

# Quantum suppression of conductance fluctuations in atomic-size contacts

*Jan van Ruitenbeek,  
Kamerlingh Onnes Laboratorium*



1

## In collaboration with

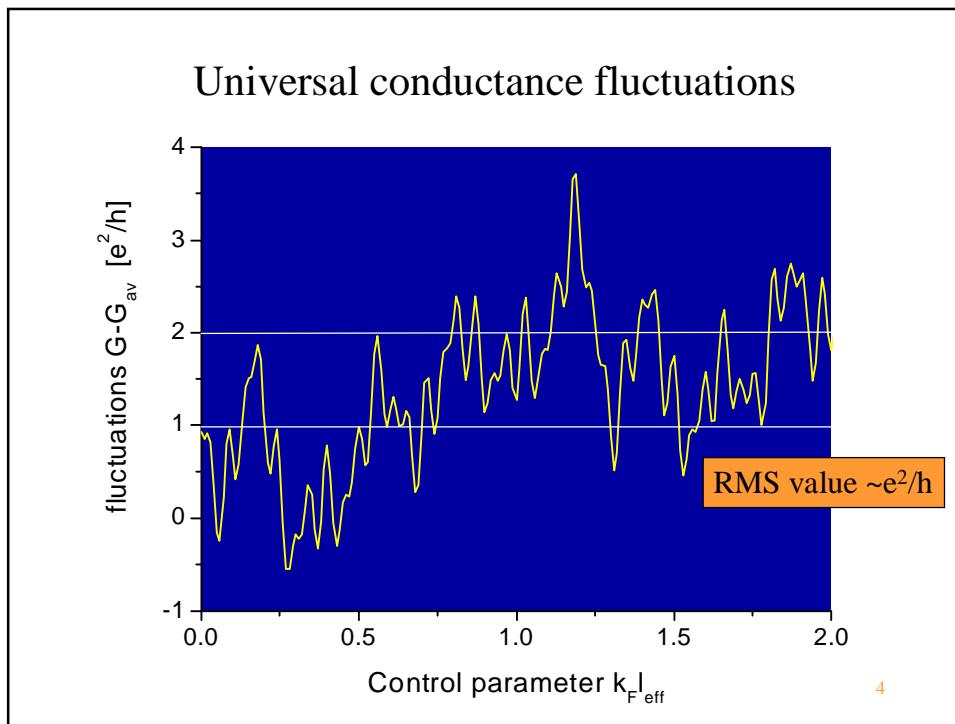
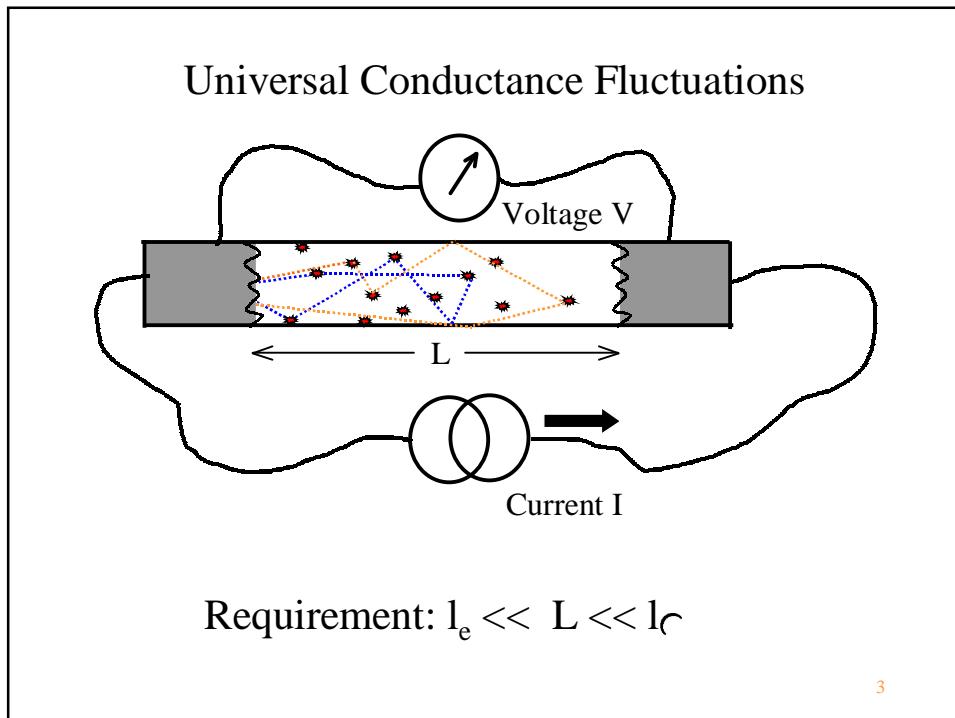
### **Leiden:**

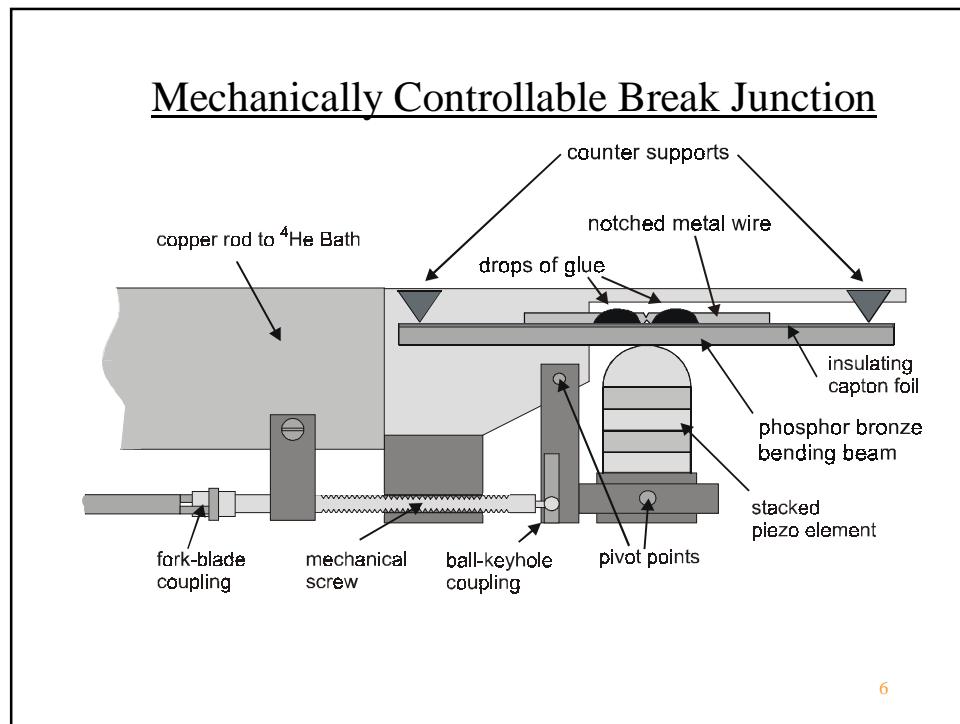
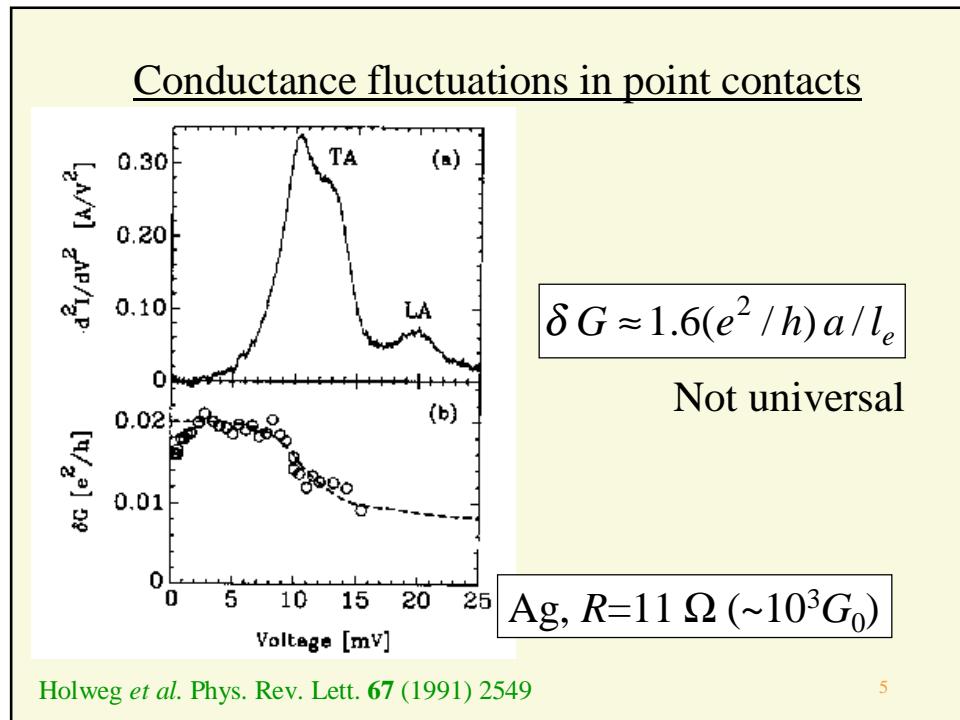
- Chris Muller
- Martijn Krans
- Niko van der Post
- Helko van den Brom
- [Bas Ludoph](#)
- Alex Yanson

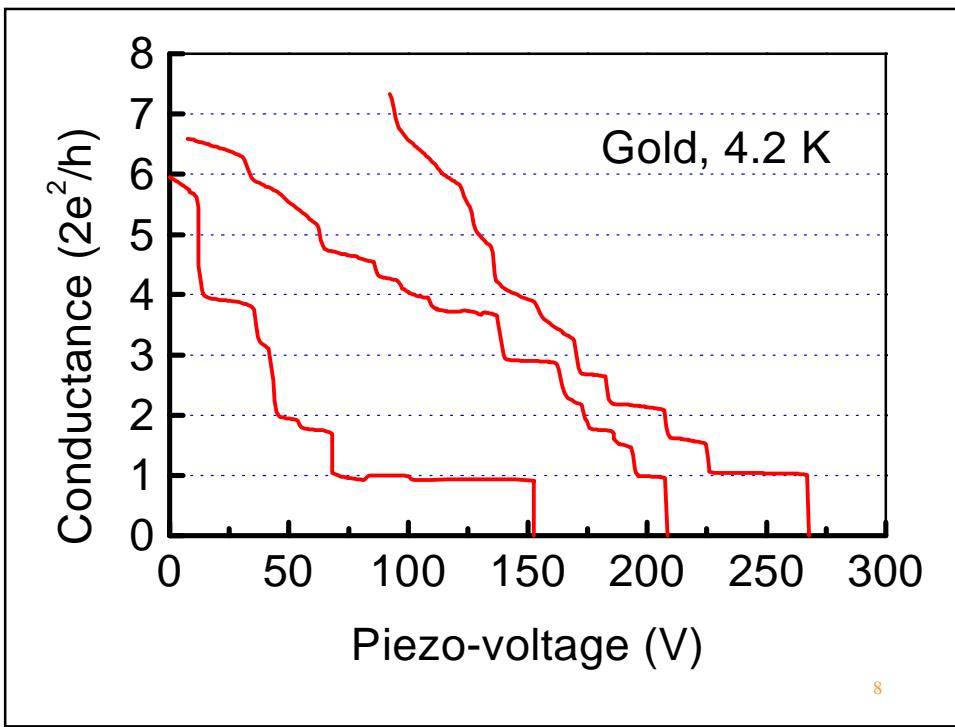
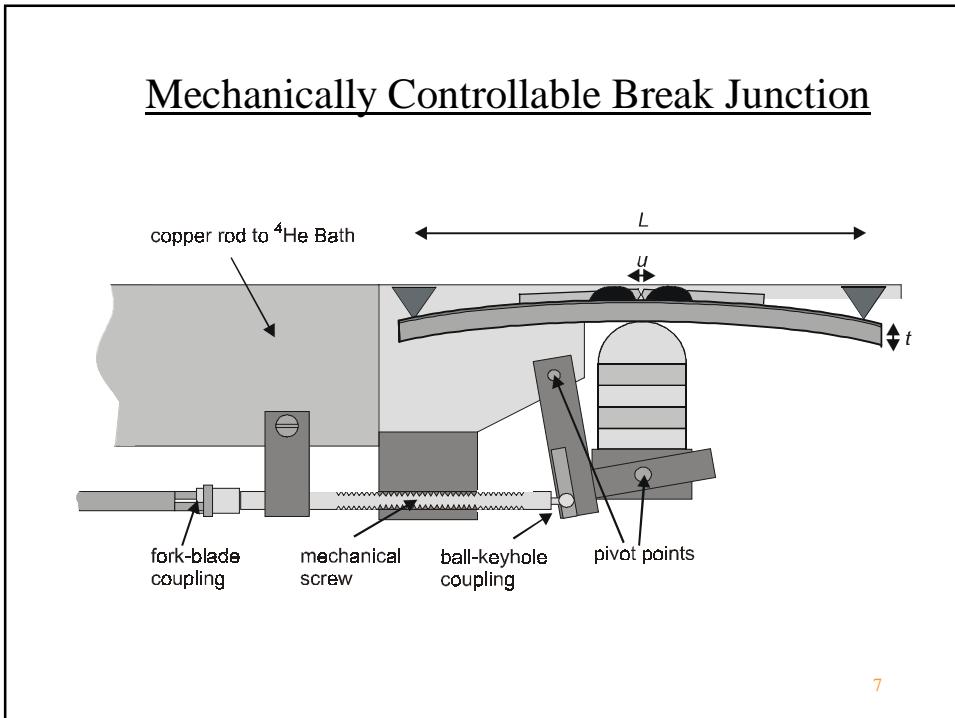
### **Saclay:**

- [Michel Devoret](#)
- [Daniel Esteve](#)
- [Cristian Urbina](#)

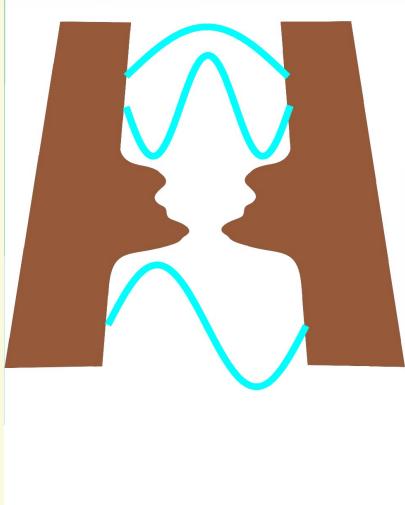
2







## Quantum conductance (2 dimensions)



Incoming and reflected modes

Scattering at the contact

Transmitted modes

9

## Conductance is transmission

Vector of incoming waves from the left, on a basis of quantum modes:

$$\vec{i}_l$$

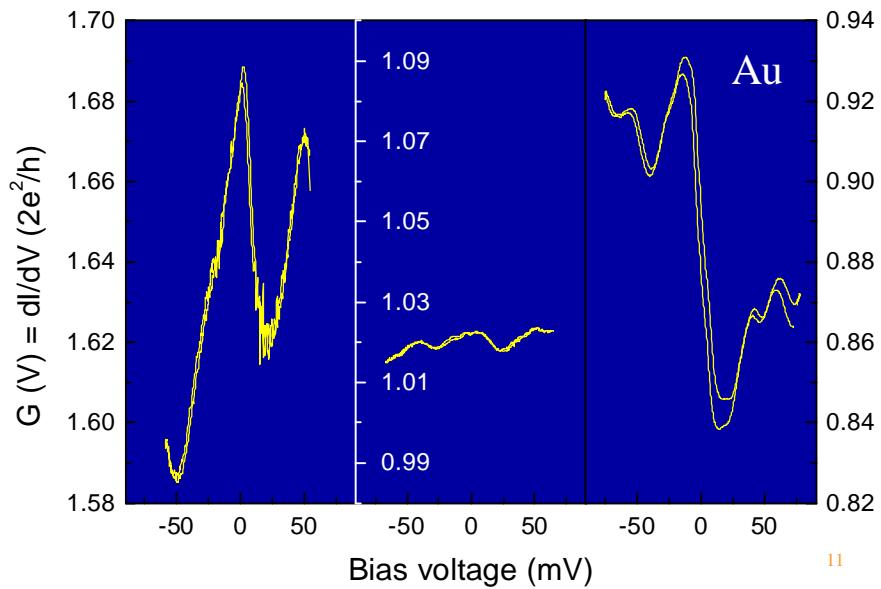
Vector of outgoing waves to the right:  $\vec{o}_r$

Matrix of transmission amplitudes:  $\vec{o}_r = \hat{t} \vec{i}_l$

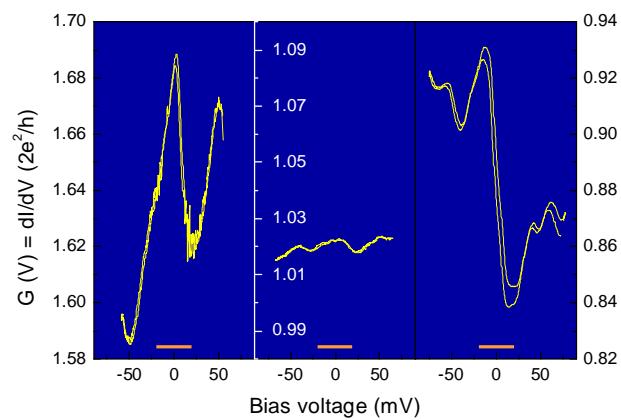
Landauer:  $G = \frac{2e^2}{h} \text{Tr}(\hat{t}^+ \hat{t}) = \frac{2e^2}{h} \sum_n T_n$

10

Examples for several settings of the conductance of the contact

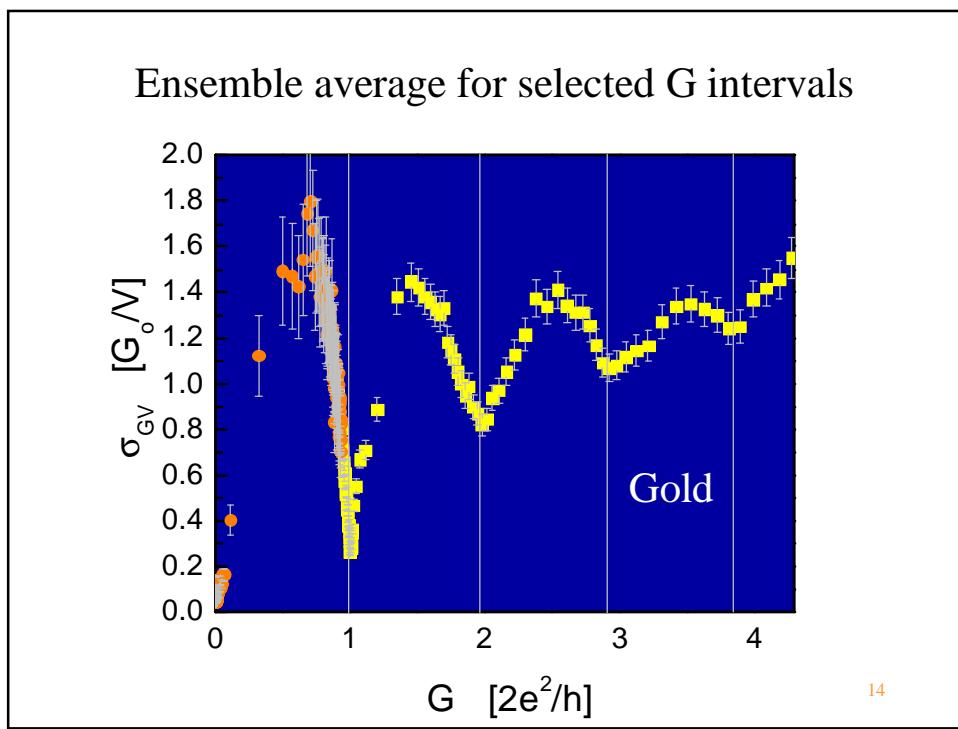
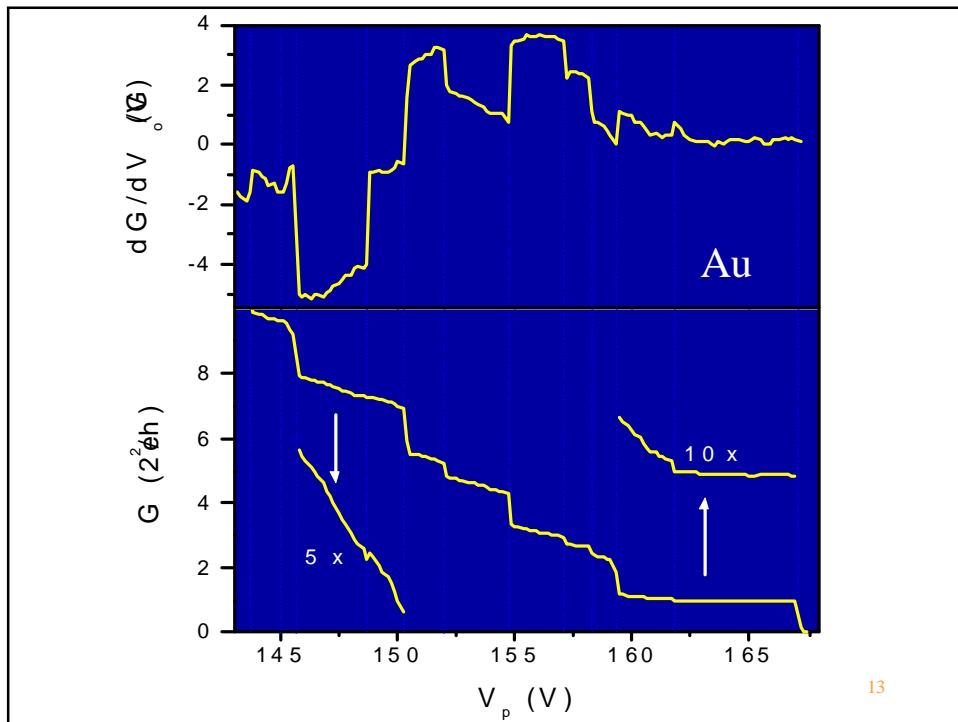


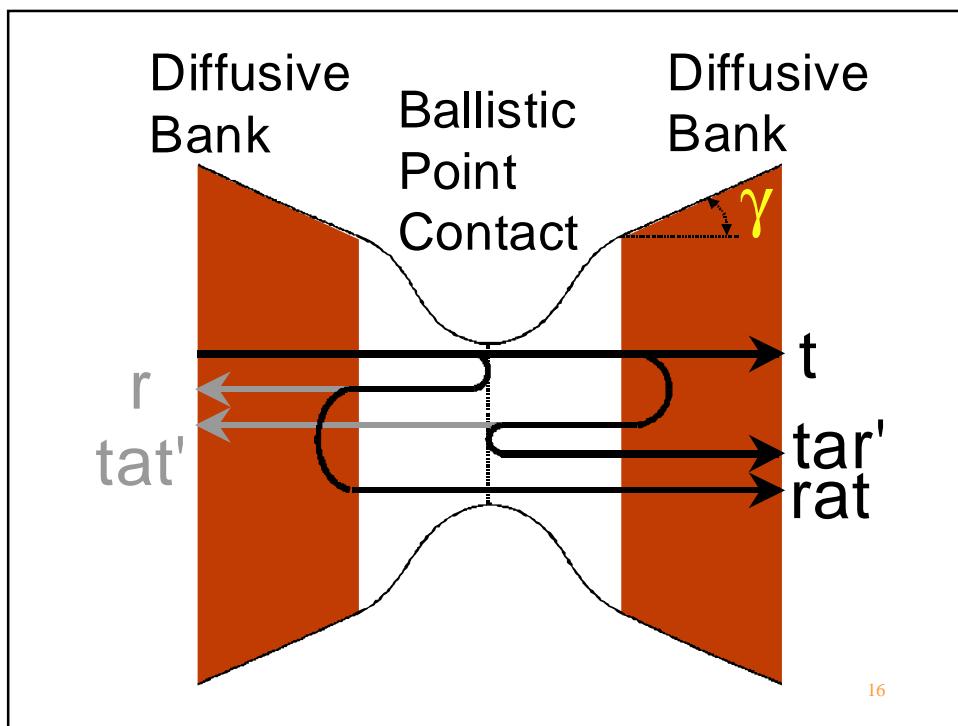
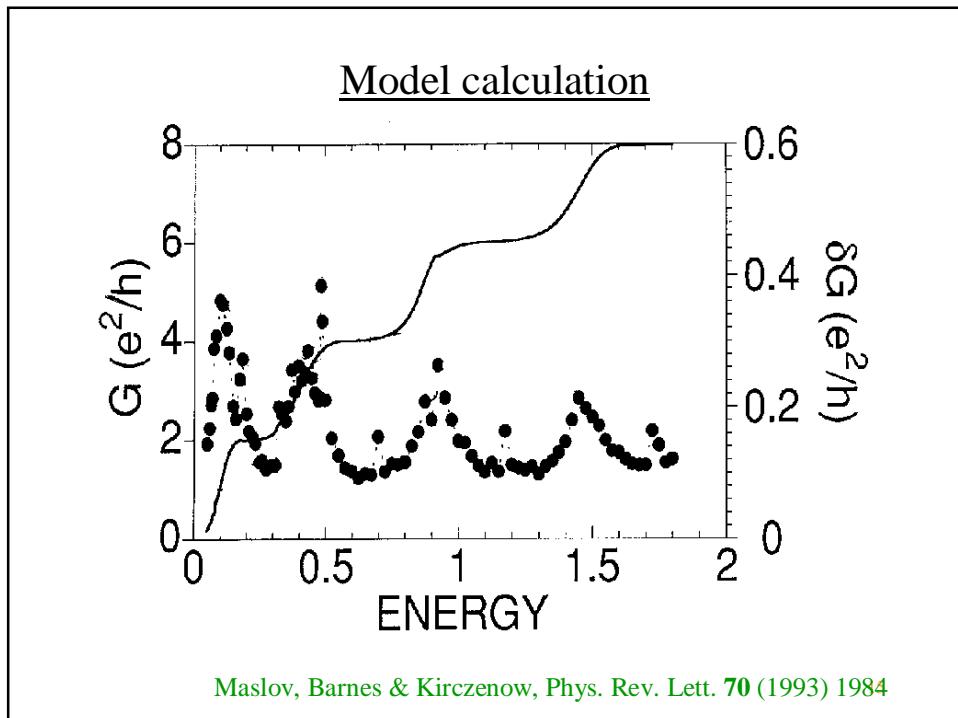
Experimental approach to ensemble average



- Large modulation voltage  $V = V_{\text{bias}} + V_{\text{mod}} \sin(\omega t)$
- Fixed bias voltage ( $V_{\text{bias}} = 0$ ) ; 20mV; 48 kHz
- Record  $G$  and  $dG/dV$  simultaneously

12





### Calculation of scattering corrections to G

Define  $\sigma_{\text{GV}} \equiv \left( \frac{\partial^2 I}{\partial V^2} \right)_{\text{RMS}}$

$$I(V_0 + V_{\text{mod}} \sin(\omega t)) = I(V_0) + \left( \frac{\partial I}{\partial V} \right)_{V_0} \sin(\omega t) - \frac{1}{4} \left( \frac{\partial^2 I}{\partial V^2} \right)_{V_0} V_{\text{mod}}^2 \cos(2\omega t) + \dots$$

$$I = \frac{2e^2}{h} \int_0^{eV} \text{Tr}(\hat{t}^\dagger \hat{t}) dE$$

17

### Calculation of scattering corrections to G (continued)

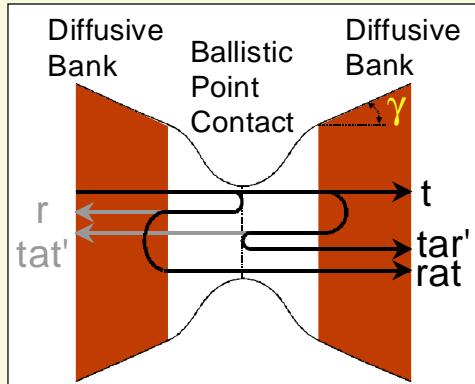
$$\begin{aligned} \text{Tr}(\hat{t}^\dagger \hat{t}) &= \sum_{n=1}^N T_n (1 + \text{Re}(a_{r_{mn}} r'_n + r_n a_{l_{nn}})) \\ \langle a_{r,l_{mn}}(E_1) a_{r,l_{nn}}^*(E_2) \rangle &= \int_0^\infty P_{\text{cl}}(\tau) e^{-i(E_1 - E_2)\tau/\hbar} d\tau \end{aligned}$$

$$\sigma_{\text{GV}} = \frac{2.71eG_0}{\hbar k_F v_F \sqrt{1-\cos\gamma}} \left( \frac{\hbar/\tau_e}{eV_{\text{mod}}} \right)^{3/4} \sqrt{\sum_{n=1}^N T_n^2 (1-T_n)}$$

18

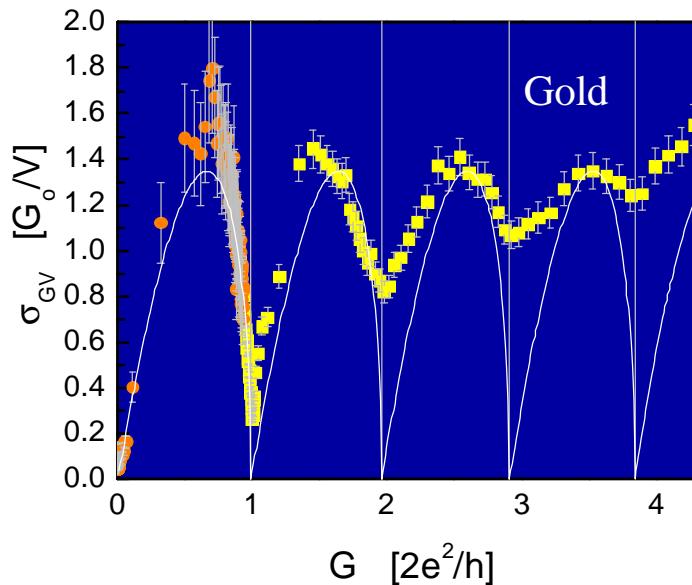
The probability per unit time to return to the contact, after a time  $\tau$ , into a given mode  $n$ , is

$$P_{\text{cl}}(\tau) = \frac{v_F}{2\sqrt{3\pi} k_F^2 (D\tau)^{3/2} (1 - \cos \gamma)}$$

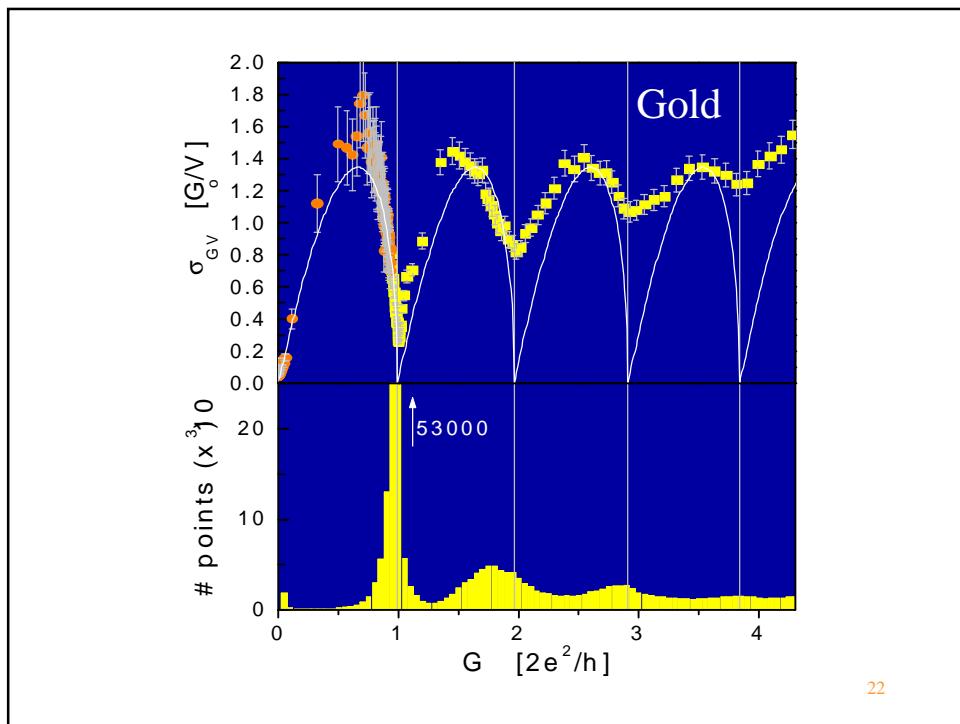
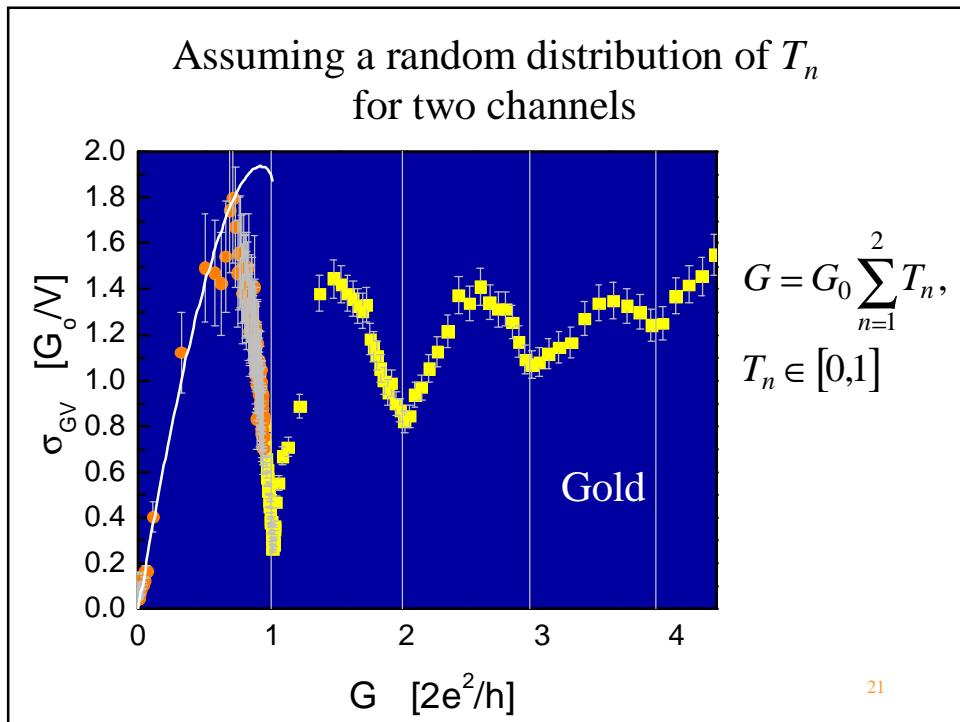


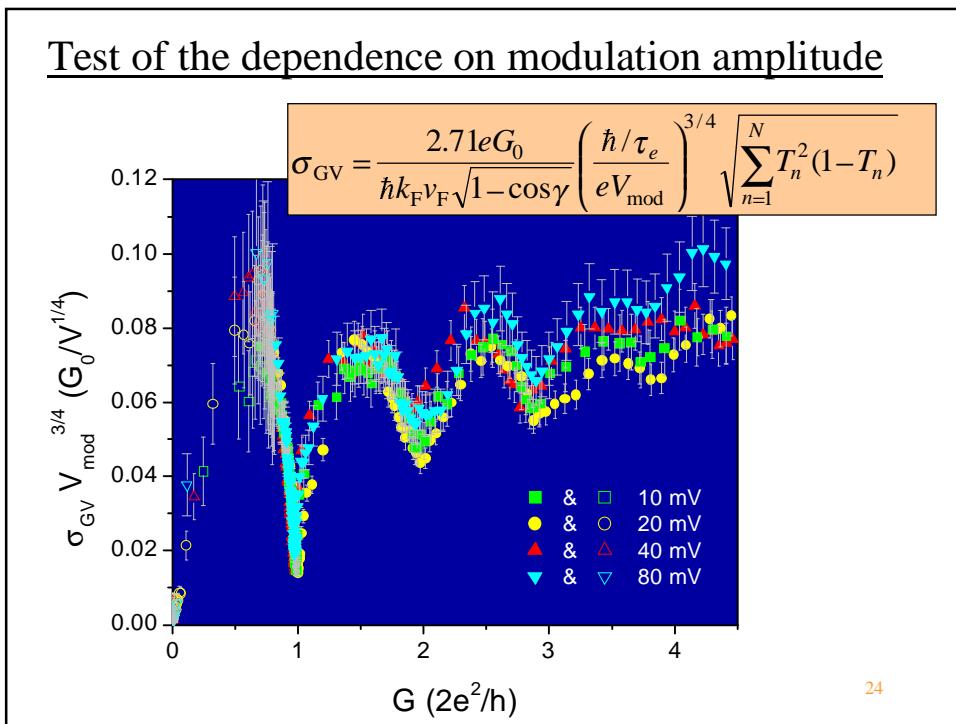
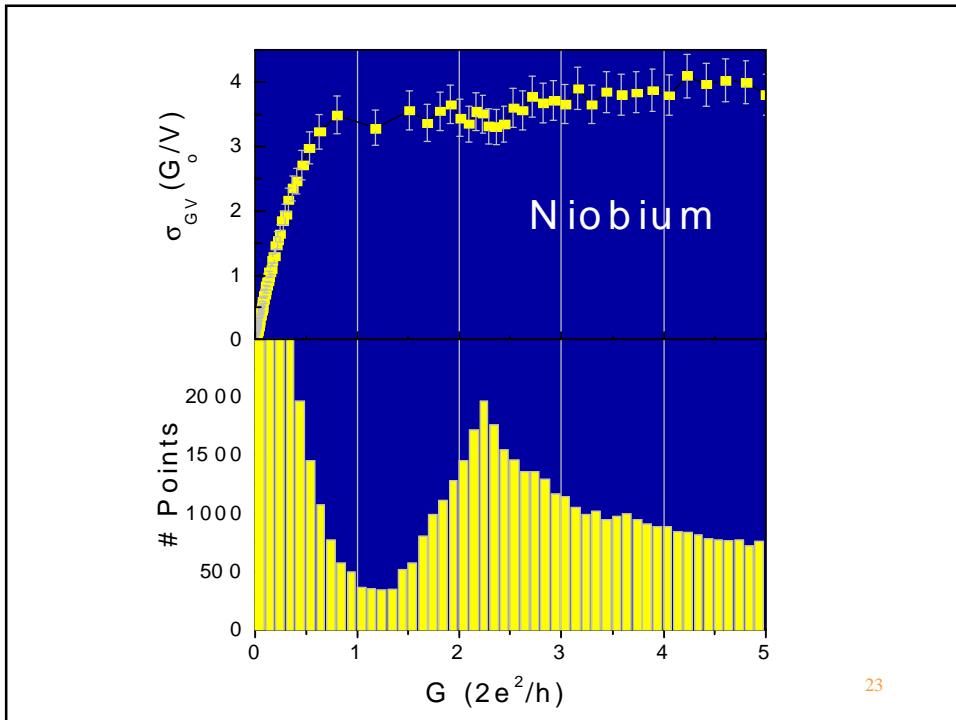
19

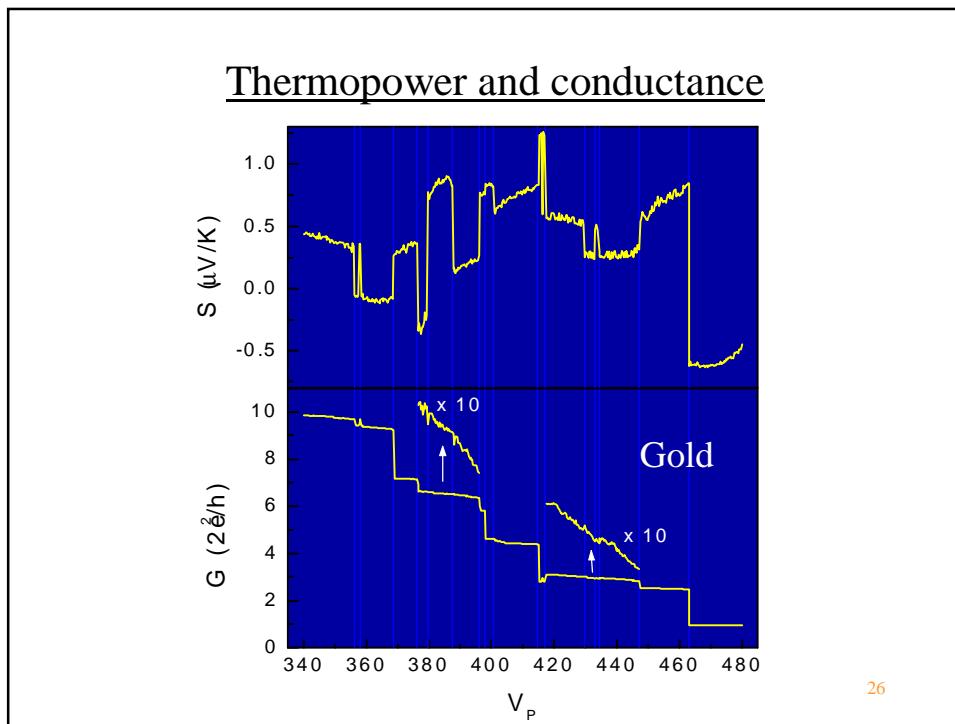
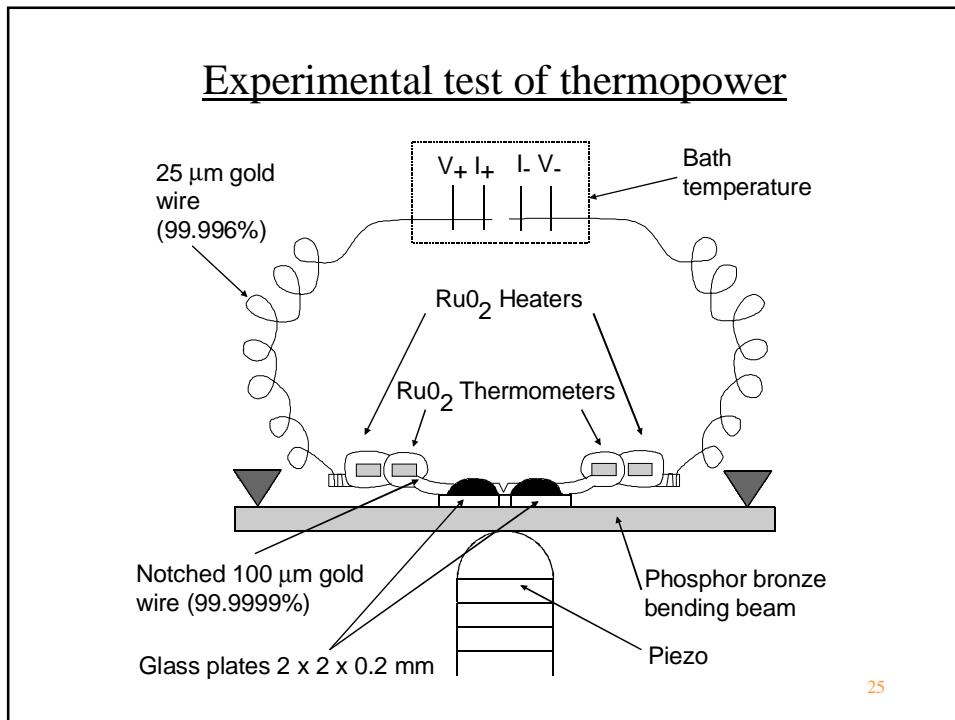
Model assuming channel saturation



20







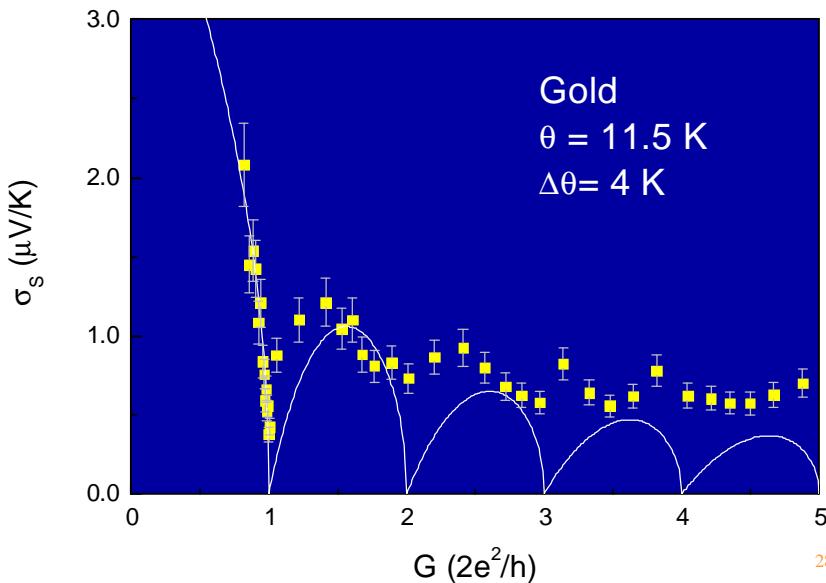
Fluctuating contribution to the thermopower  
 (analogous to conductance fluctuations)

$$\sigma_s = \frac{ck_B}{ek_F l_e \sqrt{1-\cos\gamma}} \left( \frac{k_B \theta}{\hbar v_F / l_e} \right)^{1/4} \frac{\sqrt{\sum_{n=1}^N T_n^2 (1-T_n)}}{\sum_{n=1}^N T_n}$$

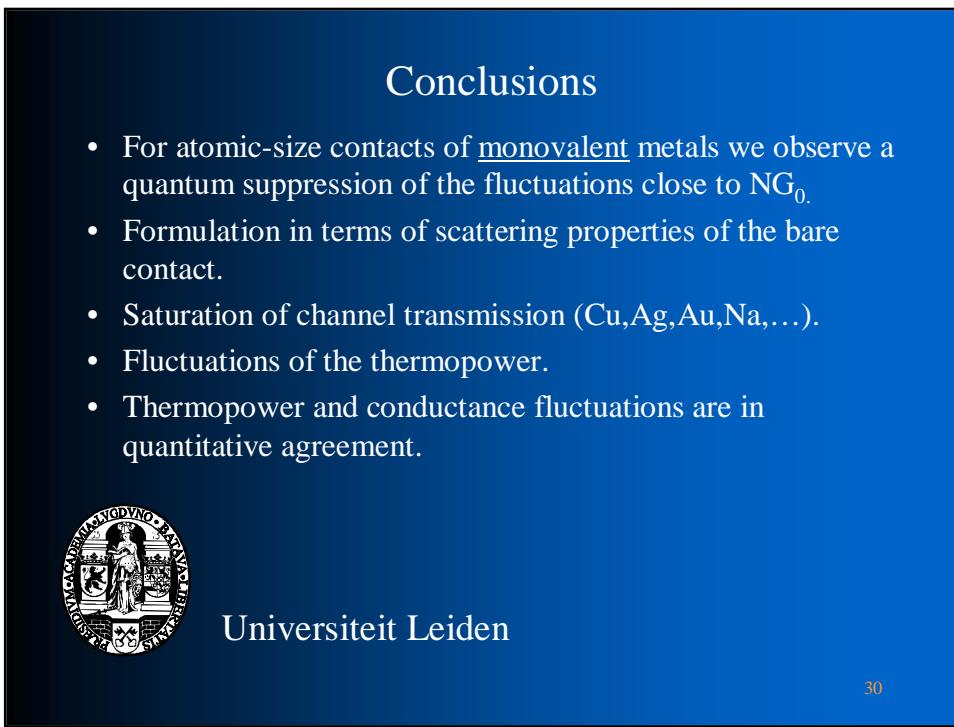
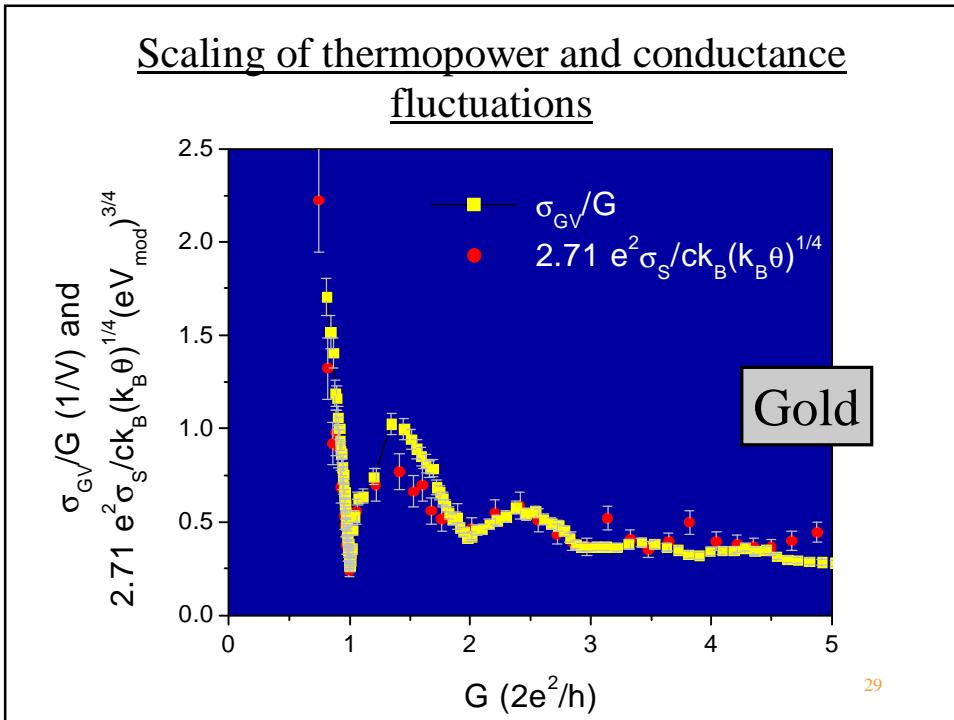
$c=5.658$  for  $\Delta\theta/\theta \rightarrow 0$

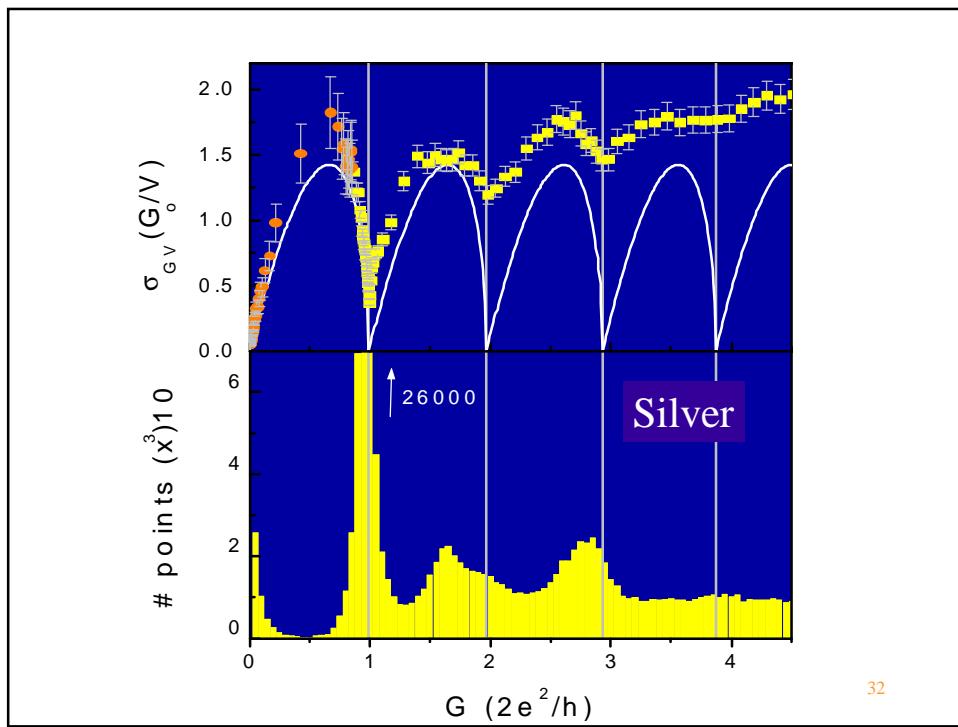
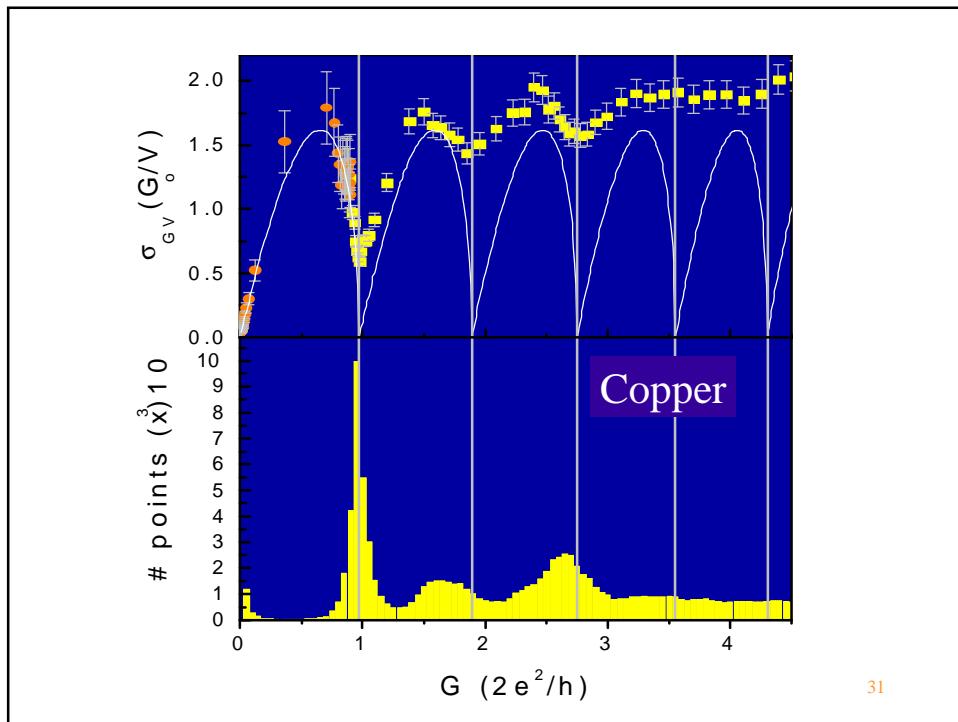
27

Standard deviation in thermopower



28





### Series resistance

To lowest order

$$\langle \text{Tr}(\hat{t}^+ \hat{t}) \rangle = \sum_{n=1}^N T_n \left[ 1 - \sum_{m=1}^N T_m \left( \langle |a_{r_{mn}}|^2 \rangle + \langle |a_{l_{mn}}|^2 \rangle \right) \right]$$

From shift :  $\langle |a_{l,r_{mn}}|^2 \rangle \approx 0.005$  for gold  $\Rightarrow \ell_e = 6 \pm 1 \text{ nm}$

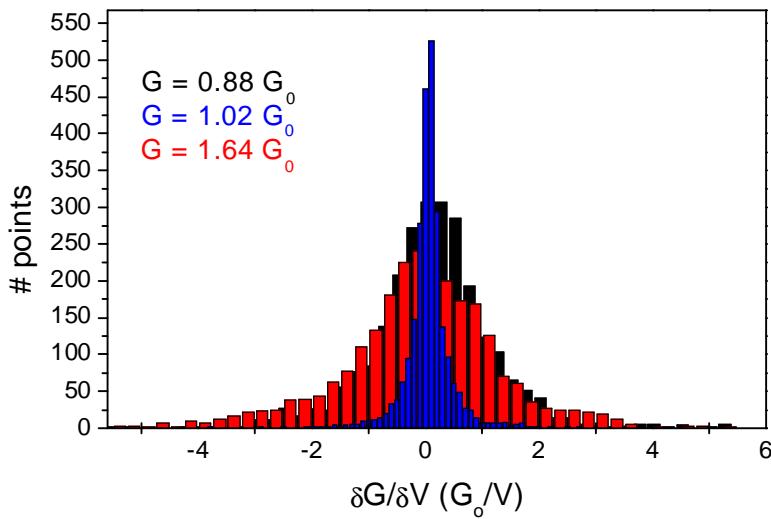
From random matrix theory, perfectly transmitted channels:

$$\frac{\langle G \rangle}{G_0} = \frac{g}{1+(g+1)r} + \frac{1}{3} \left( \frac{(g+1)r}{1+(g+1)r} \right)^3$$

(Beenakker and Melsen, PRB **50** (1994), 2450)

33

Histogram of  $\delta G / \delta V$  for 2500 points centered at given conductance value



34

Scaling of the two properties

$$\frac{\sigma_{\text{GV}}}{G} = \sigma_s \frac{2.71e^2}{ck_B(k_B\theta)^{1/4}(eV_{\text{mod}})^{3/4}}, \quad G = G_0 \sum_{n=1}^N T_n$$

35