Chaos and Complementarity in Cosmological Spacetimes

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Chaos and Complementarity in de Sitter space

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Motivation

- A lot has recently been conjectured about de Sitter space and inflation in quantum gravity **[more in KITP "Swampland" program]**.
- Can **quantum information** ideas put a bound on the lifetime of de Sitter space and inflation?
- **Black holes** are often considered the harmonic oscillator of quantum gravity. They are **fast scramblers** **[Sekino, Susskind].**
- The similarity between a BH horizon & the cosmological horizon has led **[Susskind]** to conjecture that dS is also a fast scrambler. We found interesting similarities & differences **[Aalsma, GS].**
- Recent studies of quantum chaos for BHs have offered a window into their microscopic description. We computed **out-of-time-order correlators** (OTOCs) to assess the chaotic nature of dS horizon and explore consequences for dS complementarity & inflation.

Quantum Chaos

- The exponential blueshift in energy between an asymptotic and a free-falling observer is key in making black holes chaotic.

E_0

$E_0 e^{\frac{2\pi}{\beta} t}$

β = inverse temperature of BH

- A probe of chaos in quantum systems is the **double commutator**:

$$C(t) = \langle -[V(0), W(t)]^2 \rangle = 2 - 2\,\langle V(0)W(t)V(0)W(t)\rangle \equiv 2 - 2F(t)$$

V and W are Hermitian, unitary operators; F(t) is the **out-of-time-order correlator (OTOC)**. Chaotic behavior manifests in an exponential growth of C(t) or equivalently, an exponential decay of F(t).

Quantum Chaos for Black Holes

- In some thermal systems with a large # of dof N, e.g., holographic CFTs dual to black holes **[Shenker,Stanford];[Roberts,Stanford]; [Maldacena,Shenker,Stanford]**:

$$F(t) = 1 - \frac{f_0}{N} e^{\lambda_L t} + \mathcal{O}(N^{-2}) \ , \qquad \left(\beta/2\pi \ll t \ll \lambda_L^{-1} \log(N)\right)$$

- The timescale when F(t) drops by an order 1 amount is known as the **scrambling time**:

$$t_* = \lambda_L^{-1} \log(N)$$

- The (quantum) **Lyapunov exponent** λ_L determines how fast chaos can grow and it has been argued to obey a universal bound:

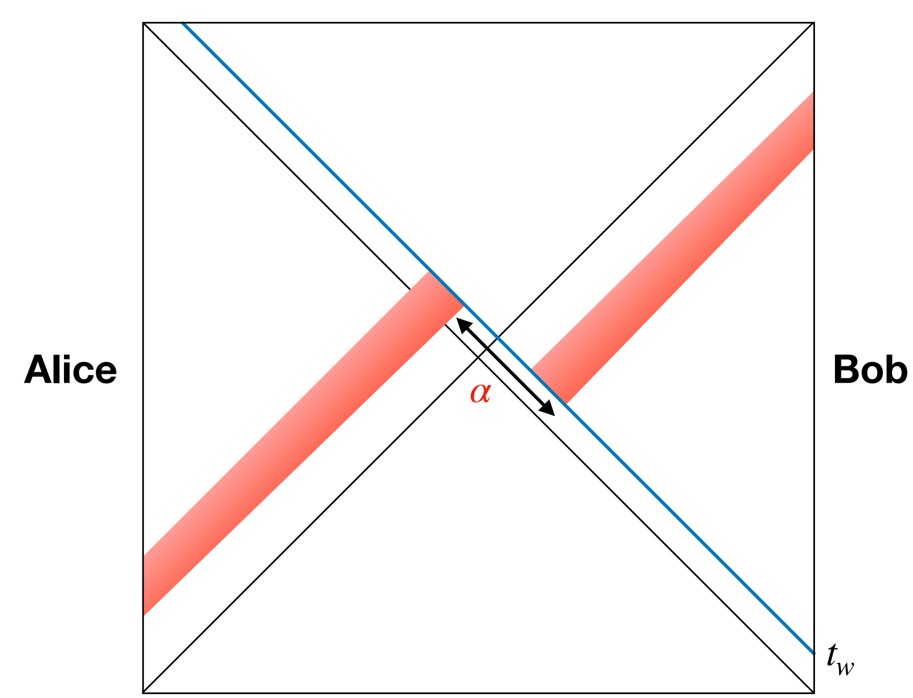
$$\lambda_L \ \leq \ 2\pi/\beta \qquad \textbf{[Maldacena, Shenker, Stanford]}$$

- Black holes saturate this bound; they are fast scramblers **[Susskind].**

Quantum Chaos for de Sitter Space

- There is similarly a blueshift in energy between an observer at the center of the static patch and a free-falling one through its horizon.
- Like in the BH case, a perturbation released a scrambling time before the t=0 slice is highly boosted, creating a **shockwave**.
- This led **[Susskind]** to conjecture that dS is also a fast scrambler. But as we'll see, there are at least two interesting differences:
    - Geodesics crossing a positive-energy shockwave (generated by matter satisfying NEC) experience a gravitational **time advance** **[Gao, Wald]** rather than a time delay.

Possible to send signals from otherwise causally disconnected regions



Quantum Chaos for de Sitter Space

- Another difference is the absence of a spatially asymptotic and non-gravitating boundary theory to probe the static patch.

- Nonetheless, we can study chaos by restricting to a single static observer. We calculated various OTOCs with operators inserted at the origin of different static patches to establish the chaotic behavior of dS & show that λ_L **saturates the chaos bound.**
- We found that the OTOC does not decay in the same way as that for BHs but behaves as **[Aalsma, GS]:**

$$F(t) \sim 1 - N^{-2} e^{2\lambda_L t}$$

- We then comment on the implications to de Sitter complementarity and the constraints on de Sitter and inflation.

de Sitter Space

- We carried out our analysis for de Sitter space, but it is straightforward to generalize our results to inflationary spacetimes.
- dS_d can be described as a hyperboloid embedded into d+1 dimensional Minkowski space using embedding coordinates:

$$\eta_{AB} X^A X^B = \ell^2$$

- In **static coordinates**, time translational symmetry is manifest:

$$ds^2 = -\left(1 - r^2/\ell^2\right) dt^2 + \left(1 - r^2/\ell^2\right)^{-1} dr^2 + r^2 d\Omega_{d-2}^2$$

where

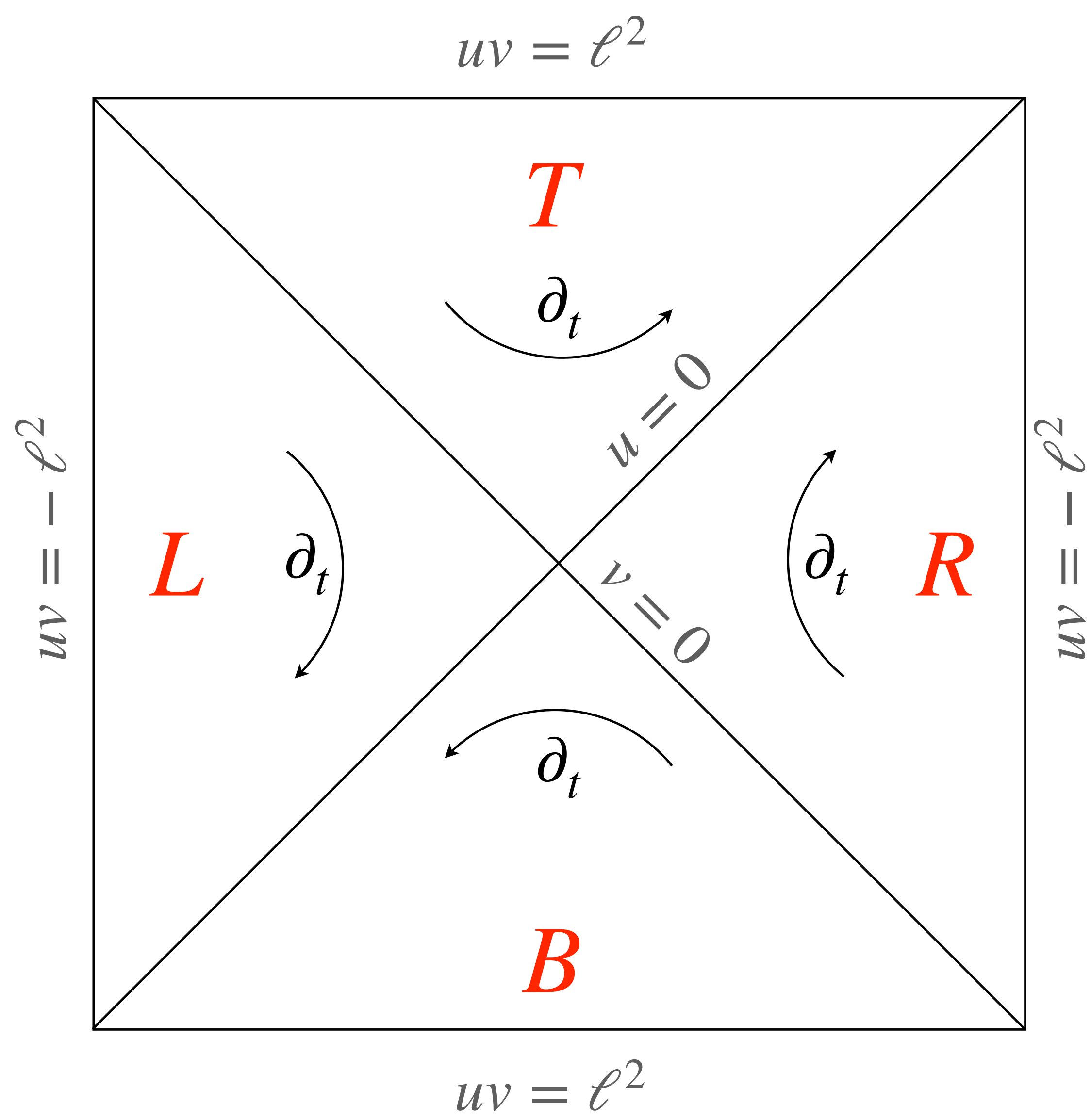
$$X^0 = \sqrt{\ell^2 - r^2} \sinh(t/\ell) \ ,$$
$$X^d = \sqrt{\ell^2 - r^2} \cosh(t/\ell) \ ,$$
$$X^i = r y^i \ .$$

- This metric only covers 1/4 of the global dS Penrose diagram, known as the **static patch**, surrounded by the horizon at r=l.

de Sitter Space

- By complexifying the time coordinates $t_x = t + i\,\varepsilon_x$, we can cover the 4 static patches of global dS space:

$$\epsilon_R = 0\ , \quad \epsilon_L = -\pi\ell\ , \quad \epsilon_T = -\frac{\pi}{2}\ell\ , \quad \epsilon_B = \frac{\pi}{2}\ell$$



In Kruskal-like coordinates that provide a global cover:

$$ds^2 = \frac{4\ell^4}{(\ell^2 - uv)^2}(-du dv) + \ell^2 \frac{(\ell^2 + uv)^2}{(\ell^2 - uv)^2} d\Omega^2_{d-2}$$

Wightman Function

- 2-point function of scalar fields in a particular vacuum state |Ω⟩:

$$W(x,y) \equiv \langle \Omega | \, \varphi(x)\varphi(y) \, | \Omega \rangle$$

- The scalar field is described by the action:

$$S = -\frac{1}{2}\int d^d x \sqrt{-g}\left(\partial_\mu\varphi\partial^\mu\varphi + m^2\varphi^2 + \xi R\varphi^2\right)$$

- For states |Ω⟩ that preserve dS isometries, W(x,y) depends only on:

$$Z(x,y) = \frac{1}{\ell^2}\eta_{AB}X^A(x)X^B(y)$$

- For the **Bunch-Davies vacuum**:

$$W(x,y) = \frac{\Gamma(h_+)\Gamma(h_-)}{\ell^d(4\pi)^{d/2}\Gamma(d/2)}\,{}_2F_1\left(h_+, h_-, \frac{d}{2}; \frac{1+Z(x,y)}{2}\right)$$

where

$$h_\pm = \frac{1}{2}\left(d - 1 \pm \sqrt{(d-1)^2 - 4\ell^2\tilde{m}^2}\right) \qquad \tilde{m}^2 = m^2 + \xi R.$$

Wightman Function

- The distinction between the real and imaginary regimes of $h_{\pm}$:

Complementary series ($h_{\pm}$ real) $\qquad 0 < \tilde{m}^2\ell^2 < \frac{(d-1)^2}{4}$

Principal series ($h_{\pm}$ complex) $\qquad \tilde{m}^2\ell^2 \geq \frac{(d-1)^2}{4}$

- W(x,y) is analytic everywhere in the complex Z plane except at a **branch cut** along the line Z ≥ 1, the correct iε prescription:

$$W(x,y) = \frac{\Gamma(h_+)\Gamma(h_-)}{\ell^d (4\pi)^{d/2}\Gamma(d/2)}\, {}_2F_1\left(h_+, h_-, \frac{d}{2}; \frac{1 + Z(x,y) + i\epsilon\, \mathrm{sgn}(x,y)}{2}\right)$$

where sgn (x,y) = +1 if x is in the future of y and sgn (x,y) = -1 if x is in the past of y **[See e.g., Einhorn, Larsen, '03]**

Shockwaves

- Consider the R patch: the relation between static & global coords.:

$$u = -\ell e^{-t/\ell}\sqrt{\frac{\ell - r}{\ell + r}} \; , \quad v = \ell e^{t/\ell}\sqrt{\frac{\ell - r}{\ell + r}}$$

- Time translation t → t + c corresponds to a boost in Kruskal coords

$$u \to e^{-c/\ell} u \; , \quad v \to e^{c/\ell} v$$

- A particle released from the origin of the static patch in the past is highly blueshifted when it crosses the t=0 slice: **shockwave geometry.**
- A shockwave traveling at the past horizon v=0 is given by the metric:

$$ds^2 = \frac{4\ell^4}{(\ell^2 - uv)^2}(-du dv) - 4\alpha\delta(v)dv^2 + \ell^2\left(\frac{\ell^2 + uv}{\ell^2 - uv}\right)^2 d\phi^2$$

- We focus on 2+1 dim though it is easy to generalize our results to higher-dim. dS shockwave geometries which are known **[Hotta, Tanaka];[Sfetos].**

Shockwaves

- This is a solution to Einstein's equations with a stress tensor:

$$T_{vv} = \frac{\alpha}{4\pi G_N \ell^2}\delta(v)$$

- The NEC enforces $\alpha > 0$, and thus geodesics crossing the past horizon at v=0 experiences a **time advance** by an amount α.
- If this shockwave is generated by a particle with a thermal energy in its rest frame $E_0 = \beta^{-1} = 1/(2\pi l)$, α is given by the blueshifted energy

$$\alpha = \frac{G_N}{2} e^{t_w/\ell} \qquad t = -t_w$$

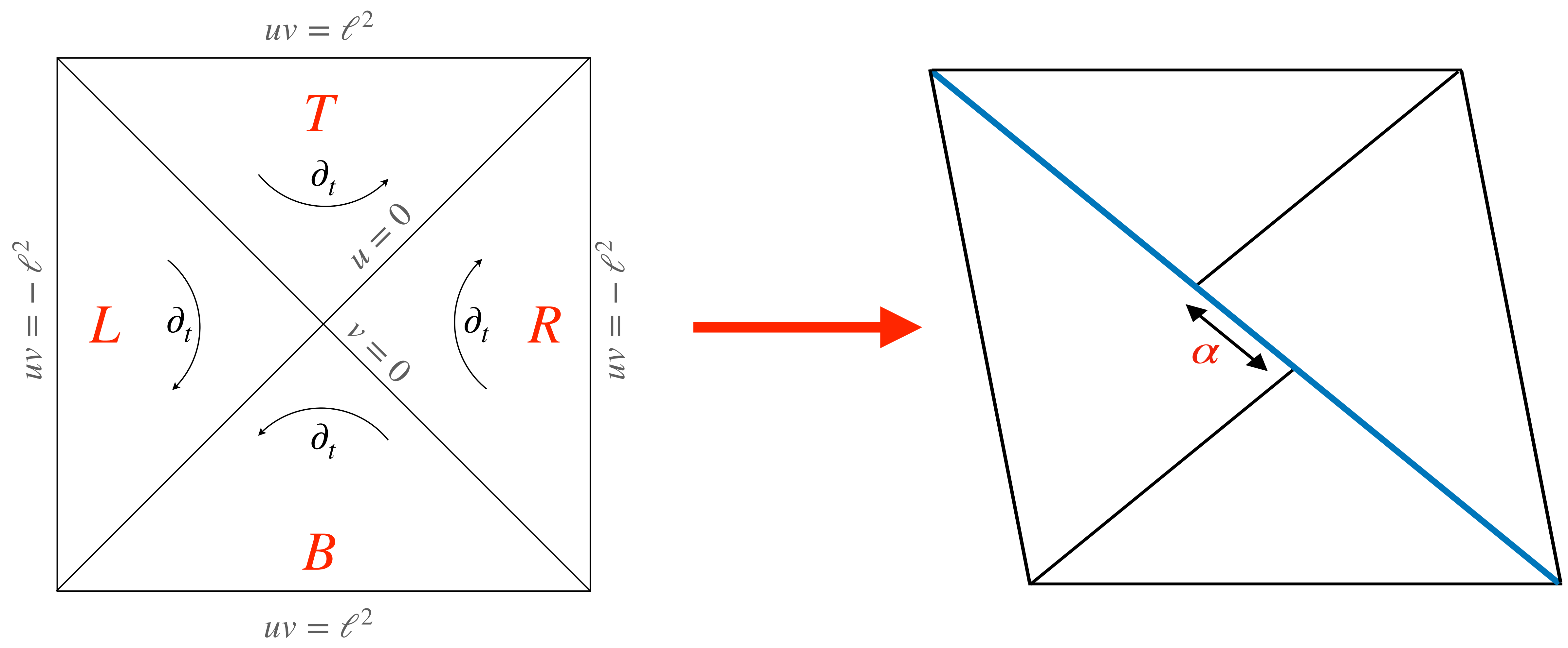
(time particle released)

- Useful to consider the coord. transformed ($u = \tilde{u} - \alpha\theta(v)$) metric:

$$ds^2 = \frac{4\ell^4}{(\ell^2 - (\tilde{u} - \alpha\theta(v))v)^2}(-d\tilde{u}dv) + \ell^2\left(\frac{\ell^2 + (\tilde{u} - \alpha\theta(v))v}{\ell^2 - (\tilde{u} - \alpha\theta(v))v}\right)^2 d\phi^2$$

Shockwaves

- A **positive energy shockwave** generates a discontinuity in the metric by an amount α that brings the L and R patch into causal contact.



- The shockwave is generated by an operator W(t) with t < 0. For inflation applications, we can think of it as the flux of inflaton energy exiting the horizon.

OTOC in the Geodesic Approximation

- We computed the OTOC that was previously studied in the context of black holes **[Shenker, Stanford]:**

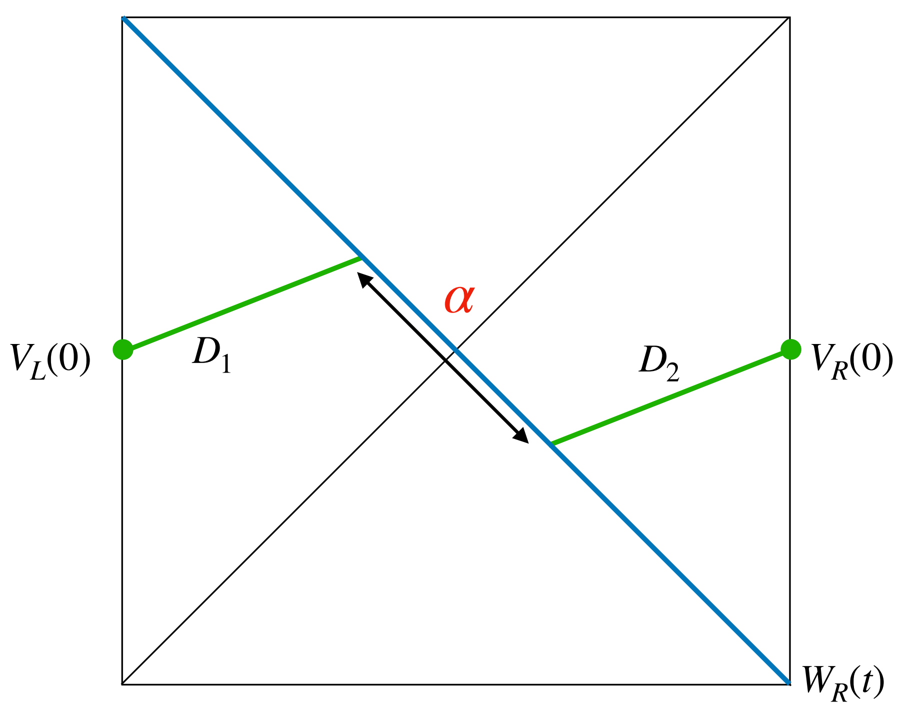
$$F(t) = \langle W_R(t) V_L(0) V_R(0) W_R(t) \rangle$$

- W_R and $V_{L,R}$ are operators inserted at the origin of a static patch indicated by the subscript; we can also view this as a purely right-sided correlator by evaluating $V_L(0) = V_R(-i\pi l)$.
- We first evaluate this OTOC in the **geodesic approximation**, valid when V corresponds to inserting a massive field with $ml >> 1$.
- F(t) is the 2-point function in the shockwave background, given by the sum of geodesics with the location of V at the endpoints:

$$F(t) \simeq \sum_{\text{geodesics}} e^{-mD} \qquad \text{with} \qquad \cos\left(\frac{D(x,y)}{\ell}\right) = Z(x,y)$$

OTOC in the Geodesic Approximation

- **Caution:** F(t) is only defined for geometries with a real analytic continuation. Shockwaves introduce non-analyticities in the metric.
- Adding the geodesic distances D_1 and D_2:



$$\cos\left(\frac{D_1}{\ell}\right) = \frac{u}{\ell}\,, \quad \cos\left(\frac{D_2}{\ell}\right) = \frac{\alpha - u}{\ell}$$

$$D = D_1 + D_2 = \ell \arccos\left(\frac{u}{\ell}\right) + \ell \arccos\left(\frac{\alpha - u}{\ell}\right)$$

Extremizing D over u gives u= α/2

- In the geodesic approximation, the OTOC behaves as:

$$F(\alpha) = e^{-2m\ell \arccos\left(\frac{\alpha}{2\ell}\right)} \xrightarrow[\alpha \ll 2\ell]{} 1 + \frac{mG_N}{2} e^{t_w/\ell} + \mathcal{O}\left(\frac{G_N}{\ell} e^{t_w/\ell}\right)^2$$

OTOC in the Geodesic Approximation

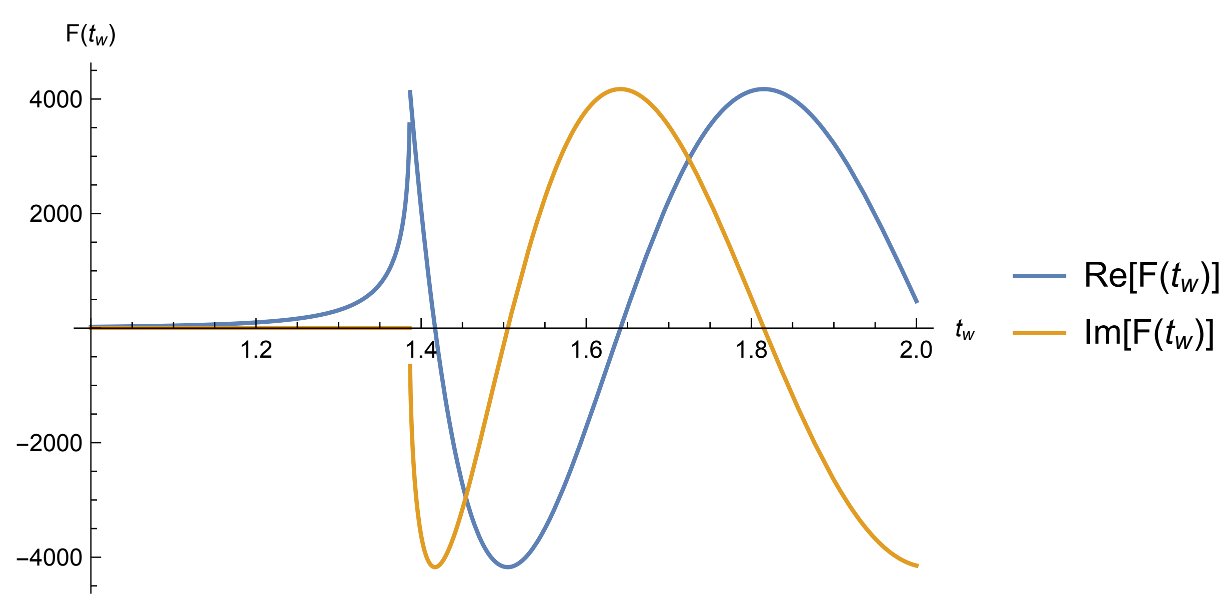
- The expansion is valid for: $t_w \ll t_* = \ell \log\left(4\ell/G_N\right)$
- We recognize the upper bound as the **scrambling time:**

$$t_* = \ell \log S \quad \text{with} \quad S = \frac{\pi \ell}{2G_N} \gg 1$$

- The OTOC does not decay but **grows exponentially**: A positive shockwave causally connects the L and R patches and so V_L and V_R are only spacelike when $\alpha < 2l$.
- Above t_*, F(t) picks up an imaginary part and starts to oscillate:

$$\mathrm{Re}(F(t_w)) = +\cos\left(2m\ell\left|\arccos\left(\frac{G_N}{4\ell}e^{t_w/\ell}\right)\right|\right)$$

$$\mathrm{Im}(F(t_w)) = -\sin\left(2m\ell\left|\arccos\left(\frac{G_N}{4\ell}e^{t_w/\ell}\right)\right|\right)$$



Beyond the Geodesic Approximation

- This OTOC does not display chaotic behavior; not a surprise as positive energy shockwaves in dS make V_L & V_R more correlated.
- The oscillatory behavior of F(t) follows from that of the Wightman function which **oscillates** for massive fields in the **principal series reps.** (in the geodesic approximation, $ml >> 1$).
- For light fields (complementary series reps.), the Wightman function doesn't oscillate. We expect qualitative different behavior of the OTOC.
- We computed the OTOC beyond the geodesic approximation, focussing on conformally coupled scalars (for analytic expressions, and also to illustrate the non-oscillatory behavior):

$$\tilde{m}^2\ell^2 = 3/4$$

- The OTOC displays **chaotic behavior**; the oscillations are absent and the imaginary part of OTOC has a nice interpretation in terms of info exchange between different static patches.

Beyond the Geodesic Approximation

- The OTOC for BHs was computed beyond the geodesic approximation in **[Shenker, Stanford]** as an overlap between:

$$|\Psi\rangle = V_R(t_3)W_L(t_4)\,|\mathrm{TFD}\rangle \;\; , \qquad |\Psi'\rangle = W_R(t_2)^\dagger V_L(t_1)^\dagger\,|\mathrm{TFD}\rangle$$

- In an elastic Eikonal approximation,

$$\langle V_{x_1}(t_1)W_{x_2}(t_2)V_{x_3}(t_3)W_{x_4}(t_4)\rangle = \frac{16}{\pi^2}\int \mathcal{D}e^{i\delta(s,|x-x'|)}\left[p_1^u\psi_1^*(p_1^u,x)\psi_3(p_1^u,x)\right]\left[p_2^v\psi_2^*(p_2^v,x')\psi_4(p_2^v,x')\right]$$

- For dS space, the result can be adopted with some modifications:

$$\psi_1(p^u,x) = \int dv e^{2ip^u v}\,\langle V(u,v,x)V_{x_1}(t_1)^\dagger\rangle\big|_{u=0} \;\; ,$$

$$\psi_2(p^v,x) = \int du e^{2ip^v u}\,\langle W(u,v,x)W_{x_2}(t_2)^\dagger\rangle\big|_{v=0} \;\; ,$$

$$\psi_3(p^u,x) = \int dv e^{2ip^u v}\,\langle V(u,v,x)V_{x_3}(t_3)\rangle|_{u=0} \;\; ,$$

$$\psi_4(p^v,x) = \int du e^{2ip^v u}\,\langle W(u,v,x)W_{x_4}(t_4)\rangle|_{v=0} \;\; .$$

$\langle \dots \rangle$ are Wightman functions in the BD vacuum instead of AdS bulk-to-boundary propagators

Beyond the Geodesic Approximation

- The Wightman function greatly simplifies for $\tilde{m}^2\ell^2 = 3/4$

$$W(x,y) = \frac{1}{4\sqrt{2}\ell^3\pi}\frac{1}{\sqrt{1 - Z(x,y) - i\epsilon\,\mathrm{sgn}(x,y)}}$$

- The **Eikonal phase** is given by the classical action:

$$\delta = \frac{1}{2}\int d^3x\sqrt{-g}\left[\frac{1}{16\pi G_N}h_{uu}\mathcal{D}^2 h_{vv} + h_{uu}T^{uu} + h_{vv}T^{vv}\right] = -\frac{1}{4}\pi G_N \ell p^u p^v \cos(\phi' - \phi'')$$

- The OTOC can be solved analytically in terms of special functions:

$$F(t) = g\left(\pi H_0(2g) + 2\mathcal{F}(g^2) + 2\log\left(-g\right) J_0(2g)\right)$$

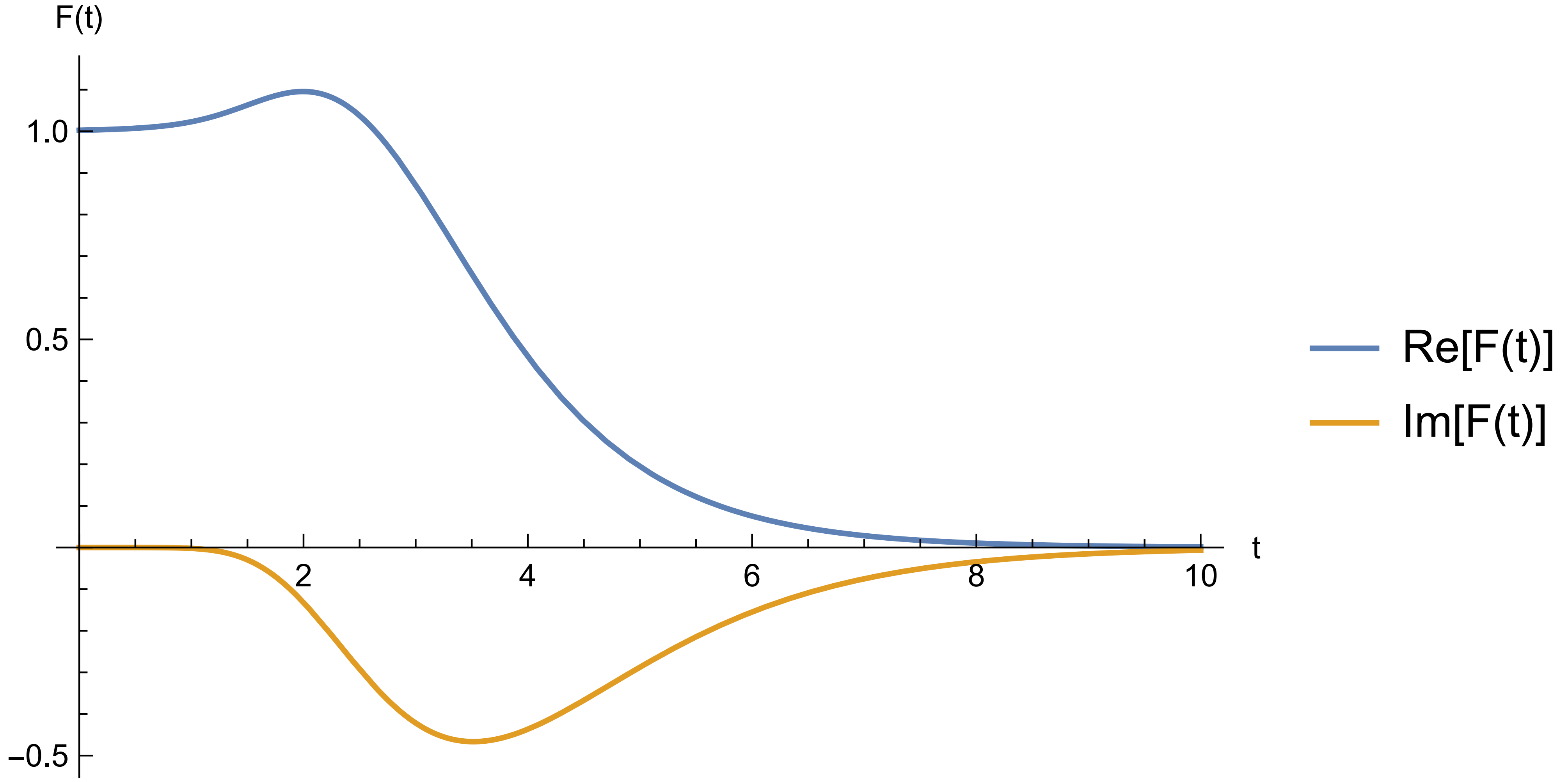
with Struve function $H_n(z)$, Bessel function of the 1st kind $J_n(z)$,

$$\mathcal{F}(z) = \lim_{a\to 1}\partial_a\left(\frac{{}_0F_1(a,-z)}{\Gamma(a)}\right) \ ,$$

$$\epsilon_{ij} = i\left(e^{i\epsilon_i/\ell} - e^{i\epsilon_j/\ell}\right) \ , \quad g(t) = \frac{8\ell\delta_{13}\epsilon_{13}\epsilon_{24}^* e^{-t/l}}{\pi G_N} \ , \quad \begin{array}{lll} \delta_{13} = +1 & \text{for} & 0 \le \arg\left(\epsilon_{13}\right) - \frac{\pi}{2} < \pi \ , \\ \delta_{13} = -1 & \text{for} & \pi \le \arg\left(\epsilon_{13}\right) - \frac{\pi}{2} < 2\pi \ . \end{array}$$

Chaotic Behavior

- To compare with the geodesic approximation, we send one of the V operators to the L patch. The OTOC as a function of t:



- The real part of the OTOC initially rises but at later times decreases and goes to zero.

Traversability

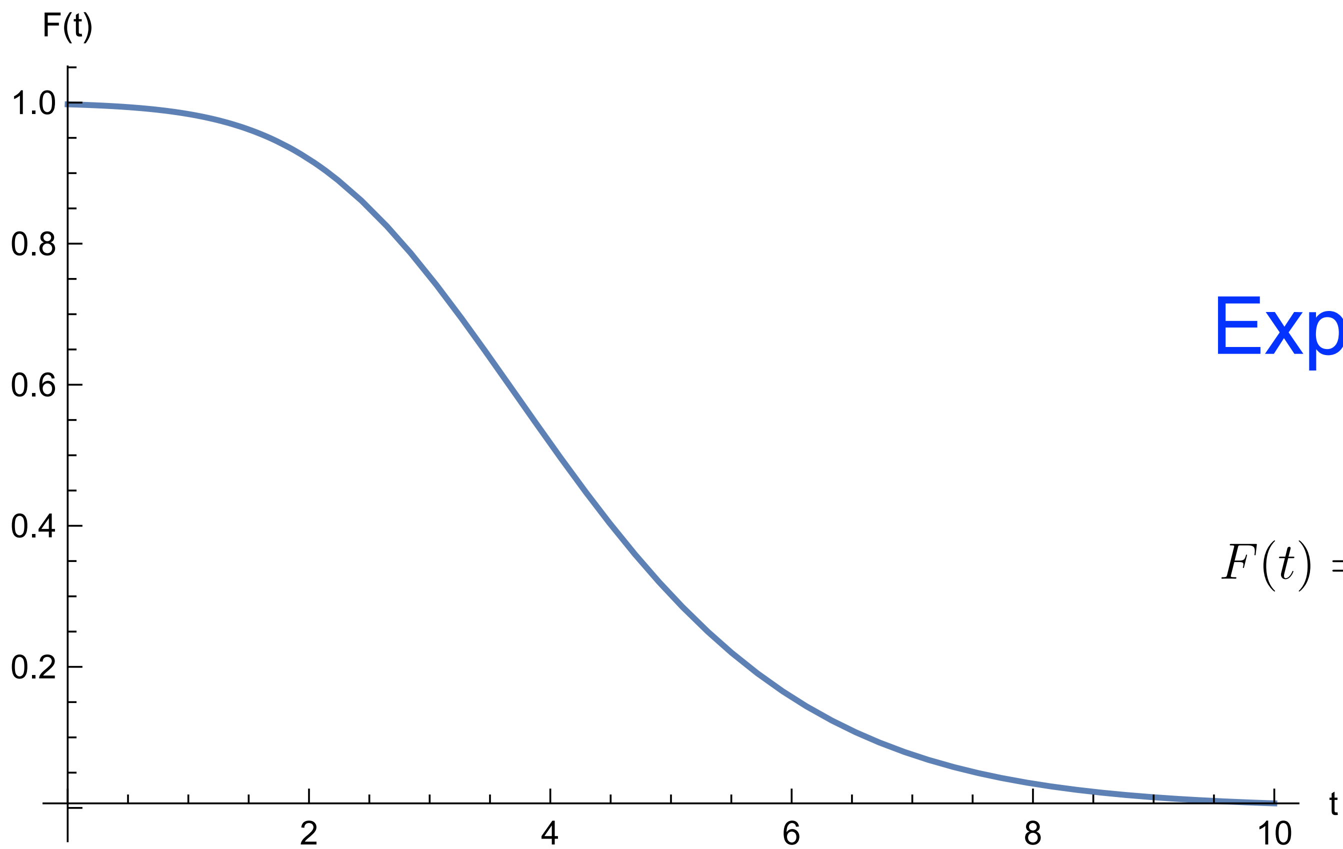
- A geodesic crossing a positive null energy shockwave in dS experiences a **time advance**: it is possible to send signals between the L and R patches. To confirm traversability, consider:

$$\langle e^{-i\epsilon_R V_L(0)} W(t) V_R(0) W(t) e^{i\epsilon_R V_L(0)} \rangle =$$
$$\langle W(t) V_R(0) W(t) \rangle + 2\epsilon_R \, \mathrm{Im}(\langle V_L(0) W(t) V_R(0) W(t) \rangle) + \mathcal{O}(\epsilon_R^2)$$

- The **imaginary part of the OTOC** is thus indicative of a signal being exchanged between the L and R patches.
- de Sitter space share similarities with **traversable wormholes** in AdS **[Gao, Jafferis, Wall]; [Maldacena, Stanford, Yang]** except that there is no need for a non-local coupling between the poles.

de Sitter as a Fast Scrambler

- We can also consider the purely right-sided OTOC:



Expanding for $|g(t)| \gg 1$

$$F(t) = 1 - \left(\frac{G_N \pi}{8\ell} e^{t/\ell}\right)^2 + \mathcal{O}\left(\frac{G_N}{\ell} e^{t/\ell}\right)^4$$

- For l < t < l log(S), this OTOC decreases exponentially. The **Lyapunov exponent** $\lambda_L = 2\pi/\beta$ saturates the **chaos bound**.
- The **scrambling time** is l log(S) but the first order term in the expansion of F(t) goes as $1/S^2$ (for BHs, this term goes as 1/S).

Stringy Corrections

- Because of the blueshift experienced by the perturbations, one might wonder if stringy effects can modify the OTOC.
- For BHs in AdS, such stringy effects were argued to be mild **[Shenker, Stanford]**. They increase the scrambling time due to the **soft UV behavior** of string amplitudes.

$$t_* = \frac{\beta}{2\pi}\left(1 + \frac{d(d+1)}{4}\frac{\ell_s^2}{\ell^2} + \dots\right)\log(S)$$

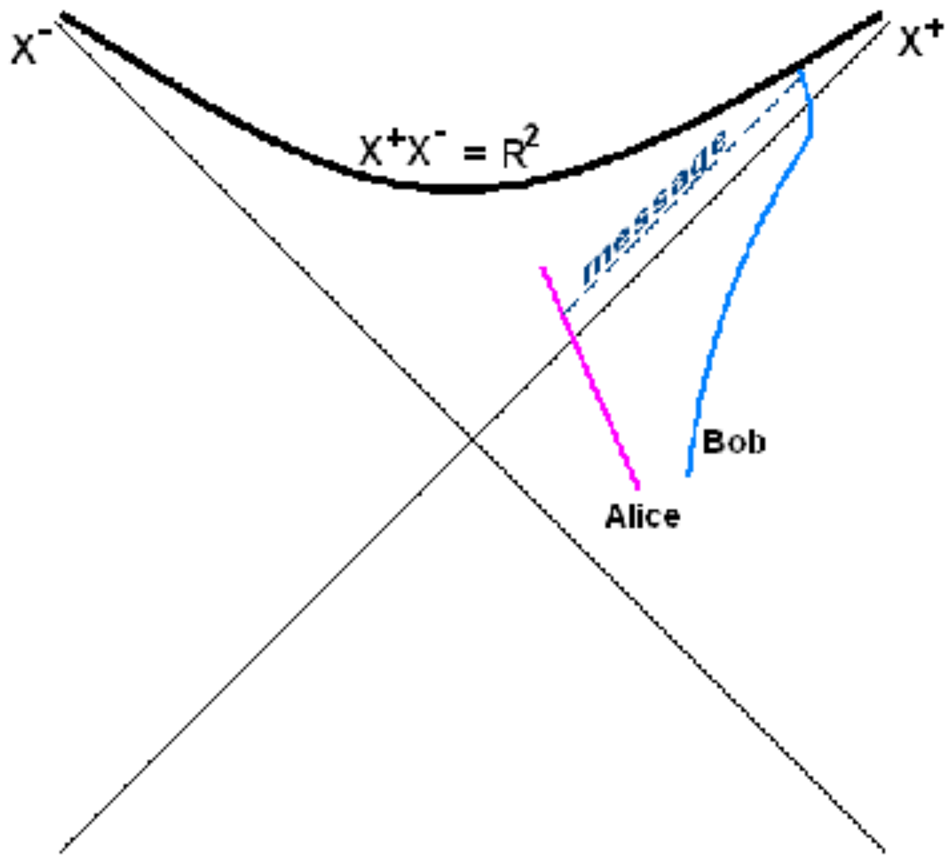
- In dS, the mass of the higher-spin states satisfy the **Higuchi bound**, in order to fall into unitary reps. of the isometry group:

$$m^2\ell^2 \geq (J-1)(d-4+J)$$

- A linear Regge trajectory $(m l_s)^2 = J$ violates this bound for $J \gtrsim (l/l_s)^2$. The higher-spin states can Reggeize the amplitude up to the Planck scale if $H \lesssim m_s^2/M_P$. The soft UV behavior may increase the scrambling time.

Black Hole Complementarity

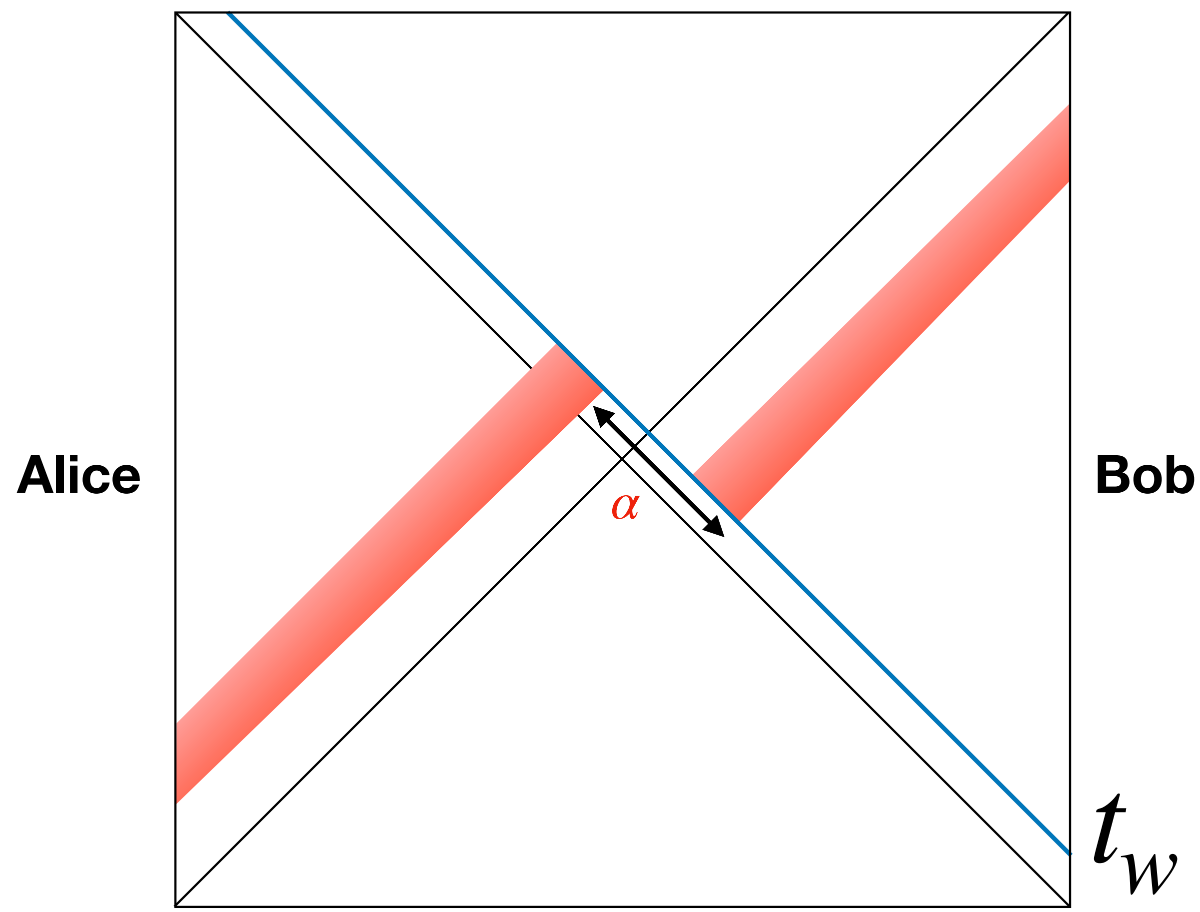
- For black holes, observer complementarity **[Susskind, Thorlacius, Uglum];['t Hooft]** suggests that the infalling and asymptotic observer have a different but "complementary" experience.



- The scrambling time is just long enough to avoid a naive violation of no-cloning **[Sekino, Susskind];[Hayden, Preskill].**

de Sitter Complementarity

- Positive energy perturbations in dS open up a wormhole connecting different static patches:



- Complementarity suggests that Bob can access at most S_{dS} states. If Bob applied the perturbation that opens the wormhole at early enough time t_w, could Alice send as much info as she likes?

Information Exchange

- The proper time during which the wormhole is open:

$$\Delta\tau = \frac{2\ell^2}{\ell^2 - uv}\sqrt{\Delta u \Delta v} \;\; = 2\alpha \sim G_N$$

- In Alice's frame, this time is blueshifted to $\Delta\tau \simeq G_N e^{t_w/\ell}$
- Complementarity suggests that $N \leq S_{dS}$.
- What goes wrong if Alice tries to send more bits?
- For the message to not backreact too strongly, the total energy:

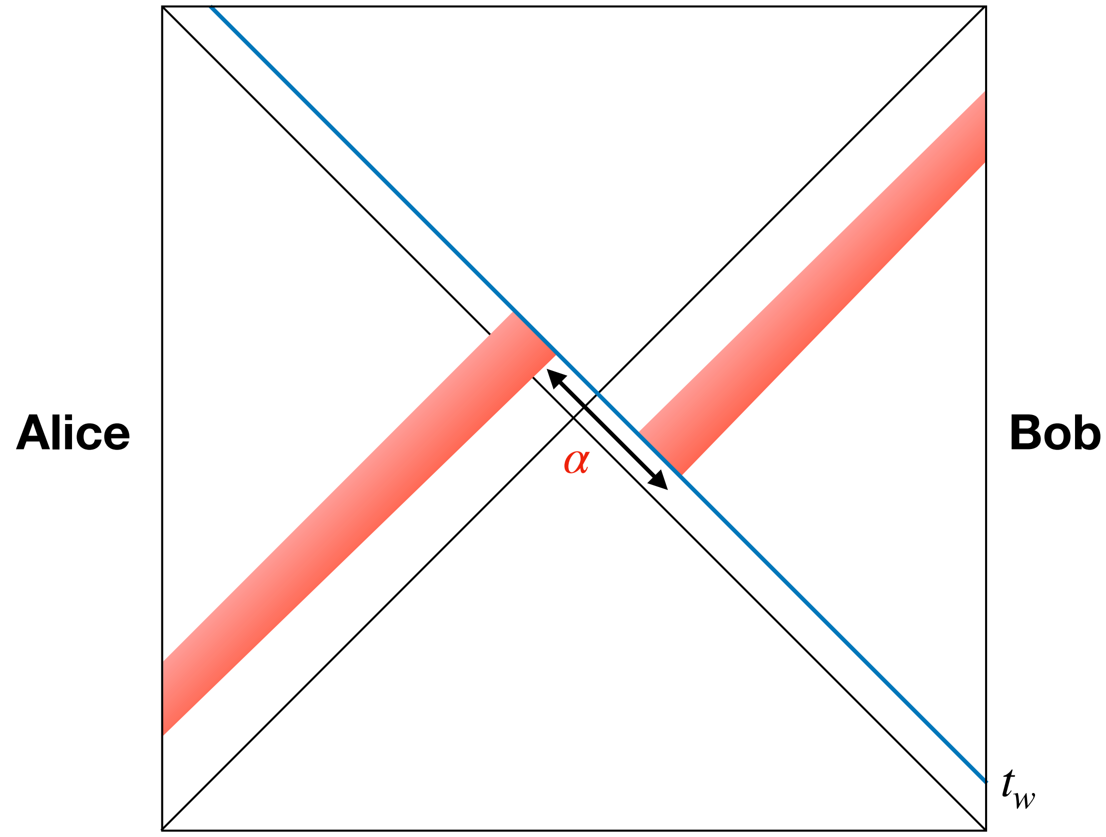
$$p^{\text{tot}} < \frac{1}{G_N}$$

- The wormhole is open for a Planckian time so the signal has to be sufficiently blueshifted to fit through:

$$p^{\nu} > \frac{1}{\Delta\tau} \sim \frac{1}{\alpha}$$

Information Exchange

- If Alice tries to send > O(1) bits to Bob, her message either does not fit through the wormhole, or backreacts too strongly.



- If she tries to send her message with K species of fields, she can at most send S_{dS} bits to Bob, while satisfying the 2 conditions and the species bound $K \lesssim l/G_N$.

Implications to Inflation

- It has recently been conjectured that trans-Planckian quantum fluctuations should remain quantum **[Bedroya,Vafa];[Bedroya, Brandenberger,LoVerde,Vafa]**, as a result the lifetime of (quasi) dS:

$$T \leq \frac{1}{H} \log\left(\frac{M_P}{H}\right)$$

- It was remarked that this bound is similar to the scrambling time.
- Whether this Trans-Planckian Censorship Conjecture is true, the scrambling time is a longer time (***double*** the # of e-folds):

$$T_{scrambling} = \frac{1}{H} log S_{dS4} = \frac{1}{H} \log\left(\frac{M_P^2}{H^2}\right)$$

- Our result also gives an interpretation of the scrambling time in an inflationary setting.

Implications to Inflation

- As the Hubble scale varies, the energy flux leaving the horizon is given by the thermodynamics relation dE = TdS **[Frolov, Kofman]:**

$$\dot{E} = \frac{\epsilon}{G_N} \qquad \text{where} \qquad \epsilon = -\frac{\dot{H}}{H^2}$$

- For simplicity, take ϵ = constant, the energy that leaves the horizon in one Hubble time 1/H is $E = \epsilon\,(G_N H)^{-1}$
- This can be described as a positive energy shockwave if E ≥ H, or

$$\epsilon \geq \frac{H^2}{8\pi M_p^2}$$

- Info can enter a Hubble patch from a previously disconnected region after $N_e \geq \log S_{dS}$ due to the shockwaves. If O(1) bit of info enters per e-fold, backreaction may become important when $N_e \geq \log S_{dS}$ for single field and $N_e \geq S_{dS}$ for maximum allowed # fields.

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Summary

- Perturbations in de Sitter that satisfy the NEC result in a shockwave geometry leading to a **time advance** for the geodesics crossing it.
- This time advance brings different static patches of dS into causal contact, much like a **traversable wormhole** in AdS.
- We computed OTOCs to assess the chaotic nature of the dS horizon; dS space is a **fast scrambler** but with differences from BHs.
- We discussed consequences of our results for dS complementarity and the implications to inflation.
- Other quantum informatic considerations may put a bound on inflation and the subsequent dark energy phase **[Aalsma, GS, work in progress]**