Arborescent Vs non-arborescent knots and links

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See our updates on colored HOMFLY-PT on knotebook.org website

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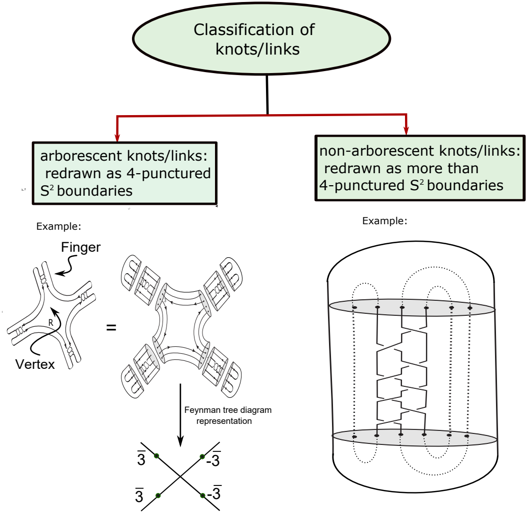
- Summary and open problems

Introduction

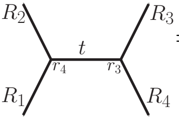


(a) $(\bar{3},\bar{3},-\bar{3},\bar{3})$ anti-parallel pretzel link

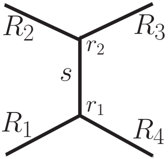
(b) $\mathbf{8_{18}}$ knot

Eigenbasis of Braiding operator B

For the four-punctured S^2 boundary, the conformal block bases are:



$$= |\phi^{(1)}_{t,r_3 r_4}(R_1, R_2, R_3, R_4)\rangle$$



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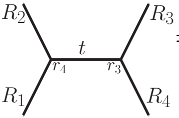

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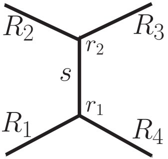

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$a^{t,r_3 r_4}_{s;r_1 r_2}\begin{bmatrix} R_1 & R_2 \\ R_3 & R_4 \end{bmatrix}$ is the duality matrix relating the two basis

Figure 8 knot invariant

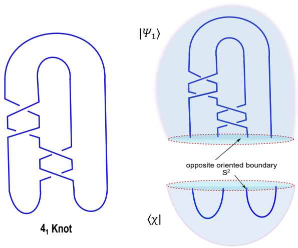
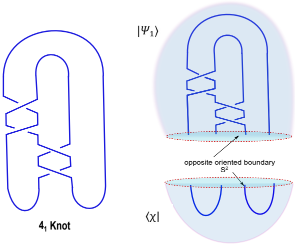
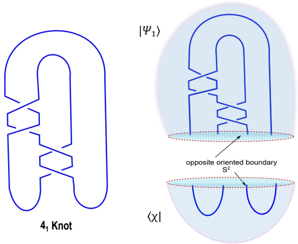


Figure 8 knot invariant



Involves braidings in middle as well as side two-strands.

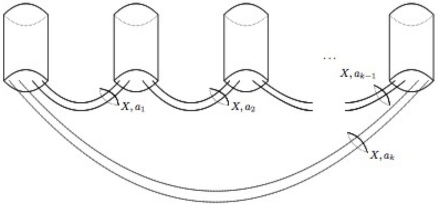
Figure 8 knot invariant



Involves braidings in middle as well as side two-strands. Duality matrix required to go from middle to side-strand basis! The invariants will involve braiding eigenvalues and duality matrices

Arborescent Knots

- The knots with more than four-strands which can be drawn as



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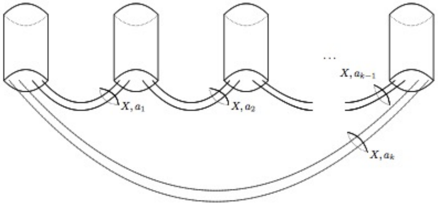

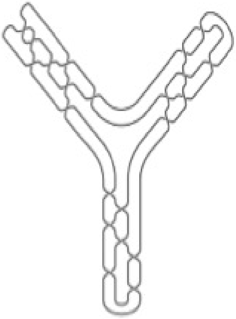

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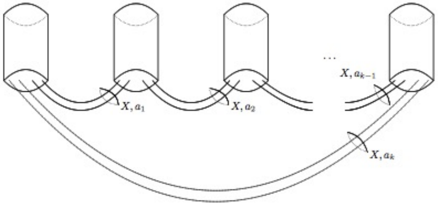
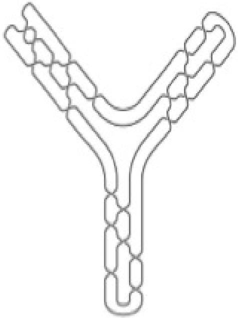
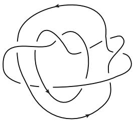


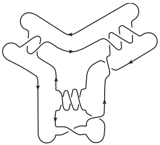


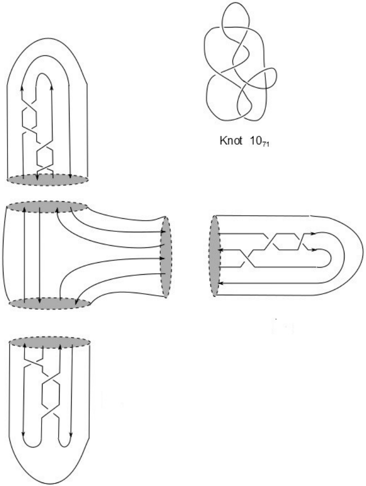
are called **Arborescent knots**.

- These knots in S^3 are obtained from gluing three-balls where some three-balls have two or more four-punctured S^2 boundaries

10_{152} and 10_{71} arborescent knots



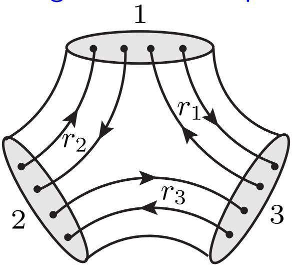




Knot 10_{71}

Building blocks

Requires the following building blocks to compute knot polynomials



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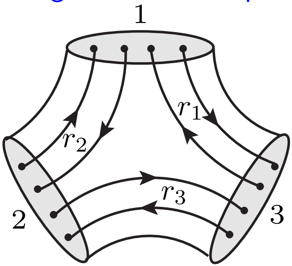

$$\nu_3 = \sum_{t,r_1,r_2,r_3} (\Omega(t, r_1, r_2, r_3) |\phi^{(1)}_{t;r_1,r_2}\rangle |\phi^{(2)}_{t;r_2,r_3}\rangle \ldots |\phi^{(3)}_{t;r_3,r_1}\rangle$$

$$\Omega(t; r_1, r_2, r_3) = \frac{\{R, \bar{R}, t, r_1\}\{R, \bar{R}, t, r_1\}\{R, \bar{R}, t, r_1\}}{\sqrt{dim_q t}}$$

Equivalent Building Blocks

- To write states of some diagrams, equivalent diagrams are shown:

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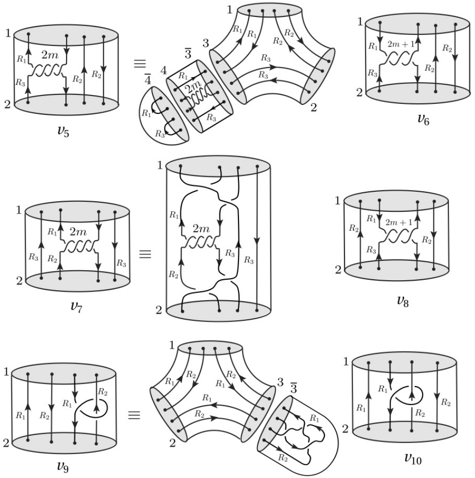
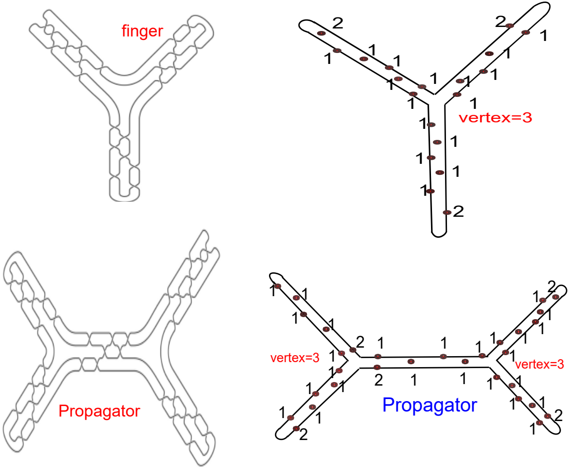


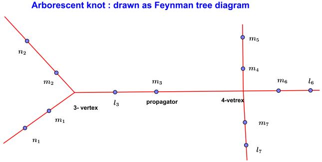
Arborescent knot- Feynman diagram analogy



Arborescent knots (Feynman tree diagram)

Family Approach: Arborescent knots

one universal invariant as a function of parameters- choice of parameters gives different knot invariants!



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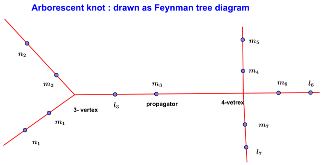

The best parametric family (for describing upto 10-crossing knots) in this class (of 4-point Feynman trees with up to 7 parameters)

A.Mironov, A. Morozov, An. Morozov, V.Singh, A. Sleptsov, PR (2016)

$d_R \sum_{X,\bar{Y}} F_{ap}(X) F_{pap}(X) T_X^n \bar{P}_{X\bar{Y}} F_{apa}(\bar{Y}) F_{aa}(\bar{Y})$

$9_{32-33}, 10_{45}, 10_{57}, 10_{62}, 10_{64}, 10_{66}, 10_{79-85}, 10_{87-91}, 10_{94}, 10_{98}, 10_{99}, 10_{139}, 10_{141}, 10_{143}, 10_{148-154}$- list not contained!

Arborescent knot invariants

- arborescent knot invariants will involve braiding eigenvalues and two types of duality matrices $a_{s;r_1,r_2}^{t;r_3,r_4}\begin{bmatrix}\bar{R} & R\\ \bar{R} & R\end{bmatrix}$ and or $a_{s_1;r_1,r_2}^{t;r_3,r_4}\begin{bmatrix}\bar{R} & R\\ R & \bar{R}\end{bmatrix}$

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- However, other duality matrices are needed for **non-arborescent knot** invariants!

Current status on the duality matrix elements

- **Duality matrices proportional to quantum Wigner 6j** (completely known for $SU(2)$ (Kirillov, Reshetikhin) and hence *we can write the polynomial form of any knot invariant (colored Jones' polynomials* $J_n(q)$*)*

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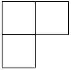
- For  , Gu-Jockers have worked out (2014)

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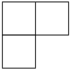
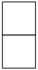
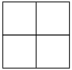
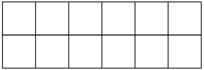
- For  , Gu-Jockers have worked out (2014)
- For ,  . . .  are known.

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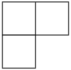
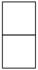
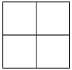
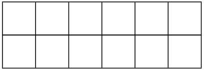
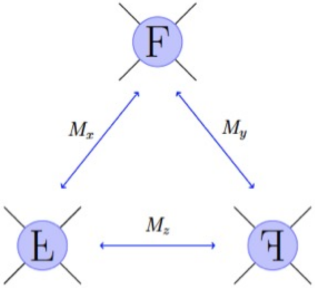
- For  , Gu-Jockers have worked out (2014)
- For ,  . . .  are known.
- Challenging open problem : to write a Kirillov-Reshetikhin type form for SU(N)

Status on mutation from our approach

- On any two tangle, mutation refers to π rotation about x or y axis (M_x, M_y)



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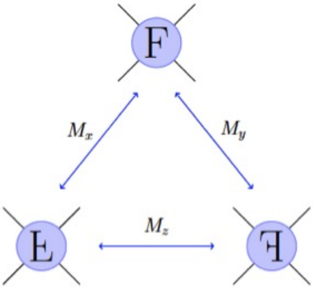

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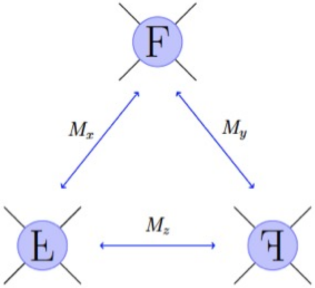

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- need to go beyond symmetric representation.

[2,1] colored HOMFLY-PT

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- Crucial input in the context of mixed representation: *multiplicity*

$$(21;0)\otimes(21;0) = (42;0)_0 \oplus (2^3;0)_0 \oplus (31^3;0)_0 \oplus (321;0)_0 \oplus (321;0)_1 \oplus (41^2;0)_0 \oplus (3^2;0)_0 \oplus (2^21^2;0)_0$$

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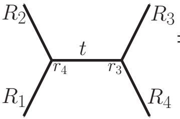
- Hence the states in the four-point conformal blocks involve multiplicity index r_i : $|\phi_{s,r_1,r_2}\rangle$



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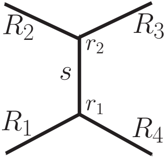
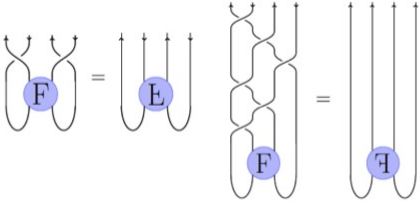


$$= |\phi^{(2)}_{s,r_1 r_2}(R_1, R_2, R_3, R_4)\rangle$$

Mutation operation on two-tangles



$$
\begin{aligned}
|\scalebox{1}[-1]{F}\rangle &= b_1^{(-)}[b_3^{(-)}]^{-1}|\mathbf{F}\rangle \\
&= \sum_{t,r_1,r_2} \{R,\bar{R},\bar{t},r_1\}\{R,\bar{R},\bar{t},r_2\}|\phi^{(1)}_{t,r_1,r_2}(R,\bar{R},R,\bar{R})\rangle\langle\phi^{(1)}_{t,r_1,r_2}(R,\bar{R},R,\bar{R})|\mathbf{F}\rangle
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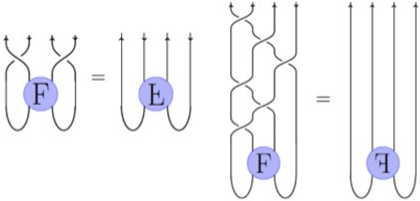

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|\reflectbox{\mathbf{F}}\rangle &= \left([b_1^{(-)}]^{-1}b_2^{(+)}[b_1^{(-)}]^{-1}\right) b_1^{(-)}[b_3^{(-)}]^{-1}\left([b_1^{(-)}]^{-1}b_2^{(+)}[b_1^{(-)}]^{-1}\right)|\mathbf{F}\rangle \\
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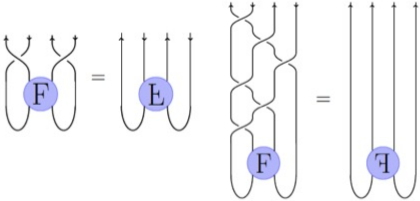

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|\text{Ⅎ}\rangle &= b_1^{(-)}[b_3^{(-)}]^{-1}|\mathbf{F}\rangle \\
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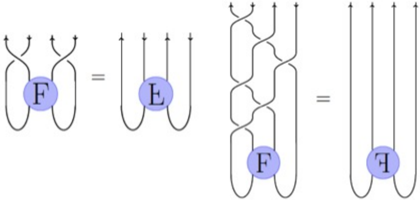
|\text{ꟻ}\rangle &= \left([b_1^{(-)}]^{-1}b_2^{(+)}[b_1^{(-)}]^{-1}\right) b_1^{(-)}[b_3^{(-)}]^{-1} \left([b_1^{(-)}]^{-1}b_2^{(+)}[b_1^{(-)}]^{-1}\right)|\mathbf{F}\rangle \\
&= \sum_{t,r_1,r_2} \{R,\bar{R},\bar{t},r_1\}\{R,\bar{R},\bar{t},r_2\}|\phi^{(1)}_{t,r_2,r_1}(R,\bar{R},R,\bar{R})\rangle\langle\phi^{(1)}_{t,r_1,r_2}(R,\bar{R},R,\bar{R})|\mathbf{F}\rangle\ .
\end{aligned}
$$

parenthesis denotes signs ± 1.

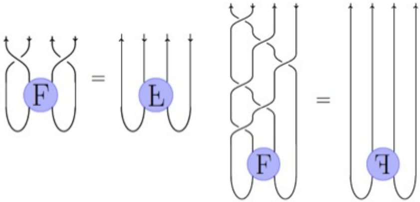
Mutation operation on two-tangles



$$
\begin{aligned}
|\scalebox{1}[-1]{\textbf{F}}\rangle &= b_1^{(-)}[b_3^{(-)}]^{-1}|\mathbf{F}\rangle \\
&= \sum_{t,r_1,r_2}\{R,\bar{R},\bar{t},r_1\}\{R,\bar{R},\bar{t},r_2\}|\phi^{(1)}_{t,r_1,r_2}(R,\bar{R},R,\bar{R})\rangle\langle\phi^{(1)}_{t,r_1,r_2}(R,\bar{R},R,\bar{R})|\mathbf{F}\rangle \\
|\reflectbox{\textbf{F}}\rangle &= \left([b_1^{(-)}]^{-1}b_2^{(+)}[b_1^{(-)}]^{-1}\right)b_1^{(-)}[b_3^{(-)}]^{-1}\left([b_1^{(-)}]^{-1}b_2^{(+)}[b_1^{(-)}]^{-1}\right)|\mathbf{F}\rangle \\
&= \sum_{t,r_1,r_2}\{R,\bar{R},\bar{t},r_1\}\{R,\bar{R},\bar{t},r_2\}|\phi^{(1)}_{t,r_2,r_1}(R,\bar{R},R,\bar{R})\rangle\langle\phi^{(1)}_{t,r_1,r_2}(R,\bar{R},R,\bar{R})|\mathbf{F}\rangle\ .
\end{aligned}
$$

parenthesis denotes signs ± 1.Notice the amplitudes of mutant tangles are related by sign when $r_1 \neq r_2$

Mutation operation on two-tangles

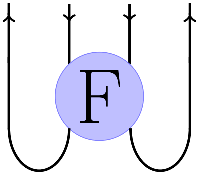


$$
\begin{aligned}
|\reflectbox{Ⅎ}\rangle &= b_1^{(-)}[b_3^{(-)}]^{-1}|\mathbf{F}\rangle \\
&= \sum_{t,r_1,r_2} \{R,\bar{R},\bar{t},r_1\}\{R,\bar{R},\bar{t},r_2\}|\phi^{(1)}_{t,r_1,r_2}(R,\bar{R},R,\bar{R})\rangle\langle\phi^{(1)}_{t,r_1,r_2}(R,\bar{R},R,\bar{R})|\mathbf{F}\rangle \\
|\text{ꟻ}\rangle &= \left([b_1^{(-)}]^{-1}b_2^{(+)}[b_1^{(-)}]^{-1}\right) b_1^{(-)}[b_3^{(-)}]^{-1}\left([b_1^{(-)}]^{-1}b_2^{(+)}[b_1^{(-)}]^{-1}\right)|\mathbf{F}\rangle \\
&= \sum_{t,r_1,r_2} \{R,\bar{R},\bar{t},r_1\}\{R,\bar{R},\bar{t},r_2\}|\phi^{(1)}_{t,r_2,r_1}(R,\bar{R},R,\bar{R})\rangle\langle\phi^{(1)}_{t,r_1,r_2}(R,\bar{R},R,\bar{R})|\mathbf{F}\rangle\ .
\end{aligned}
$$

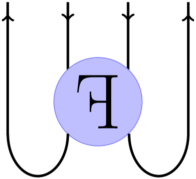
parenthesis denotes signs ± 1.Notice the amplitudes of mutant tangles are related by sign when $r_1 \neq r_2$ (occurs only for irreps with multiplicity)

Tangle and its M_y mutation

- The mutation operation (M_y) on $|\mathbf{F}\rangle$ which gives $|\mathbf{Ⅎ}\rangle$ whose state can also be obtained.



$$= \sum_{s,r_1,r_2} f_{s,r_1,r_2} |\phi^{(1)}_{s,r_1,r_2}(R,\bar{R},\bar{R},R)\rangle \ ,$$

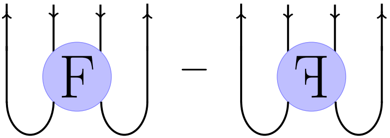


$$= \sum_{s,r_1,r_2} \tilde{f}_{s,r_1,r_2} |\phi^{(1)}_{s,r_1,r_2}(R,\bar{R},\bar{R},R)\rangle$$

- The coefficients are related by mutation operation :

$$\tilde{f}_{s,r_1,r_2} = (-1)^{r_1+r_2} f_{s,r_2,r_1} \ .$$

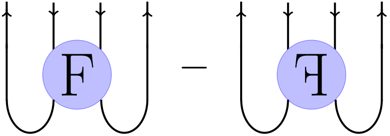
Difference between tangle F and mutant tangle of F



$$|\mathbf{F}\rangle - |\mathbf{Ⅎ}\rangle = (f_{(1;1),0,1} + f_{(1;1),1,0}) \sum_{r_1 \neq r_2} |\phi^{(1)}_{(1;1),r_1,r_2}(R, \bar{R}, \bar{R}, R)\rangle \ .$$

For some mutants, these coefficients could be zero(**for example, pretzel mutant knot pairs with odd antiparallel braidings**.)

Difference between tangle F and mutant tangle of F



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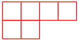
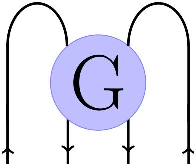
For some mutants, these coefficients could be zero(**for example, pretzel mutant knot pairs with odd antiparallel braidings**.)

We require duality matrix for $R =$  (two-row representations) with multiplicity more than two to compute difference between such antiparallel pretzel mutants

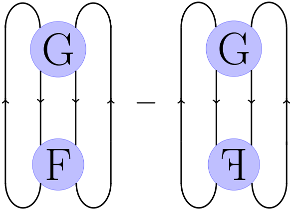
Knot and its mutant invariant

Let us cap each of these tangles with a tangle $\langle \mathbf{G}|$, which we write



$$= \sum_{s,r_1,r_2} g_{s,r_1,r_2} \langle \phi^{(1)}_{s,r_1,r_2}(R, \bar{R}, \bar{R}, R)| \ .$$

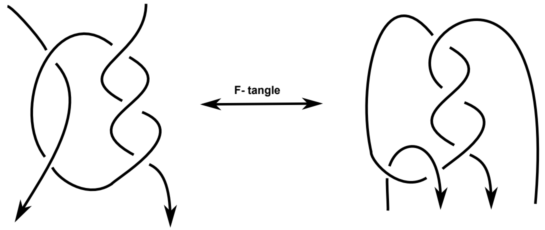
Then, the difference between the invariants of the mutant pairs arising from these 2-tangles will be

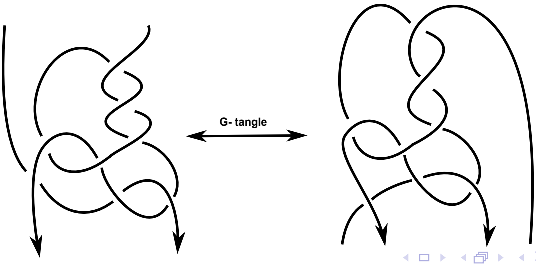


$$= (f_{(1;1),0,1} + f_{(1;1),1,0})(g_{(1;1),0,1} + g_{(1;1),1,0})$$

Kinoshita-Terasaka and Conway mutants

- This mutant pair is made of the following *F* and *G*-tangle





Knot invariant for the mutant pair

The explicit expression for the coefficient for tangle G turns out to be

$$g_{t,r_{10},r_{11}} = \dim_q R \sum \Omega(i, r_1, r_2, r_3)\Omega(j, r_6, r_7, r_8)\ {\lambda^{+}_{l;r_5}}^2 a^{*0;0,0}_{l;r_5,r_5}\begin{bmatrix} \bar{R} & R \\ R & \bar{R} \end{bmatrix}$$
$$a^{*i;r_2,r_3}_{l;r_5,r_5}\begin{bmatrix} \bar{R} & R \\ R & \bar{R} \end{bmatrix} {\lambda^{+}_{k;r_4}}^{-3} a^{0;0,0}_{k;r_4,r_4}\begin{bmatrix} \bar{R} & R \\ R & \bar{R} \end{bmatrix} a^{i;r_1,r_2}_{k;r_4,r_4}\begin{bmatrix} \bar{R} & R \\ R & \bar{R} \end{bmatrix} (\lambda^{-}_{s;r_9})^2$$
$$a^{*0;0,0}_{s;r_9,r_9}\begin{bmatrix} R & \bar{R} \\ R & \bar{R} \end{bmatrix} a^{*j;r_7,r_6}_{s;r_9,r_9}\begin{bmatrix} R & \bar{R} \\ R & \bar{R} \end{bmatrix} a^{t;r_{10},r_{11}}_{j;r_8,r_9}(\lambda^{-}_{t;r_{10}})^{-1}\begin{bmatrix} R & \bar{R} \\ R & \bar{R} \end{bmatrix}$$
$$a^{i;r_1,r_3}_{j;r_8,r_6}\begin{bmatrix} R & \bar{R} \\ R & \bar{R} \end{bmatrix}$$

Similarly, the coefficients in the tangle F state is

$$f_{t,r_{10},r_{11}} = \sum_{w,u}\sum_{r_{14},r_{13},r_{12}} \Omega(t, r_{10}, r_{11}, r_{12})\ (\lambda^{+}_{w;r_{14}})^3 a^{*0;0,0}_{w;r_{14},r_{14}}\begin{bmatrix} \bar{R} & R \\ R & \bar{R} \end{bmatrix}$$
$$a^{t;r_{11},r_{12}}_{w;r_{14},r_{14}}\begin{bmatrix} \bar{R} & R \\ R & \bar{R} \end{bmatrix} (\lambda^{-}_{u;r_{13}})^{-2} a^{0;0,0}_{u;r_{13},r_{13}}\begin{bmatrix} R & \bar{R} \\ R & \bar{R} \end{bmatrix} a^{*t;r_{12},r_{10}}_{u;r_{13},r_{13}}\begin{bmatrix} R & \bar{R} \\ R & \bar{R} \end{bmatrix}$$

Non-Arborescent Knots

Other methods to obtain these knot invariants -

Non-Arborescent Knots

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Non-Arborescent Knots

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$$a_{ij}\begin{bmatrix}[r_1] & [r_2] \\ [r_3] & \overline{[\ell_\nu - n_\nu, m_\nu - n_\nu]}\end{bmatrix} = a_{ij}^{(sl_2)}\begin{bmatrix}(r_1 - n_\nu)/2 & (r_2 - n_\nu)/2 \\ (r_3 - n_\nu)/2 & (\ell_\nu - m_\nu)/2\end{bmatrix}$$

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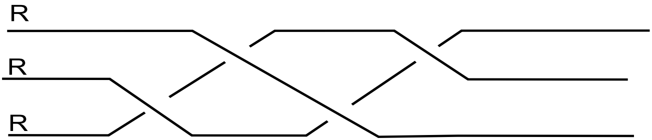
enabling invariants for links from 3-strand braids carrying different symmetric colors.

Colored HOMFLY-PT from quantum $\mathcal{R}$ matrices

- For m=3 strand and each strand carrying representation R, parameterized by a sequence of integers (a_1, b_1, a_2, b_2) (H.Itoyama, A. Mironov, A. Morozov, And. Morozov arXiv:1209.6304v1)
  As a example: sequence of integers (-1,-1,-1,-1)



- colored HOMFLY-PT using quantum $\mathcal{R}$ matrices will be

$$H_R = Tr\{(\mathcal{R} \otimes \mathcal{I})^{a_1}(\mathcal{I} \otimes \mathcal{R})^{b_1}(\mathcal{R} \otimes \mathcal{I})^{a_2}(\mathcal{I} \otimes \mathcal{R})^{b_2}\}$$

- Instead of working in tensor space $R^{\otimes 3}$, it is simpler to work using the irreducible representation

For example.,

$$
\begin{aligned}
H_{[1]} &= \sum_{[111],[21],[3]} tr\{(\mathcal{R}_1^Q)^{a_1}(\mathcal{R}_2^Q)^{b_1}(\mathcal{R}_1^Q)^{a_2}(\mathcal{R}_2^Q)^{b_2}\} \\
&= q^{a_1+b_1+a_2+b_2} S^*_{[3]} + q^{-(a_1+b_1+a_2+b_2)} S^*_{[111]} + \\
&\quad tr\{(\mathcal{R}_1^{[21]})^{a_1}(U_{[21]}\mathcal{R}_1^{[21]}U_{[21]})^{b_1}(\mathcal{R}_1^{[21]})^{a_2}(U_{[21]}\mathcal{R}_1^{[21]}U_{[21]})^{b_2}\} S^*_{[21]}
\end{aligned}
$$

where S^*_Q are the quantum dimensions of the representation Q.

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$$\begin{aligned} H_{[1]} &= \sum_{[111],[21],[3]} tr\{(\mathcal{R}_1^Q)^{a_1}(\mathcal{R}_2^Q)^{b_1}(\mathcal{R}_1^Q)^{a_2}(\mathcal{R}_2^Q)^{b_2}\} \\ &= q^{a_1+b_1+a_2+b_2} S^*_{[3]} + q^{-(a_1+b_1+a_2+b_2)} S^*_{[111]} + \\ & tr\{(\mathcal{R}_1^{[21]})^{a_1}(U_{[21]}\mathcal{R}_1^{[21]}U_{[21]})^{b_1}(\mathcal{R}_1^{[21]})^{a_2}(U_{[21]}\mathcal{R}_1^{[21]}U_{[21]})^{b_2}\} S^*_{[21]} \end{aligned}$$

where S^*_Q are the quantum dimensions of the representation Q.

• quantum $\mathcal{R}_1$ is diagonalisable and there is a unitary transformation U_Q to obtain $\mathcal{R}_2 = U_Q \mathcal{R}_1 U_Q$.

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- U_Q is non-trivial when paths to obtain Q from $R^{\otimes 3}$ is two or more.

For example.,

$$H_{[1]} = \sum_{[111],[21],[3]} tr\{(\mathcal{R}_1^Q)^{a_1}(\mathcal{R}_2^Q)^{b_1}(\mathcal{R}_1^Q)^{a_2}(\mathcal{R}_2^Q)^{b_2}\}$$

$$= q^{a_1+b_1+a_2+b_2} S^*_{[3]} + q^{-(a_1+b_1+a_2+b_2)} S^*_{[111]} +$$

$$tr\{(\mathcal{R}_1^{[21]})^{a_1}(U_{[21]}\mathcal{R}_1^{[21]}U_{[21]})^{b_1}(\mathcal{R}_1^{[21]})^{a_2}(U_{[21]}\mathcal{R}_1^{[21]}U_{[21]})^{b_2}\}S^*_{[21]}$$

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- Highest weight method is one method which enables determining these U matrices.

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where S^*_Q are the quantum dimensions of the representation Q.

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• U_Q is non-trivial when paths to obtain Q from $R^{\otimes 3}$ is two or more.

• Highest weight method is one method which enables determining these U matrices.

• The procedure is straightforward for $m = 4$ or more strands but will involve new unitary matrices.

Highest Weight Method(HWM)

• Co-multiplication Δ and the action of the lowering & raising operators in the $SU_q(N)$ context are defined as follows:

$$\begin{aligned}\Delta(T_i^+) &= \mathbb{I} \otimes T_i^+ + T_i^+ \otimes q^{-2H_i} \\ \Delta(T_i^-) &= q^{2H_i} \otimes T_i^- + T_i^- \otimes \mathbb{I}.\end{aligned}$$

$$T_i^- V_i = V_{i-1}; \qquad T_i^+ V_{i-1} = V_i.$$
$$H_i V_i = +\frac{1}{2} V_i; \qquad H_i V_{i-1} = -\frac{1}{2} V_{i-1}.$$

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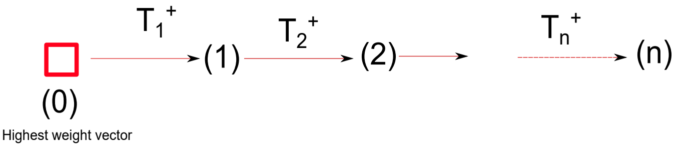
$$H_i V_i = +\frac{1}{2} V_i; \qquad H_i V_{i-1} = -\frac{1}{2} V_{i-1}.$$

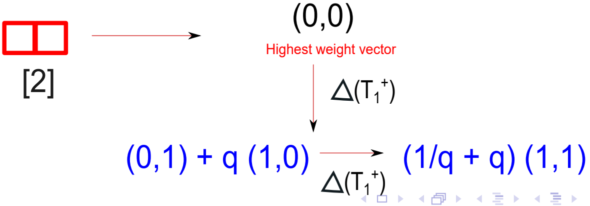
where V_i is an i-th vector of the fundamental representation, and T_i^+, T_i^- and q^{H_i} are generators of $SU_q(N)$.

HWM contd

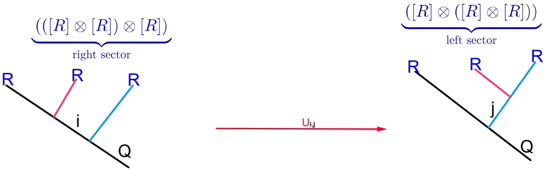
Action of raising operators T_i^+ on representation of $SU_q(N)$



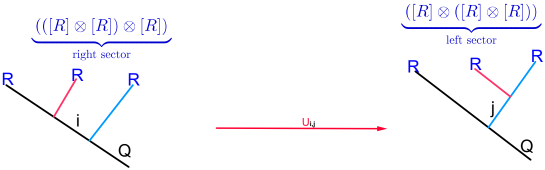
More Examples:

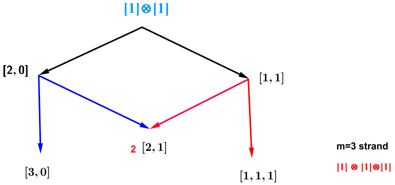


HWM contd


$(([R] \otimes [R]) \otimes [R])$
right sector
R
R
R
i
Q
$U_{i,j}$
$([R] \otimes ([R] \otimes [R]))$
left sector
R
R
R
j
Q

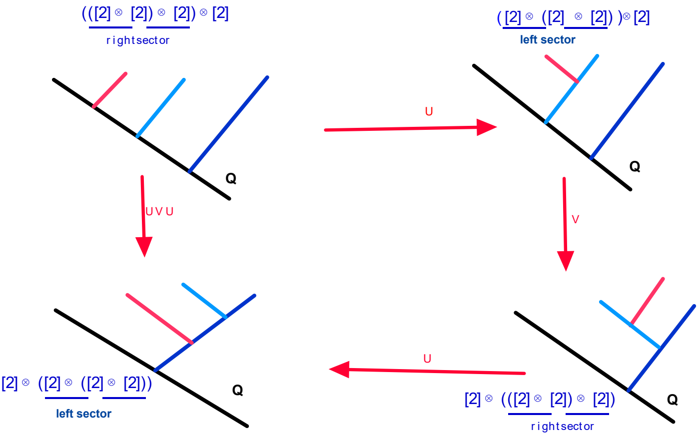
HWM contd



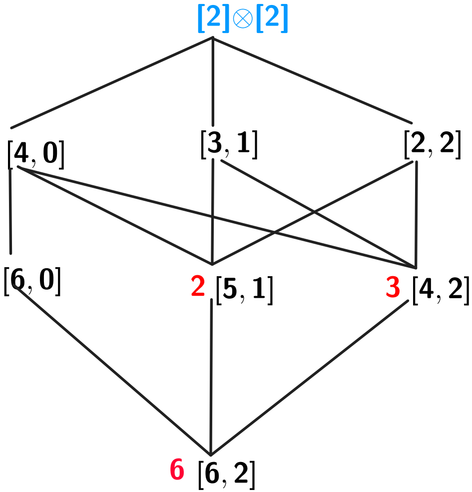


$$U_{[2,1]} = \begin{pmatrix} \frac{1}{[2]} & \frac{\sqrt{[3]}}{[2]} \\ \frac{\sqrt{[3]}}{[2]} & -\frac{1}{[2]} \end{pmatrix}$$

HWM contd


(([2] ⊗ [2]) ⊗ [2]) ⊗ [2]
rightsector
([2] ⊗ ([2] ⊗ [2]))⊗ [2]
left sector
U
Q
UVU
V
[2] ⊗ ([2] ⊗ ([2] ⊗ [2]))
left sector
[2] ⊗ (([2] ⊗ [2]) ⊗ [2])
rightsector

HWM contd



$[2]^{\otimes 2}$ m=2 strand

$[2]^{\otimes 3}$ m=3 strand

$[2]^{\otimes 4}$ m=4 strand

Eigen value hypothesis approach

- 1. The diagonal elements of quantum $\mathcal{R}_i$ are governed by the characteristic equation: $\prod_{Q}(\mathcal{R}_i - \lambda_j) = 0.$

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- Here $j \in R \otimes R$ and the braiding eigenvalue $\lambda_j = \epsilon_j q^{\varkappa_j}$ where $\varkappa_j = 1/2 \sum_i j_i(j_i + 1 - 2i)$ associated with the Young diagram j.

Eigen value hypothesis approach

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- 3. Also the commutativity relation applicable when we have more than 3-strand braids: $\mathcal{R}_i \mathcal{R}_j = \mathcal{R}_j \mathcal{R}_i, \quad i \neq j \pm 1$.
- Using the three properties, eigenvalue hypothesis claims that the U, V matrix elements can be determined in terms of the eigenvalues λ_j's.

Eigen value hypothesis contd

- For 2 strand braids, we have only one $\mathcal{R}$ obeying characteristic equation.
- For 3 strand braids, we have $\mathcal{R}_1$ and $\mathcal{R}_2$ which are related by a unitary matrix U :

$$\mathcal{R}_2 = U\mathcal{R}_1 U^\dagger \ .$$

  Characteristic equation and Yang-Baxter equation enables the form of U matrix elements as functions of λ_j's.
- For 4 strand braids, $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{R}_3$ related by two unitary matrices U and V. The relation between $\mathcal{R}_3$ and $\mathcal{R}_1$ is

$$\mathcal{R}_3 = UVU\mathcal{R}_1 U^\dagger V^\dagger U^\dagger \ .$$

- The matrix elements U and V for matrices upto order 6×6 were deduced from the three properties obeyed by quantum $\mathcal{R}_i$ matrices (recent paper-1711.10952
- The procedure appears straightforward for higher strand braids (*need to explore!*)

HOMFLY-PT Calculation

HOMFLY-PT polynomial for knots from 3-strand braid with braiding sequence $(a_1, b_1, c_1, a_2, b_2, c_2 \ldots)$

$$\mathcal{H}_{[2]}^{a_1,b_1,c_1,a_2,b_2,c_2,\ldots\ldots} = \sum S_Q . Tr(\prod_i R_{1Q}^{a_i} U_Q R_{1Q}^{b_i} V_Q U_Q R_{1Q}^{c_i} U_Q^\dagger V_Q^\dagger U_Q^\dagger)$$

where S_Q is the quantum dimension of representation $Q \in R^{\otimes 4}$ and a_i, b_i, c_i are the power of braiding operators

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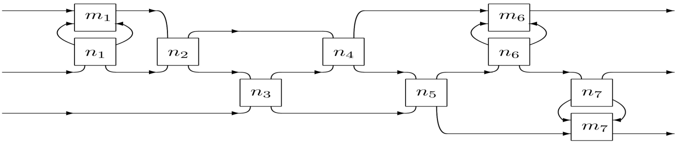
(S. Dhara, A. Mironov, A. Morozov, An.Morozov, **PR**, VKS, A.Sleptsov, arXiv:1711.10952)

Hybrid approach

•By combining methods applicable to arborescent and non-arborescent knots, colored HOMFLY-PT is obtainable for some non-arborescent knots drawn below:



List of the non-arborescent knots										
Knot	n_1	n_2	n_3	n_4	n_5	n_6	n_7	m_1	m_6	m_7
9_{34}	-2	1	3	1	-1	0	1	2	2	-2
9_{39}	2	-1	-1	1	-1	0	1	2	2	+2
9_{41}	0	1	1	-1	-3	2	1	2	2	+2
9_{47}	0	-1	3	1	-1	0	1	2	2	+2
9_{49}	0	1	1	-1	-3	0	-1	2	2	-2

(A. Mironov, A. Morozov arXiv:1506.00339),(Mironov, A. Morozov, An.Morozov, PR, VKS, A.Sleptsov, arXiv:1601.04199)

An example using hybrid method

The explicit invariant will be

$$d_{[1]} H_{[1]}^{(n_1,\ldots,n_7|m_1,m_6,m_7)} = d_{[3]} \cdot K_{[2]}^{n_1,m_1} \cdot \left(\prod_{i=2}^{5} P_{[2]}^{(n_i)} \right) K_{[2]}^{n_6,m_6} \bar{K}_{[2]}^{(m_7,n_7)} +$$
$$d_{[111]} \cdot K_{[11]}^{(m_1,n_1)} \cdot \left(\prod_{i=2}^{5} P_{[11]}^{(n_i)} \right) K_{[11]}^{n_6,m_6} \bar{K}_{[11]}^{(m_7,n_7)}$$
$$+ d_{[21]} \cdot Tr_{2\times 2} \{ M_{2\times 2} \}$$

where,

$$P_X^{(n)} = \frac{(\bar{S}\,\bar{T}^n S)_{0,X}}{S_{0,X}}, K_X^{n,m} = \frac{(S T^m S^\dagger \bar{T}^n S)_{0,X}}{S_{0,X}}, \bar{K}_X^{(m_7,n_7)} = \frac{(\bar{S}\,\bar{T}^{m_7} \bar{S}\,\bar{T}^{n_7} S)_{0,X}}{S_{0,X}}$$

Example contd

$$M_{2\times 2} = \begin{pmatrix} K_{[2]}^{n_1,m_1} & 0 \\ 0 & K_{[11]}^{n_1,m_1} \end{pmatrix} \left(\prod_{i=2}^{5} L_{2\times 2}^{i}\right) \begin{pmatrix} K_{[2]}^{n_6,m_6} & 0 \\ 0 & K_{[11]}^{n_6,m_6} \end{pmatrix}$$
$$\begin{pmatrix} \frac{1}{[2]} & \frac{\sqrt{[3]}}{[2]} \\ \frac{\sqrt{[3]}}{[2]} & -\frac{1}{[2]} \end{pmatrix} \begin{pmatrix} \bar{K}_{[2]}^{(m_7,n_7)} & 0 \\ 0 & \bar{K}_{[11]}^{(m_7,n_7)} \end{pmatrix} \begin{pmatrix} \frac{1}{[2]} & \frac{\sqrt{[3]}}{[2]} \\ \frac{\sqrt{[3]}}{[2]} & -\frac{1}{[2]} \end{pmatrix},$$

where

$$L_{2\times 2}^{i} = \begin{pmatrix} P_{[2]}^{(n_i)} & 0 \\ 0 & P_{[11]}^{(n_i)} \end{pmatrix} \begin{pmatrix} \frac{1}{[2]} & \frac{\sqrt{[3]}}{[2]} \\ \frac{\sqrt{[3]}}{[2]} & -\frac{1}{[2]} \end{pmatrix}$$

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- All our results are updated from time to time in the knotebook.org website. This includes integrality checks conjectured within topological string context.

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- Probably all knot invariants (including universal invariant) must be rewritable in q-Pocchhamer form to attempt Piotr's knot-quiver correspondence.

Thank You