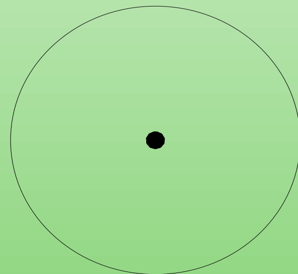
- The fuzzball conjecture offers a promising approach to the black hole information paradox
- So far, only geometries for two-charge microstates have been found. These geometries have classically vanishing horizon area.
- Want to find geometries that could be microstates for three-charge black holes and the new black ring solutions, which have classically finite horizons.
- Want to find geometries that could be microstates for four-dimensional black holes
- Connect the old picture of D-brane state counting with the new fuzzball picture
- Offer fresh insights into ideas surrounding quantum foam and geometric transitions
- Find new smooth, stable SUGRA backgrounds

Outline

1. The fuzzball hypothesis
2. The Bena-Warner ansatz
3. Solving the equations for a three-charge system and global constraints
4. Reduction to IIA
5. Reduction to 4D and special geometry
6. Scaling and quivers
7. Discussion and conclusions

The fuzzball hypothesis

- In the usual black hole picture we have a horizon, empty space and all interesting physics concentrated at the singularity



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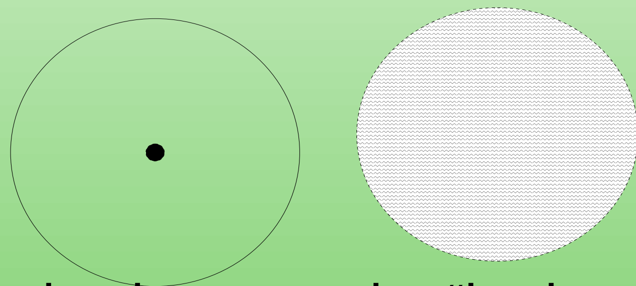

- In the fuzzball, the region between the "horizon" and the singularity is not empty. Instead there is interesting geometry and physics in this region

- The singular black hole geometry with a horizon is an emergent phenomenon that results from coarse graining. Each microstate is smooth and horizon free.

More on fuzzballs

- Each microstate looks the same asymptotically. Closer in we see differences
- Our three-charge solutions will replace a core region of singular brane sources with a geometric transition to a bubbling foam of two-cycles threaded by flux
- The intricate geometry of these cycles will distinguish individual microstates
- Along the way we will find rules for arranging the cycles
- We will reduce to 4D and show how to connect the picture of D-brane state counting with microstates via a smooth running of g_s

The Bena-Warner ansatz

- We utilize an ansatz due to Bena-Warner for 3-charge, 1/8 BPS solutions in 5D
- The setup is M-theory on a T^6 with 3 stacks of M2-branes wrapped on each 2-cycle. These will induce M5-brane dipole charge
- The 5D space is time fibred over a hyperkahler base space, HK

$$ds_{11}^2 = -(Z_1 Z_2 Z_3)^{-2/3}(dt+k)^2 + (Z_1 Z_2 Z_3)^{1/3} ds_{HK}^2 + ds_{T^6}^2,$$

$$\begin{aligned} ds_{T^6}^2 &= (Z_1 Z_2 Z_3)^{1/3}\left(Z_1^{-1}(dz_1^2 + dz_2^2) + Z_2^{-1}(dz_3^2 + dz_4^2) + Z_3^{-1}(dz_5^2 + dz_6^2)\right). \\ ds_{HK}^2 &= H^{-1}\sigma^2 + H(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2), \\ \sigma &= d\tau + f_a dx^a,\ \star_3 d\sigma = dH,\ \tau \sim \tau + 4\pi \end{aligned}$$

- The C-field is given by

$$\begin{aligned} C_{(3)} &= -(dt+k)\left(Z_1^{-1}\, dz_1 \wedge dz_2 + Z_2^{-1}\, dz_3 \wedge dz_4 + Z_3^{-1}\, dz_5 \wedge dz_6\right) \\ &\quad +2\, a^1 \wedge dz_1 \wedge dz_2 + 2\, a^2 \wedge dz_3 \wedge dz_4 + 2\, a^3 \wedge dz_5 \wedge dz_6. \end{aligned}$$

Bena-Warner ansatz continued

Define $G^i = da^i$. The BW ansatz solves the EOM if

$$
\begin{aligned}
G^i &= \star G^i, \\
d \star dZ_i &= 2 s_{ijk} G^j \wedge G^k, \\
dk + \star dk &= 2 G^i Z_i.
\end{aligned}
$$

Where $s^{ijk} = |\epsilon^{ijk}|$ is the symmetric tensor and the Hodge dual is only on HK.

Solving the EOM

- We can solve the EOM using 8 harmonic functions ($r_p = |\vec{x} - \vec{x_p}|$, $i = 1 \ldots 3$)

$$H = \sum_{p=1}^{N} \frac{n_p}{r_p}, \qquad M_i = 1 + \sum_{p=1}^{N} \frac{Q_i^p}{4r_p}, \qquad K = l_0 + \sum_{p=1}^{N} \frac{l_p}{r_p}, \qquad h^i = \sum_{p=1}^{N} \frac{d_p^i}{4r_p}$$

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- Notice that the poles of each harmonic function overlap, this is necessary for the solution to be smooth
- With these harmonic functions we can solve for all quantities relevant for the SUGRA solution

$$\begin{aligned}
Z_i &= M_i + 2s_{ijk}h^j h^k/H, \\
a^i &= (h^i/H)\sigma + a^i_a dx^a,\ d(a^i_a dx^a) = -\star_3 dh^i, \\
k &= k_0\,\sigma + k_a\,dx^a, \qquad k_0 = K + 8H^{-2}\,h^1\,h^2\,h^3 + H^{-1}\,M_i\,h^i \\
d(k_a dx^a) &= H \star_3 dK - K \star_3 dH + h^i \star_3 dM_i - M_i \star_3 dh^i
\end{aligned}$$

Constraints

- For our solutions to be smooth, the various charges in the harmonic functions cannot all be independent
- To ensure smoothness we must have

$$Q_i^p = -s_{ijk}\frac{d_p^j d_p^k}{2n_p}, \quad l_p = \frac{d_p^1 d_p^2 d_p^3}{16 n_p^2}, \quad l_0 = -\sum_i \frac{s^i}{4}, \quad s^i = \sum_p d_p^i$$

- We must also insure that $d^2(k_a dx^a) = 0$. To display this condition, its easiest to use the new variables

$$\tilde{\lambda}_p^i = (d_p^i/n_p - s^i), \qquad \Gamma_{pq} = \frac{\prod_i (n_p d_q^i - n_q d_p^i)}{n_p^2 n_q^2}$$

- This condition can then be written (we call this the “bubble equation”)

$$4\sum_i n_p \tilde{\lambda}_p^i + \sum_{q=1}^{N} \frac{\Gamma_{pq}}{r_{pq}} = 0, \qquad p = 1...(N-1)$$

Zeros of the Z_i

- To avoid singularities we need the determinant of the metric and its inverse to be well defined and non-vanishing

$$\sqrt{-g_{11}} = (Z_1 Z_2 Z_3)^{1/3} \, H \, \sqrt{g_{\mathbf{R}^3}}$$

- We see that to avoid singularities we need $Z_i \neq 0$. Our simple tactic for enforcing this is to everywhere demand

$$Z_i H > 0 \qquad \forall i \in 1, 2, 3$$

CTCs

- To exclude CTCs in our 5D reduced space we require our spacetime to be *stably causal*
- For a spacetime to be stably causal it must admit a globally defined, smooth function whose gradient is everywhere timelike. We call this a *time function*
- Our candidate function is simply the coordinate t, which is a time function if

$$-g^{\mu\nu}\partial_\mu t\partial_\nu t = -g^{tt} = (Z_1Z_2Z_3)^{-1/3}\,H^{-1}\Big((Z_1Z_2Z_3)H - H^2\,k_0^2 - g^{ab}_{\mathbf{R}^3}k_ak_b\Big) > 0$$

- In general, this is a complicated function and we have not analyzed this in detail. It is possible that this will place further constraints on the relative pole positions
- This condition implies $Z_iH > 0$ and also guarantees there are no horizons
- Avoiding Dirac strings will lead to quantization of the d^i_p

Asymptotic charges

- By looking at the asymptotic behavior of the metric and C-field we can read off the expressions for the total membrane charge and $SU(2)_L \times SU(2)_R$ angular momenta

$$Q_i = -\frac{1}{2}\sum_{p=1}^{N} n_p s_{ijk} \lambda_p^j \lambda_p^k, \qquad J_R = \sum_{p=1}^{N} n_p \lambda_p^1 \lambda_p^2 \lambda_p^3,$$

$$J_L = 4|\sum_{p=1}^{N}\sum_i n_p \lambda_p^i \vec{x}_p| = \frac{1}{2}\left|\sum_{pq} \Gamma_{pq} \frac{\vec{x}_p - \vec{x}_q}{|\vec{x}_p - \vec{x}_q|}\right|$$

- Note that while J_L depends on the position of the poles, J_R does not. This is due to the $U(1)_R$ isometry generated by ∂_τ. Later on, we'll use this isometry to reduce our solutions to 4D

General features

- The geometry is characterized by a set of regular 2-cycles S_{pq} coming from the fiber σ over each interval from $\vec{x}_p$ to $\vec{x}_q$. The bubble equation tells us how these bubbles can be arranged based on the flux through them.
- All brane sources have vanished and been replaced by flux on cycles $\Rrightarrow$ geometric transition
- A generic microstate will have a large number of poles. The geometry will be a foam of 2-cycles with an overall expected size of the representative black hole horizon (this needs to be worked out!)
- We have solved the EOM and insured smoothness

Summary of conditions

- Our solution is completely parameterized by a set of poles on $\mathbf{R}^3$ with quantized residues n_p and quantized fluxes d_p^i
- These and the quantities that depend on them must satisfy the following conditions for us to have a smooth (up to orbifold points) and regular solution free of CTCs and horizons to 11D SUGRA with three membrane charges and 4 supersymmetries:

$$1) \quad 4\sum_i n_p \tilde{\lambda}_p^i + \sum_{q=1}^{N} \frac{\Gamma_{pq}}{r_{pq}} = 0,$$

$$2) \quad (Z_1 Z_2 Z_3) H - H^2 k_0^2 - g_{\mathbf{R}^3}^{ab} k_a k_b > 0$$

Reduction to IIA

- We can reduce to 4D along the τ direction by placing the geometry in Taub-Nut (this insures we have a finite circle at infinity). We do this by adding a constant to H

$$H \rightarrow H + \delta H, \quad \delta H = 4/L^2, \quad L = g_s l_s$$

- We can also add constants to the 7 other harmonic functions ($\delta M_i,\ \delta h^i,\ \delta K$), not all of which will be independent since we must make sure that the metric and C-field have the right asymptotic behavior
- We can now reduce along τ to a 10D IIA solution in 4 non-compact directions

The reduction

Defining dimesionless harmonic functions and new radial coordinate $\rho = 2r/L$

$$M_0 = -HL^2/4, \qquad K^0 = 4K/L, \qquad K^i = Lh^i$$

the reduction gives (ds_3^2 is now in the conventional form)

$$ds^2_{IIA} = -J_4^{\frac{-1}{2}}(dt + k_a dx^a)^2 + J_4^{1/2}\Big(ds_3^2 + (-Z_i M_0)^{-1} ds^2_{T_i}\Big)$$

$$e^{2\Phi} = (J_4)^{3/2}(-ZM_0)^{-3}, \qquad B_2 = -\left(\frac{K^i}{M_0} + \frac{2k_0}{L\, Z_i}\right) dV_i$$

$$C_1 = \frac{L}{2} f_a dx^a - \frac{2M_0^2 k_0}{LJ_4}(dt + k_a dx^a)$$

$$C_3 = \left[- Z_i^{-1}(dt + k_a dx^a) + 2\vec{a}^i - \left(\frac{K^i}{M_0} + \frac{2k_0}{L\, Z_i}\right)\frac{L}{2} f_a dx^a\right] \wedge dV_i$$

The reduction cont.

- J_4 is the quartic invariant of $E_{7(7)}$

$$
\begin{aligned}
J_4 &= M_0K^0(M_iK^i) + M_1K^1(M_2K^2 + M_3K^3) + M_2K^2M_3K^3 \\
&- \frac{1}{4}(M_\alpha K^\alpha)^2 - M_0M_1M_2M_3 - K^0K^1K^2K^3, \qquad \alpha \in 0 \ldots 3
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- The reduction alters the bubble equation. It becomes

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\psi_p + \frac{2}{L}\sum_q \frac{\Gamma_{pq}}{\rho_{pq}} = 0, \quad \psi_p = \sum_i n_p\lambda_p^i - \frac{1}{L^2n_p^2}\prod_i n_p\lambda_p^i, \quad \lambda_p^i = \frac{d_p^i}{n_p} - L^2\,\delta h^i
$$

Asymptotic charges

We now have a solution of IIA in 4 non-compact directions with $0, 2, 4, 6$-brane charges. We can read off the angular momentum and quantized charges

$$
\begin{aligned}
J &= \frac{1}{2}|\sum_{p,q} \Gamma_{pq}\hat{r}_{pq}|, \\
Q_0^{D6} &= \frac{L}{2}\sum_p (-n_p) = \frac{g_s l_s}{2} N_6, \quad Q_i^{D2} = -\frac{1}{2L}\sum_p s_{ijk}\frac{d_p^j d_p^k}{2n_p} = -\frac{4G_4\, V_i}{4\pi^2 g_s l_s^3} N_{i\,D2}, \\
Q_{D0}^0 &= \frac{1}{2L^2}\sum_p \frac{d_p^1 d_p^2 d_p^3}{n_p^2} = \frac{4G_4}{g_s l_s} N_0, \quad Q_{D4}^i = \frac{1}{2}\sum_p d_p^i = 2\pi^2\frac{g_s l_s^3}{V_i} N_4^i
\end{aligned}
$$

At each point p we can interpret the charges as arising from a D6-brane with fluxes on it. Each of these is 1/2-BPS, the aggregate is 1/8-BPS.

Reduction to 4D and special geometry

- We can further reduce to 4D and obtain solutions to $\mathcal{N} = 8$ SUGRA
- This theory has an $E_{7(7)}$ duality group. The three D2-brane charges and the D6-brane charge transform in an electric **28** of the maximal compact subgroup $SU(8)/\mathbf{Z_2}$. The three D4-brane charges and the D0-brane transform in the magnetic **28**. Together they transform in the **56** of $E_{7(7)}$
- We can write a charge vector

$$\Gamma_p = (Q_0^p,\, Q_i^p;\, Q_p^0,\, Q_p^i), \quad \Gamma = \sum_p \Gamma_p = (Q_0,\, Q_i;\, Q^0,\, Q^i)$$

and define the E_7 symplectic product

$$< \Gamma_p, \Gamma_q > = Q_p^0 Q_0^q - Q_q^0 Q_0^p + Q_p^i Q_i^q - Q_i^q Q_i^p = \frac{\Gamma_{pq}}{4L}$$

- We can also think of our 8 harmonic functions as part of a single one valued in the **56**. We can rewrite all our EOM and constraints in this language.

Which black holes?

- These solutions are candidate microstates for 4D black holes. The area of the associated black hole is

$$A = 2\pi\sqrt{J_4(\Gamma)}$$

- Writing this in terms of the charges

$$J_4(\Gamma) = \frac{1}{4}(Q^i Q_i + Q^0 Q_0)^2 - (Q^0 \prod_i Q^i + Q_0 \prod_i Q_i) - \frac{1}{2}(\sum_i (Q^i Q_i)^2 + (Q^0 Q_0)^2)$$

- To get a finite area, we need to turn on at least 4 charges, which we can easily do. An example is the D2-D2-D2-D6 black hole with area $A = 2\pi\sqrt{-Q_0 \prod_i Q_i}$
- In general we can find microstates for finite area 4D black holes

More on BPSness

- For the black hole solution $J_4(\mathcal{H})$ falls off like ρ^{-4} at a pole since the metric goes like

$$J_4^{1/2} d\rho^2, \quad J_4 = M_0 Z_1 Z_2 Z_3$$

$$Z_i = 1 + \frac{Q_i}{\rho}, \quad M_0 = 1 + \frac{Q_0}{\rho}$$

- This is typical of 1/8-BPS solutions with finite area. As we turn off charges the solution goes first to a 1/8-BPS solution of vanishing area with falloff ρ^{-3}, and then to 1/4-BPS (ρ^{-2}) and 1/2-BPS (ρ^{-1})

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- This is typical of 1/8-BPS solutions with finite area. As we turn off charges the solution goes first to a 1/8-BPS solution of vanishing area with falloff ρ^{-3}, and then to 1/4-BPS (ρ^{-2}) and 1/2-BPS (ρ^{-1})
- For our microstates though, they *always* fall off like ρ^{-1} at a pole even though they are 1/8-BPS
- This occurs because our solution is multicentered. Each “atom” (charge center) is 1/2-BPS, but the “molecule” is 1/8-BPS

The new bubble equation and a scaling relation

- The reduction alters the bubble equation. It becomes

$$\psi_p + \frac{2}{L}\sum_q \frac{\Gamma_{pq}}{\rho_{pq}} = 0, \quad \psi_p = \sum_i n_p \lambda_p^i - \frac{1}{L^2 n_p^2}\prod_i n_p \lambda_p^i, \quad \lambda_p^i = \frac{d_p^i}{n_p} - L^2\,\delta h^i$$

- This equation has a novel scaling behavior. If we scale all coordinates by $(t, \rho, z^i) \to \alpha(t, \rho, z^i)$ the bubble equation remains invariant. This corresponds to scaling $l_P^{11} \to \alpha^{1/3} l_P^{11}$

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- We can interpret this scaling as taking $g_s \to \alpha g_s$, while holding the torus volume and l_s fixed in string units
- Since the pole locations are $D6$-branes wrapping the torus, we see that as we vary g_s we alter the distance between the branes

The open string picture and quivers (in progress!)

- We see that as we reduce g_s, the branes will get closer together. When they are within a string length the open string picture becomes the more valid description. The open string picture is given by a quiver theory.

- The quiver is given as follows. Each individual charge vector Γ_p (atom) gives a $U(1)$ factor. If $\Gamma_p = N_p \hat{\Gamma}_p$ is still appropriately quantized we have a $U(N_p)$ factor.

- The number of bifundamentals between gauge groups p and q is given by the intersection number Γ_{pq}

Quiver transitions

- When the open string picture is first valid, the system is described by a quiver gauge theory in the Coulomb phase. The chiral multiplet scalars are massive with masses proportional to the brane separation

- As we further lower g_s the scalars would become tachyonic. This moves our quiver theory onto the Higgs branch

- Taking $g_s \to 0$ collapses all the branes on top of each other. This is the picture of a D-brane ground state, and is the starting point for the Strominger-Vafa counting

The picture and quantum mechanics

- Flipping the picture around we find that going from zero to strong coupling takes us on the path: D-brane vacuum state $\longrightarrow$ quiver theory in Higgs phase $\longrightarrow$ quiver theory in Coulomb phase $\longrightarrow$ 10D BPS particles (wrapped branes) $\longrightarrow$ 11D spacetime foam

- Quantum mechanically, we will have a wave function that is peaked in different phases depending on g_s, the transitions should be smooth

- We anticipate this to be the connection between the older picture of microstate counting and the fuzzball geometries

Summing up

- We have demonstrated a solution generating technique for general $U(1)$ invariant, BPS, three-charge microstates and shown how to reduce them to 4D
- These solutions replaced a singular core region with an intricate geometry of two-cycles threaded by electric and magnetic flux
- After reduction the solutions are interpreted as D-branes in IIA
- These solutions are candidate microstates for 4D, finite area black holes
- We demonstrated a novel scaling behavior and conjectured a relation to D-brane ground states
- This scaling transitions us from a spacetime foam in 5D through a quiver gauge theory in 4D and down to D-brane ground states

Open questions

- Can all microstates be written in terms of 1/2-BPS atoms?
- How do we invert our conditions so that we can find and count all microstates for given conserved charges?
- What are the dual CFT states? How can the CFT encode our microscopic variables?
- What are the relations to the OSV conjecture on the black hole partition function and topological strings?
- The solutions organize themselves nicely with the $E_{7(7)}$ (and also E_8) U-duality groups. Can we use this to generate more general solutions? Can we then lift back up?
- Quivers with closed loops generate superpotentials. How does this affect our story?