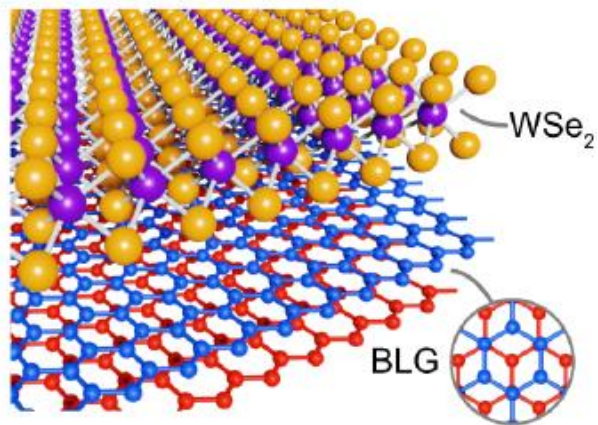


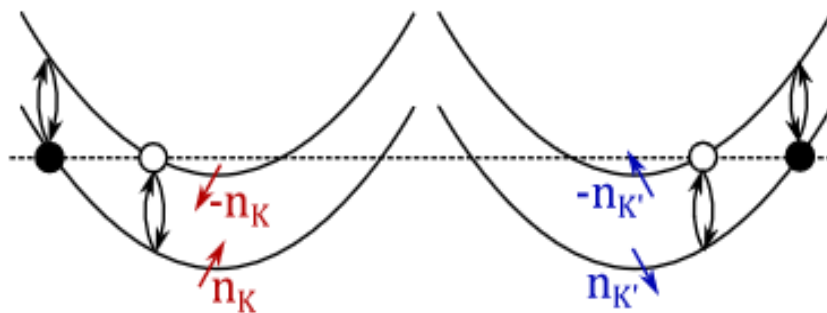
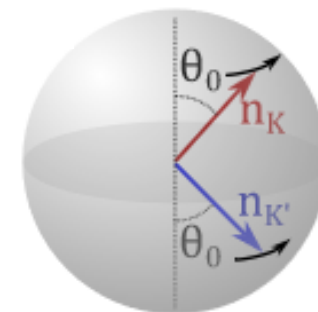
Rhombohedral graphene:

spin canting, collective modes and superconductivity



Étienne Lantagne-Hurtubise
KITP – Tunable 2D materials program

Sep 10, 2024



Acknowledgments

Experiments:

Yiran Zhang

Robert Polski

Haoxin Zhou

Stevan Nadj-Perge

Caitlin Patterson

Owen Sheekey

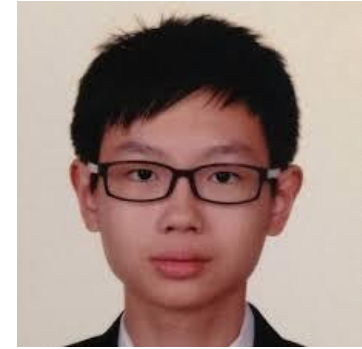
Trevor Arp

Ludwig Holleis

Youngjoon Choi

Andrea Young

Theory collaborators:



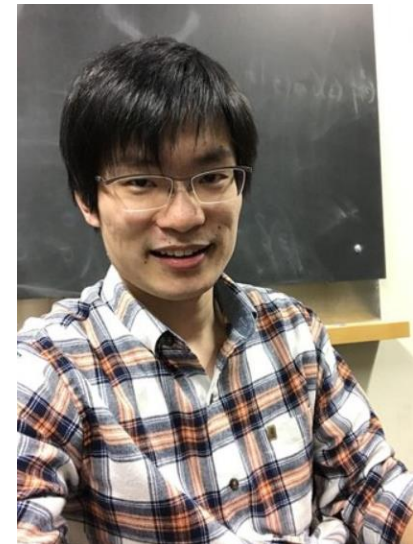
Jin Ming Koh

(Caltech undergrad -> Singapore A*STAR -> Harvard PhD)

Zhiyu Dong

Alex Thomson (UC Davis)

Jason Alicea



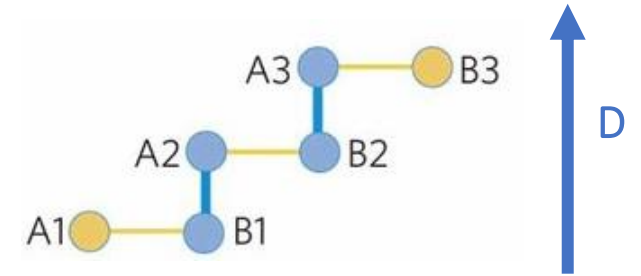
Caltech



Rhombohedral graphene multilayers

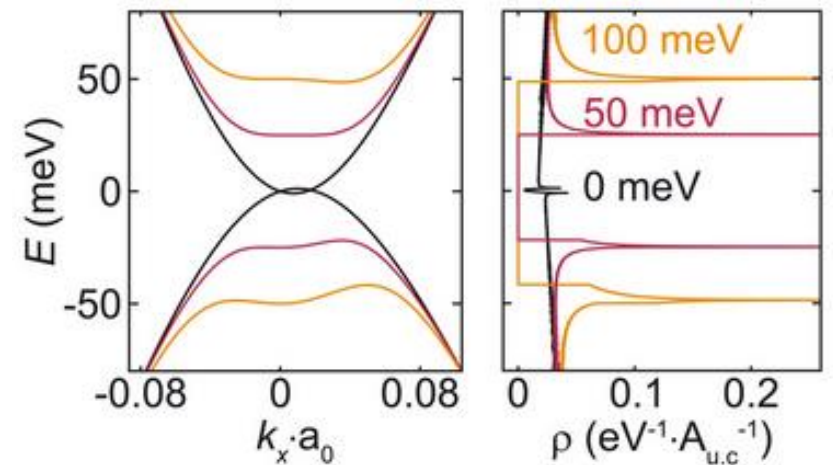
“ABC” stacking

Guinea, Castro Neto, & Peres, PRB **73**, 245426 (2006)



Perpendicular electric displacement field D **opens a gap** and **tunes the density of states**

- *In situ* tuning of doping level and correlation strength
- Extremely clean! Mean free paths $\sim 10 \mu\text{m}$



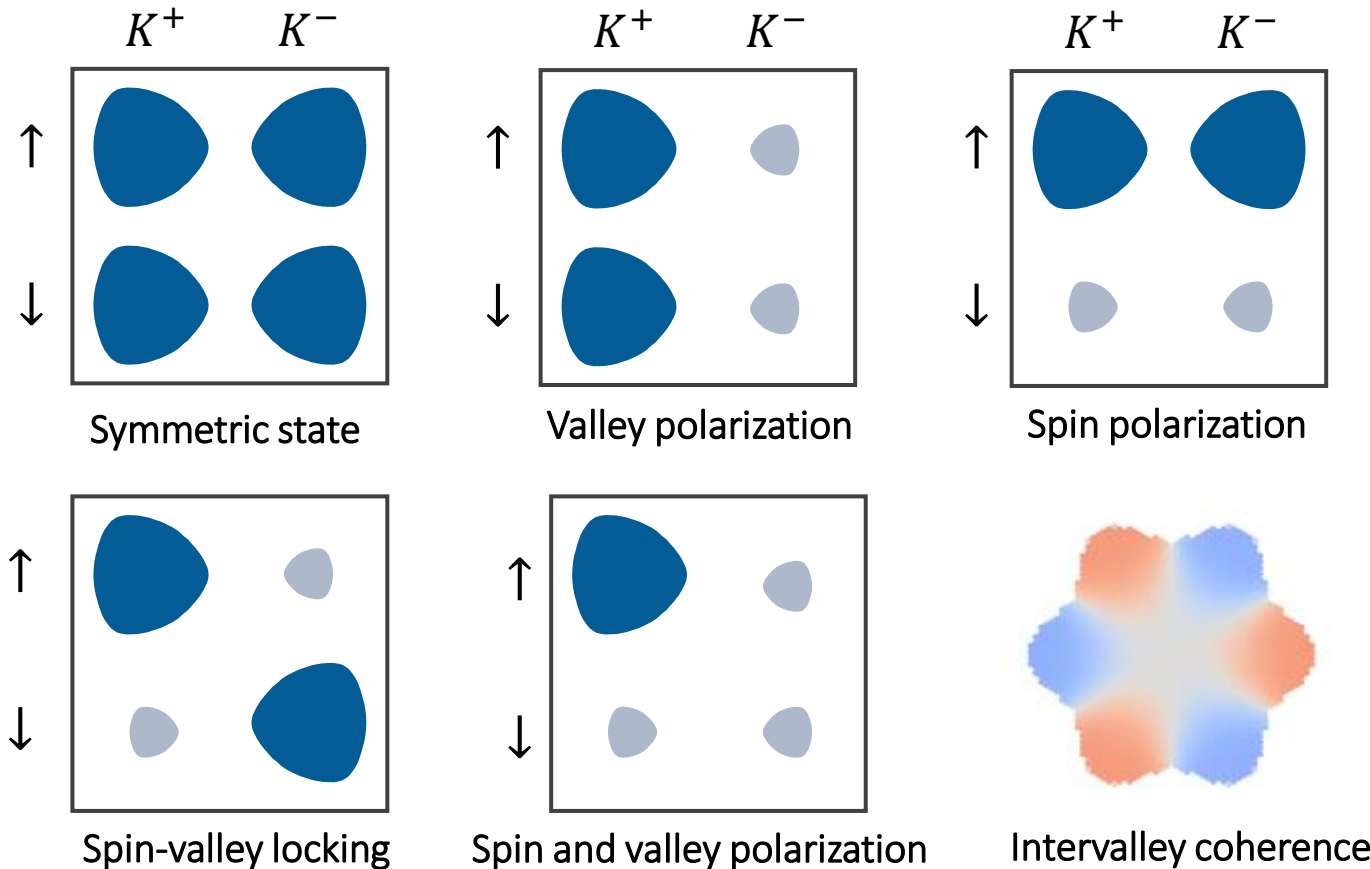
Bilayer graphene

Zhou et al., Science (2022)

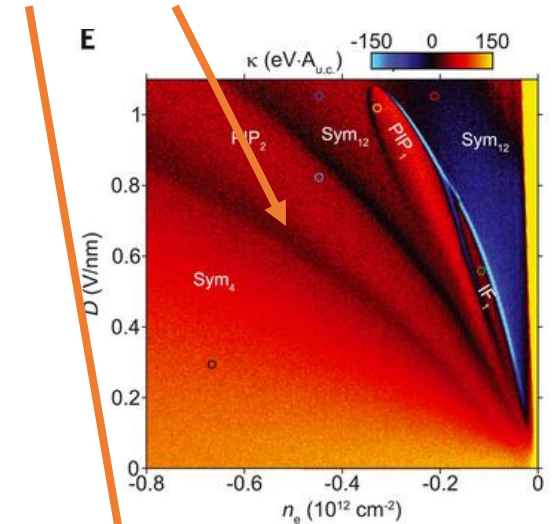
Flavor polarization

Four-fold degenerate band structure: valleys (K, K') + spins (\uparrow, \downarrow)

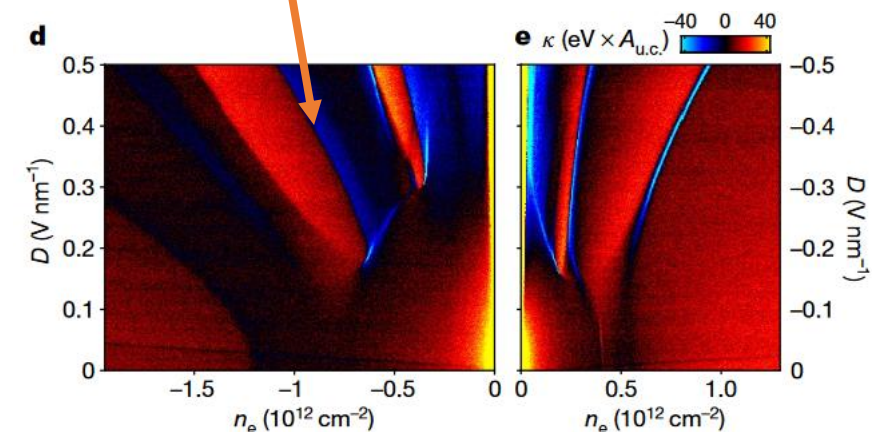
Prone to **symmetry breaking** (Stoner-type) instabilities



Symmetry breaking phase transitions between metals



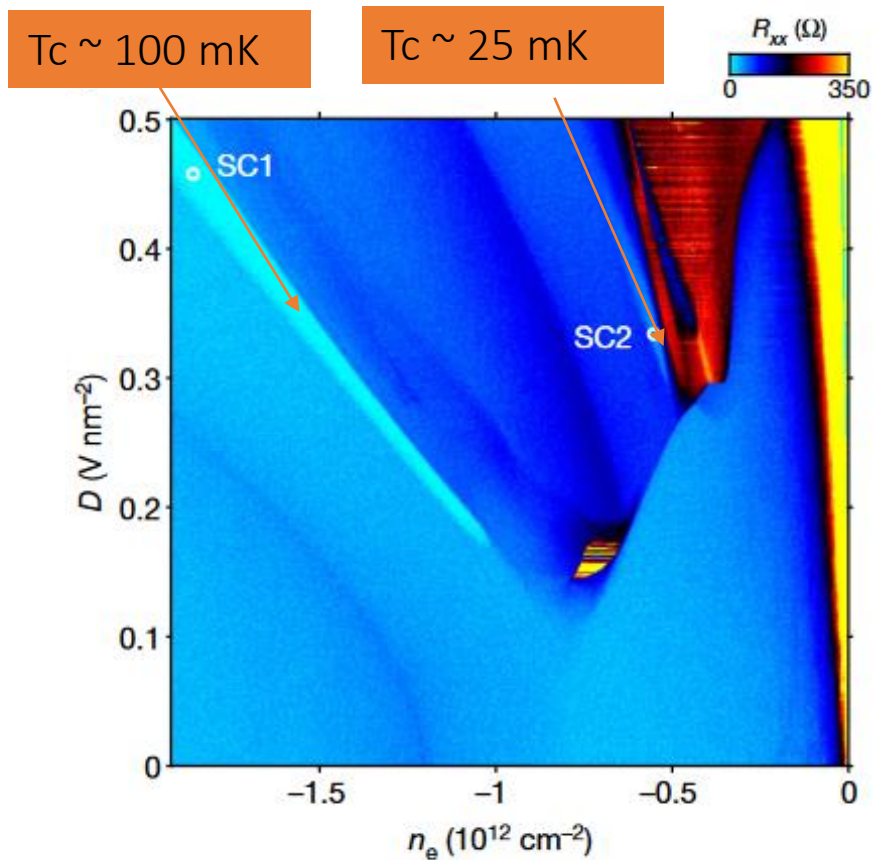
Bilayers: Zhou et al, Science (2022)



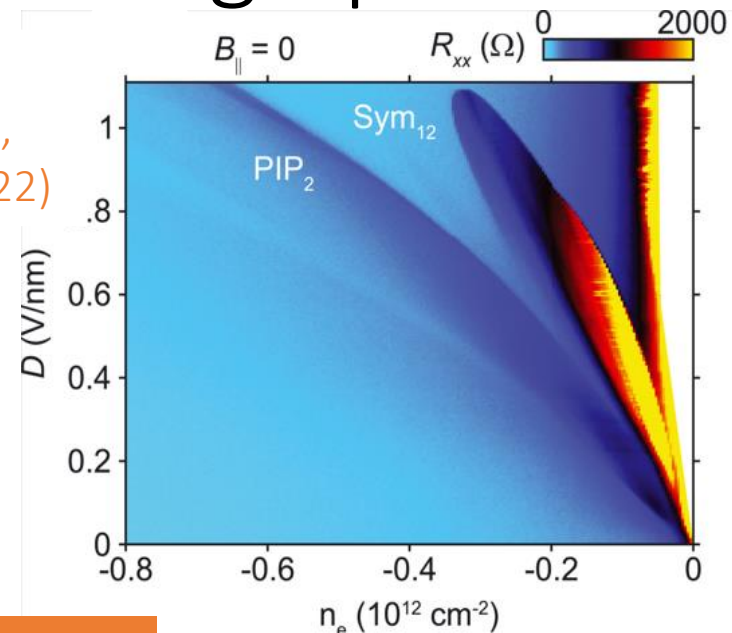
Trilayers: Zhou et al, Nature (2021)

Superconductivity in rhombohedral graphene

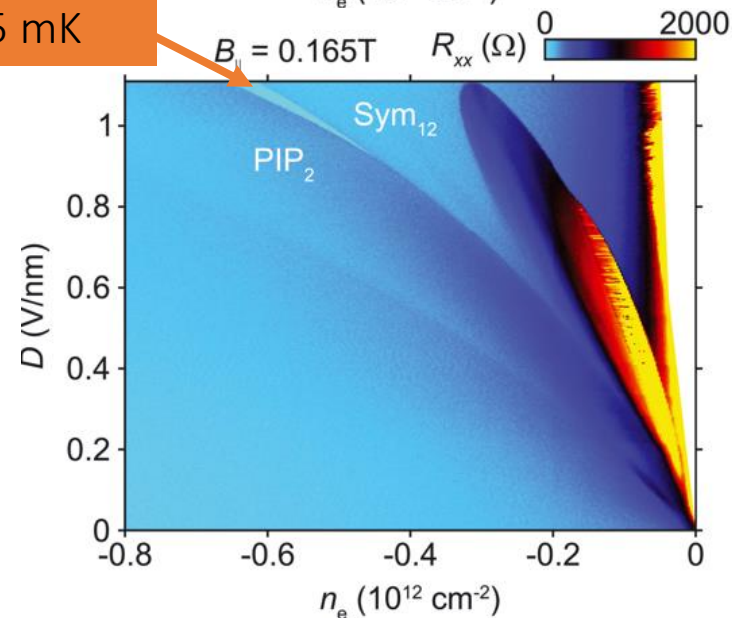
Trilayers:
Zhou et al.,
Nature
(2021)



Bilayers:
Zhou et al,
Science (2022)



$T_c = 25 \text{ mK}$



SC induced by in-plane B field!

Pairing mechanism?

- Order parameter fluctuations close to phase transitions

Chatterjee, Wang, Berg & Zaletel, Nat. Comm. **13**, 6013 (2022)

Dong, Lee & Levitov, PNAS **120** (2023)

Dong, Chubukov & Levitov, PRB **107**, 174512 (2023)

Dong, Levitov & Chubukov, PRB **108**, 134503 (2023)

- Kohn-Luttinger (over-screened Coulomb interactions)

Ghazaryan, Holder, Serbyn & Berg, PRL **127**, 247001 (2021)

Szabó & Roy, PRB **105**, L081407 (2022)

You & Vishwanath, PRB **105**, 134524 (2022)

Cea, Pantaleón, Phong, Guinea, PRB **105**, 075432 (2022)

Qin, Huang, Wolf, Wei, Blinov & MacDonald, PRL **130**, 146001 (2023)

Li, Kuang, Jimeno-Pozo, Sainz-Cruz, Zhan, Yuan & Guinea, PRB **108**, 045404 (2023)

Lu, Wang, Chatterjee & You, PRB **106**, 155115 (2022)

Dai, Ma, Zhang, Guo & Ma, PRB **107**, 245106 (2023)

Wagner, Kwan, Bultinck, Simon & Parameswaran, arXiv:2302.00682

Son, Hsu & Kim, arXiv:2405.05442

- Phonon mediated

Chou, Wu, Sau & Das Sarma, PRL **127**, 187001 (2021)

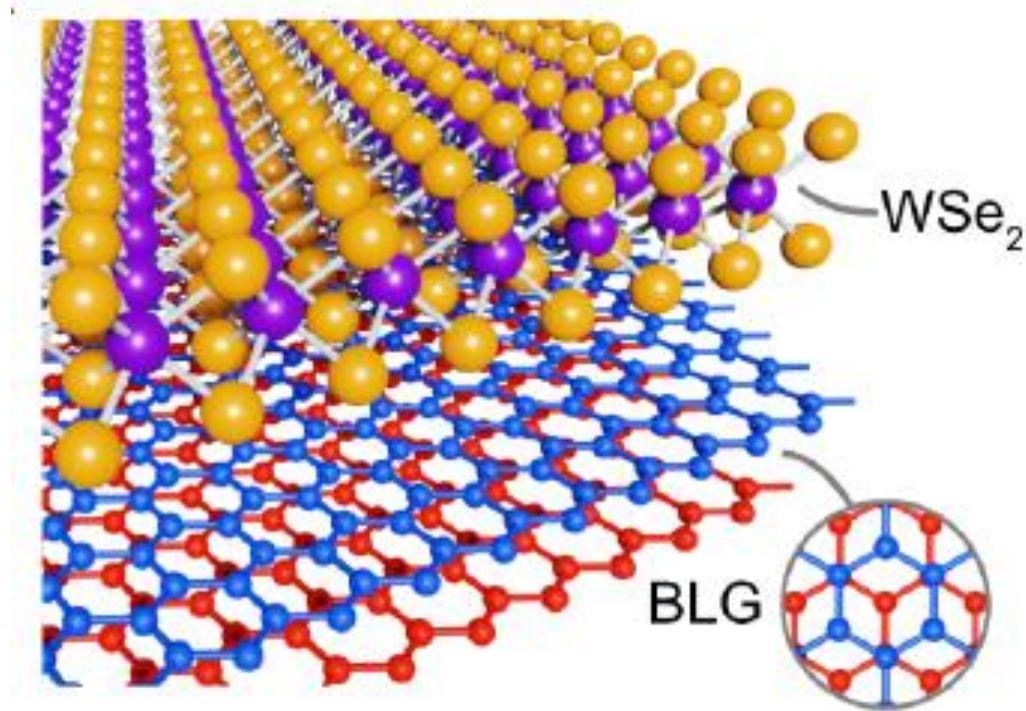
Chou, Wu, Sau & Das Sarma, PRB **105**, L100503 (2022)

Chou, Wu, Sau & Das Sarma, PRB **106**, 024507 (2022)

Vinas Bostrom, Fischer, Profe, Zhang, Kennes & Rubio, npj Computational Materials **10** (2024).

And more...

1. Enhanced superconductivity in BLG-WSe₂



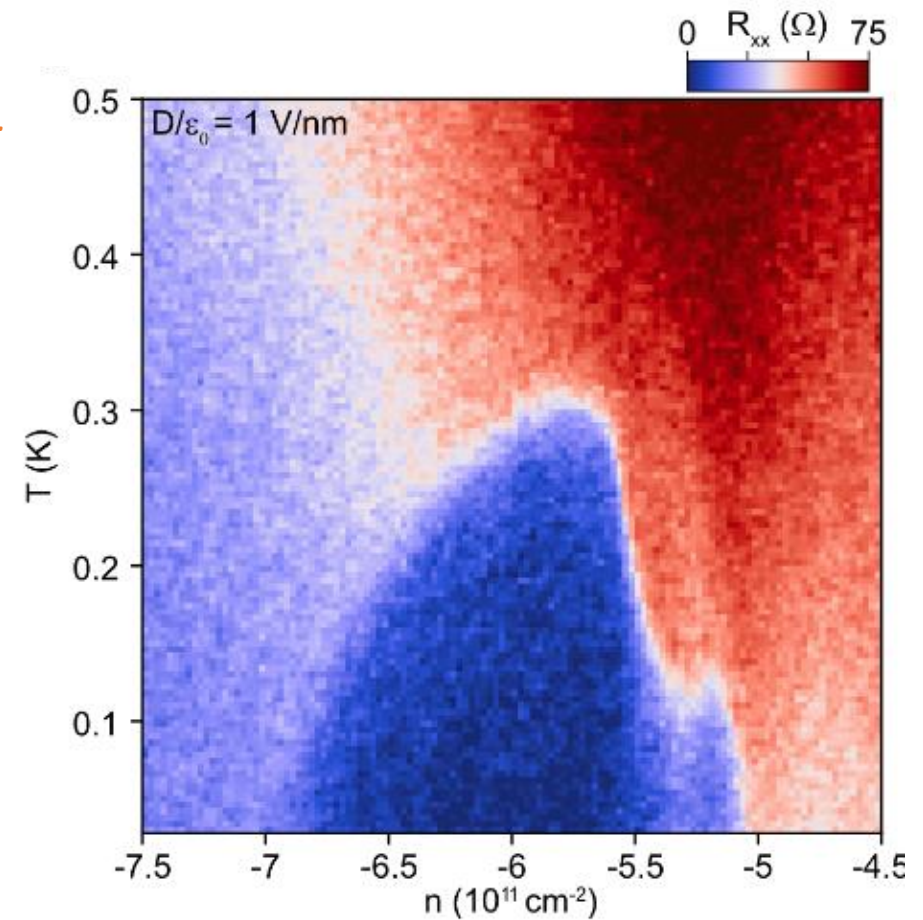
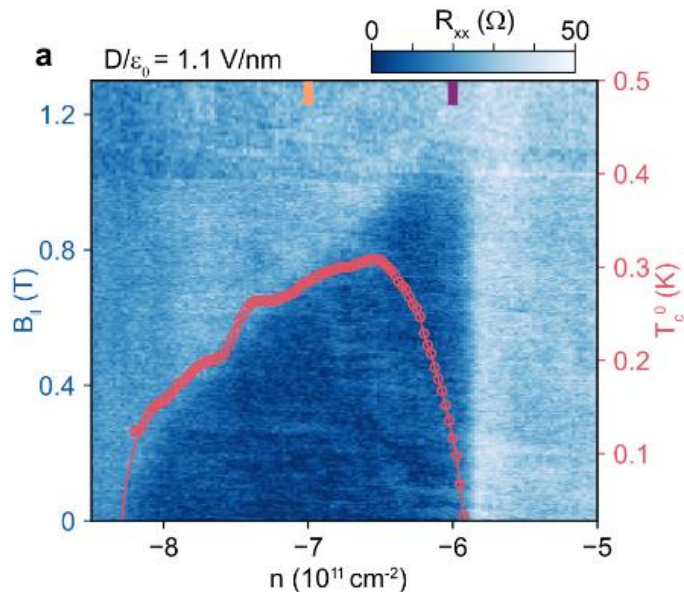
A surprise!

Zhang, Polski, Thomson, *ÉLH*, *et al.*
Nature **613**, 268-273 (2023)

Large SC region observed at $\mathbf{B} = 0$

Only for $D > 0$: proximity to WSe_2 favors SC

Strongly **density-dependent** Pauli limit violation (singlet vs triplet like response)



$T_c = 300 \text{ mK}$
Pairing strongly enhanced!

Spin-orbit coupling in BLG

Intrinsically weak ($\sim 50 \mu\text{eV}$). Can be enhanced by proximity effect

- Two main contributions: Ising and Rasha SOC

$$\frac{\lambda_I}{2} \tau_z s_z, \quad \frac{\lambda_R}{2} (\tau_z \sigma_x s_y - \sigma_y s_x)$$

$(\tau, \sigma, s) = (\text{valley}, \text{sublattice}, \text{spin})$

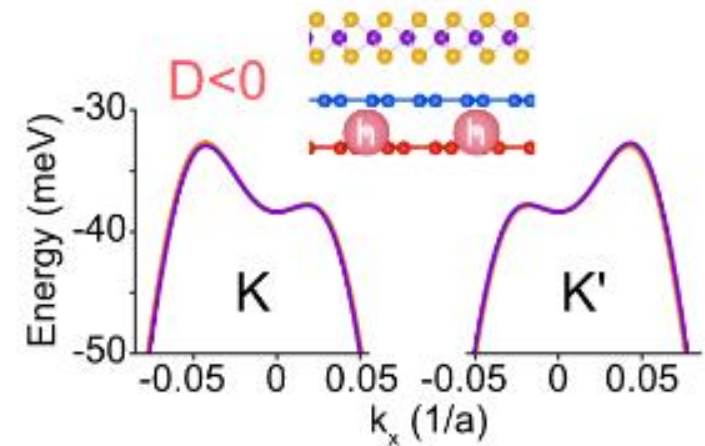
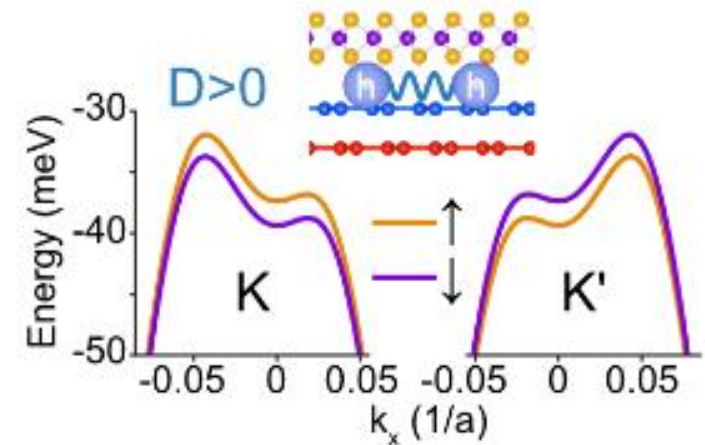
- Ising SOC couples asymmetrically with D field: “spin-orbit valve”

Gmitra and Fabian, PRL (2017)

Khoo, Morpurgo and Levitov, Nano Letters (2017)

- Quantum Hall measurements:

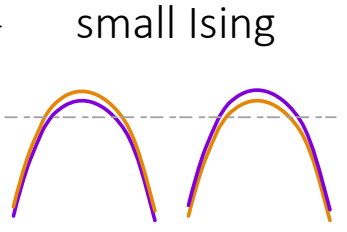
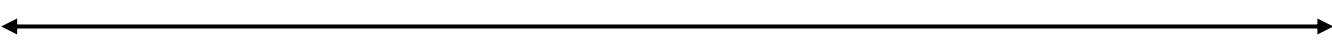
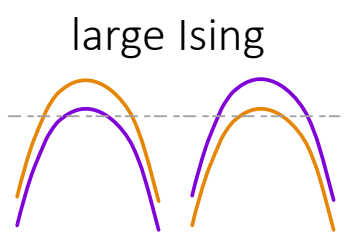
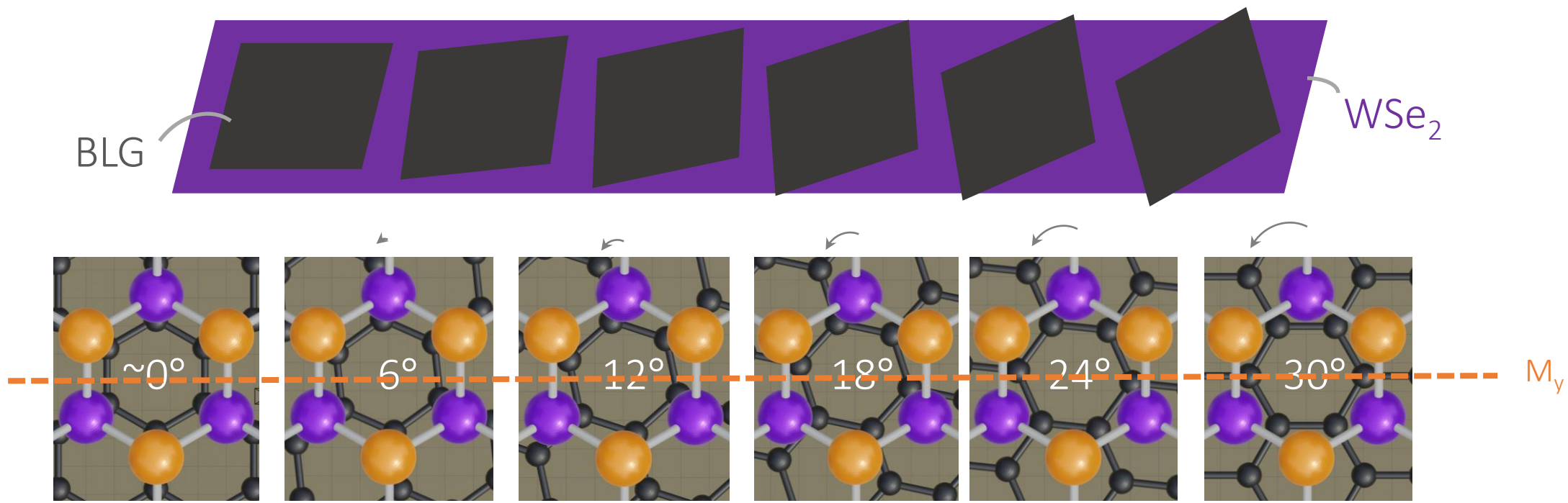
$$\lambda_I \sim 0.6 - 1.6 \text{ meV}$$



Tuning SOC with interfacial twist



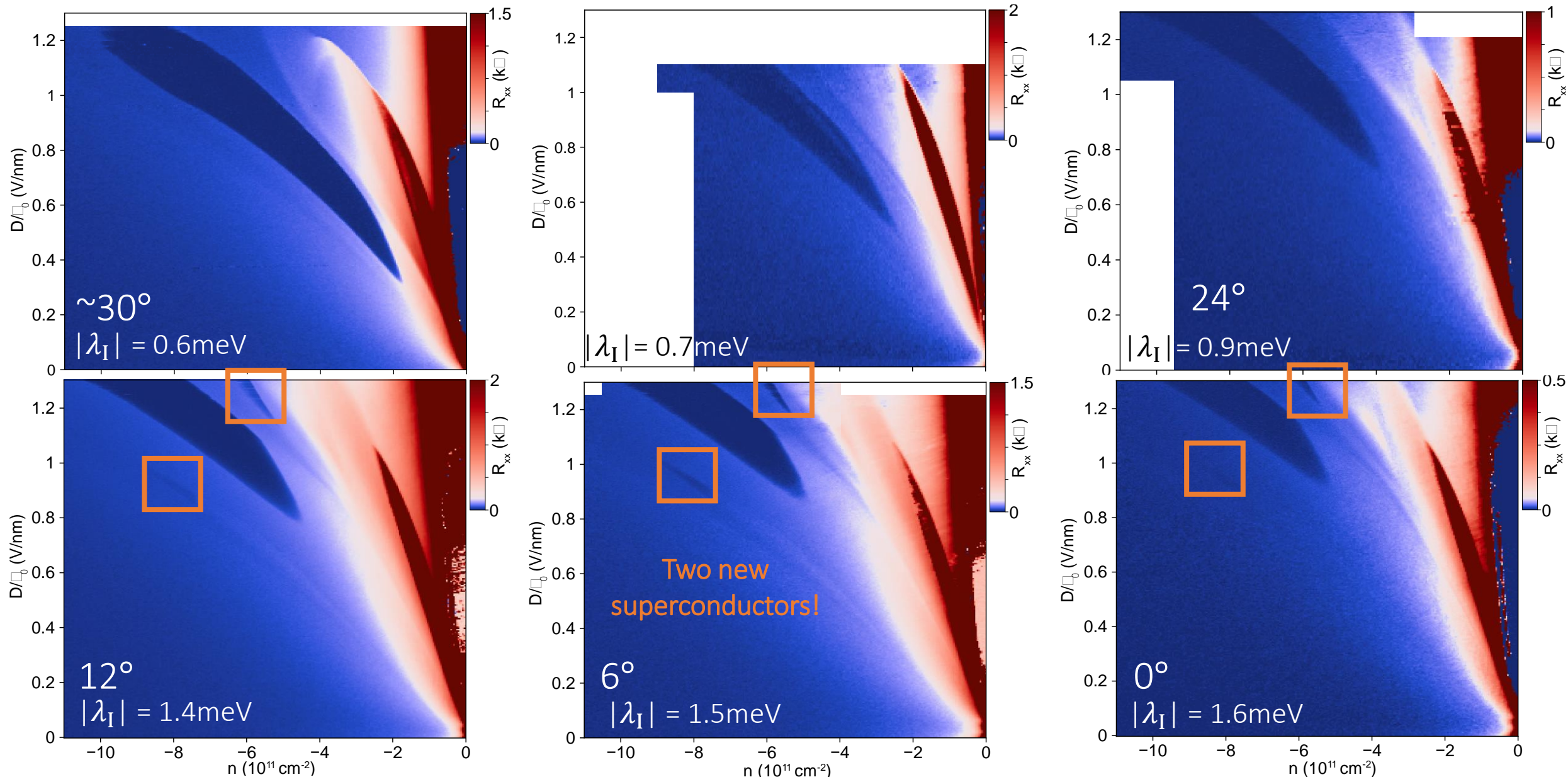
Tuning SOC with interfacial twist



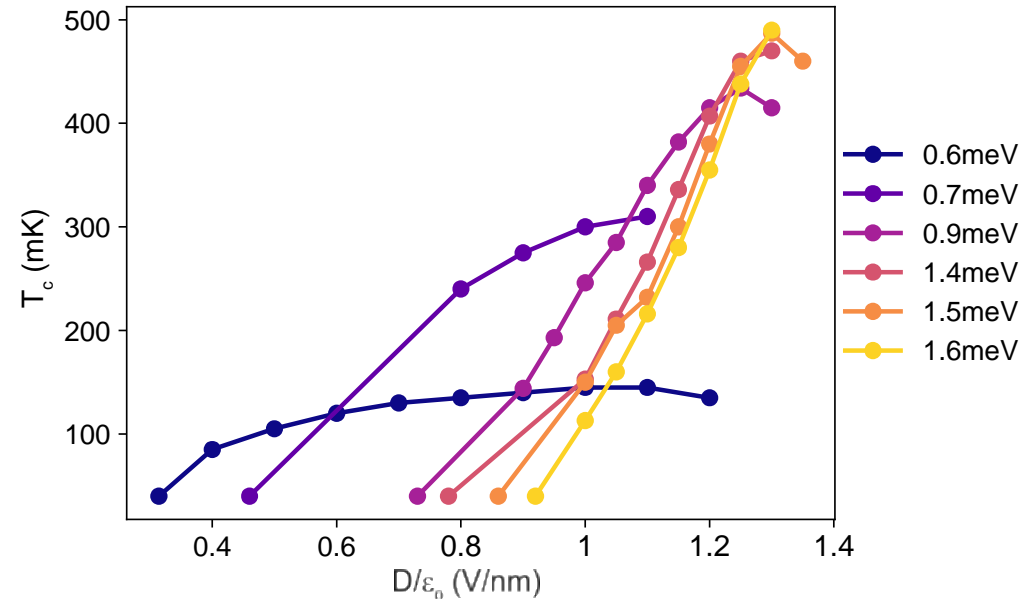
Ising SOC controlled by breaking M_y

Li & Koshino, PRB **99**, 075438 (2019)
David *et al.*, PRB **100**, 085412 (2019)

Phase diagrams for different Ising SOC



Trends (large SC pocket)



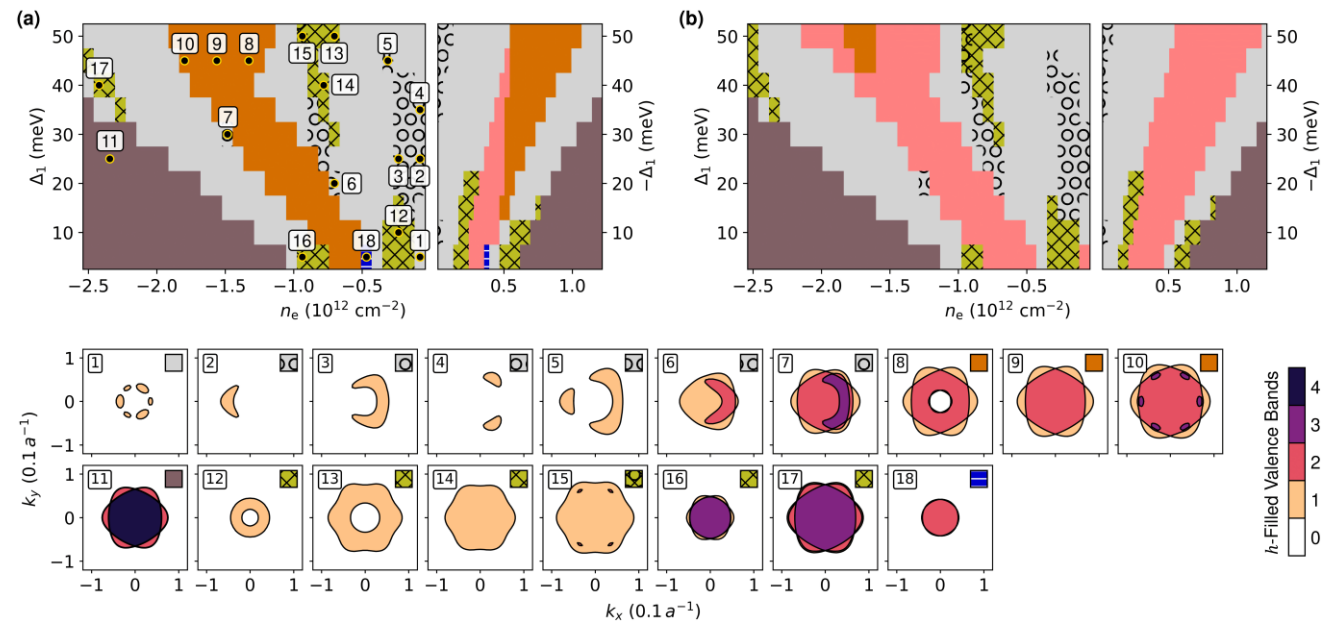
Increasing Ising SOC

- Superconductivity is pushed to higher D field
 - Higher critical temperature

These opposing trends provide non-trivial constraints for theory!

- Understand the parent normal state
- Pairing mechanism?

2. Phase diagrams of spin-orbit coupled graphene multilayers



Trilayers: Koh, Alicea and \acute{E} LH, Phys. Rev. B **109**, 035113 (2024)

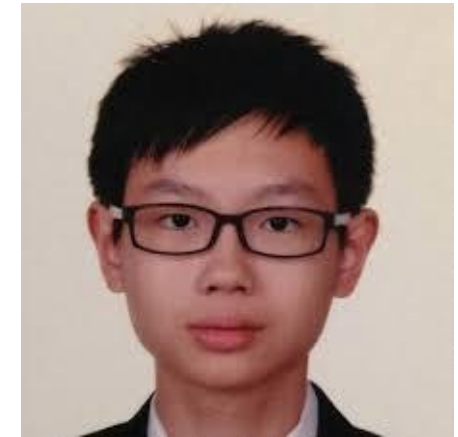
Bilayers: Koh, Thomson, Alicea and \acute{E} LH, arXiv:2407.09612

Xie & Das Sarma, PRB **107**, L201119 (2023)

Zhumagulov, Kochan & Fabian, PRB **110**, 045427 (2024)

Wang, Vila, Zaletel & Chatterjee, PRL **132**, 116504 (2024)

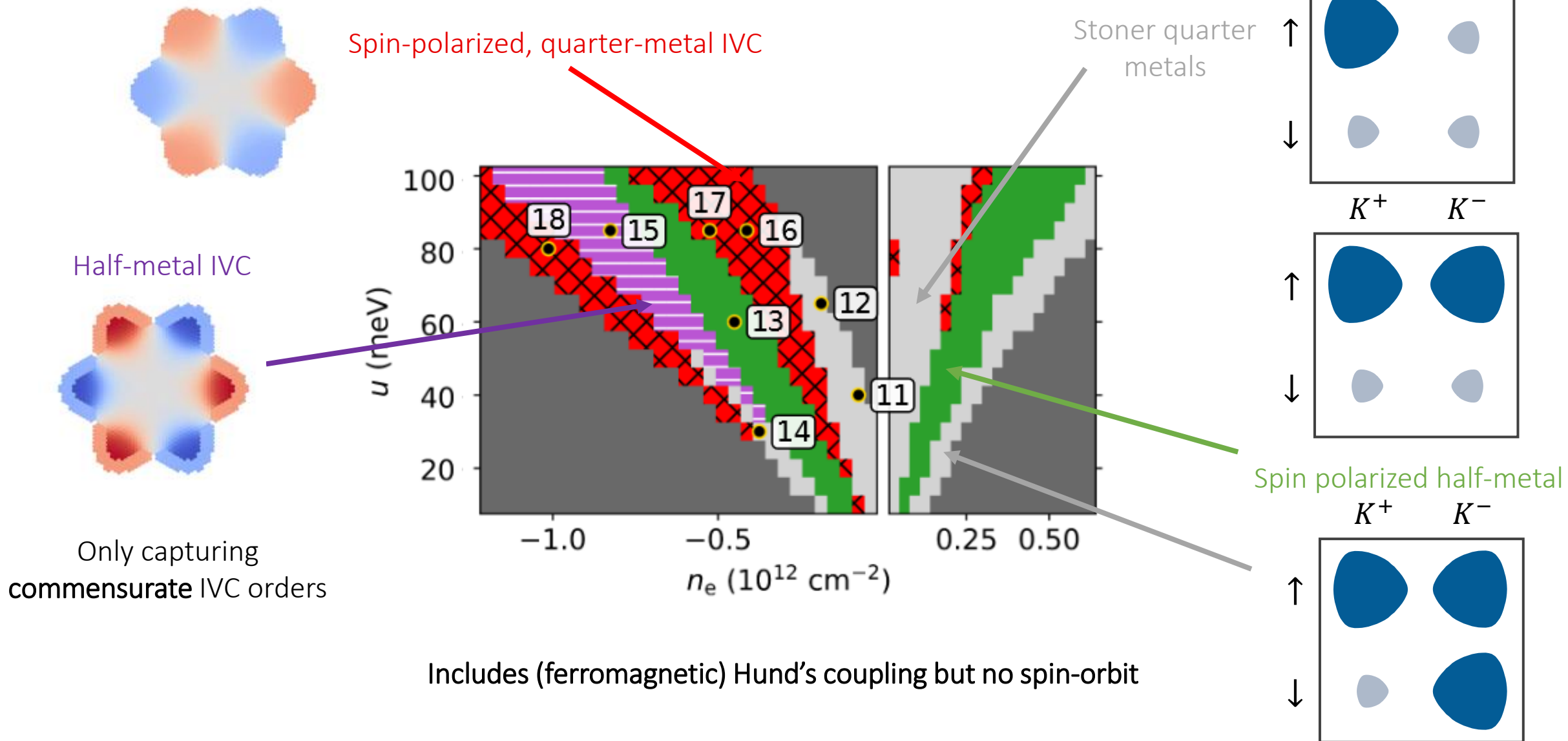
Zhumagulov, Kochan & Fabian, PRL **132**, 186401 (2024)



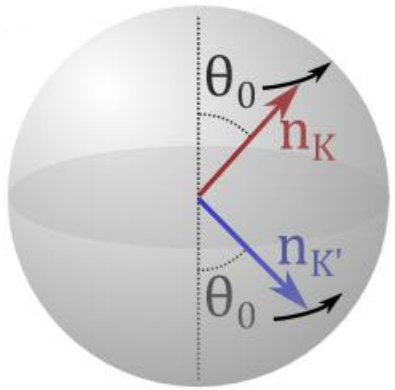
Jin Ming Koh

See also:

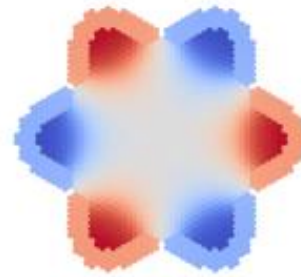
Bilayer graphene phase diagram



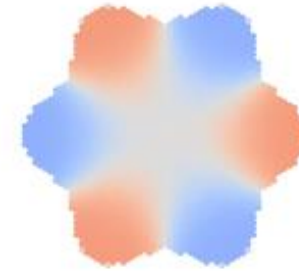
Adding Ising SOC



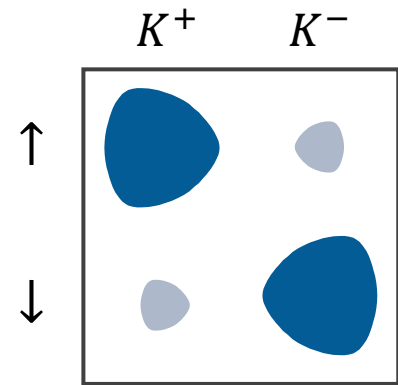
Spin-canted half-metal



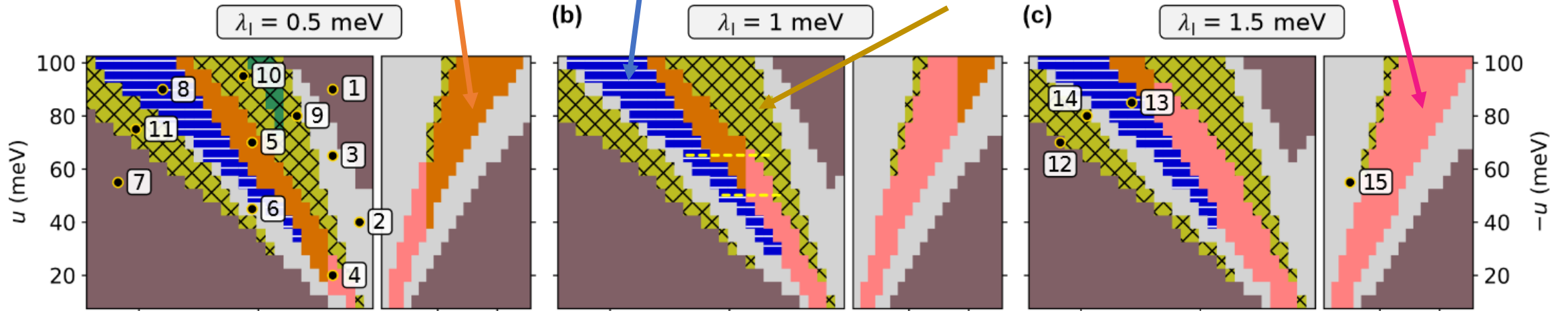
Half-metal IVC



Spin-valley-locked, quarter metal IVC

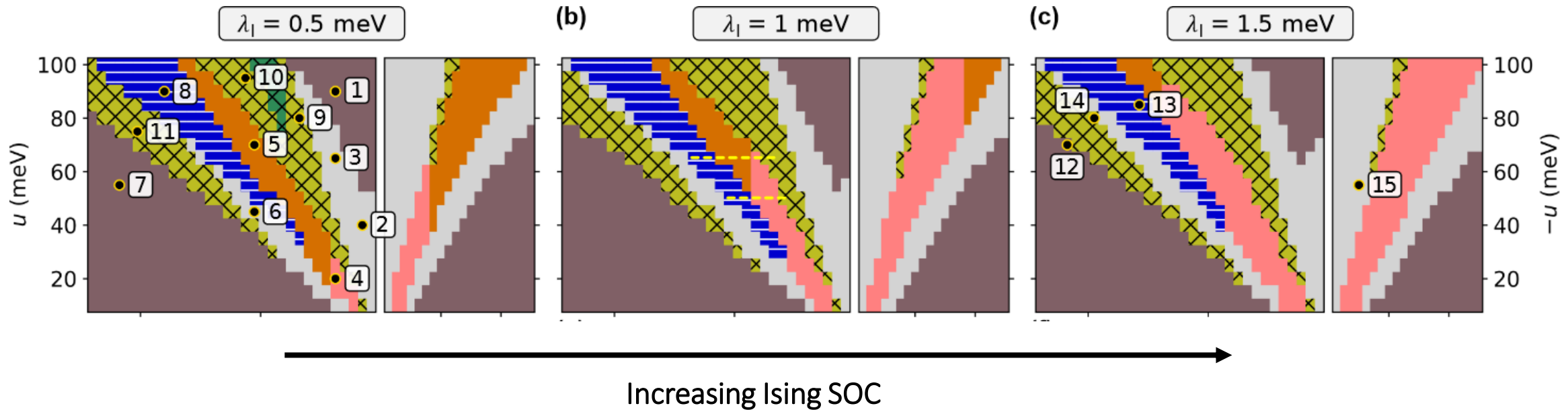
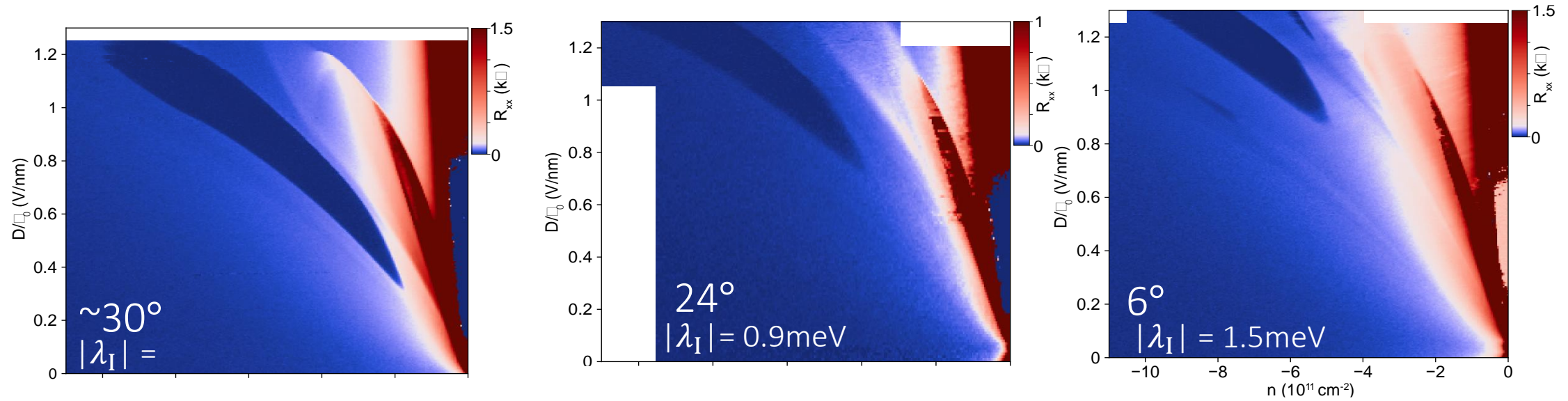


Spin-valley-locked half-metal



Increasing Ising SOC

Comparing with experiments



Identifying the normal state

Hartree-Fock suggests two candidate half-metal states suppressed by Ising SOC:

- **Spin-canted phase:** breaks $U(1)$ spin rotations along the z-axis
- **Inter-valley coherent state:** breaks $U(1)$ valley conservation



Similar ideas for IVC fluctuations

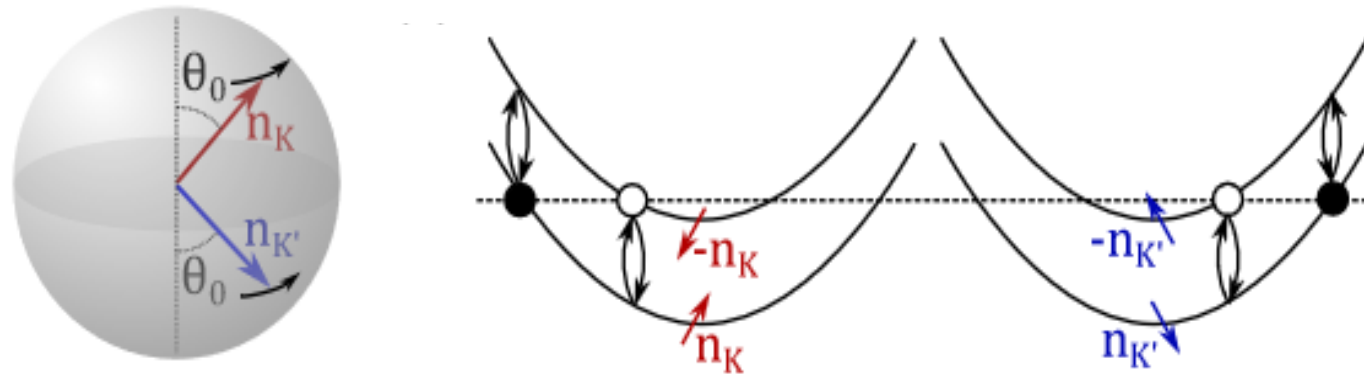
Chatterjee et al, Nature Communications (2022)
You and Vishwanath, PRB (2022)
Dong, Lee, Levitov, PNAS (2023)
Vituri, Xiao, Pareek, Holder, Berg, arXiv:2408.10309



Low-energy collective modes (Goldstone magnons)
could mediate Cooper pairing

Dong, ÉLH & Alicea, arXiv:2406.17036

3. Pairing from magnons?



Dong, ÉLH and Alica, arXiv:2406.17036

+ Collaboration with Andrea Young's group

C. Patterson, O. Sheekey, T. Arp, L. Holleis, J.M. Koh, ... ,

ÉLH, J. Alica and A. Young, arXiv:2408.10190

Spin canting order

Competition between Ising SOC and Hund's coupling

$$\mathcal{F} = \frac{\kappa}{2}(\mathbf{n}_K^2 + \mathbf{n}_{K'}^2) - J\mathbf{n}_K \cdot \mathbf{n}_{K'} + \frac{\lambda}{2}(n_K^z - n_{K'}^z) + \dots$$

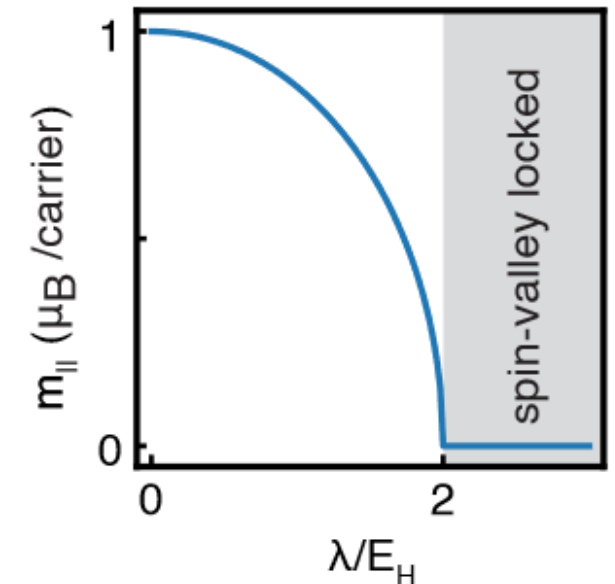
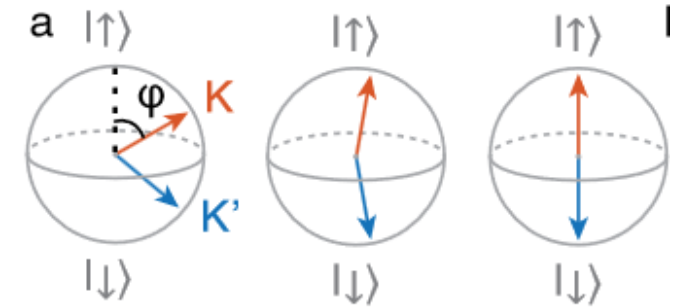
Spin polarization in each valley

Controls onset of Stoner ferromagnetism
 (Coulomb + kinetic energy)

Ferromagnetic Hund's coupling arises from
 lattice-scale electron-electron repulsion

Two valley-balanced solutions: spin-valley locked & spin canted

Spin canting breaks continuous U(1) spin rotations
 -> Goldstone modes

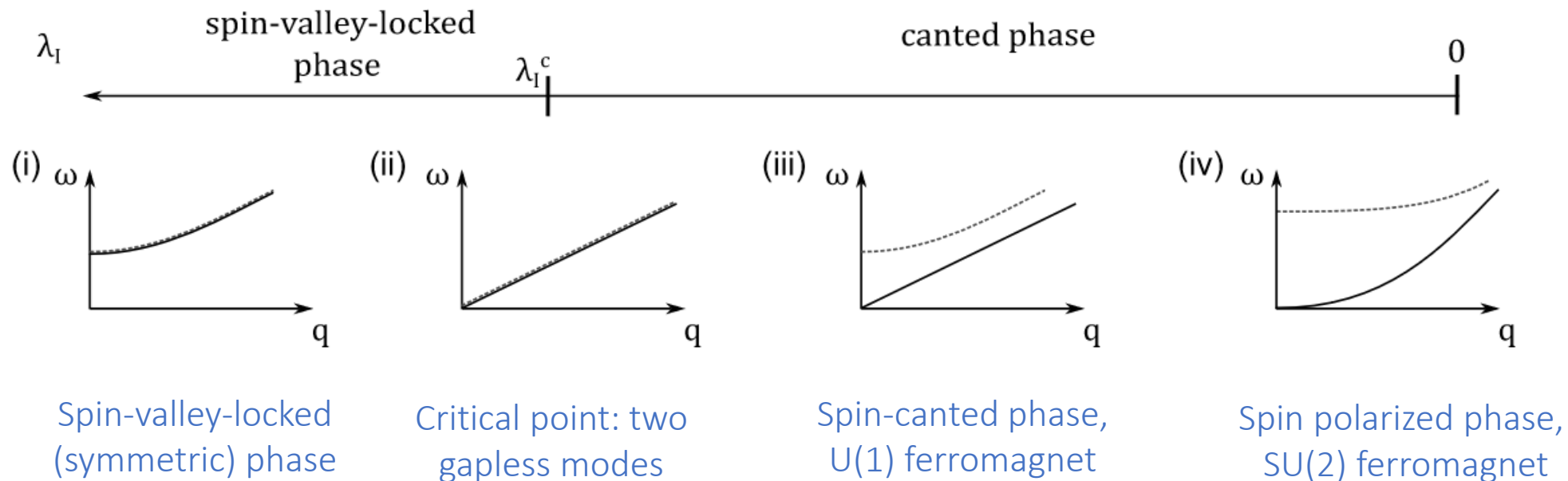
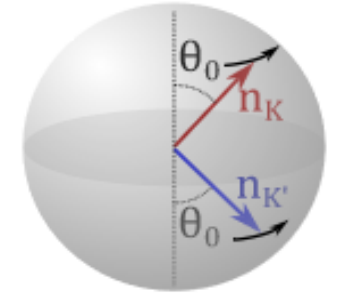


$$E_H = J|\mathbf{n}_{K,K'}|$$

Low-energy magnons

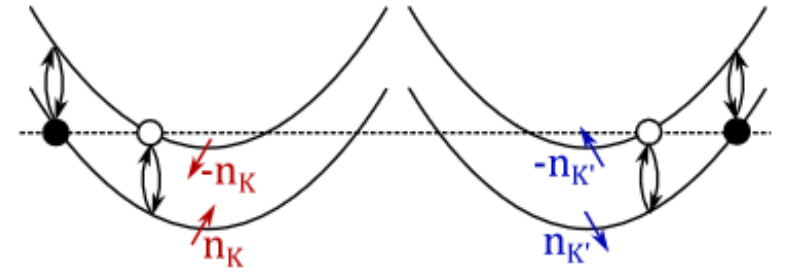
Goldstone mode: valley-symmetric azimuthal fluctuations

Critical mode: valley-antisymmetric polar fluctuations



Magnon mediated interactions

- Spin-flipping processes
- Qualitatively different scaling in two regimes:

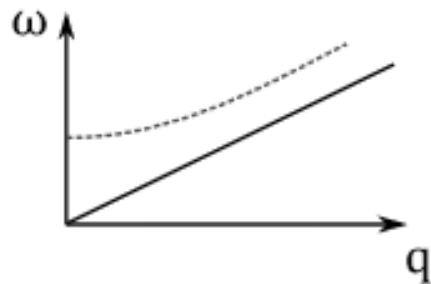


band indices

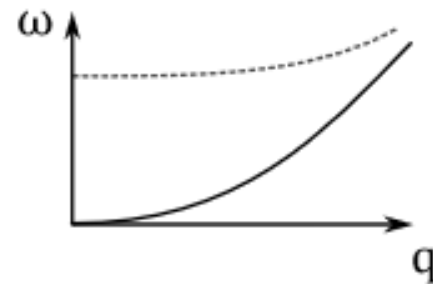
$$g_{\xi_1, \xi_2}^{-+, + -}(\nu, \mathbf{q}) \sim -\frac{\cos^2 \theta_0}{2z_s \kappa} \frac{1}{\nu^2 + c_s^2 \mathbf{q}^2}$$

valley indices

$$g_{\xi_1, \xi_2}^{-+, + -}(\nu, \mathbf{q}) \sim -\frac{1}{-i\gamma\nu + D\mathbf{q}^2}$$



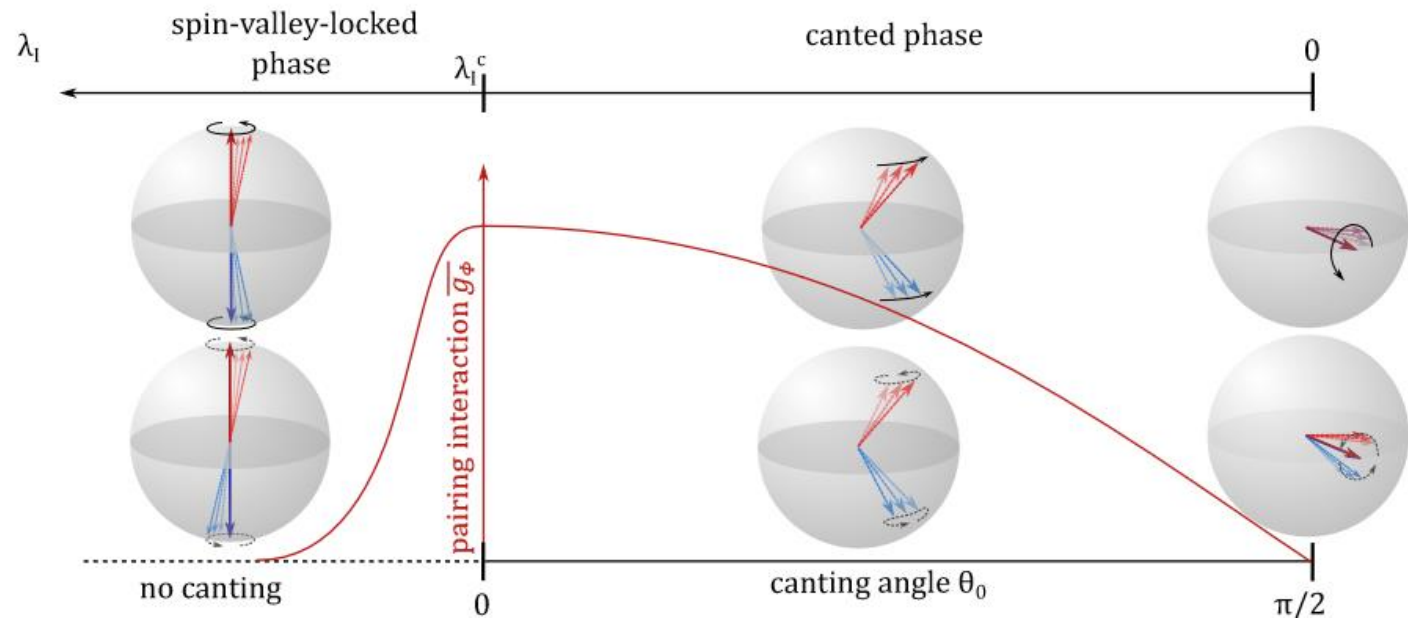
Spin-canted phase,
U(1) ferromagnet



Spin polarized phase,
SU(2) ferromagnet

Pairing from magnons

Leading-order diagrams:

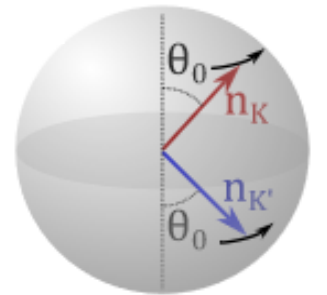


Pairing interaction (integrated over the Fermi surface):

Sign refers to majority/minority Fermi surfaces

$$\overline{g_{\pm}^{(2)}}(\nu) \sim \frac{\cos \theta_0}{k_F^{\pm}} \ln \frac{|\nu|}{\Lambda}$$

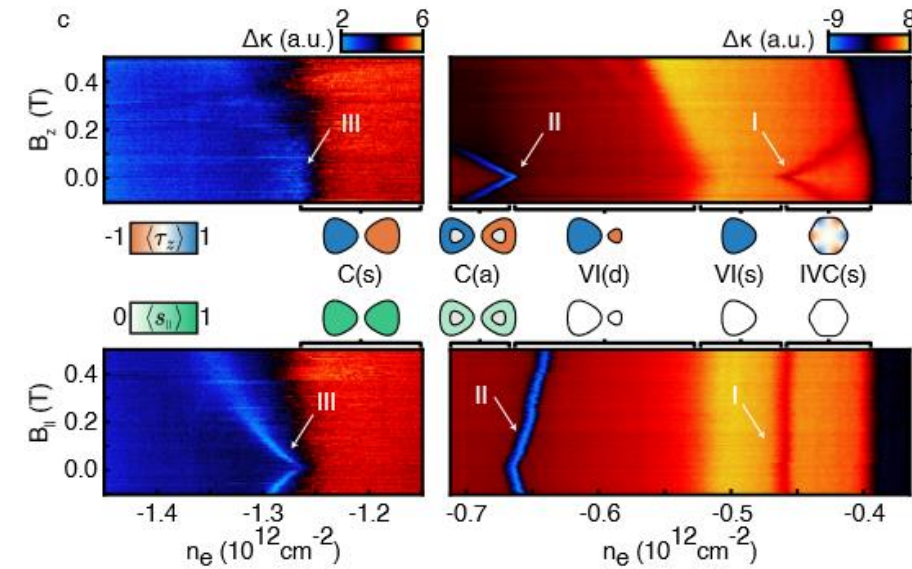
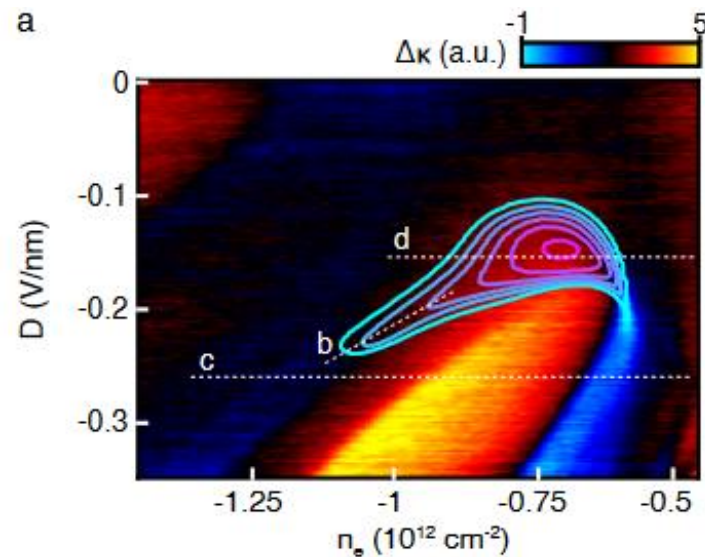
- Crucially depends on SOC: vanishes in the FM limit, $\theta_0 \rightarrow \pi/2$
- Low-frequency divergence: strong retardation effects
- Is enhanced for small Fermi pockets



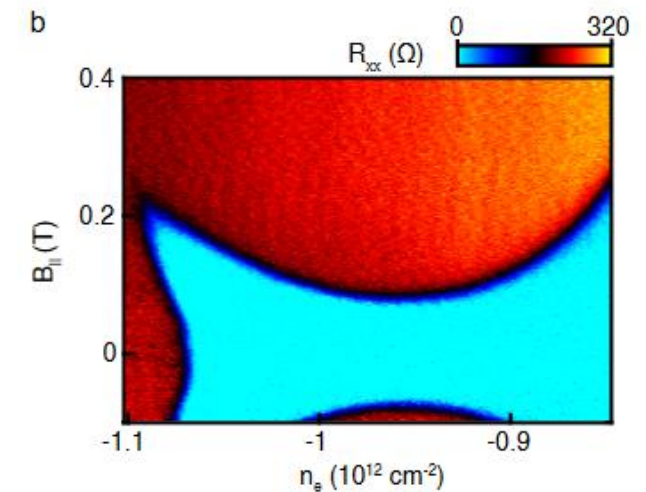
Pairing from magnons

Our theory predicts that the normal state hosting SC should host a **spontaneously-generated, in-plane magnetic moment**

- Could be resolved in magnetic scanning probes (SQUID on tip) or with magneto-transport/compressibility
- Collaboration with Andrea Young's group on trilayers/WSe2



C. Patterson, O. Sheekey, T. Arp, L. Holleis, J.M. Koh, ... , ÉLH, J. Alicea and A. Young
arXiv:2408.10190



Outlook

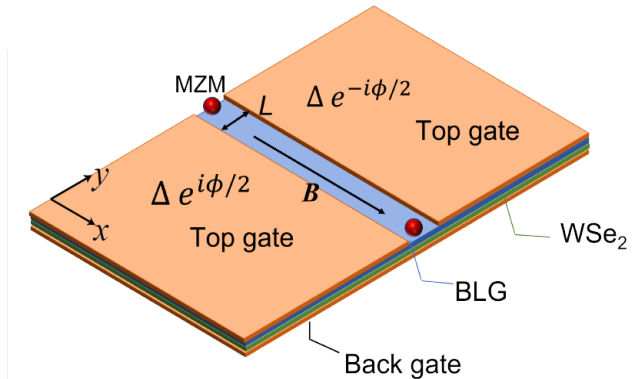
Rhombohedral graphene: exciting interplay of correlations, SOC and superconductivity deep in the clean limit

- Tunable and rich phase diagram. Control over microscopic parameters (e.g. band dispersion and induced SOC) highly constrains theory
- Multiple unconventional superconductors, possibly exotic pairing mechanisms
- **How to experimentally access collective modes and probe their (spin/valley) structure?**
- **Can constrain pairing mechanisms?** (e.g. gapping out magnon modes due to applied magnetic field. Analog for IVC fluctuations?)
- Investigate **dynamical** and/or **beyond mean-field** methods...

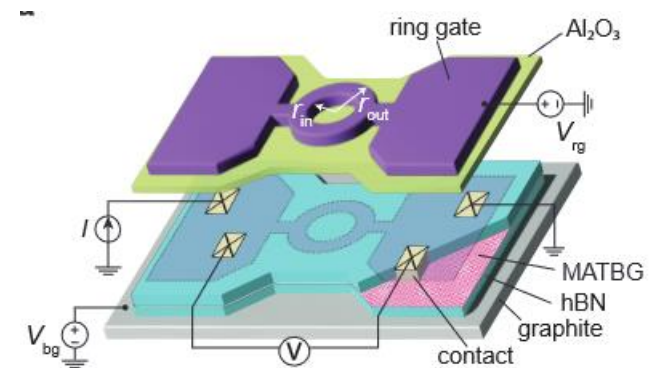
Vituri, Xiao, Pareek, Holder, Berg, arXiv:2408.10309
Wolf, Wei, Zhou, Huang, arXiv:2408.15884

c.f. discussion by Agnes Valenti last week

- **Gate-defined** architectures for quantum devices?



Xie, ÉLH, Young, Nadj-Perge & Alicea, Phys. Rev. Lett. **131**, 146601 (2024)



Iwakiri *et al.*, Nat. Comms. **15**, 390 (2024)

Extras

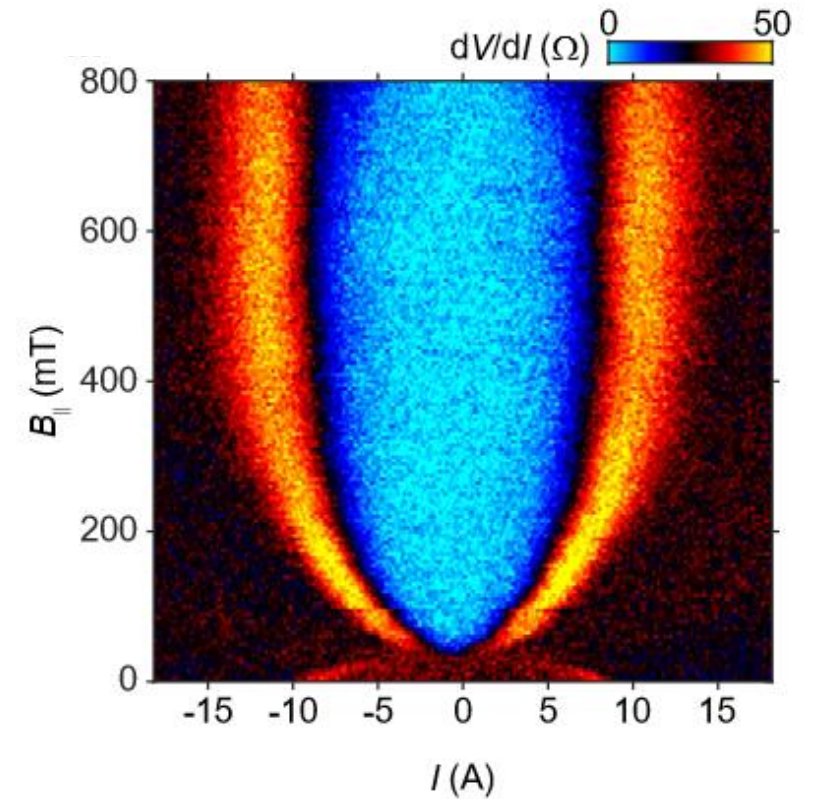
SC induced by a magnetic field?

Large (> 20) violation of Pauli limit:

$$B_p = \frac{\Delta}{\sqrt{2}\mu_B} = \frac{1.76k_B T_c}{\sqrt{2}\mu_B} \approx 40 \text{ mT}$$

Ginzburg-Landau coherence length $\xi_{\text{GL}} \ll l$ Electronic mean free path

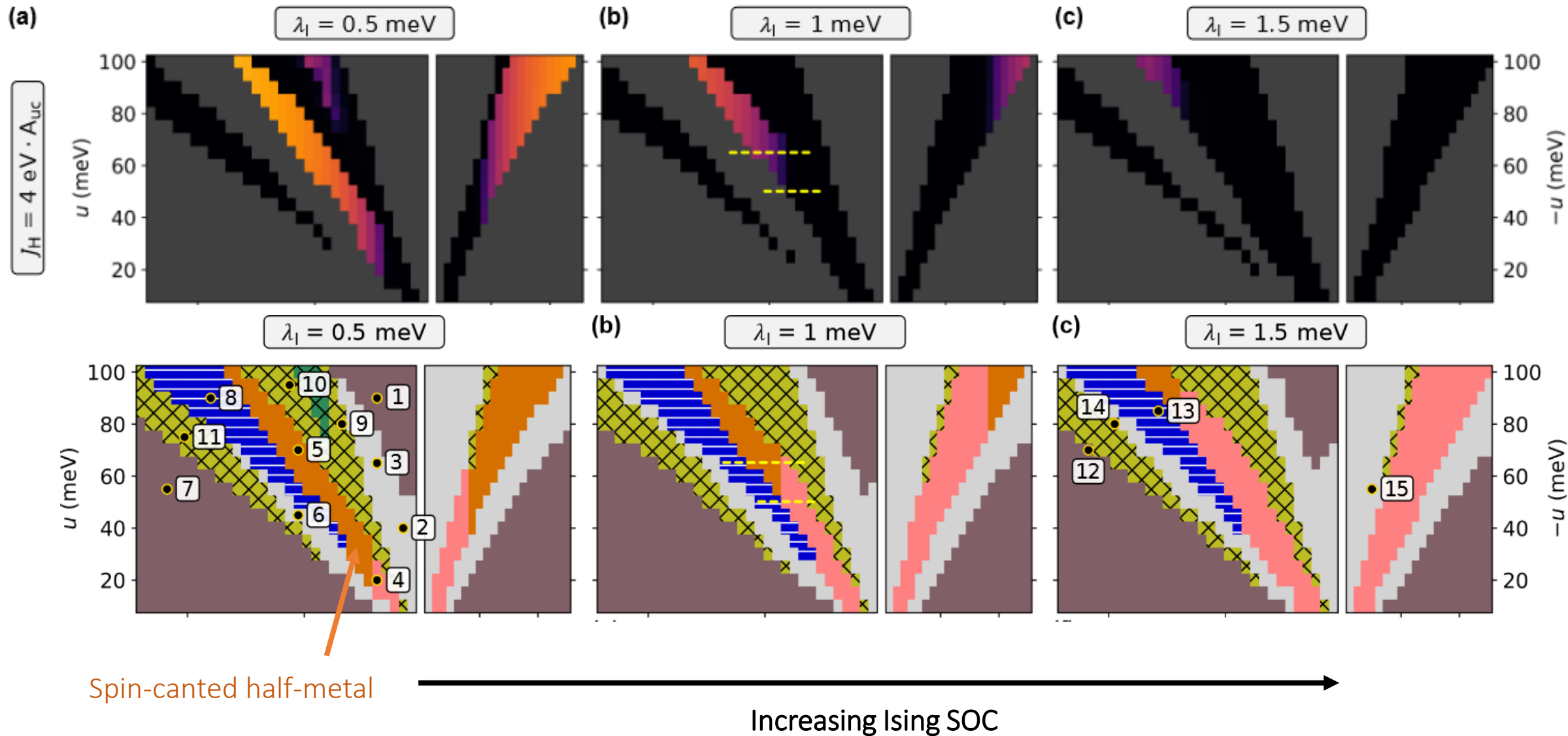
$$\xi_{\text{GL}} \sim 250 \text{ nm} \quad l \sim 10 \mu\text{m}$$



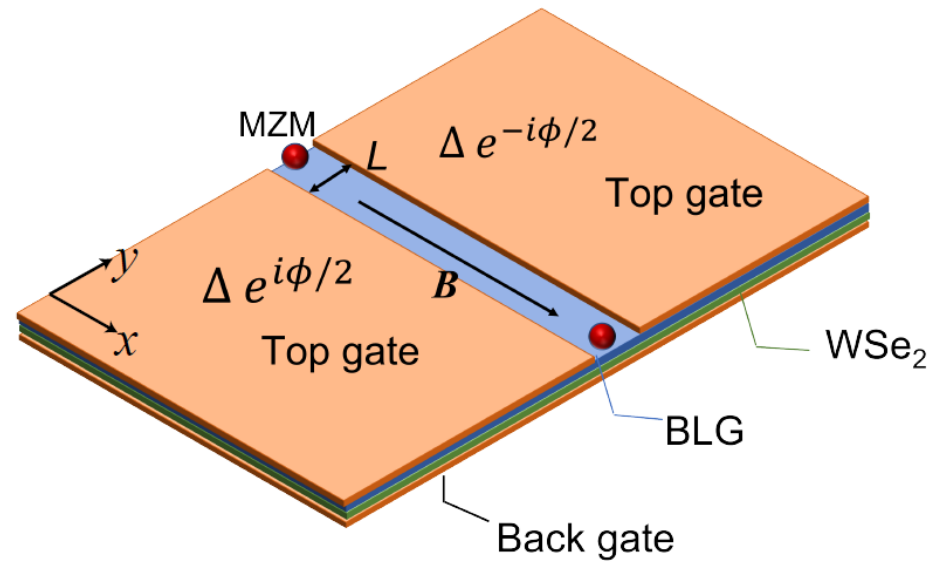
Anderson theorem: singlet superconductors are stable against T-preserving disorder

P. W. Anderson, Theory of dirty superconductors, J. Phys. Chem. Solids. 11, 26–30 (1959)

Canting angle



4. Gate-defined topological Josephson junctions



Xie, ÉLH, Young, Nadj-Perge & Alicea, Phys. Rev. Lett. **131**, 146601 (2024)

Topological SC and the Kitaev model

- Cooper pairing in a 1D chain of **spinless fermions** can lead to topological SC with unpaired **Majorana zero-modes**

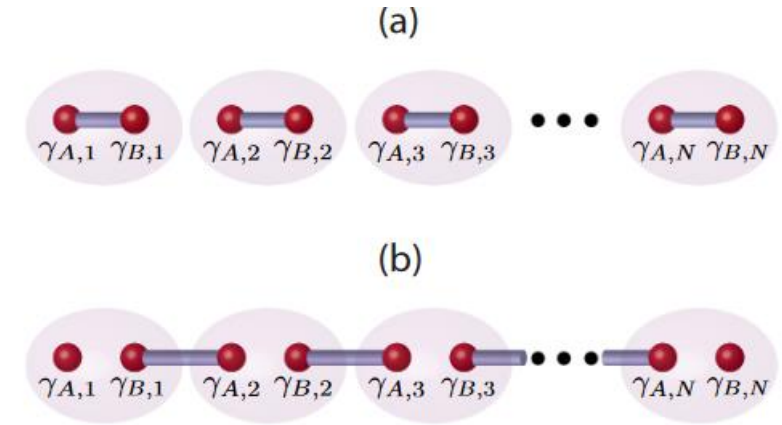
Kitaev (2001)

- Various theoretical proposals to create effectively spinless fermions

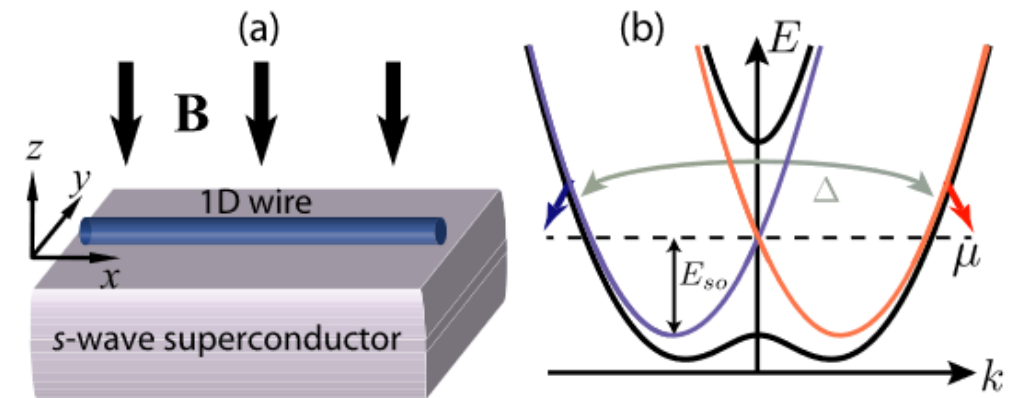
- Proximitized spin-orbit coupled nanowires at the heart of a multi-million \$ effort!

Oreg, Refael and von Oppen, PRL (2010)

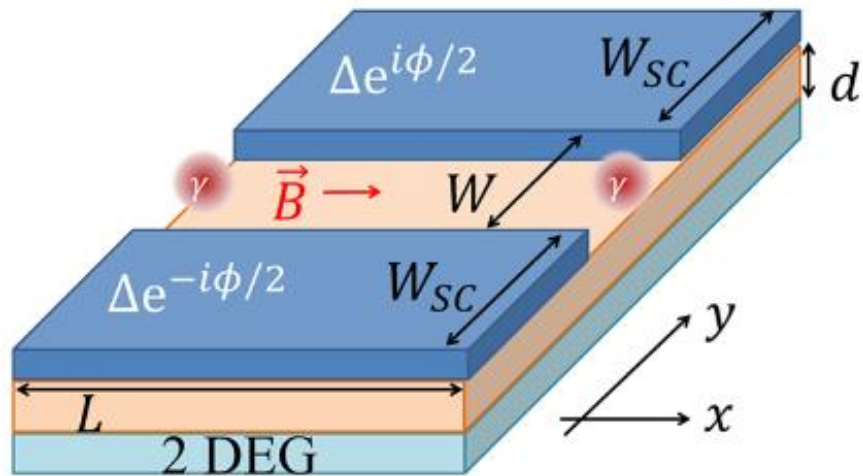
Lutchyn, Sau and das Sarma, PRL (2010)



Alicea, Rep. Prog. Phys (2012)

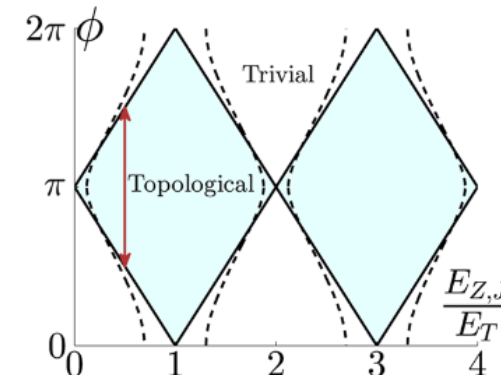


Topological SC in planar Josephson junctions



- SOC in 2D electron gas
- Proximity-induced SC
- In-plane Zeeman field
- Phase difference

Pientka, Keselman, Berg, Yacoby, Stern & Halperin, PRX (2017)
Hell, Leijnse & Flensberg, PRL (2017)



Topological SC in planar Josephson junctions

Current experiments rely on **complex heterostructures**:

- SC film (Al or NbTiN)
- 2DEG (HgTe, InAs or InSb quantum wells)

Ren *et al.*, Nature **569**, 93 (2019)

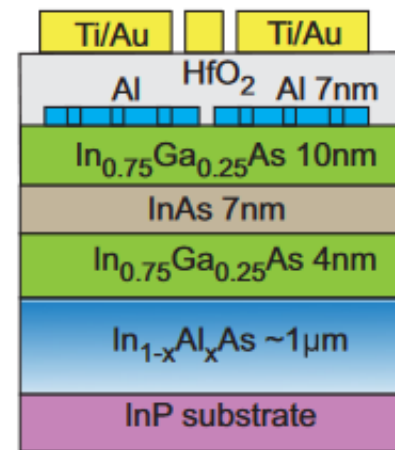
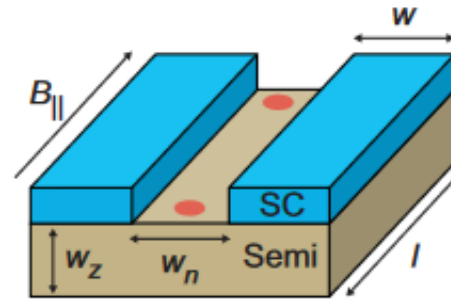
Fornieri *et al.*, Nature **569**, 89 (2019)

Ke *et al.*, Nat. Commun. **10**, 3764 (2019)

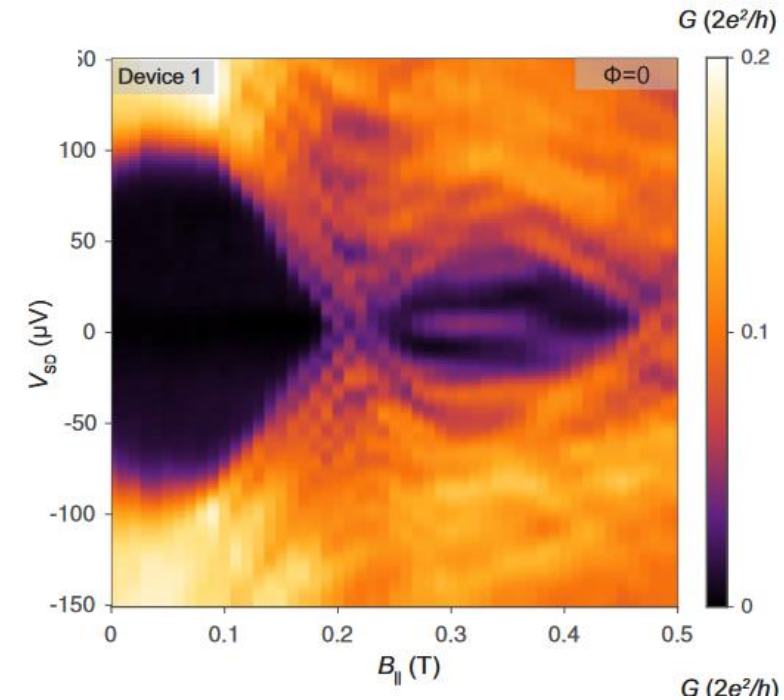
Dartailh *et al.*, PRL **126**, 036802 (2021)

Banerjee *et al.*, PRB **107**, 245304 (2023)

Can we realize this physics in
an **intrinsic** junction?



Banerjee *et al.*,
PRB **107**, 245304 (2023)

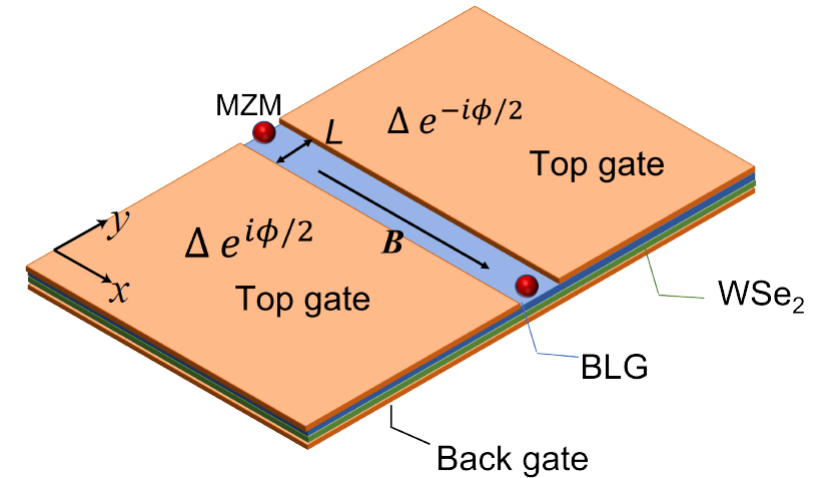


Tunneling conductance at the
junction ends reveals gap closing
followed by “zero-bias” features

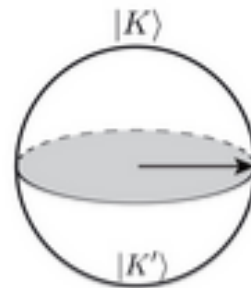
Topological SC in planar Josephson junctions

Superconductivity in BLG+WSe₂ is **gate tunable**

- Intrinsic JJ by patterning with gates!
- Induced SOC + Zeeman field lifts the spin degeneracy
- Graphene entails an additional headache: the valley



Valley degeneracy can be lifted by interactions: inter-valley coherence



- Potential payoffs:
- No material interfaces
 - (Almost) no orbital coupling of B

Minimal model

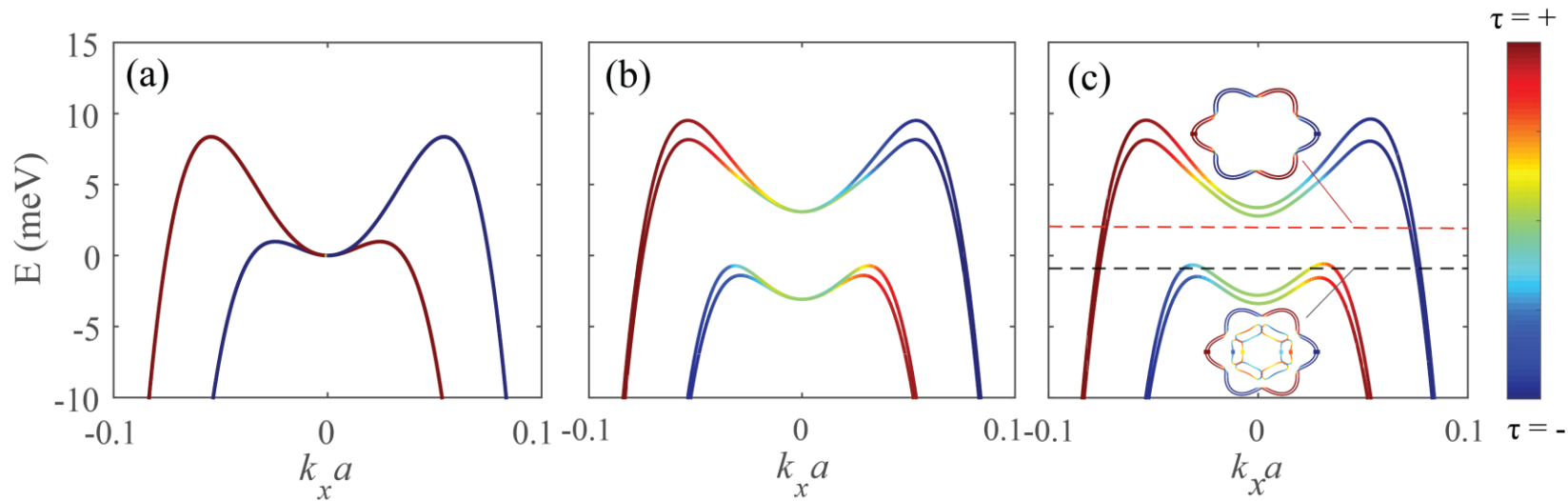
$$H(\mathbf{k}) = h_0(\mathbf{k}) + \beta_I \tau_z s_z + \alpha_R (k_x s_y - k_y s_x) + \lambda_0 \tau_x + \mathbf{h} \cdot \mathbf{s}$$

Dispersion of BLG

Induced Ising and Rashba SOC

Inter-valley coherence

Zeeman coupling to in-plane field



BLG dispersion

Adding IVC and SOC

Applying Zeeman field

Topological SC relies on inter-valley coherence
Can be turned into a diagnostic tool

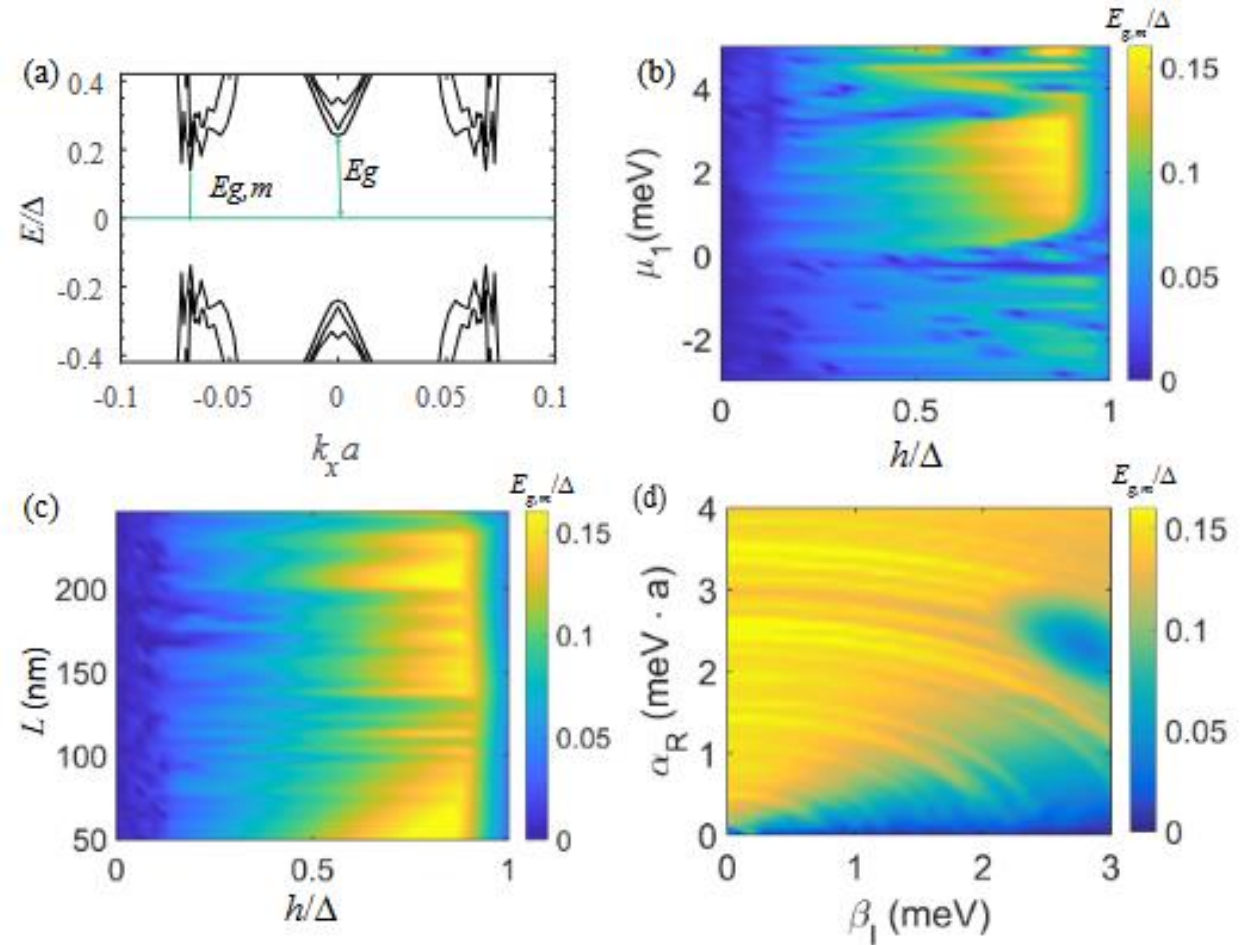
Optimizing the minimal topological gap

Minimal gap across the junction generically occurs for non-zero k_x

Optimal regime:

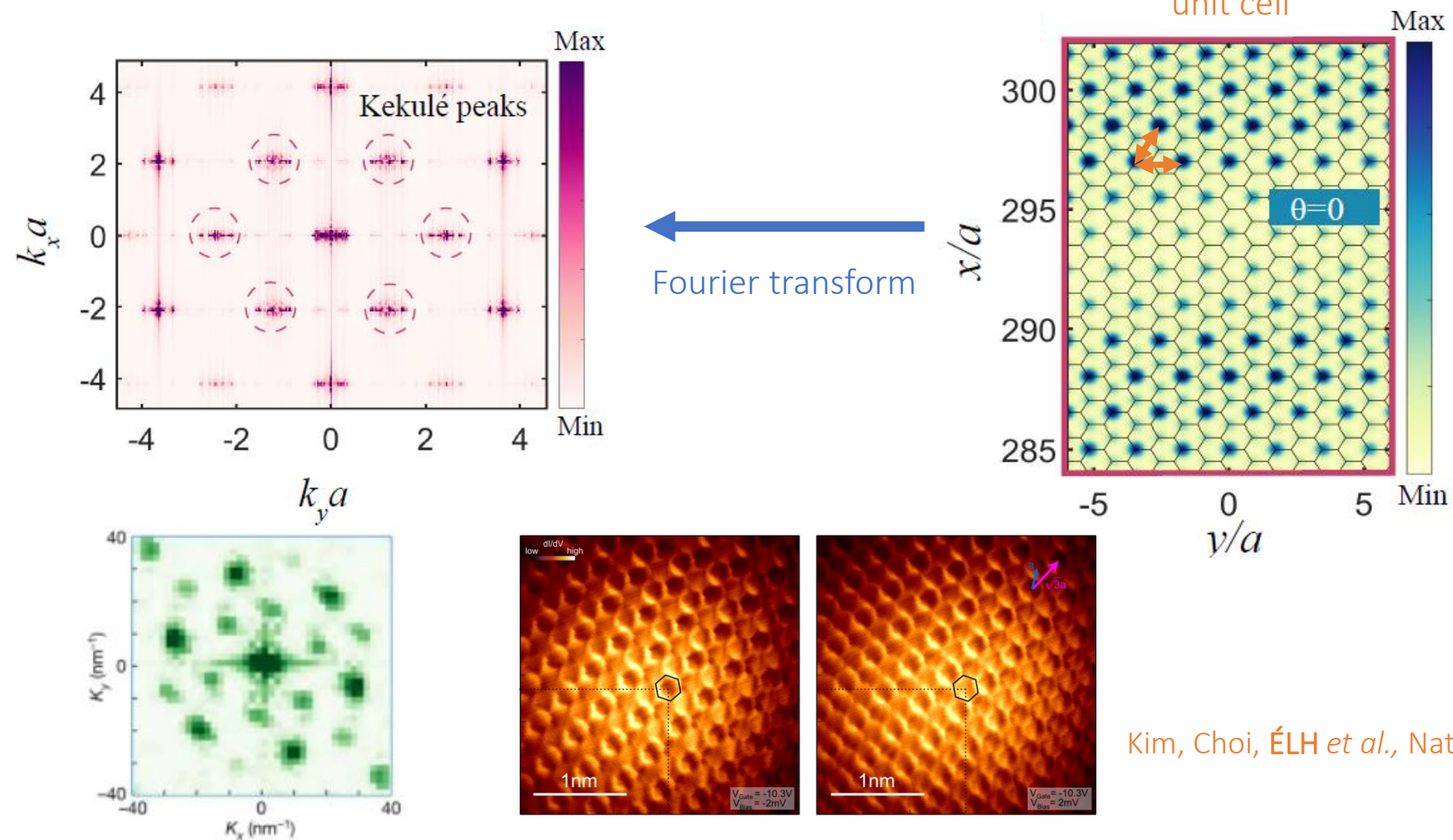
- Doping level without “polluting” Fermi pockets
- Short junction limit
- Effective SOC in low-energy bands dominated by Rashba contribution

$$\tilde{\alpha}_R = \frac{\lambda_0}{\sqrt{\lambda_0^2 + \beta_I^2}} \alpha_R, \quad \tilde{\hbar} = \frac{\lambda_0}{\sqrt{\lambda_0^2 + \beta_I^2}} \hbar$$



Identifying IVC order by tunneling

Majorana wavefunctions inherit Kekulé pattern:



$\sqrt{3} \times \sqrt{3}$
enlargement of
unit cell

Kim, Choi, ÉLH *et al.*, Nature 623, 942 (2023)

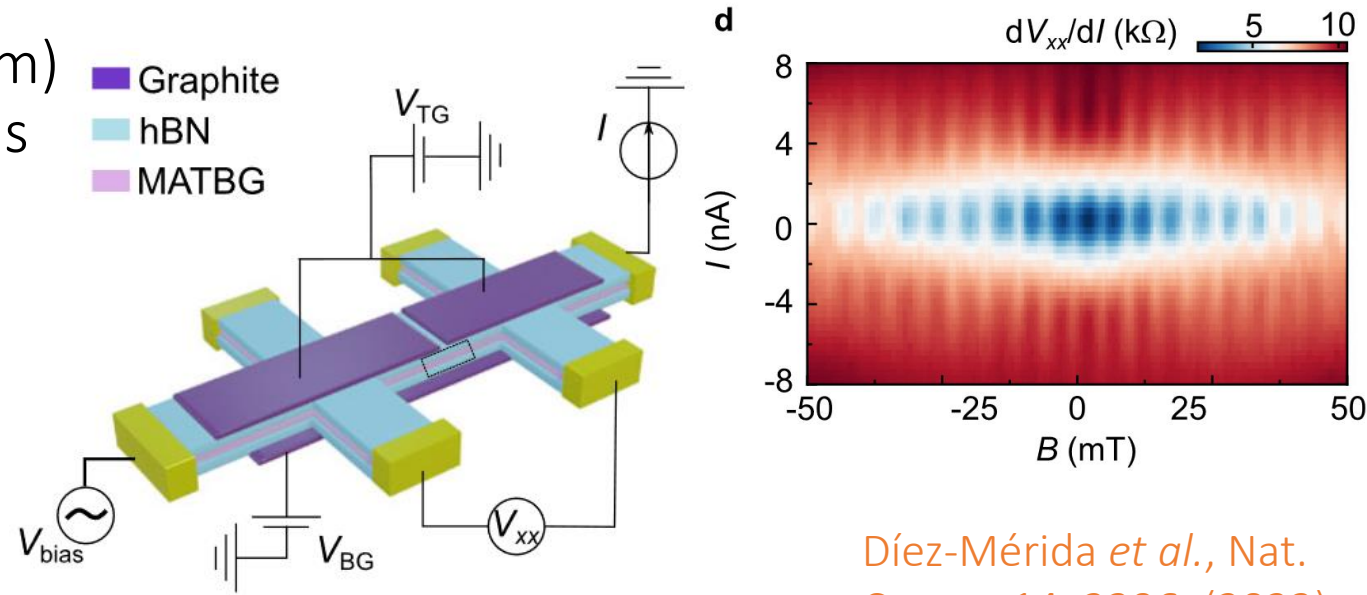
Gate-defined experiments probing phase coherence

- Gate-defined Josephson junctions (~ 100 nm) can image Fraunhofer interference patterns

de Vries *et al.*, Nat. Nanotech. **16**, 760 (2021)

Rodan-Legrain *et al.*, Nat. Nanotech. **16**, 769 (2021)

Díez-Mérida *et al.*, Nat. Comm. **14**, 2396, (2023)



Díez-Mérida *et al.*, Nat. Comm. **14**, 2396, (2023)

- Little-Parks experiment with gate-defined superconducting loops (~ 1 μm)

Iwakiri *et al.*, arXiv:2308.07400

