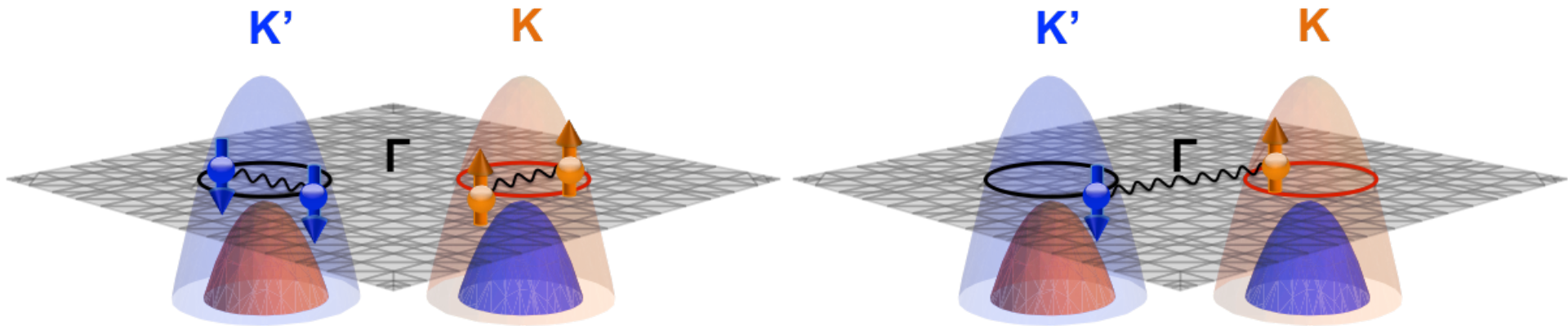


# Exotic superconductivity in monolayer transition metal dichalcogenides



Yi-Ting Hsu

(Cornell/Maryland)

## Discovery of superconductivity

The Nobel Prize in Physics 1913 was awarded to Heike Kamerlingh Onnes "for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium".

## Discovery of He<sup>3</sup> superfluidity

The Nobel Prize in Physics 1996 was awarded jointly to David M. Lee, Douglas D. Osheroff and Robert C. Richardson "for their discovery of superfluidity in helium-3".

## Discovery of heavy fermion sc

E 52, NUMBER 8      PHYSICAL REVIEW LETTERS      20 FEBRUARY 1983

### Possibility of Coexistence of Bulk Superconductivity and Spin Fluctuations in UPt<sub>3</sub>

G. R. Stewart, Z. Fisk, J. O. Willis, and J. L. Smith  
Los Alamos National Laboratory, Los Alamos, New Mexico 87545  
(Received 24 October 1983)

## Discovery of superfluidity

The Nobel Prize in Physics 1978 was divided, one half awarded to Pyotr Leonidovich Kapitsa "for his basic inventions and discoveries in the area of low-temperature physics", the other half jointly to Arno Allan Penzias and Robert Woodrow Wilson "for their discovery of cosmic microwave background radiation".

## Discovery of sc in cuprate

The Nobel Prize in Physics 1987 was awarded jointly to J. Georg Bednorz and K. Alexander Müller "for their important breakthrough in the discovery of superconductivity in ceramic materials".

## Discovery of sc in Sr<sub>2</sub>RuO<sub>4</sub>

### letters to nature

Nature 372, 532 - 534 (08 December 1994); doi:10.1038/372532a0

### Superconductivity in a layered perovskite without copper

Y. MAENO\*, H. HASHIMOTO\*, K. YOSHIDA\*, S. NISHIZAKI†, T. FUJITA\*, J. G. BEDNORZ† & F. LICHTENBERG†‡

## Discovery of Fe-based sc

J. Am. Chem. Soc., 2008, 130 (11), pp 3296–3297

Iron-Based Layered Superconductor La[O<sub>1-x</sub>F<sub>x</sub>]FeAs (x = 0.05–0.12) with

T<sub>c</sub> = 26 K

Yoichi Kamihara,\*† Takumi Watanabe,‡ Masahiro Hirano,†§ and Hideo Hosono †§

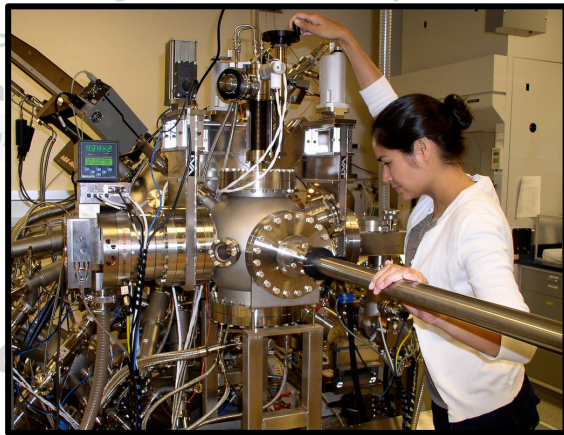
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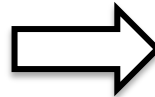
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## Discovery of Fe-based superconductivity

E 52, NUMBER 8

20 FEBRUARY 2008

### Possibility of Coexistence of Bulk Superconductivity and Spin Fluctuations in UPT<sub>3</sub>

## Theoretical predictions

Y. Kamihara, T. Watanabe, M. Hirano, and H. Hosono  
 Los Alamos National Laboratory, Los Alamos, New Mexico 87545  
 (Received 24 October 1983)

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## Theoretical puzzles & understandings

### Discovery of sc in cuprate

The Nobel Prize in Physics 1987 was awarded to Georg Bednorz and Alex Müller "for their discovery of high-temperature superconductivity in layered perovskite materials".



### Discovery of sc in Sr<sub>2</sub>RuO<sub>4</sub>

letters to nature

Nature 372, 532 - 534 (08 December 1994); doi:10.1038/372532a0

## Superconductivity in a layered perovskite without copper

Y. MAENO\*, H. HASHIMOTO\*, K. YOSHIDA\*, S. NISHIZAKI†, T. FUJITA†, J. G. BEDNORZ† & F. LICHTENBERG†‡

## Far less examples of Fe-based sc

J. Am. Chem. Soc., 2008, 130 (11), pp 3296–3297

## since making prediction for sc is challenging!

Yoichi Kamihara, \*† Takumi Watanabe, ‡ Masahiro Hirano, †§ and Hideo Hosono †§

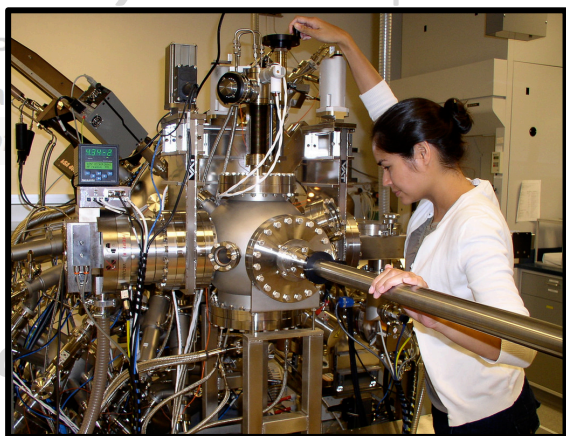
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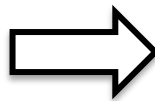
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## Discovery of superconductivity in sc

NATURE, NUMBER 8 20 FEBRUARY 1983

### Possibility of Coexistence of Bulk Superconductivity and Spin Fluctuations in UPT<sub>3</sub>

**Predict where to find exotic superconductivity**  
G. G. Lonzarich, J. K. Freerick, and J. van den Brink  
Los Alamos National Laboratory, Los Alamos, New Mexico 87545  
(Received 24 October 1983)

## Discovery of sc in Sr<sub>2</sub>RuO<sub>4</sub>

letters to nature  
Nature 372, 532 - 534 (08 December 1994); doi:10.1038/372532a0

**Realization of new exotic superconductors**  
Y. MAENO\*, H. HASHIMOTO\*, K. YOSHIDA\*, S. NISHIZAKI†, T. FUJITA†, J. G. BEDNORZ† & F. LICHTENBERG†‡

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Iron-Based Layered Superconductor La[O<sub>1-x</sub>F<sub>x</sub>]FeAs (x = 0.05–0.12) with T<sub>c</sub> = 26 K

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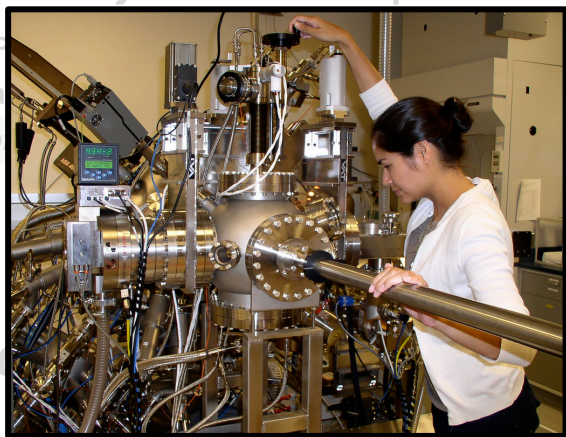
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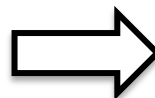
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## Theoretical puzzles

### Discovery of sc in cuprate

The Nobel Prize in Physics 2003 was awarded to Bednorz, Johannes and Müller, Georg for their discovery of high-temperature superconductivity in cuprate materials.



to Johannes Bednorz and Georg Müller for their discovery of high-temperature superconductivity in cuprate materials.

# Discovery of high-Tc superconductivity

PHYSICAL REVIEW LETTERS, NUMBER 8

20 FEBRUARY 1983

## Possibility of Coexistence of Bulk Superconductivity and Spin Fluctuations in UPT<sub>3</sub>

**Predict where to find exotic superconductivity**  
G. G. Lonzarich, D. M. Baskin, and A. I. Lichtenberg  
Los Alamos National Laboratory, Los Alamos, New Mexico 87545  
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LETTERS TO NATURE

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## Superconductivity in a layered perovskite without copper

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## Realization of new exotic superconductors

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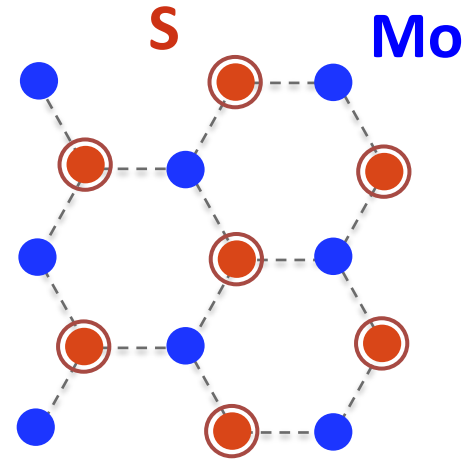
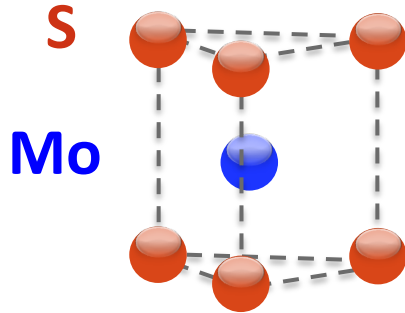
Iron-Based Layered Superconductor La[O<sub>1-x</sub>F<sub>x</sub>]FeAs (x = 0.05–0.12) with High T<sub>c</sub>

Yoichi Kamihara, \*† Takumi Watanabe, ‡ Masahiro Hirano, †§ and Hideo Hosono †§

↑  
**My goal here!**

# Monolayer transition metal dichalcogenides

- Group IV *monolayer* TMDs:  $\text{MX}_2$  (M: Mo, W, X: S, Se)
  - 2D direct-gap semiconductors
  - Lattice *breaks inversion symmetry*

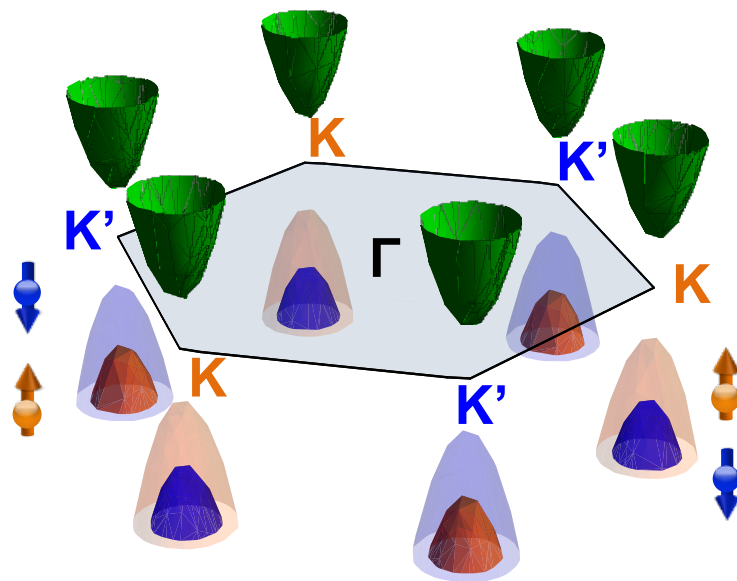


# Group IV monolayer TMDs

## ➤ Spin-orbit coupling:

- ❖ Align spins in z direction (Ising SOC)
- ❖ **Orbital selective**: doesn't act on  $d_{z^2}$  (of transition metal)

=> Huge impact on low energy band structure



$d_{z^2}$

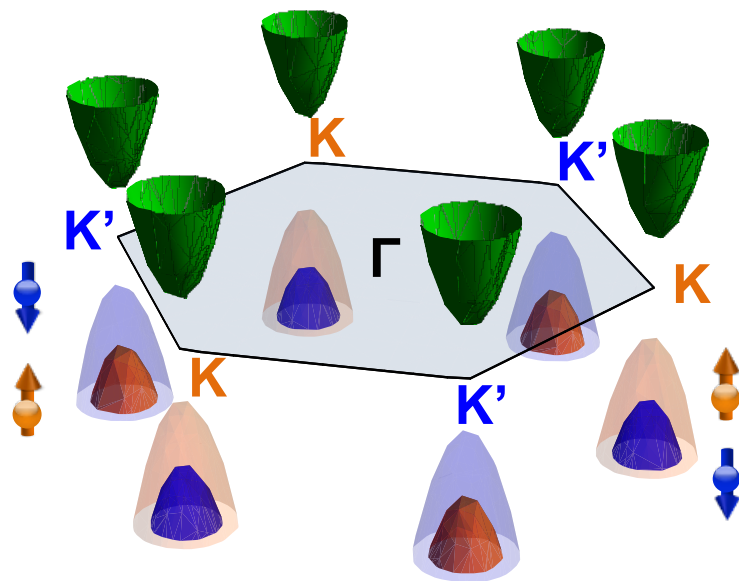
$d_{x^2-y^2}, d_{xy}$

# Group IV monolayer TMDs

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Nearly spin-degenerate

Large spin-split with  
time-reversal symmetry

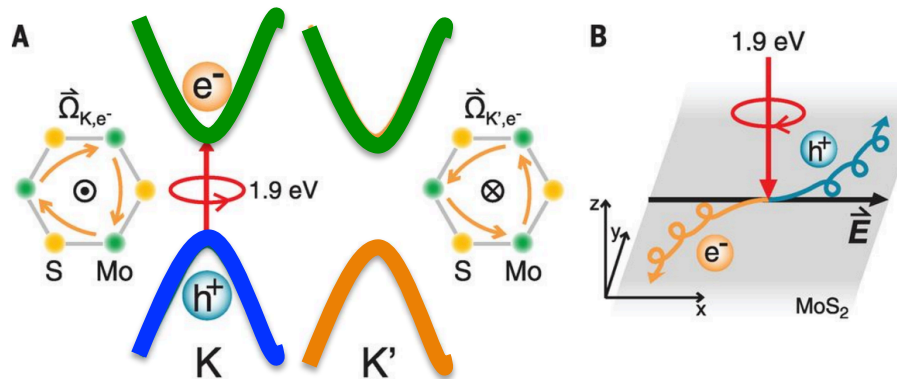


# TMDs: intriguing properties

- ◆ **Transport, optical** properties due to **large spin-split** (**spin & valley DOFs coupled**)

*E.g.* Spin & valley Hall effects

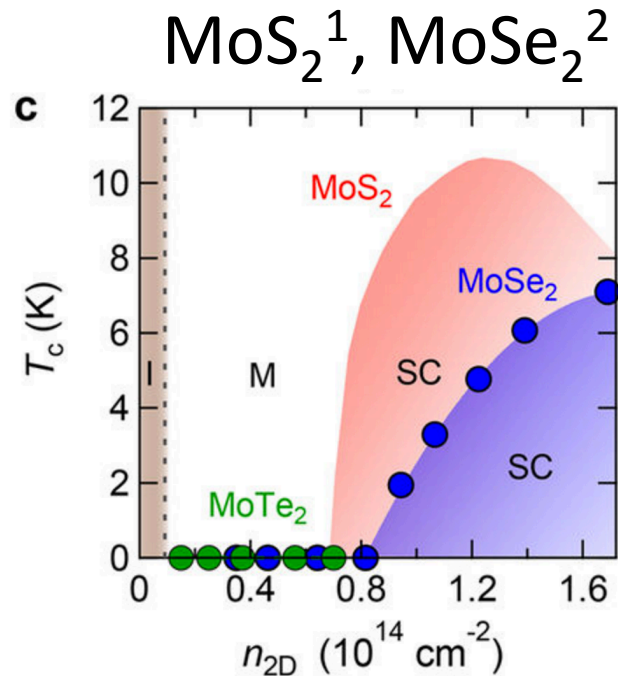
*E.g.* **Circular polarized light** => selectively control spin/valley DOFs



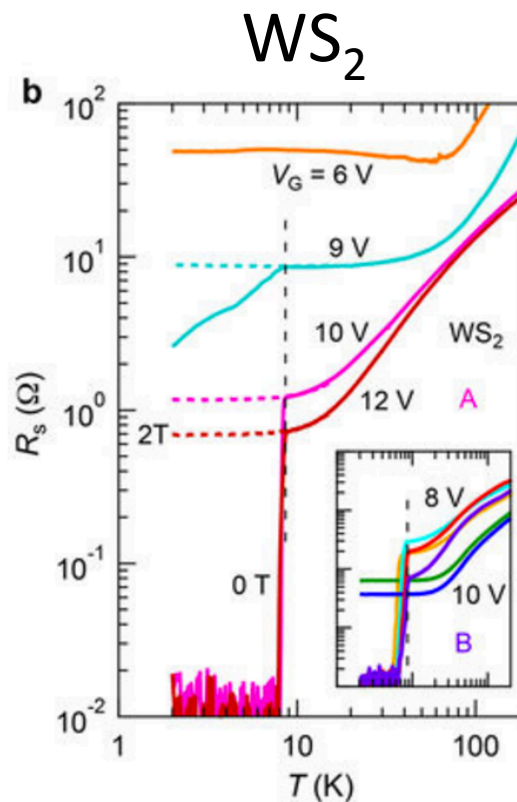
Thy: Xiao et al., PRL (2012)  
Exp: Mak et al., Science (2014)

# TMDs: superconductivity

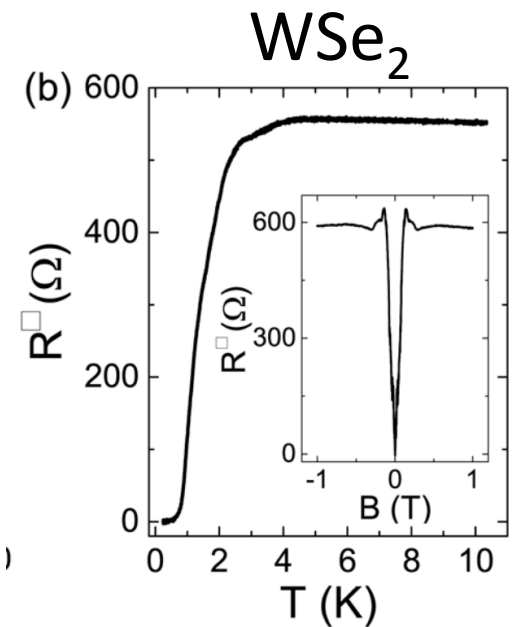
## ◆ Superconducting upon electron doping:



1: Ye et al., Science (2012)  
2: Shi et al., Sci Rep(2015)



Shi et al., Sci Rep(2015)



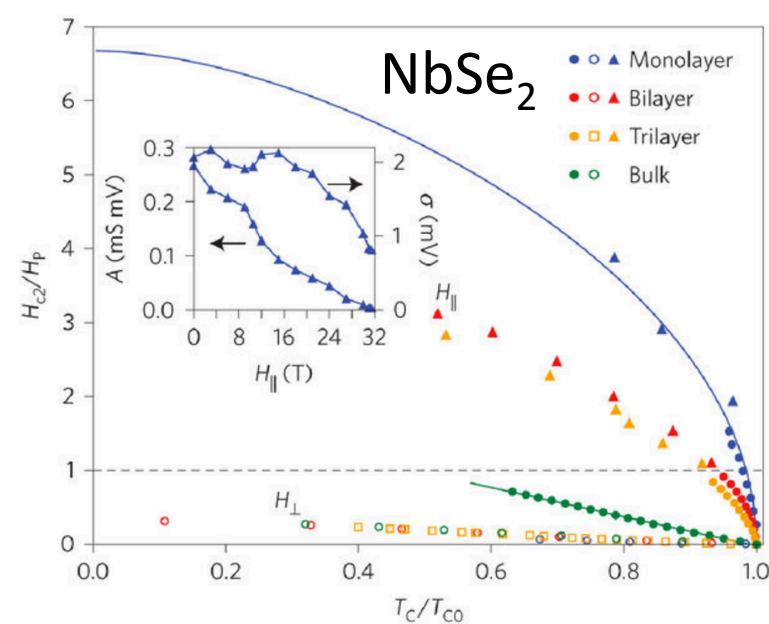
Jo et al., Nano Lett (2015)

# TMDs: superconductivity

◆ Superconductivity upon electron doping:

What do we know?

❖ Large in-plane  $H_{c2}$ : demonstration of Ising SOC



Xi et al., Nat Phys (2016)

# TMDs: superconductivity

## ◆ Superconducting upon electron doping: what do we know?

❖ Large in-plane  $H_{c2}$ : demonstration of Ising SOC

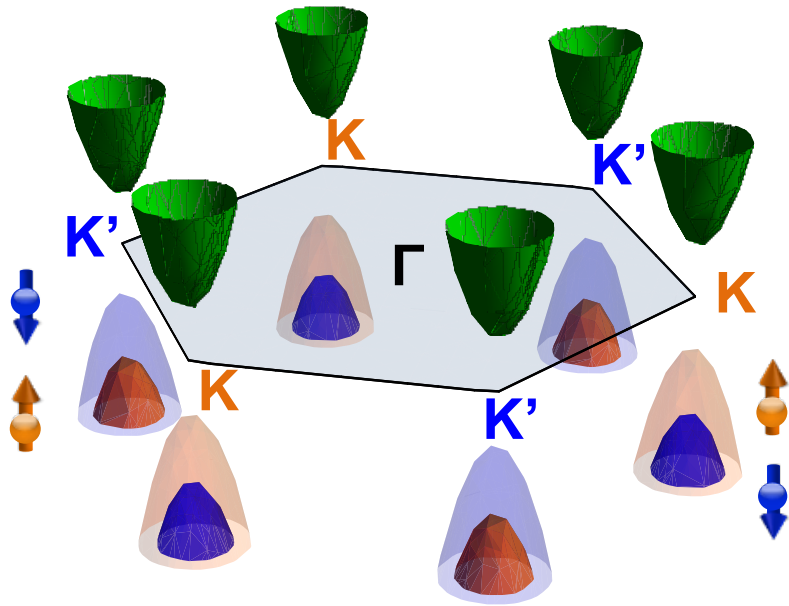
❖ Pairing symmetry:

Not a lot has been known

Parity-mixing allowed

$C_{3v}$	$\Gamma$	Singlet	Triplet
	$A_1$	s-wave	Nodeless f-wave
$A_2$			Nodal f-wave
$E$	$(d_{x^2-y^2}, d_{xy})$		$(p_x, p_y)$





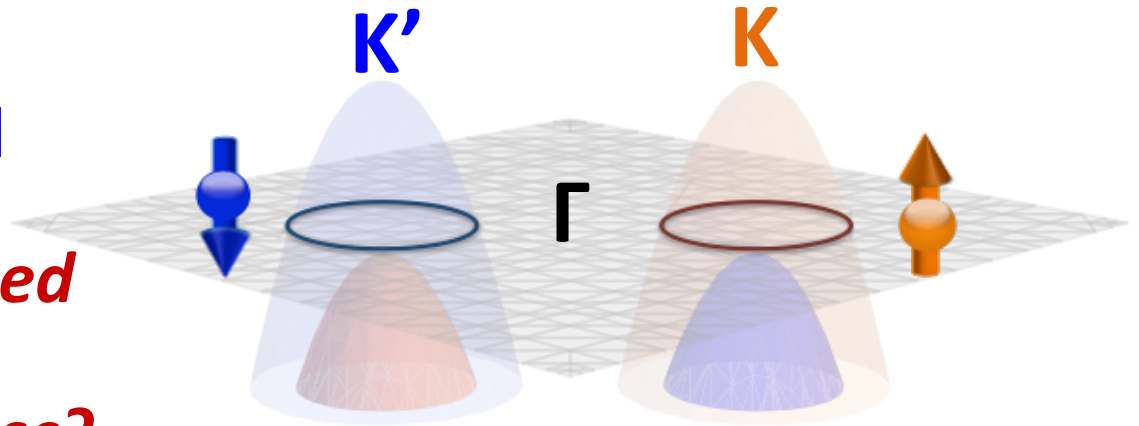
So far observed sc:  
all in the *electron-doped side*

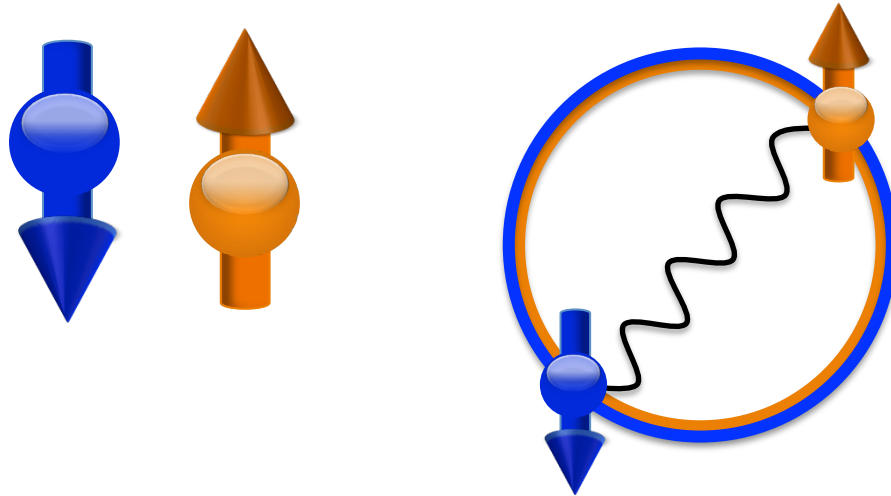
←  $\mu$  within spin-split?

Lightly hole-doped

*FS: Spin-valley locked*

*=> Good for exotic sc?*





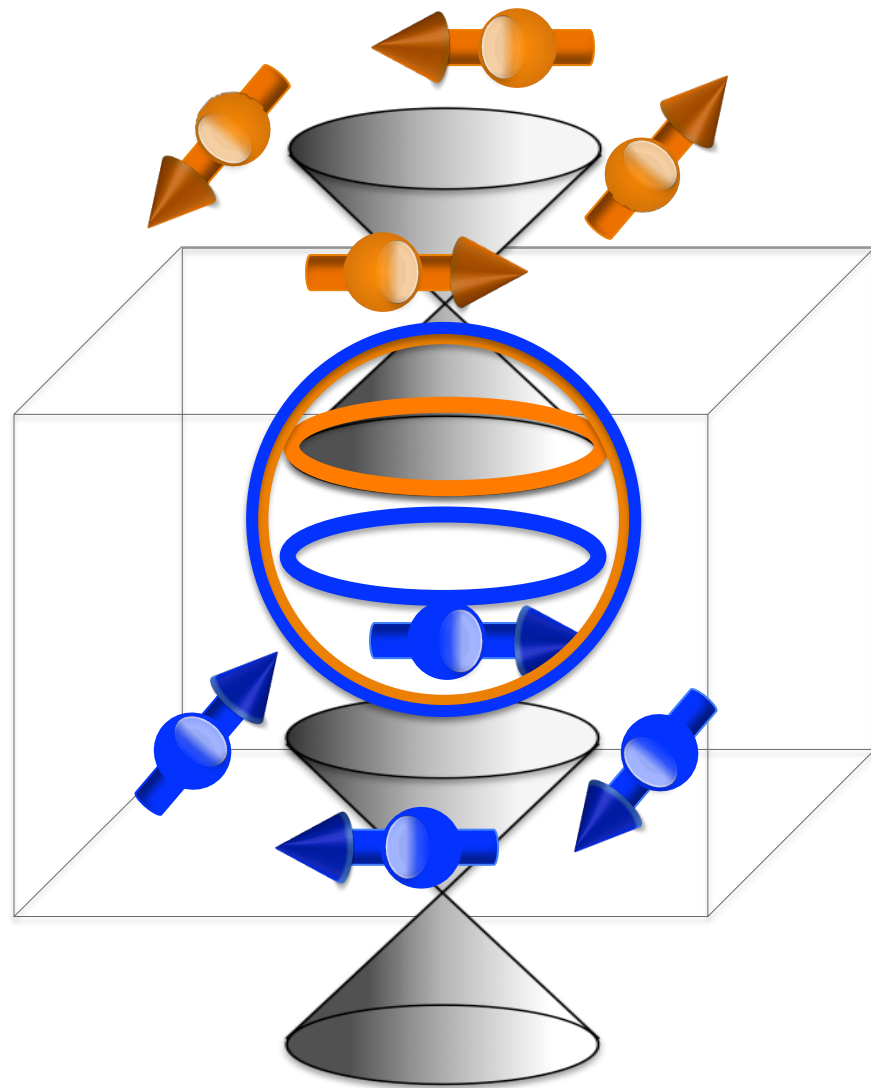
- Spin-degenerate Fermi Surfaces  
=> Likely **singlet** superconductivity

# Fu-Kane proposal

Split FS spin-degeneracy in  
**real space**

e.g. TI surface states

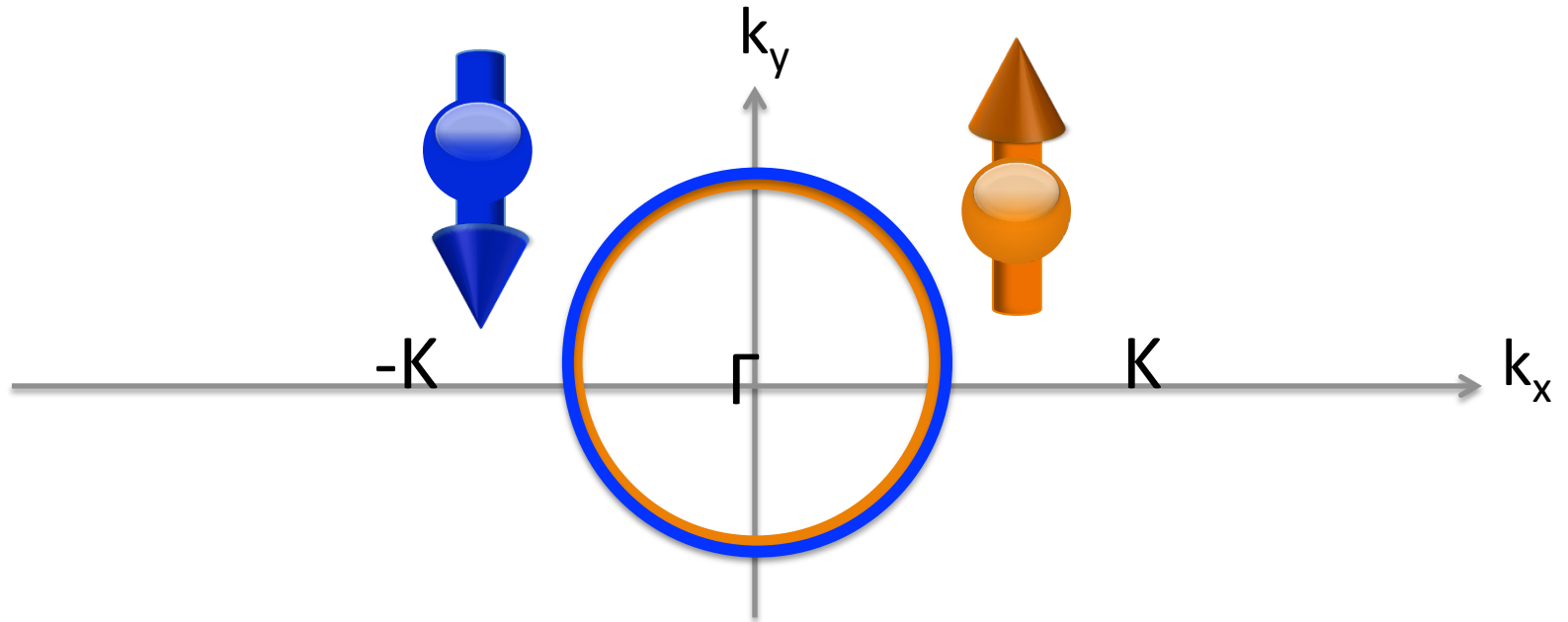
⇒ Real-space-separated  
**'spinless' fermions**



Fu & Kane:

One surface of TI + s-wave sc => **Topological sc**

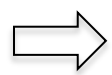
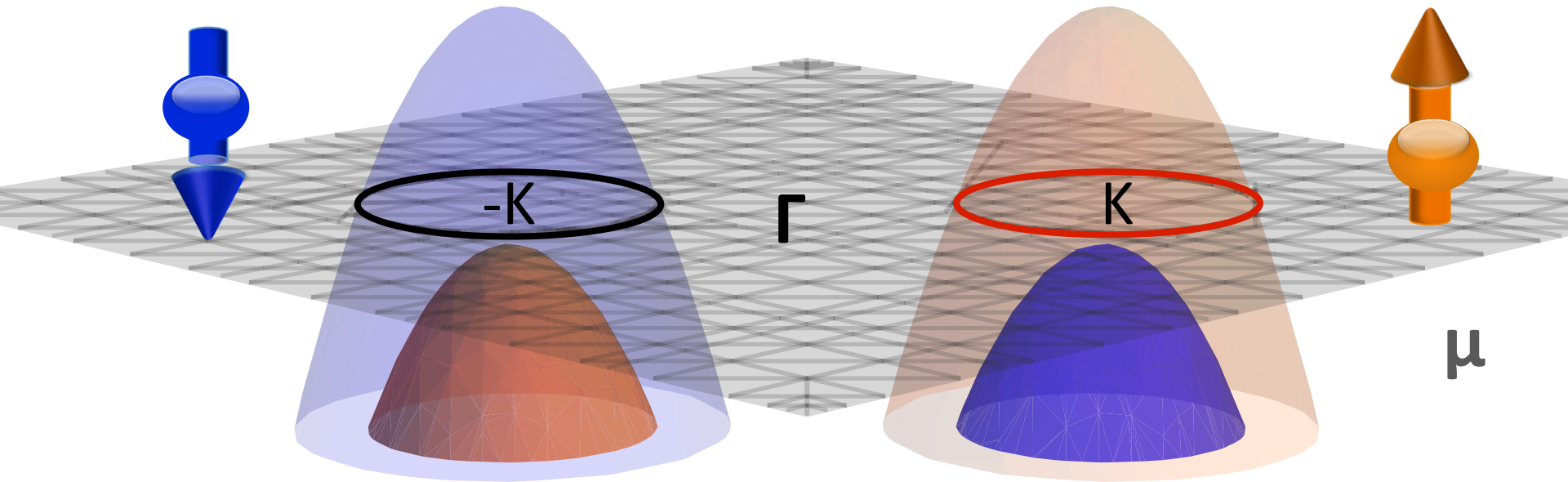
# Our proposal



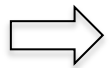
*Split spin-degeneracy in momentum space*



# Lightly hole-doped monolayer TMDs



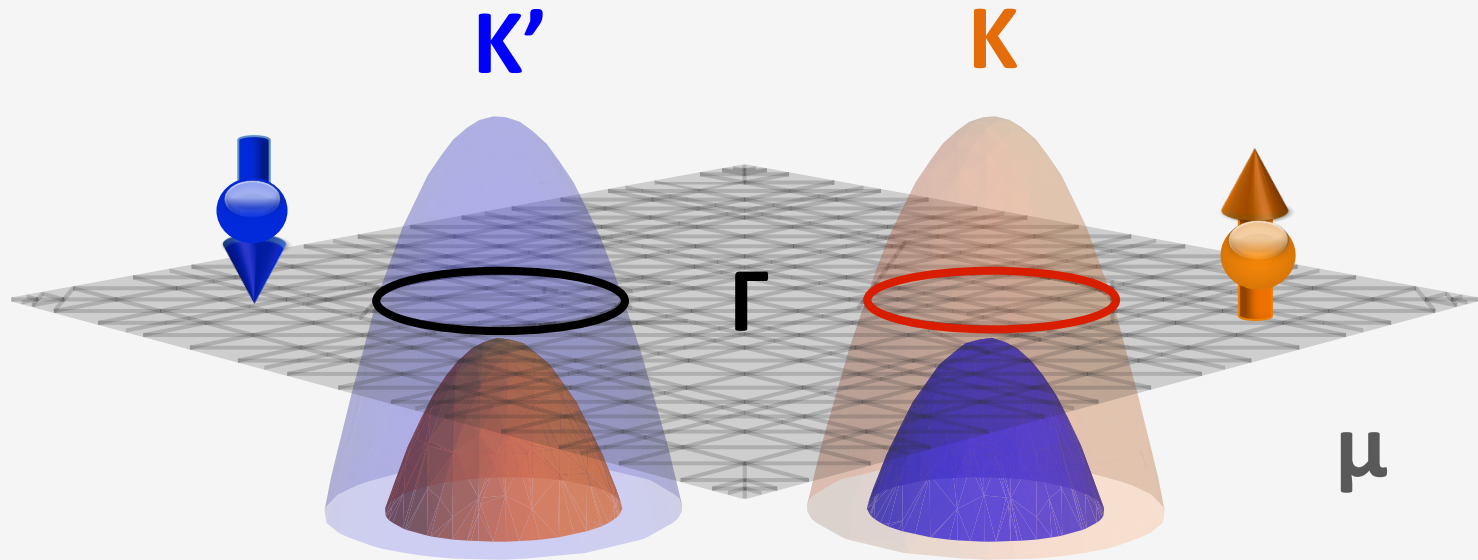
*Momentum-separated **spinless** fermions*



**Topo** sc?

Finite-momentum pairs => **Modulated** sc?

# Exotic pairing in **lightly hole-doped** TMDs



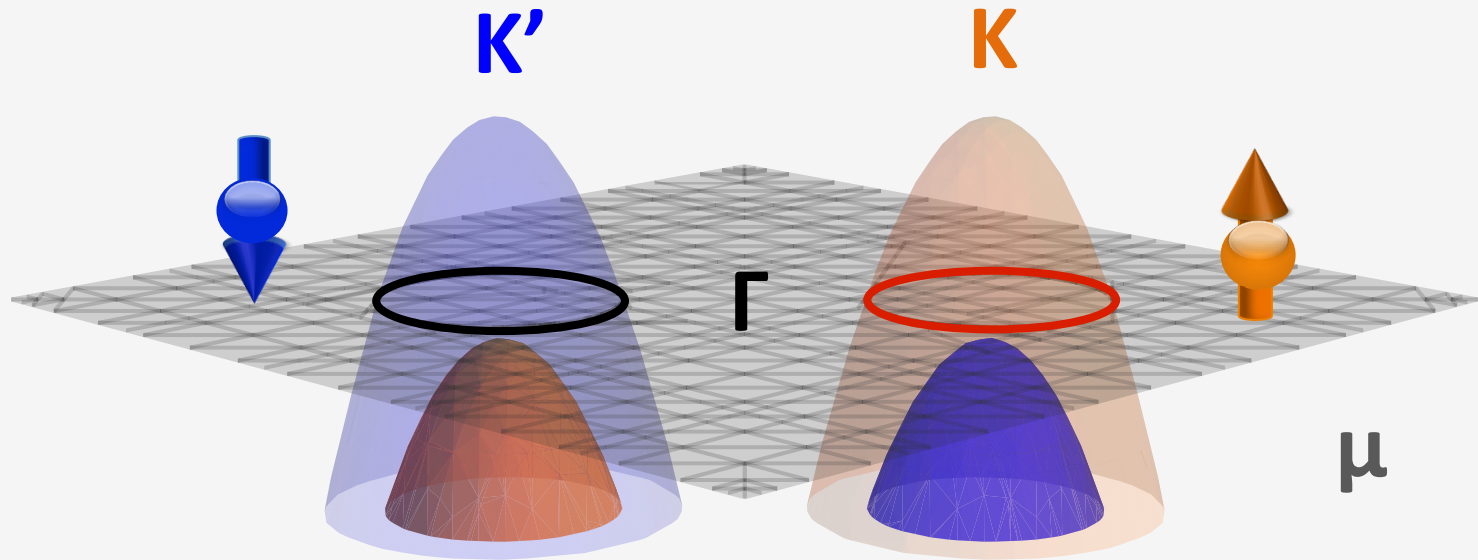
## Intrinsic pairing

YTH et al., Nat Comm (2017)

## Proximity-induced pairing

In preparation

# Exotic pairing in **lightly hole-doped** TMDs



## Intrinsic pairing

YTH et al., Nat Comm (2017)

## Proximity-induced pairing

In preparation



Abolhassan Vaezi  
Stanford University



Mark Fischer  
Weizmann Institute



Eun-Ah Kim  
Cornell University



***Want to predict:  
The *pairing symmetry* of intrinsic  
superconductivity***

# Pairing mechanism

□ d-electrons: moderate electronic repulsion

=> could be the dominant source for sc

Roldan, Cappelluti, Guinea, PRB (2013)

◆ Kohn-Luttinger mechanism => Unconventional sc

◆ Theoretical tool:

Two-step weak-coupling RG

Raghu, Kivelson, Scalapino, PRB (2010)

(Had been recently applied on ruthenates<sup>1,2,3</sup>)

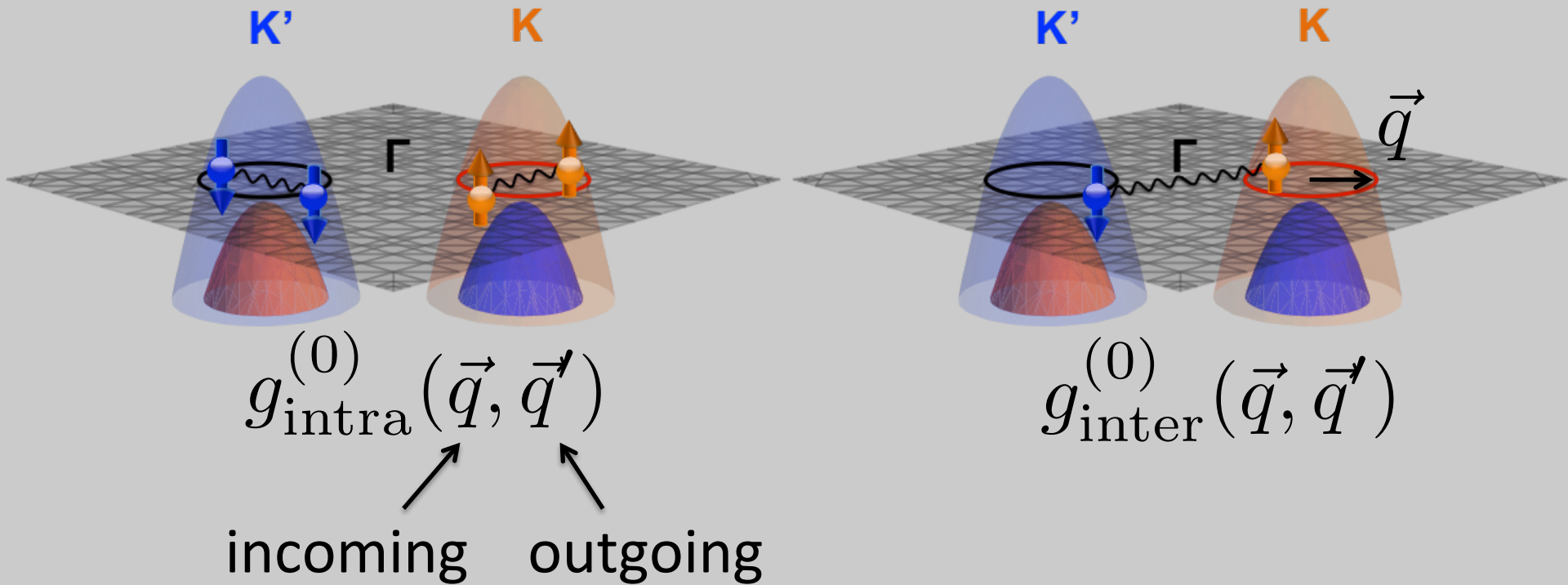
1: Raghu et al, PRL (2010)

2: YTH et al, PRB (2016)

3. Scaffidi et al, PRB (2014)

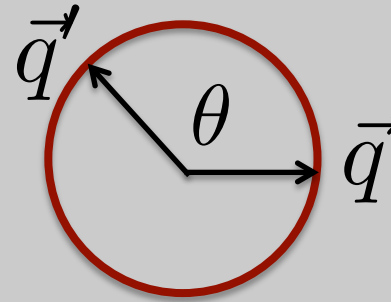
# Effective interactions in Cooper channel

- ◆ Near the Fermi surface:



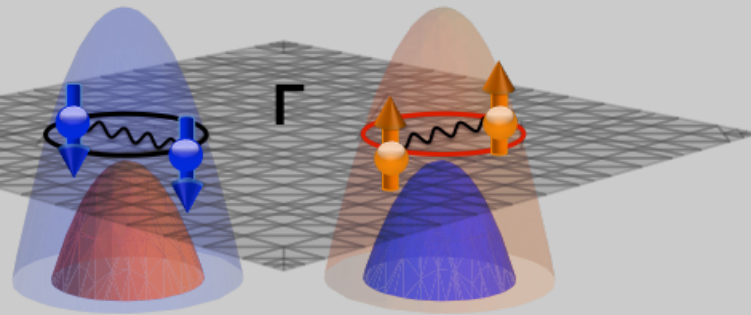
# Effective interactions in Cooper channel

Light doping:  
Circular pockets



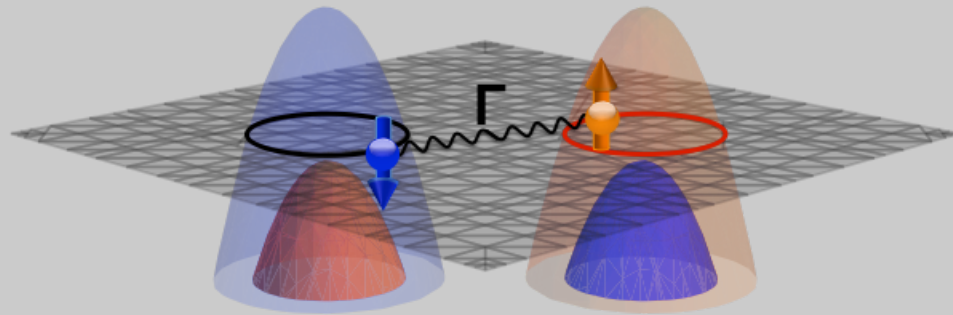
K'

K



K'

K



$$g_i^{(0)}(\vec{q}, \vec{q}') \longrightarrow g_i^{(0)}(\theta) = \sum_{\tilde{l}} \lambda_i^{\tilde{l},(0)} \cos(\tilde{l}\theta)$$

$i = \text{inter, intra}$

Interaction in partial-wave  
channel  $\tilde{l}$

# Derive effective interactions $\lambda_i^{\tilde{l},(0)}$

**Inter:**

$$g_{\text{inter}}^{(0)}(\vec{q}, \vec{q}') = \text{(1a)} + \text{(2a)} + \text{(3a)} + \text{(3b)} + \text{(3c)} + \text{(3d)} + \text{(3e)} + \text{(3f)}$$

**Intra:**

$$g_{\text{intra}}^{(0)}(\vec{q}, \vec{q}') = \text{(2b)} + \text{(3g)} + \text{(3h)} + \text{(3i)} + \text{(3j)}$$

**Two-loop contributions**

# Result: Dominant pairing symmetry

$$\lambda_{\text{inter}}^{|\tilde{l}|=1,(0)} = \lambda_{\text{intra}}^{|\tilde{l}|=1,(0)} < 0 \text{ are the largest attractions!}$$



◆ Dominant pairing channels:

inter- & intra-pocket  $|\tilde{l}| = 1$  paired states

# Result: Dominant pairing symmetry

$$\lambda_{\text{inter}}^{|\tilde{l}|=1,(0)} = \lambda_{\text{intra}}^{|\tilde{l}|=1,(0)} < 0 \text{ are the largest attractions!}$$



◆ Dominant pairing channels:

**inter- & intra-pocket  $|\tilde{l}| = 1$  paired states**

(degenerate: artifact due to the circular pockets)

◆ Inter  $|\tilde{l}| = 1$ : **p/d-wave** (2-dim irrep)

Intra  $|\tilde{l}| = 1$ : **p-wave**

} ***chiral***  
*(energetics)*



# Result: Dominant pairing symmetry

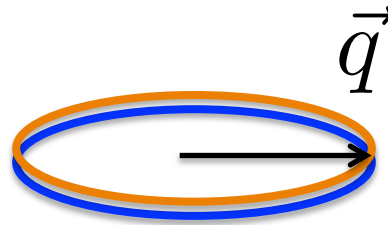
$$\lambda_{\text{inter}}^{|\tilde{l}|=1,(0)} = \lambda_{\text{intra}}^{|\tilde{l}|=1,(0)} < 0 \text{ are the largest attractions!}$$



- ◆ Dominant pairing channels:  
inter- & intra-pocket  $|\tilde{l}| = 1$  paired states
- ◆ Both are *chiral*  
 $\Rightarrow$  Both are *topological!*

# p-wave pairing in ordinary 2D Fermi liquid

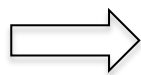
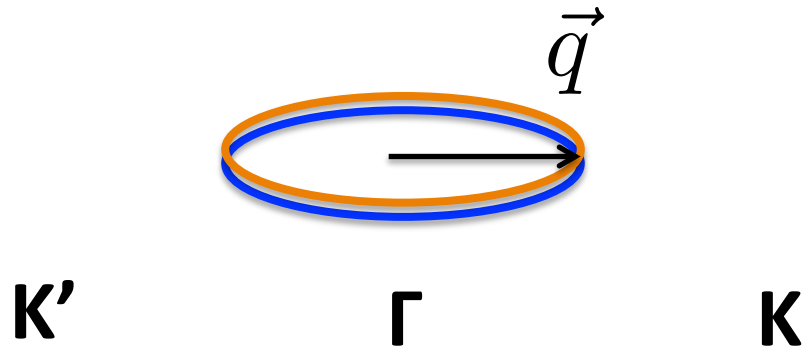
- Single **spin-degenerate** pocket + repulsive Hubbard  $U^1$



⇒ *Largest attraction appears in **p-wave** channel ( $l=1$ )<sup>1</sup>*

# Pairings in p-doped TMDs

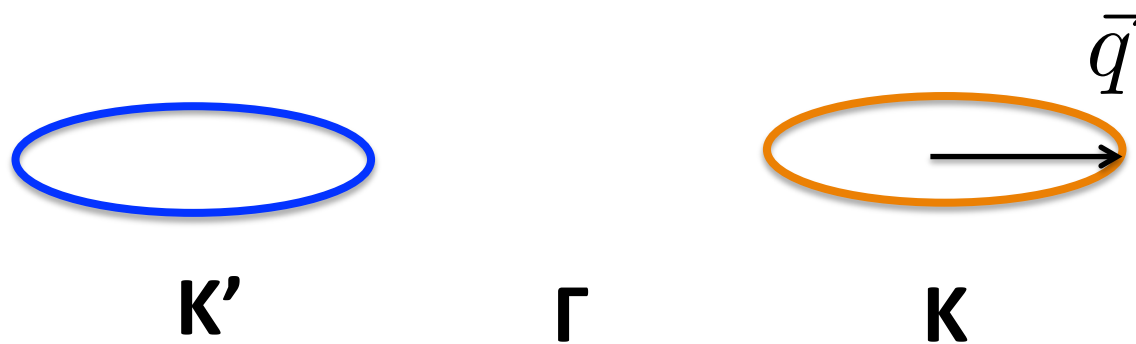
- Two **spinless** pockets centered **at K and K'**  
+ repulsive Hubbard U



1-1 correspondence for each possible pairing channel

# Pairings in p-doped TMDs

- Two **spinless** pockets centered at **K and K'**  
+ repulsive Hubbard U

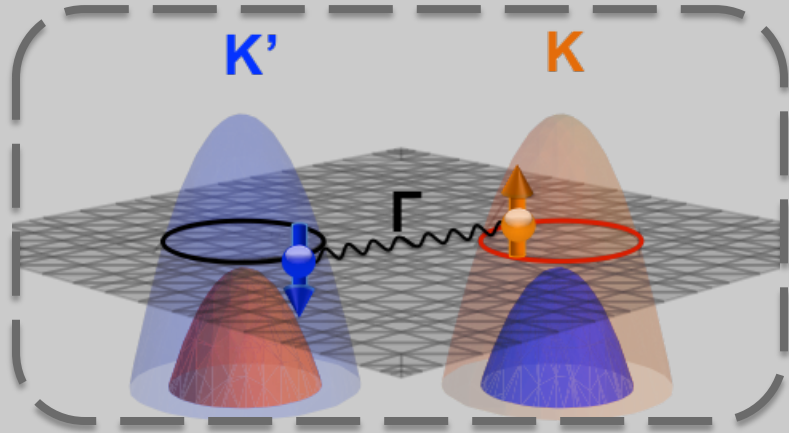


*Largest attraction appears in*

**Inter- & intra-pocket  $\tilde{l} = 1$  channel**

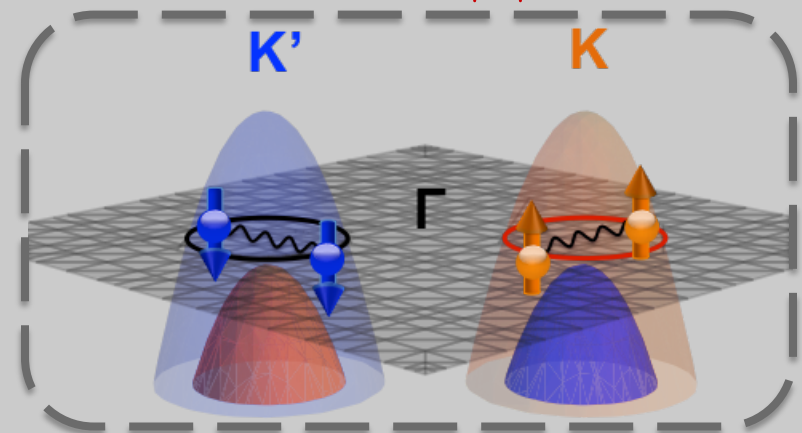
# Dominant topological paired states

Inter-pocket  $|\tilde{l}| = 1$  pair



- Mixture of singlet and triplet
- Chiral (p/d)-wave
- **Topo**:  $C=2$

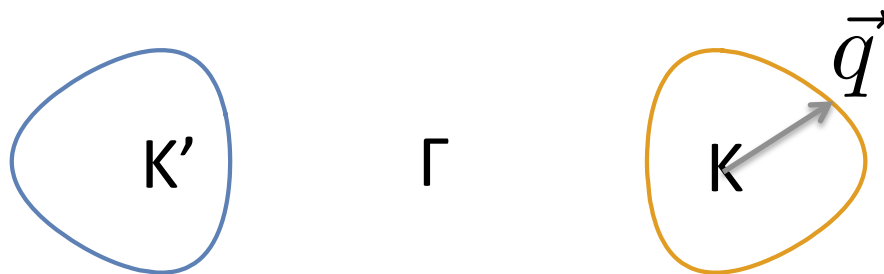
Intra-pocket  $|\tilde{l}| = 1$  pair



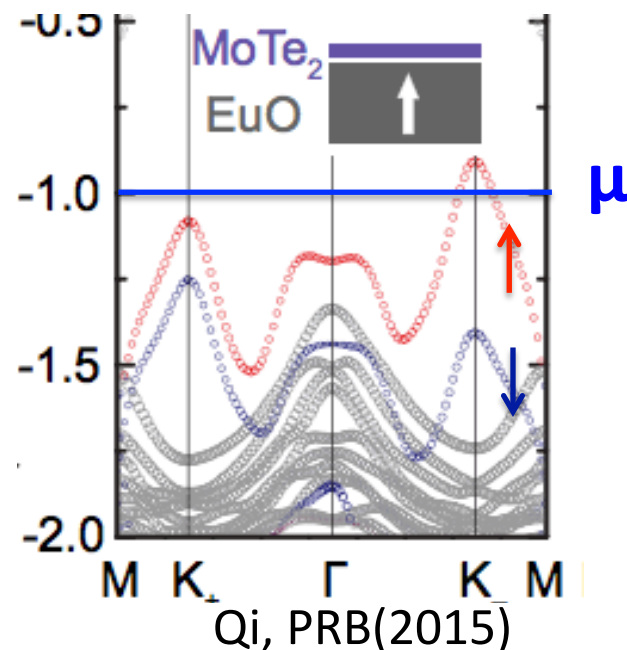
- Spin-triplet
- Chiral p-wave
- **Topo**:  $C=\pm 1$  per pocket
- **Phase modulated at  $\pm 2K$**

# Balance between Inter- and Intra-

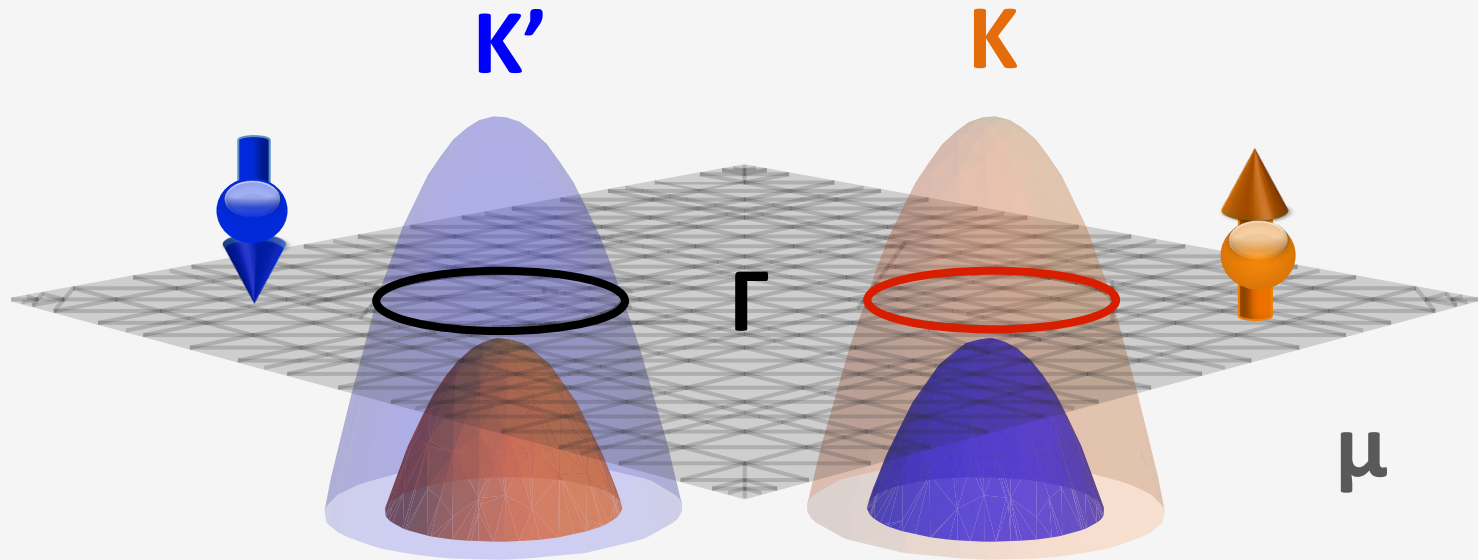
- Trigonal warping prefers inter-pocket pairing



- Ferromagnetic substrate prefers intra-pocket pairing



# Induced pairing in **lightly hole-doped** TMDs



- **Intrinsic pairing: topological**

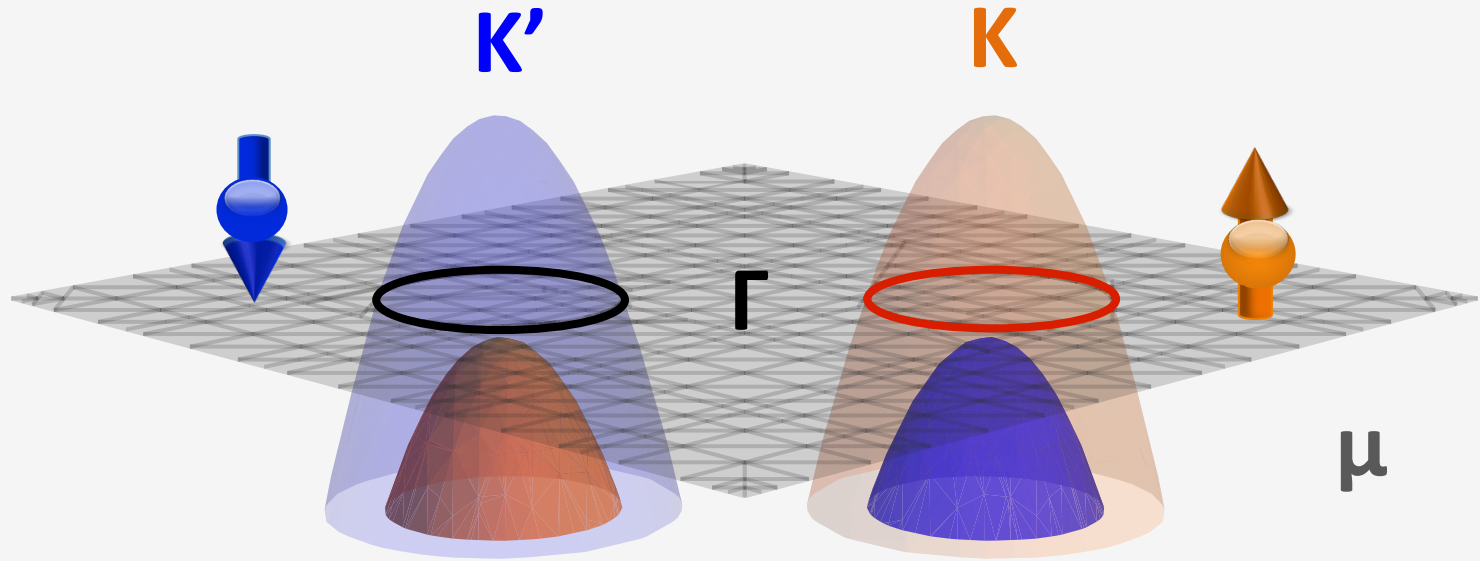
Nature Communication **8**, 14985

- **Proximity-induced pairing**

In preparation



# Induced pairing in **lightly hole-doped** TMDs



Intrinsic pairing

Nature Communication 8, 14985

**Proximity-induced** pairing

In preparation



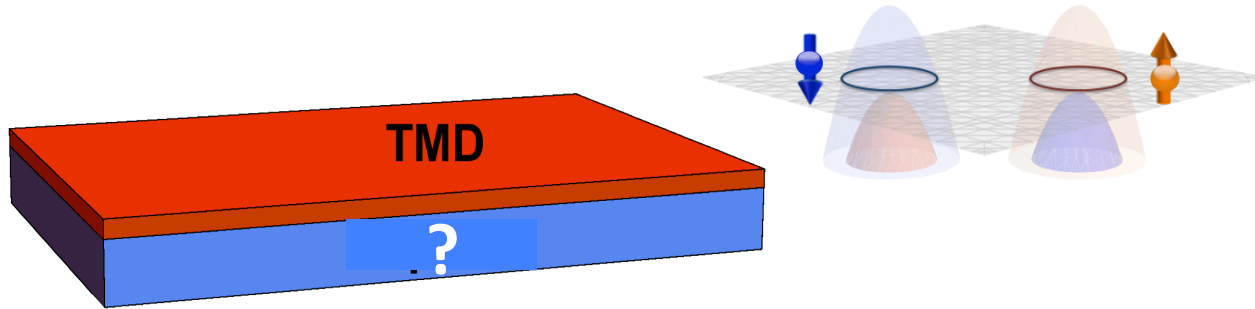
Kyungmin Lee  
Ohio State University



Eun-Ah Kim  
Cornell University

# Choose the superconductor

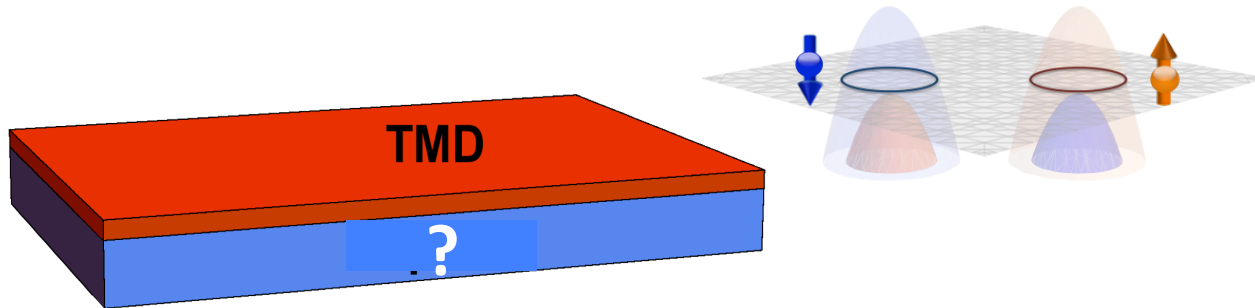
◆ Lightly hole-doped TMD with ? sc



$C_{3v}$	$\Gamma$	Singlet	Triplet
	$A_1$		s-wave
$A_2$			Nodal f-wave
$E$		$(d_{x^2-y^2}, d_{xy})$	$(p_x, p_y)$

# Choose the superconductor

◆ Lightly hole-doped TMD with ? sc

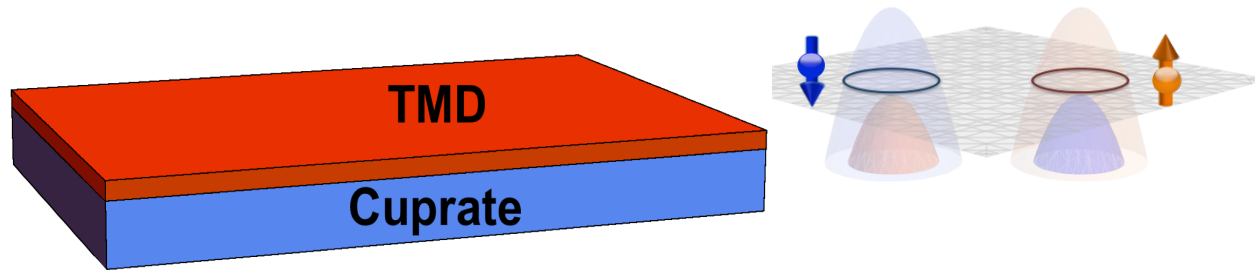


$C_{3v}$	$\Gamma$	Singlet	Triplet
	$A_1$	s-wave	Nodeless f-wave
	$A_2$		Nodal f-wave
	$E$	$(d_{x^2-y^2}, d_{xy})$	$(p_x, p_y)$

=> Use cuprate!

# Our proposal

## ◆ Lightly hole-doped TMD with cuprate



❖ Induce p-wave component?

❖ Possible high  $T_c$

Recent exp: High- $T_c$  sc induced by cuprate in

Graphene<sup>1</sup>, TaS<sub>2</sub><sup>2</sup>

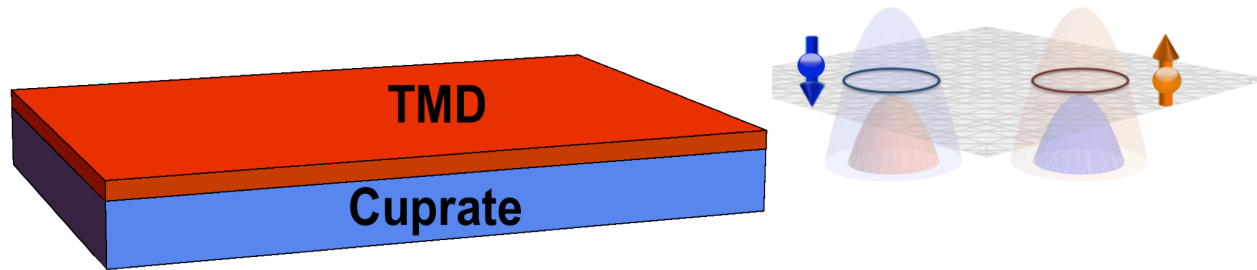
***=> High- $T_c$  odd-parity pairing?***

1: Bernardo et al, Ncomms (2017)

2: Li et al, arxiv:1703.00867 (2017)

# Our proposal

- ◆ Lightly hole-doped TMD with cuprate

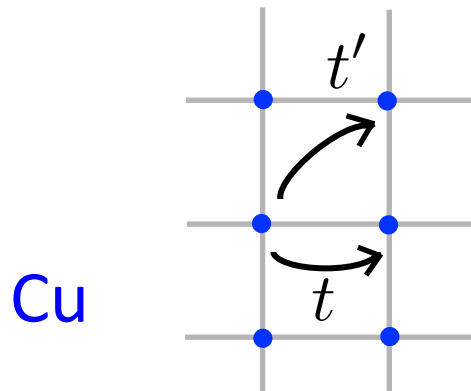


- ❖ Induce p-wave component?

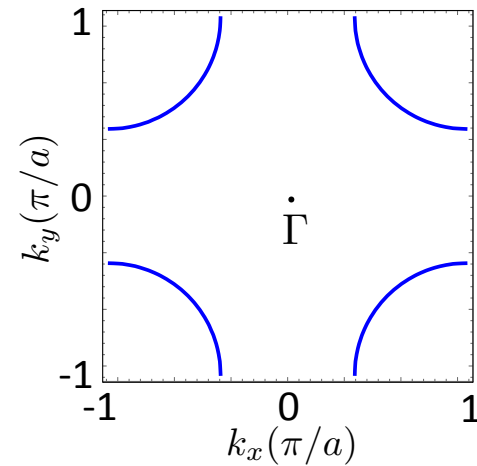
***Confirm the guess: do a self-consistent calculation***

# Model: kinetic terms

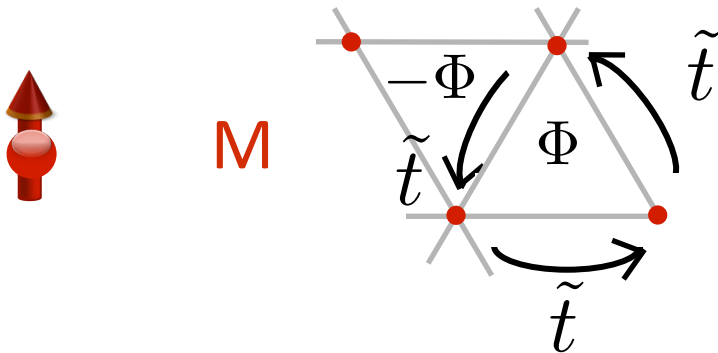
## ◆ Cuprate layer



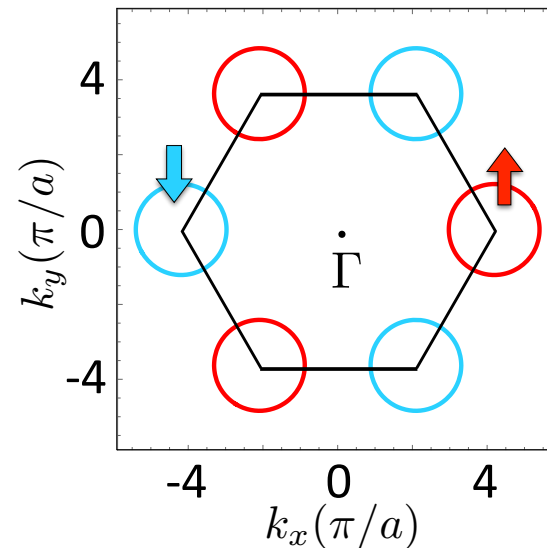
## Spin-degenerate



## ◆ monolayer TMD: two band



## Spin-valley locked

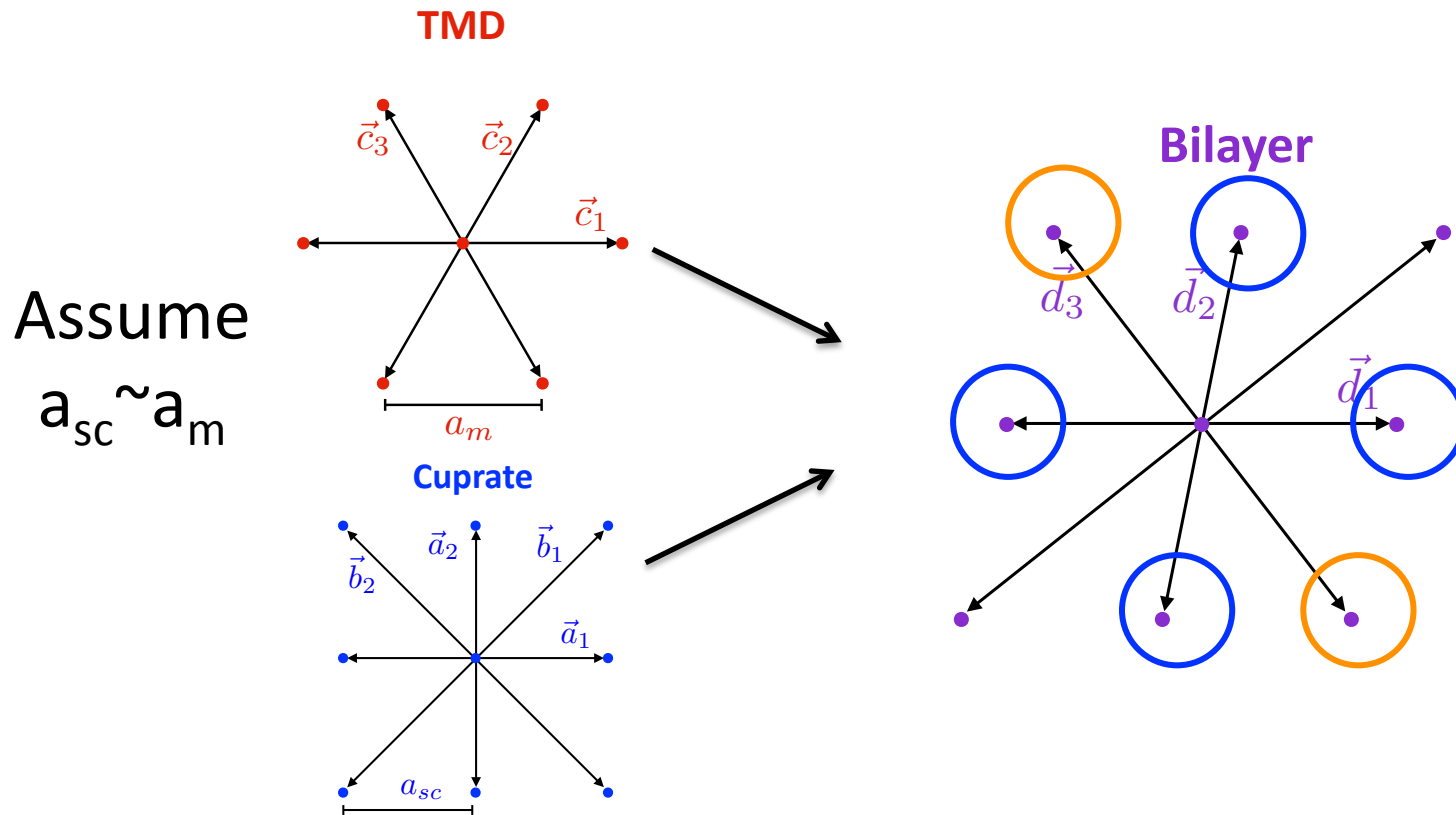


$$\Phi = 0.9\pi \quad H_{\downarrow} = H_{\uparrow}^* \Rightarrow \text{TRS}$$



# Mutual lattice straining

- ◆ Inter-layer tunneling: onsite, nn, nnn

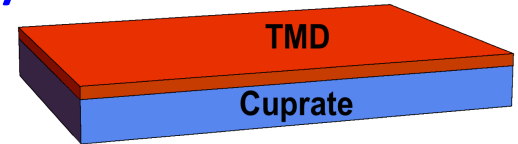


# Mean-field Hamiltonian

- ◆ Mean-field Hamiltonian for the **bilayer**:

$\psi$  : cuprate

$\phi$  : TMD



$$H_{\text{BdG}} = \sum_{\mathbf{x}, \mathbf{y}} \left( \begin{array}{cccc} \psi_{\uparrow \mathbf{x}}^\dagger & \phi_{\uparrow \mathbf{x}}^\dagger & \psi_{\downarrow \mathbf{x}} & \phi_{\downarrow \mathbf{x}} \end{array} \right) \begin{pmatrix} t_{\mathbf{xy}}^{sc} & \lambda_{\mathbf{xy}} & \Delta_{\mathbf{xy}}^{sc} & 0 \\ \lambda_{\mathbf{yx}}^* & t_{\mathbf{xy}}^m & 0 & \Delta_{\mathbf{xy}}^m \\ \Delta_{\mathbf{yx}}^{sc*} & 0 & -t_{\mathbf{yx}}^{sc} & -\lambda_{\mathbf{xy}}^* \\ 0 & \Delta_{\mathbf{yx}}^{m*} & -\lambda_{\mathbf{yx}} & -t_{\mathbf{yx}}^{m*} \end{pmatrix} \begin{pmatrix} \psi_{\uparrow \mathbf{y}} \\ \phi_{\uparrow \mathbf{y}} \\ \psi_{\downarrow \mathbf{y}} \\ \phi_{\downarrow \mathbf{y}} \end{pmatrix}$$

# Mean-field Hamiltonian

## ◆ Mean-field Hamiltonian for the bilayer:

$\psi$  : cuprate

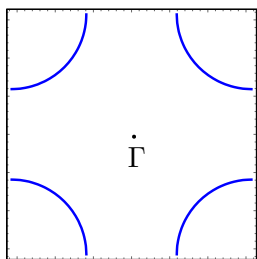
$\phi$  : TMD

$$H_{\text{BdG}} = \sum_{\mathbf{x}, \mathbf{y}} \left( \psi_{\uparrow \mathbf{x}}^\dagger \quad \phi_{\uparrow \mathbf{x}}^\dagger \quad \psi_{\downarrow \mathbf{x}} \quad \phi_{\downarrow \mathbf{x}} \right)$$

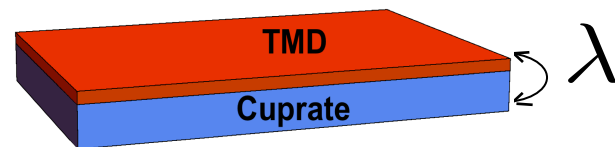
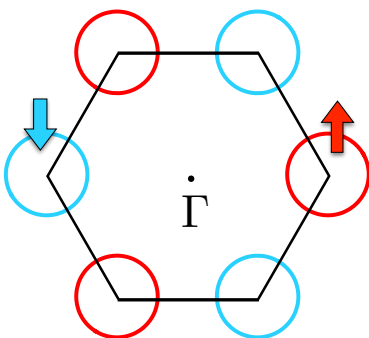
Normal

$$\begin{pmatrix} t_{xy}^{sc} & \lambda_{xy} & \Delta_{xy}^{sc} & 0 \\ \lambda_{yx}^* & t_{xy}^m & 0 & \Delta_{xy}^m \\ \Delta_{yx}^{sc*} & 0 & -t_{yx}^{sc} & -\lambda_{xy}^* \\ 0 & \Delta_{yx}^{m*} & -\lambda_{yx} & -t_{yx}^{m*} \end{pmatrix} \begin{pmatrix} \psi_{\uparrow \mathbf{y}} \\ \phi_{\uparrow \mathbf{y}} \\ \psi_{\downarrow \mathbf{y}} \\ \phi_{\downarrow \mathbf{y}} \end{pmatrix}$$

Cuprate



TMD



# Mean-field Hamiltonian

## ◆ Mean-field Hamiltonian for the bilayer:

$\psi$  : cuprate

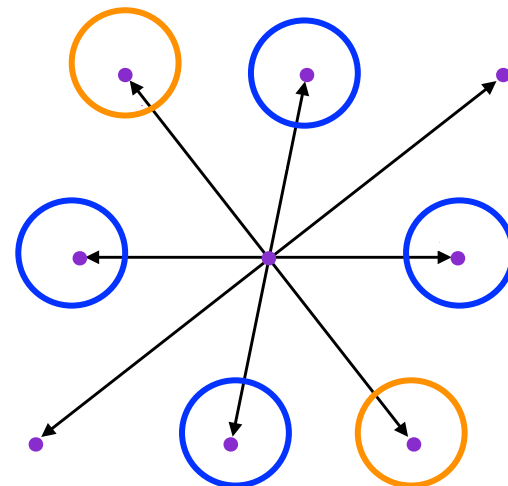
$\phi$  : TMD

$$H_{\text{BdG}} = \sum_{\mathbf{x}, \mathbf{y}} \begin{pmatrix} \psi_{\uparrow \mathbf{x}}^\dagger & \phi_{\uparrow \mathbf{x}}^\dagger & \psi_{\downarrow \mathbf{x}} & \phi_{\downarrow \mathbf{x}} \end{pmatrix} \begin{pmatrix} t_{\mathbf{xy}}^{sc} & \lambda_{\mathbf{xy}} & \Delta_{\mathbf{xy}}^{sc} & 0 \\ \lambda_{\mathbf{yx}}^* & t_{\mathbf{xy}}^m & 0 & 0 \\ \Delta_{\mathbf{yx}}^{sc*} & 0 & -t_{\mathbf{yx}}^{sc} & -\lambda_{\mathbf{xy}}^* \\ 0 & 0 & -\lambda_{\mathbf{yx}} & -t_{\mathbf{yx}}^{m*} \end{pmatrix} \begin{pmatrix} \psi_{\uparrow \mathbf{y}} \\ \phi_{\uparrow \mathbf{y}} \\ \psi_{\downarrow \mathbf{y}} \\ \phi_{\downarrow \mathbf{y}} \end{pmatrix}$$

Pairing

Cuprate

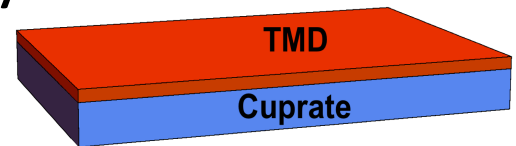
$\Delta_{\mathbf{xy}}^{sc}$  : nn & nnn pairing



# Calculation

- ◆ Mean-field Hamiltonian for the bilayer:

$\psi$  : cuprate



$\phi$  : TMD

$$H_{\text{BdG}} = \sum_{\mathbf{x}, \mathbf{y}} \begin{pmatrix} \psi_{\uparrow \mathbf{x}}^\dagger & \phi_{\uparrow \mathbf{x}}^\dagger & \psi_{\downarrow \mathbf{x}} & \phi_{\downarrow \mathbf{x}} \end{pmatrix} \begin{pmatrix} t_{\mathbf{xy}}^{sc} & \lambda_{\mathbf{xy}} & \Delta_{\mathbf{xy}}^{sc} & 0 \\ \lambda_{\mathbf{yx}}^* & t_{\mathbf{xy}}^m & 0 & \Delta_{\mathbf{xy}}^m \\ \Delta_{\mathbf{yx}}^{sc*} & 0 & -t_{\mathbf{yx}}^{sc} & -\lambda_{\mathbf{xy}}^* \\ 0 & \Delta_{\mathbf{yx}}^{m*} & -\lambda_{\mathbf{yx}} & -t_{\mathbf{yx}}^{m*} \end{pmatrix} \begin{pmatrix} \psi_{\uparrow \mathbf{y}} \\ \phi_{\uparrow \mathbf{y}} \\ \psi_{\downarrow \mathbf{y}}^\dagger \\ \phi_{\downarrow \mathbf{y}}^\dagger \end{pmatrix}$$

- ◆ Self-consistently solve gap equation

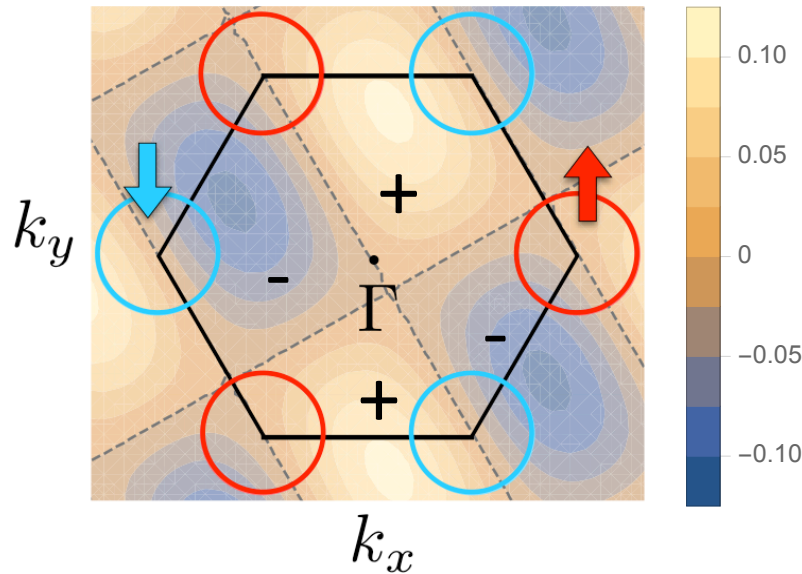
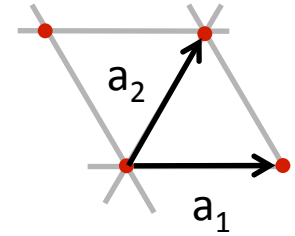
Cuprate:  $\Delta_{\mathbf{xy}}^{sc} = U \langle \psi_{\downarrow \mathbf{y}} \psi_{\uparrow \mathbf{x}} \rangle \quad U < 0$

=> Obtain finite TMD pair amplitude from Andreev reflection

# Result

- ◆ The induced pairing amplitude in TMD:

$$F(\mathbf{k}) = 2\eta_s(\cos k_1 - \cos k_2) - 2\eta_t(\sin k_1 + \sin k_2) \quad k_i = \mathbf{k} \cdot \mathbf{a}_i$$

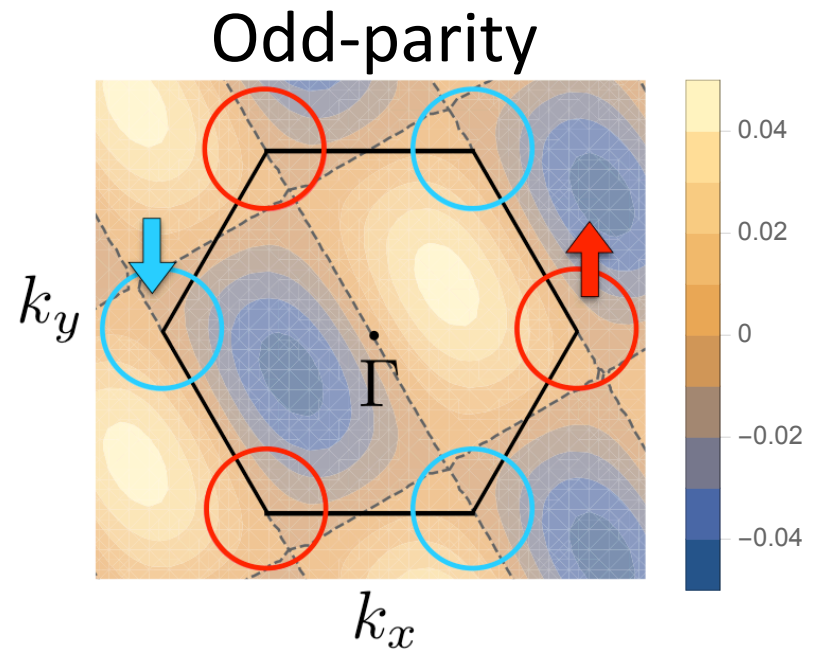
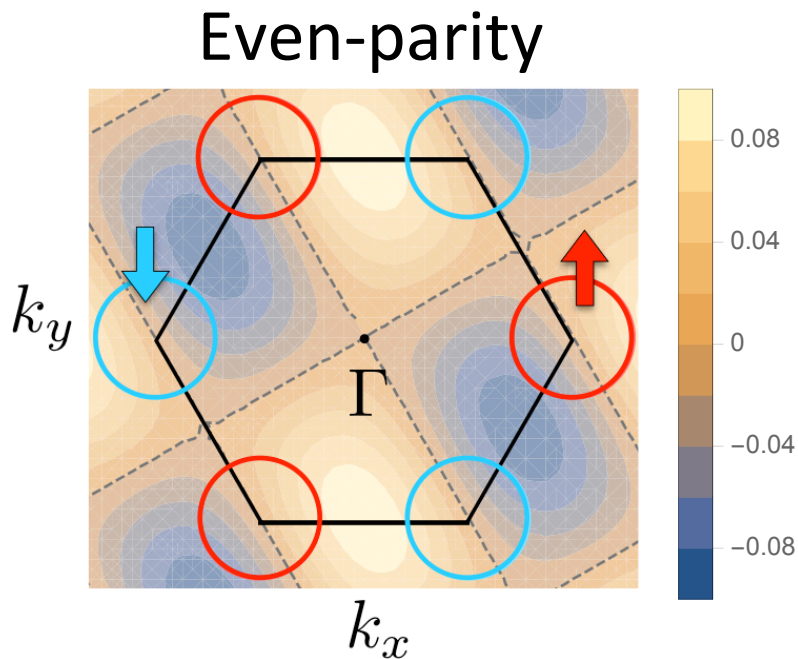


Coloring: pairing amplitude

- Exists finite odd-parity component ( $\eta_s \sim 3\eta_t$ )
- **Nodal:** partial-wave  $|\tilde{l}| = 1$  about K and K'

# Induced pairing symmetry

- ◆ The induced pairing in TMD: **Nematic (p+d)-wave**
  - Even: d-wave    Odd: p-wave
  - Nematic: single component



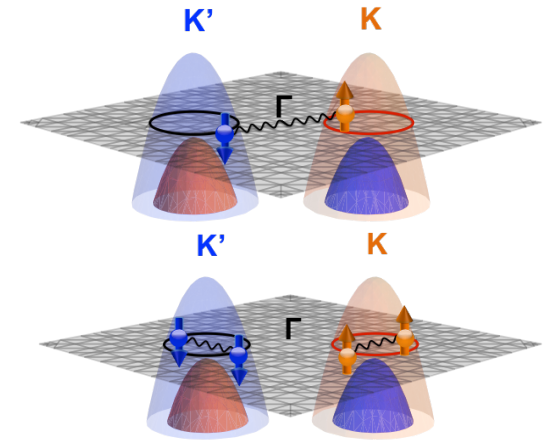


# Summary: Lightly p-doped monolayer TMDs

➤ Two types of **intrinsic topological** superconductivity: (Nat Comm **8**, 14985)

❖ Inter: Chiral (p+d)-wave  
Favored by trigonal warping

❖ Intra: Chiral p-wave & **Modulated**  
Favored by ferro substrate

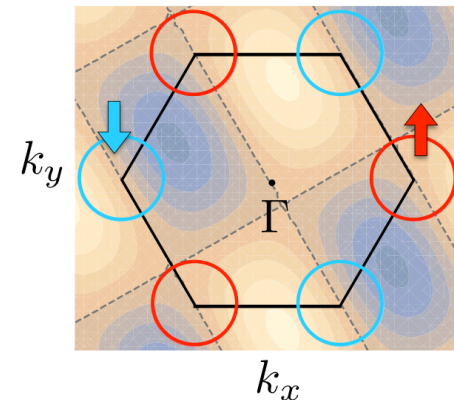
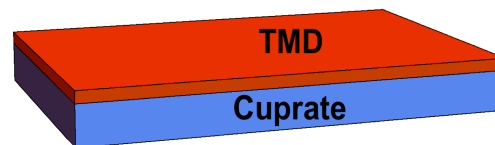


➤ **Proximity**-induced pairing by **cuprate**: (In preparation)

❖ Sizable **p-wave component** with **possibly higher-Tc**

❖ Nematic

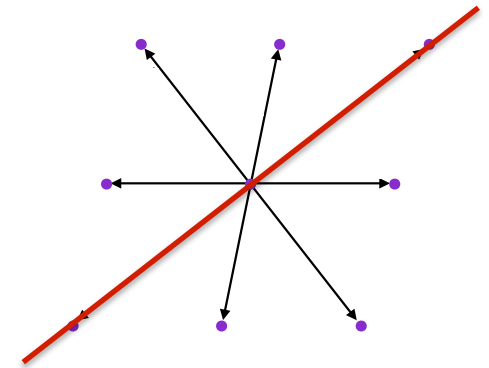
❖ Nodal



# Induced pairing symmetry

- ◆ The point group symmetry of the bilayer Hamiltonian:  $C_s$

Mirror (x+y)

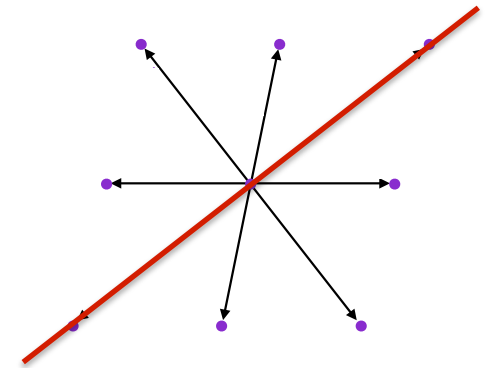


$C_s$	$\Gamma$	Singlet	Triplet
	$A_1$	s-wave etc.	f-wave etc.
	$A_1'$	$d_{x^2-y^2}, d_{xy}$ etc.	$p_x, p_y$ etc.

# Induced pairing symmetry

- ◆ The bilayer Hamiltonian has point group symmetry  $C_s$

Mirror (x+y)

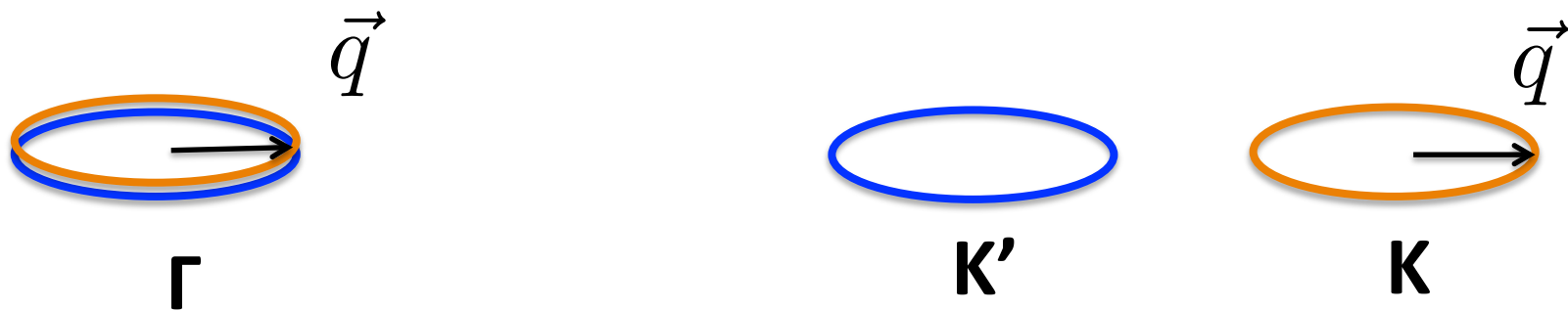


- ◆ The ‘nematic (p+d)-wave’ pairing belongs to **nontrivial irrep  $A_1'$**

$C_s$	$\Gamma$	Singlet	Triplet
	$A_1$	s-wave etc.	f-wave etc.
	$A_1'$	$d_{x^2-y^2}, d_{xy}$ etc.	$p_x + p_y, p_x - p_y$ etc.

# Result: Pairings in p-doped TMDs

- Two **spinless** pockets centered at **K and K'**  
+ repulsive Hubbard U



- ◆  $l$  even, spin-singlet  $\longleftrightarrow$  Inter,  $\tilde{l}$  even
  - ◆  $l$  odd, spin-triplet  $s_z = \pm 1$   $\longleftrightarrow$  Intra,  $\tilde{l}$  odd
  - $s_z = 0$   $\longleftrightarrow$  Inter,  $\tilde{l}$  odd
- } degenerate

# Experimental Detection

# Intrinsic : inter-pocket $\tilde{l} = 1$ pairing

## □ Inter-pocket pairing

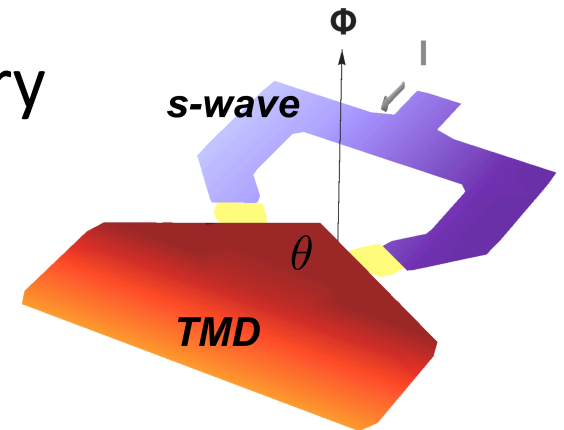
◆ Existence of triplet component:  
in-plane field dependence (?)

◆ Anisotropy: SQUID interferometry  
 $I_c(\Phi)$  is  $\theta$ -dependent

◆ Topological with  $C=2$ :

- Quantized thermal Hall conductivity
- Two chiral Majorana edge states

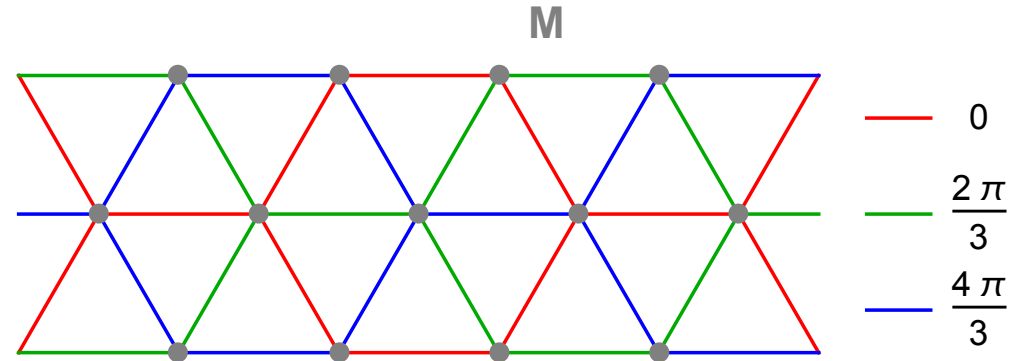
◆ Chiral: Polar Kerr effect



# Intrinsic : intra-pocket $\tilde{l} = 1$ pairing

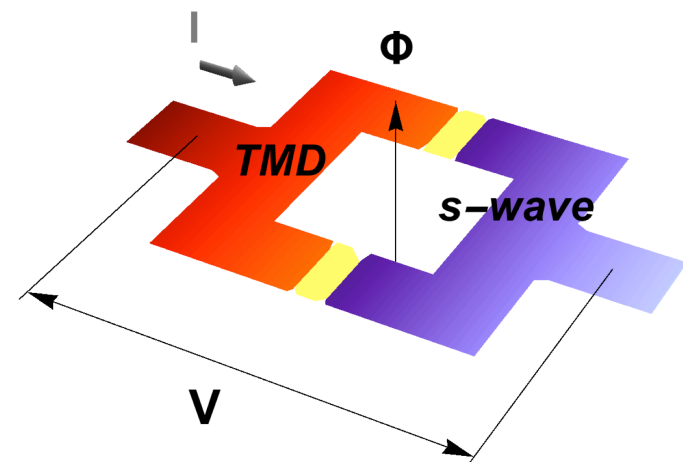
□ Intra-pocket pairing

◆ Phase modulation



◆ SQUID loop in resistive mode:

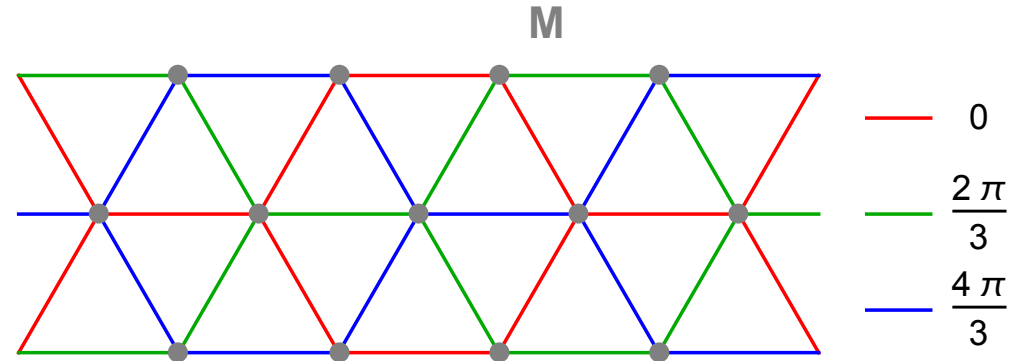
- If pairing in TMD were **uniform**  
=>  $V$  oscillates in  $\Phi$  with a period  $hc/(2e)$



# Intrinsic : intra-pocket $\tilde{l} = 1$ pairing

□ Intra-pocket pairing

◆ Phase modulation



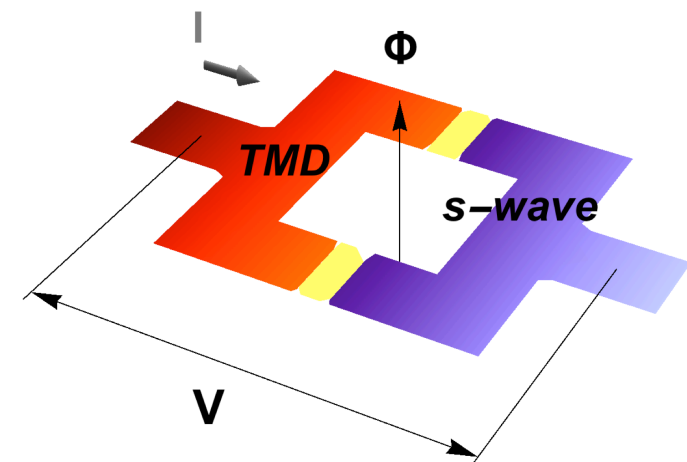
◆ SQUID loop in resistive mode:

- If TMD has **modulated** pairing

↑↑: pair momentum =  $2K$

↓↓: pair momentum =  $-2K$

*Match two sides' momenta  
=> Two pairs tunnel together*

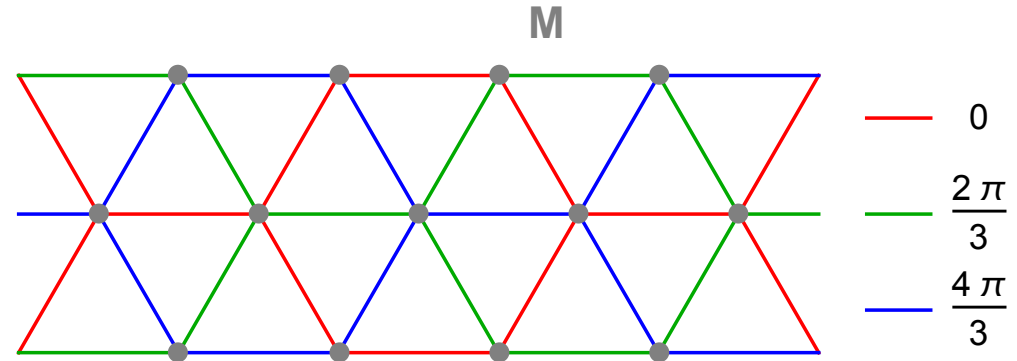




# Intrinsic : intra-pocket $\tilde{l} = 1$ pairing

□ Intra-pocket pairing

◆ Phase modulation



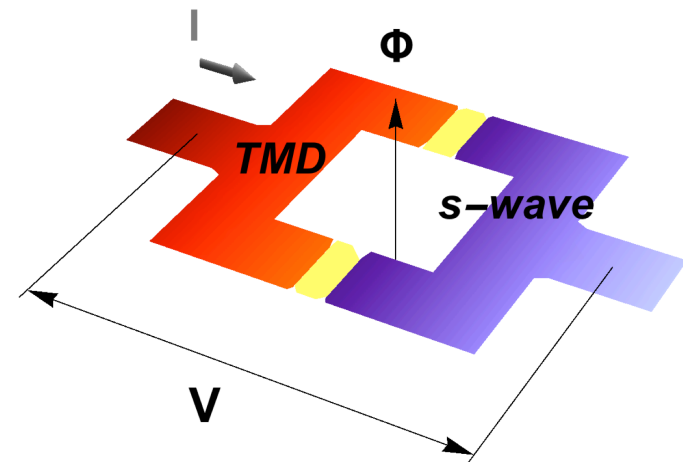
◆ SQUID loop in resistive mode:

- If TMD has **modulated** pairing

↑↑: pair momentum =  $2K$

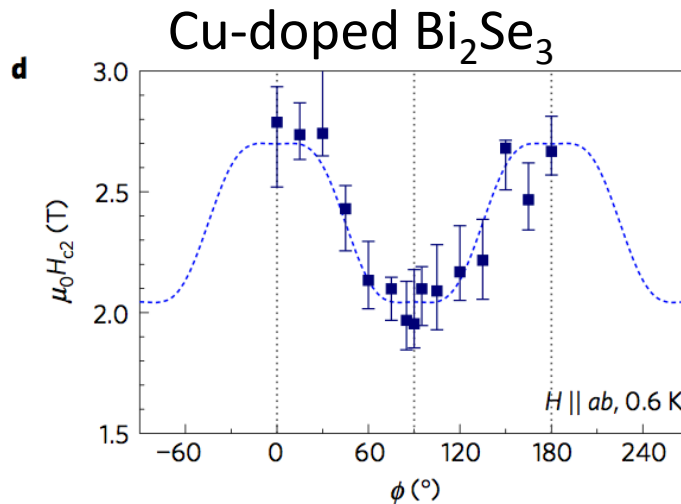
↓↓: pair momentum =  $-2K$

*Period becomes  $hc/(4e)$  !*



# Proximity-induced pairing

- ◆ **Nematic:** angular dependence of the in-plane  $H_{c2}$  should be two-fold despite trigonal lattice



Yonezawa et al, Nphys (2016)

- ◆ **Existence of triplet component:**  
in-plane field dependence (?)