Lattice Calculation of Neutron and Proton EDMs

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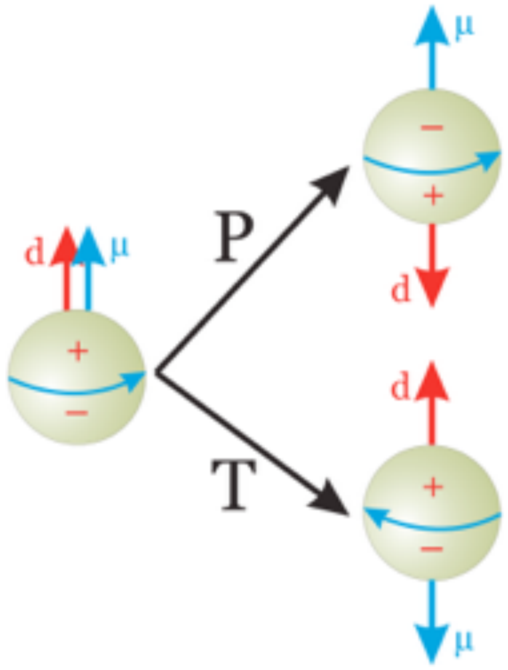
Outline

- Lattice basics
- nEDM induced by θ-term
- nEDM induced by quark chromo-EDM
- EDM in Background Electric Field

Neutron and Proton EDMs from quark-gluon CPv



$$\vec{d}_N = d_N \frac{\vec{S}}{S}$$

$$\mathcal{H} = -\vec{d}_N \cdot \vec{E}$$

Motivations to search for new CP-odd interactions

- Extensions of SM
- Required for baryogenesis
- Strong CP problem

Lattice QCD : connect quark/gluon-level effective operators to hadron/nuclei matrix elements and interactions

$$\mathcal{L}_{eff} = \sum_n \frac{c_n}{\Lambda^{d_n-4}} \mathcal{O}_n^{(d_n)}$$

$$\left\{ \begin{array}{ll} \mathcal{L}^{(4)} & = \theta \frac{g^2}{32\pi^2} G\tilde{G} \\ \mathcal{L}^{(5)} & = \sum_q \left[d_q \, \bar{q}(F\cdot\sigma)\gamma_5 q + \tilde{d}_q \bar{q}(G\cdot\sigma)\gamma_5 q \right] \\ \dots & \end{array} \right.$$

$$\boxed{\begin{array}{c} d_{n,p} \\ F_3^{n,p}(Q^2) \end{array}}$$

Hadron Structure in Lattice QCD

Lattice Field Theory ⇔ Numerical evaluation of the Path Integral

$$\langle q_x \bar{q}_y \ldots\rangle = \int \mathcal{D}\Big(Glue\Big)\int \mathcal{D}\Big(Quarks\Big)\; e^{-S_{Glue}-\bar{q}\left(\not{D}+m\right)q}\;\left[q_x \bar{q}_y \ldots\right]$$

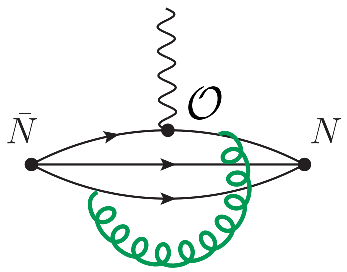
Grassmann integration

$$= \int \underbrace{\mathcal{D}\Big(Glue\Big)\; e^{-S_{Glue}}\;\mathrm{Det}\left(\not{D}+m\right)}_{\text{Hybrid Monte Carlo}}\;\left[\left(\not{D}+m\right)^{-1}_{x,y}\ldots\right]$$

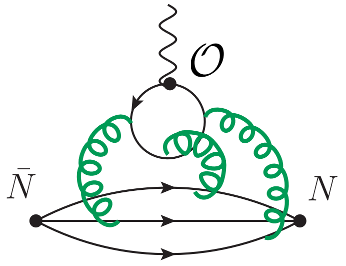
$(\not{D}+m)\cdot q = 0$
quark motion in
gluon background

Hadron Matrix Elements:

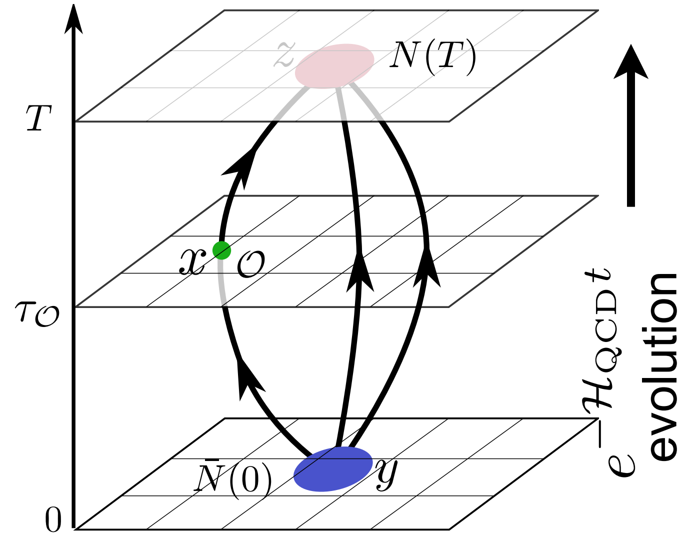
$$C^{\mathcal{O}}_{3\mathrm{pt}}(T) = \langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle =$$



"connected"



"disconnected"



$$\langle N(T)\mathcal{O}(\tau)N(0)\rangle = \sum_{n,m} Z_m e^{-E_n(T-\tau)}\langle n|\mathcal{O}|m\rangle e^{-E_m\tau} Z_n^*$$

$$\xrightarrow[T\to\infty]{} Z_{00}e^{-M_N T}\left[\langle P'|\mathcal{O}|P\rangle + \mathcal{O}\Big(\underbrace{e^{-\Delta E_{10}T}, e^{-\Delta E_{10}\tau}, e^{-\Delta E_{10}(T-\tau)}}_{\text{excited states}}\Big)\right]$$

Each quark line = $(\not{D}+m)^{-1}\cdot\psi$

Ground state
form factors

Excited states contribute to correlators and may (and do) bias results

CP-odd Interaction on a Lattice

- Linearizing in CP-odd interaction, e.g. with θ-term

$$e^{-S_{QCD}-i\theta Q} = e^{-S_{QCD}}\left[1 - i\theta Q + O(\theta^2)\right]$$

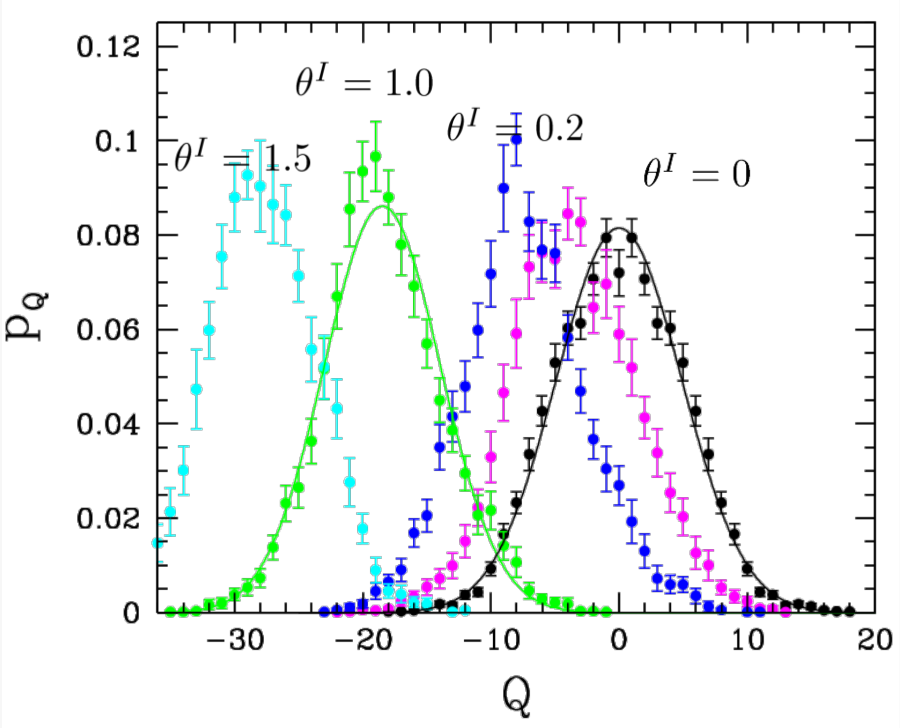
$$\langle \mathcal{O}\ldots\rangle_{\cancel{CP}} = \langle\mathcal{O}\ldots\rangle_{CP-even} - i\theta\langle Q\cdot\mathcal{O}\ldots\rangle_{CP-even} + O(\theta^2)$$

- Simulating with CP-odd term(s)

$$\langle\mathcal{O}\ldots\rangle_\theta \sim \int \mathcal{D}U\, e^{-S-\theta^I Q}\,(\mathcal{O}\ldots)$$

continued to Imag. θ
to avoid sign problem

[T.Izubuchi et al (2007) ;
R.Horsley et al (2008) ;
F.K.Guo et al (2015)]

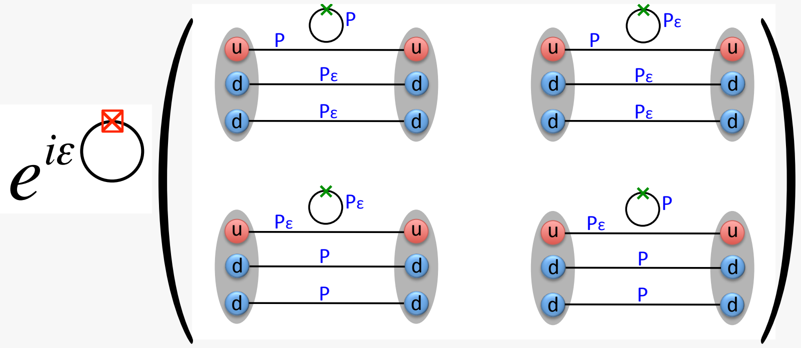


+ better sampling of Q≠0
– linearity needs check
– need new ensemble

- Reweighing with CP-odd term(s)

$$\bar q\left[\not{D} + m_q + i\epsilon(G\cdot\sigma)\gamma_5\right]q$$

quark operator with cEDM
[T.Bhattacharya et al(LANL)]



EDM from Spectrum vs. Form Factors

- Nucleon spectrum in the background electric field [S.Aoki et al '89 ; E.Shintani et al '06]

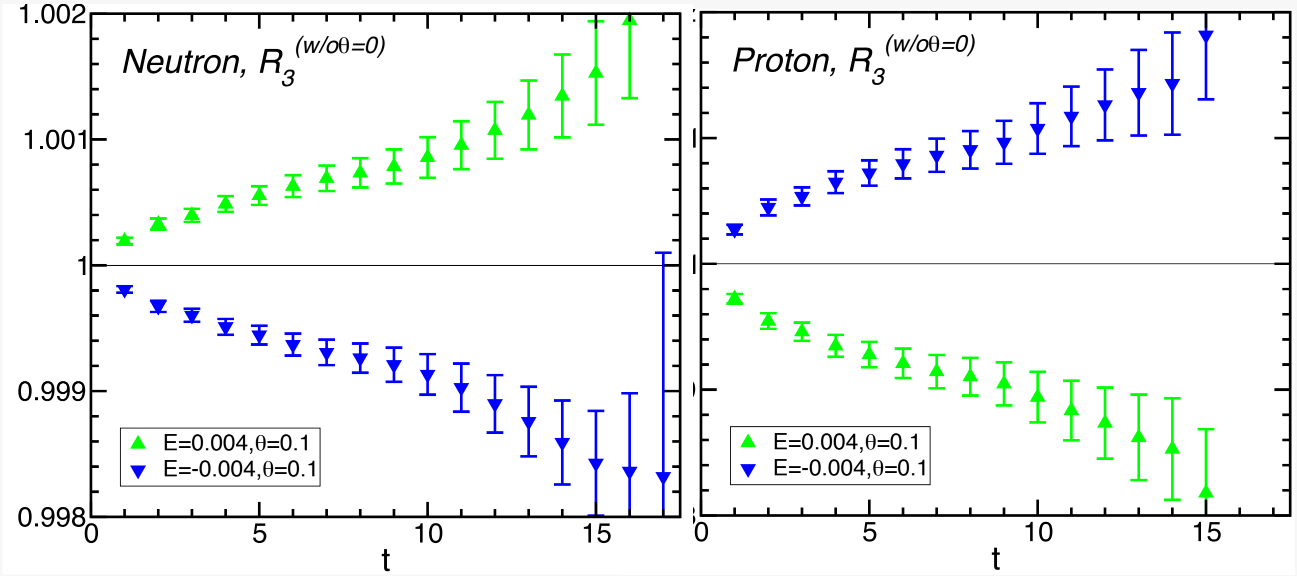
$$\langle N(t)\bar{N}(0)\rangle_{\theta,\vec{E}} \sim e^{-(E\pm\vec{d}_N\cdot\vec{E})t}$$

$$\frac{\langle N_\uparrow(t)\bar{N}_\uparrow(0)\rangle_{\theta,E_z}}{\langle N_\downarrow(t)\bar{N}_\downarrow(0)\rangle_{\theta,E_z}} \sim e^{2d_N E_z t} \approx 1 + 2d_N E_z t$$

Wick rotation: $\vec{E} \rightarrow i\vec{E}$

SU(3) g.f. link $U_z \rightarrow U_z e^{iE_z t} \sim e^{-E_z t}$

non-periodic with real(Minkowski) *E_z*



[E.Shintani et al, PRD75, 034507(2007)]

- P,T-odd Form Factor *$d_N=F_3(0)/2m$*

[E.Shintani et al '05, '15 ; F.Berruto et al '05 ; A.Shindler et al '15 ; C.Alexandrou et al'15]

$$\langle N|J^\mu|\bar{N}\rangle_{\cancel{CP}} = \bar{u}\Gamma^\mu_{CP-even}u + \bar{u}\Gamma^\mu_{CP-odd}u$$

$$F_1\gamma^\mu + F_2\frac{i\sigma^{\mu\nu}q_\nu}{2m} \qquad \hat{F}_3\frac{\gamma_5\sigma^{\mu\nu}q_\nu}{2m}$$

Need either extrapolation *$F_3(Q^2\rightarrow 0)$*,
or smart tricks [C.Alexandrou's talk]

Nucleon spinors are parity-mixed

$$\langle N(t)\bar{N}(0)\rangle_{\cancel{CP}} \sim \frac{-i\not{p} + m e^{2i\alpha_N\gamma_5}}{2m_N}e^{-E_N t}$$

CP-odd matrix elements require subtraction of $F_{1,2}$ contributions:

$$\langle Q\cdot N J^\mu \bar{N}\rangle \sim \mathcal{K}^\mu_3 F_3 + \alpha(\mathcal{K}^\mu_1 F_1 + \mathcal{K}^\mu_2 F_2)$$

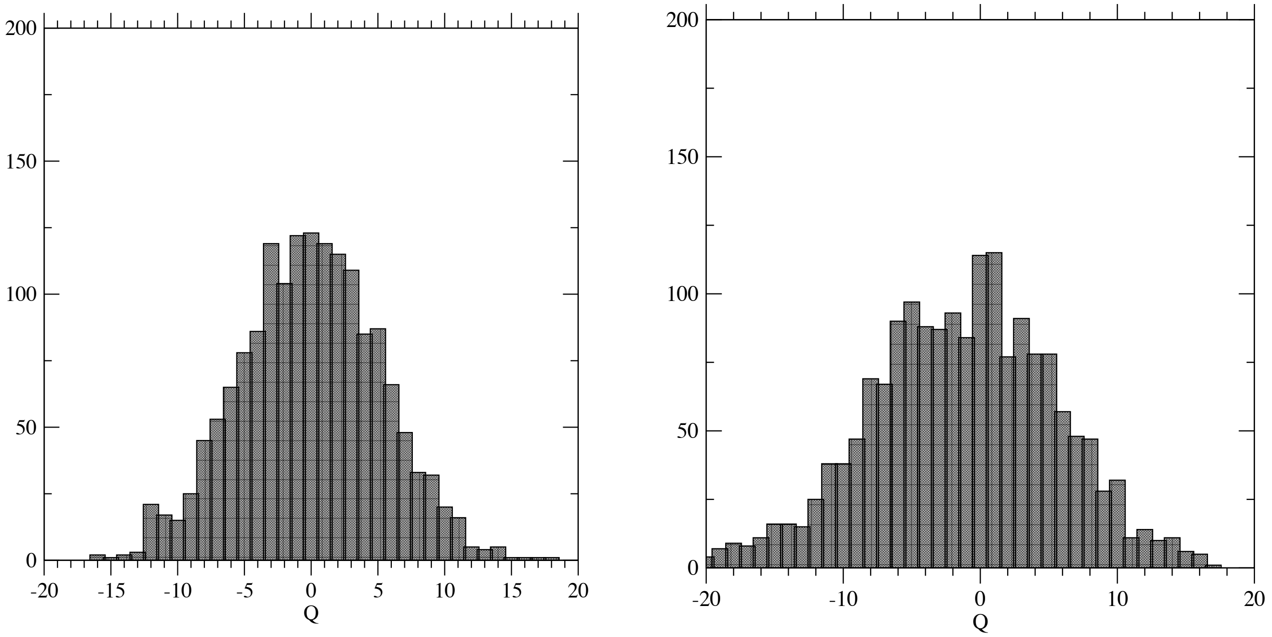
Calculation with Chirally-Symmetric Quarks

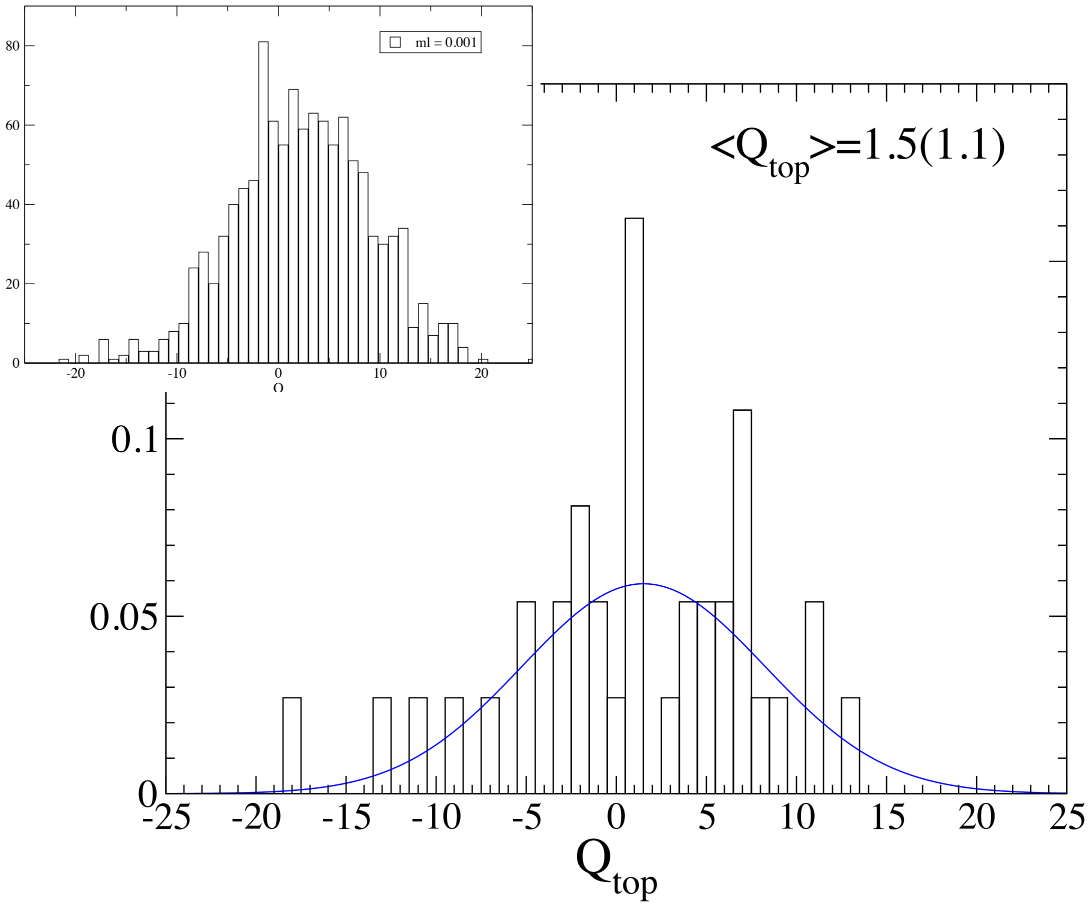
[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- 1/a = 1.73 GeV
- V=(2.7 fm)^3
- Mpi = 330, 400 MeV
- 750 configurations

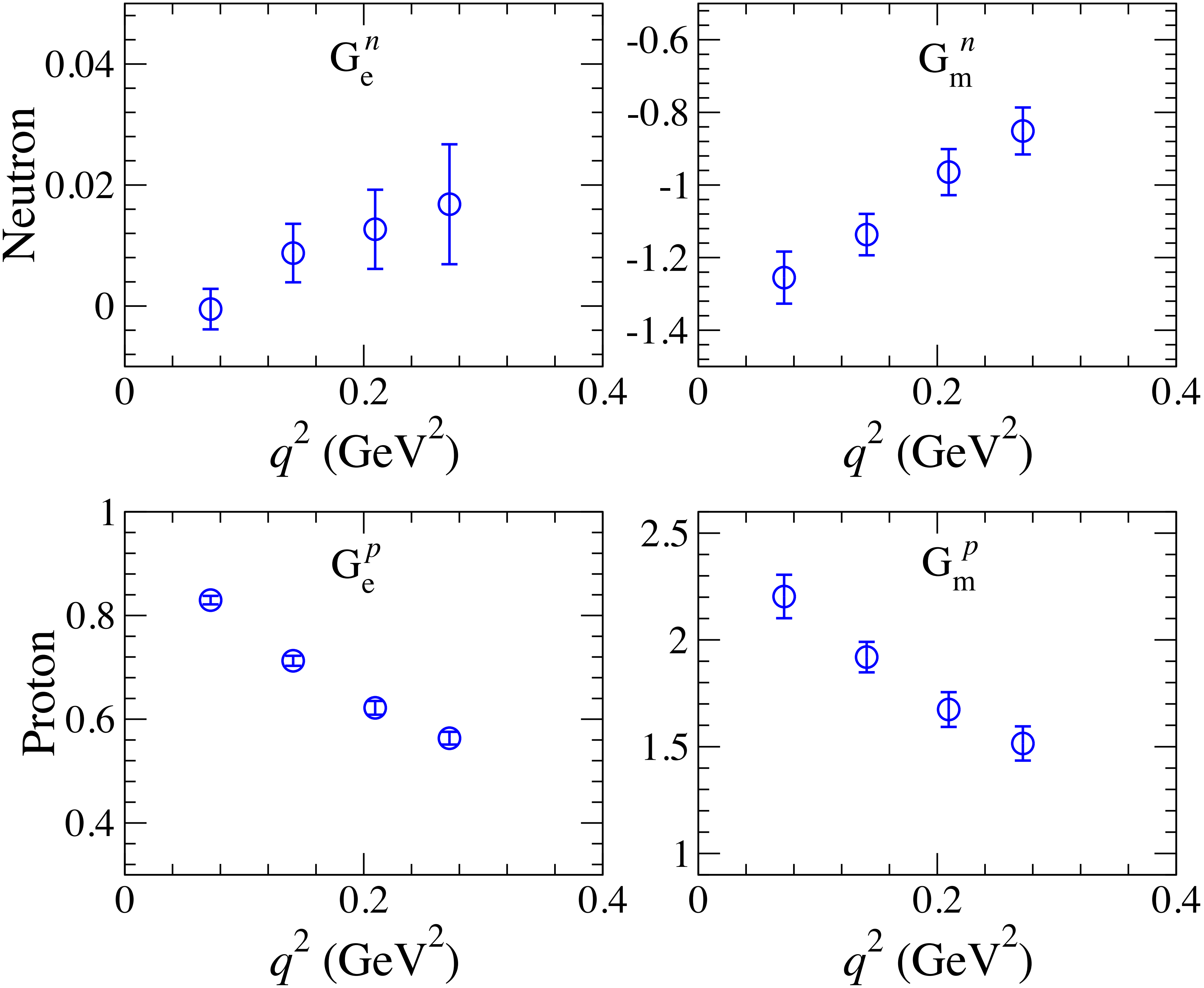
- 1/a = 1.37 GeV
- V=(4.6 fm)^3
- Mpi = 170 MeV
- 39 configurations

Topological charge





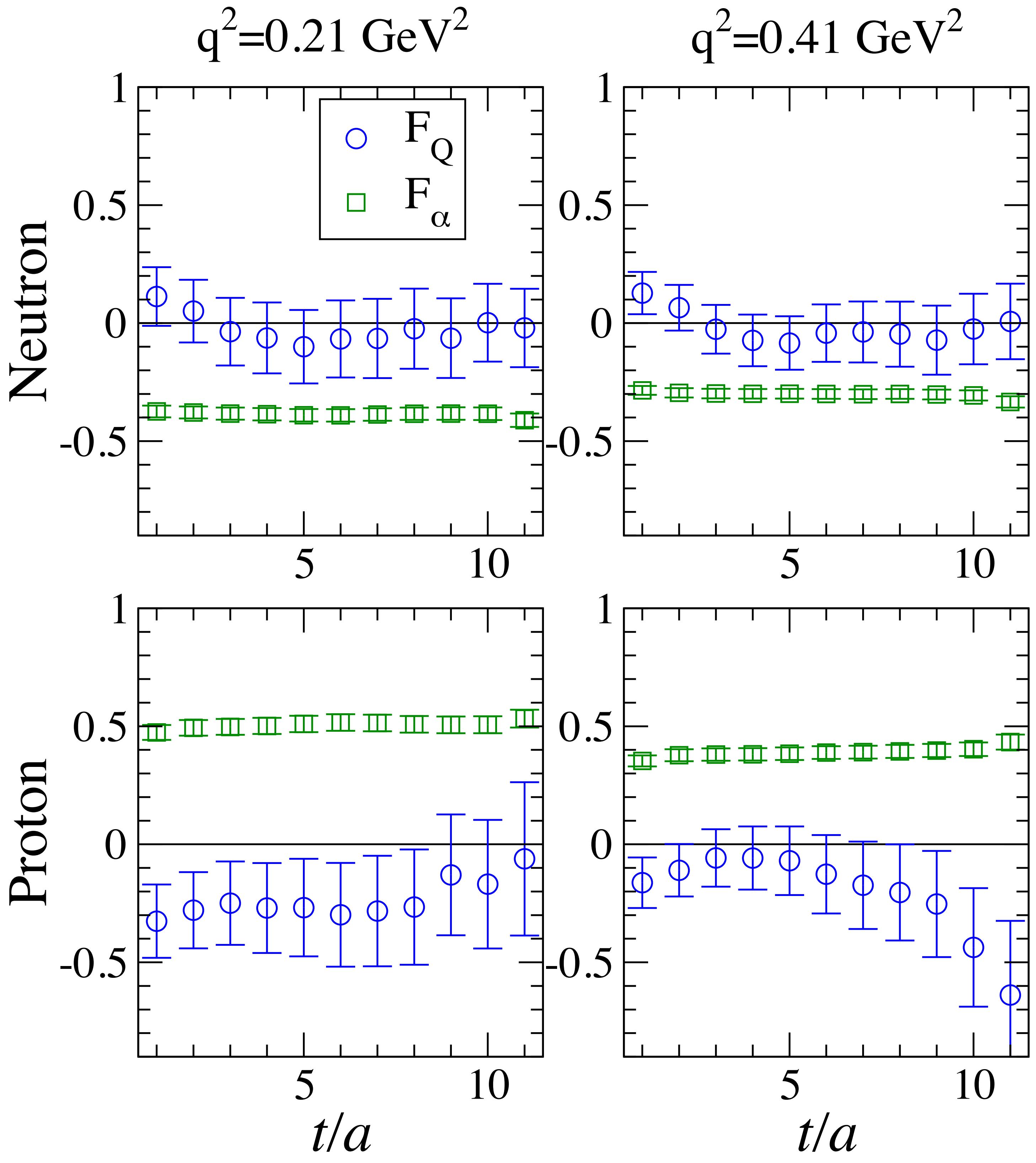
Electric and Magnetic Form Factors



[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- (4.6 fm)^{3}x(9.2 fm) box
- mπ=170 MeV

CP-odd Form Factors



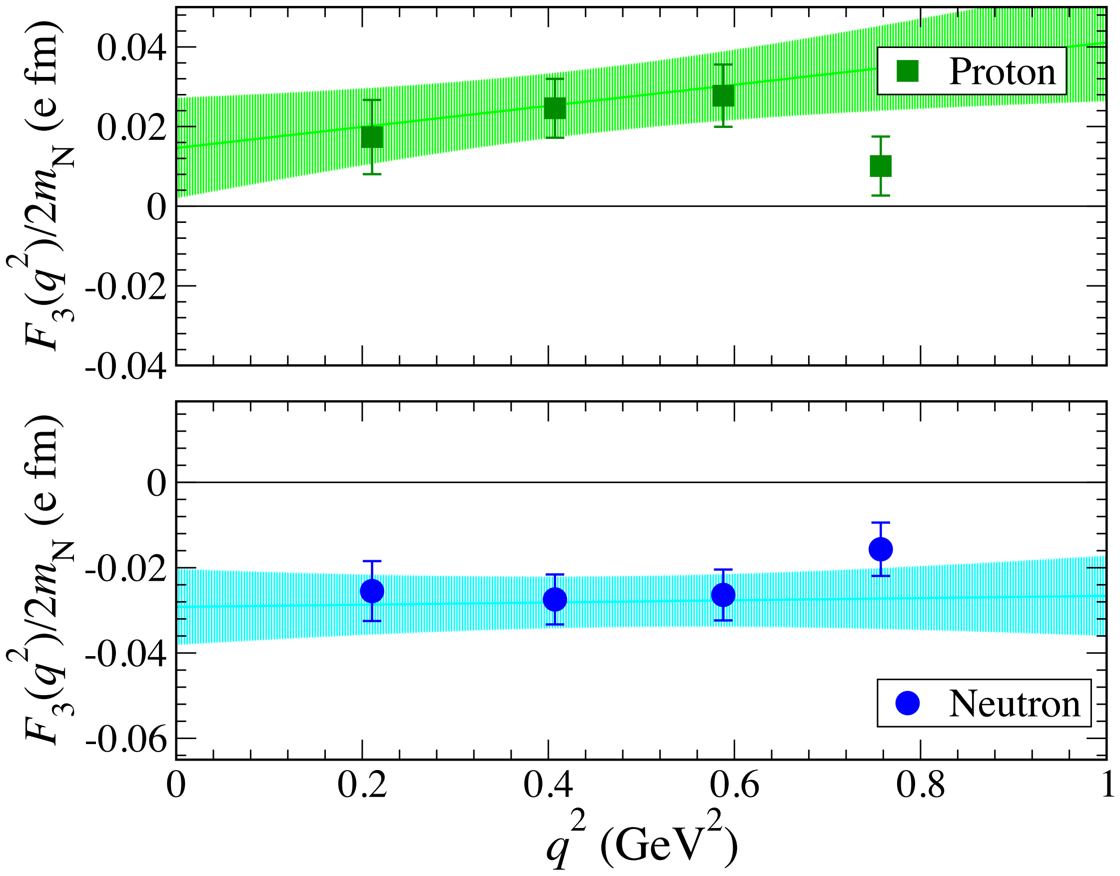
[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- (2.7 fm)^{3}x(7.3 fm) box
- mπ=330 MeV

$$F_3 = \underbrace{F_Q}_{\text{Lattice CP-odd f.f.}} + \underbrace{F_\alpha}_{\text{CP-mixing correction}}$$

Q^2-Dependence of F_3

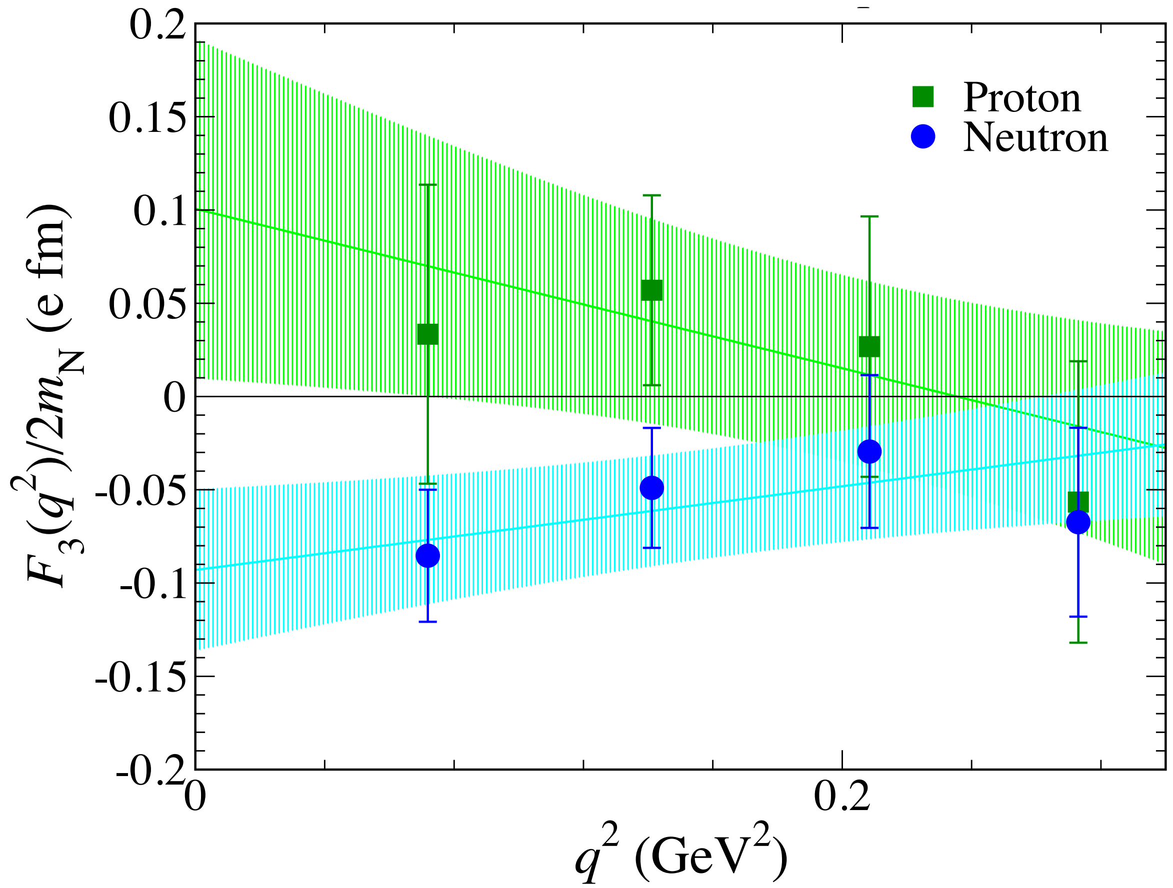
- (2.7 fm)^{3}x(7.3 fm) box
- mπ=330 MeV



$$S'_p = -11(21) \cdot 10^{-4}\, e \cdot \mathrm{fm}^3$$

$$S'_n = 24(14) \cdot 10^{-4}\, e \cdot \mathrm{fm}^3$$

- (4.6 fm)^{3}x(9.2 fm) box
- mπ=170 MeV



$$S'_p = -170(150) \cdot 10^{-4}\, e \cdot \mathrm{fm}^3$$

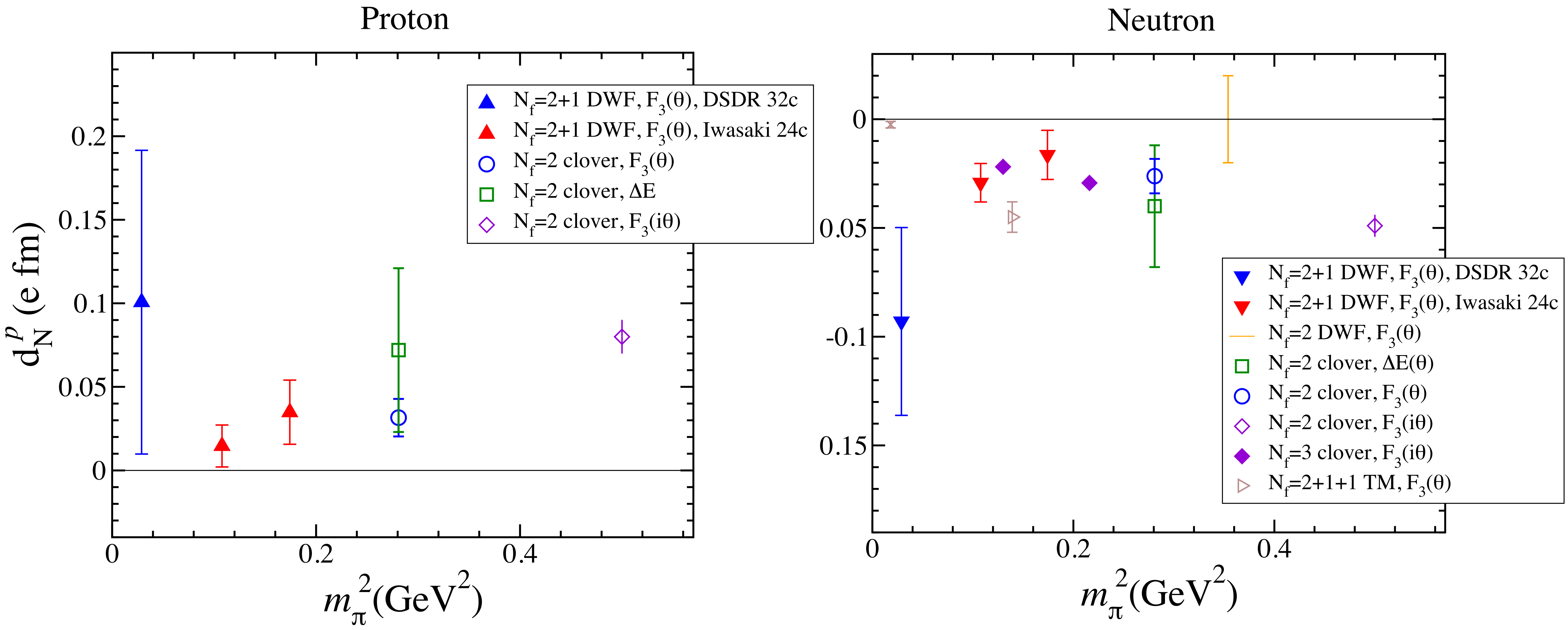
$$S'_n = 87(94) \cdot 10^{-4}\, e \cdot \mathrm{fm}^3$$

- Schiff moments from linear fit

$$\frac{1}{2m_N} F_3(Q^2) = d_N + S' Q^2 + O(Q^4)$$

[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

EDM vs. Pion Mass



- Substantial MC noise due to extensive nature of top.charge

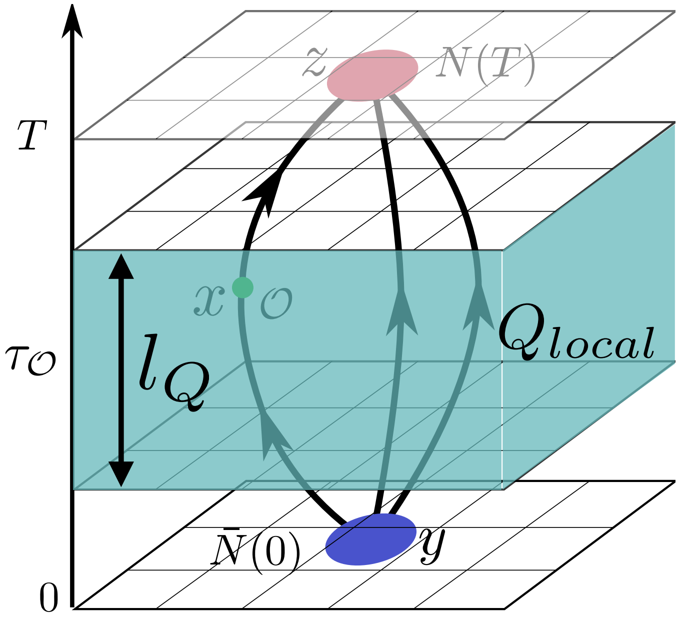
[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

Localized Sampling of Q=F̃F

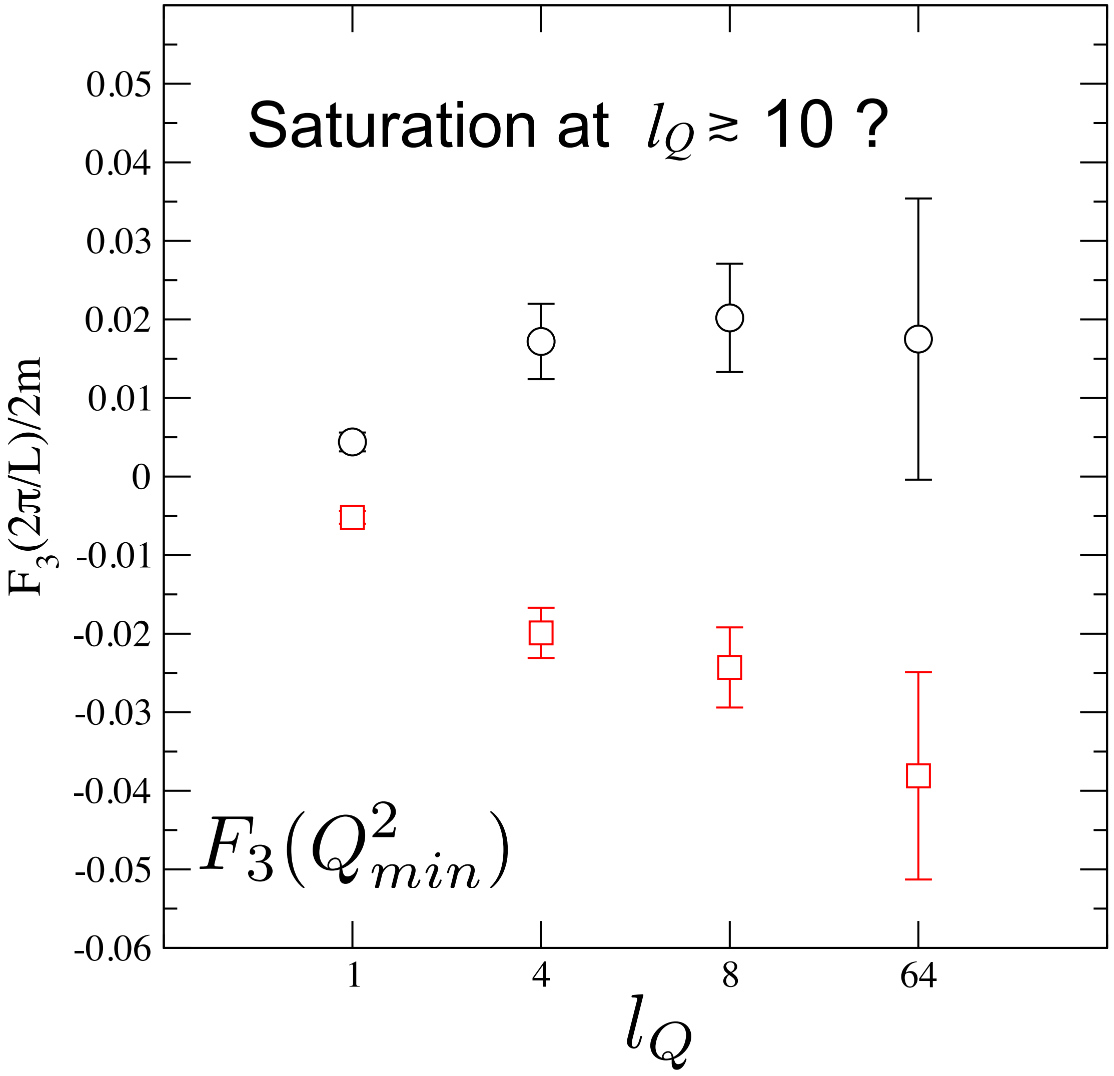
$$\langle \tilde{F}F(x)\tilde{F}F(0)\rangle \sim e^{-m_{\eta'}|x|}$$

[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- Overcome noise from top.charge fluctuations: sample F̃F locally



$$Q_{local}(\tau, l_Q) \sim \int_{\tau-l_Q/2}^{\tau+l_Q/2} dt\, dV\, \tilde{F}F$$



- (2.7 fm)^{3}x(7.3 fm) box
- mπ=330 MeV

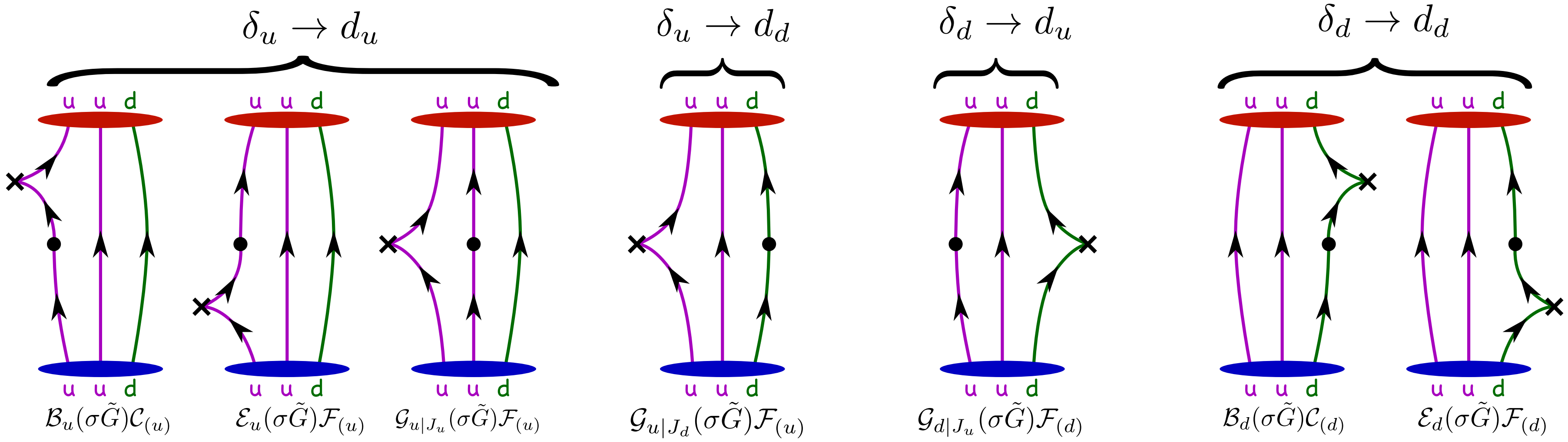
Quark Chromo-EDM

$$\begin{tikzcd}
 & \langle N(y)\,\bar{N}(0)\,\int d^4x(\tilde{G}\cdot\sigma)\rangle \\
\mathcal{L}^{(5)} = \sum_q \tilde{d}_q\,\bar{q}(G\cdot\sigma)\gamma_5 q \arrow[ur] \arrow[dr] & \\
 & \langle N(y)\,[\bar{q}\gamma^\mu q](z)\,\bar{N}(0)\,\int d^4x(\tilde{G}\cdot\sigma)\rangle
\end{tikzcd}$$

Quark-Gluon EDM: Insertions of dim-5 Operators

$$\begin{tikzcd}
& \langle N(y)\,\bar{N}(0)\,\int d^4x(\tilde{G}\cdot\sigma)\rangle \\
\mathcal{L}^{(5)} = \sum_q \tilde{d}_q\,\bar{q}(G\cdot\sigma)\gamma_5 q \arrow[ur] \arrow[dr] & \\
& \langle N(y)\,[\bar{q}\gamma^\mu q](z)\,\bar{N}(0)\,\int d^4x(\tilde{G}\cdot\sigma)\rangle
\end{tikzcd}$$

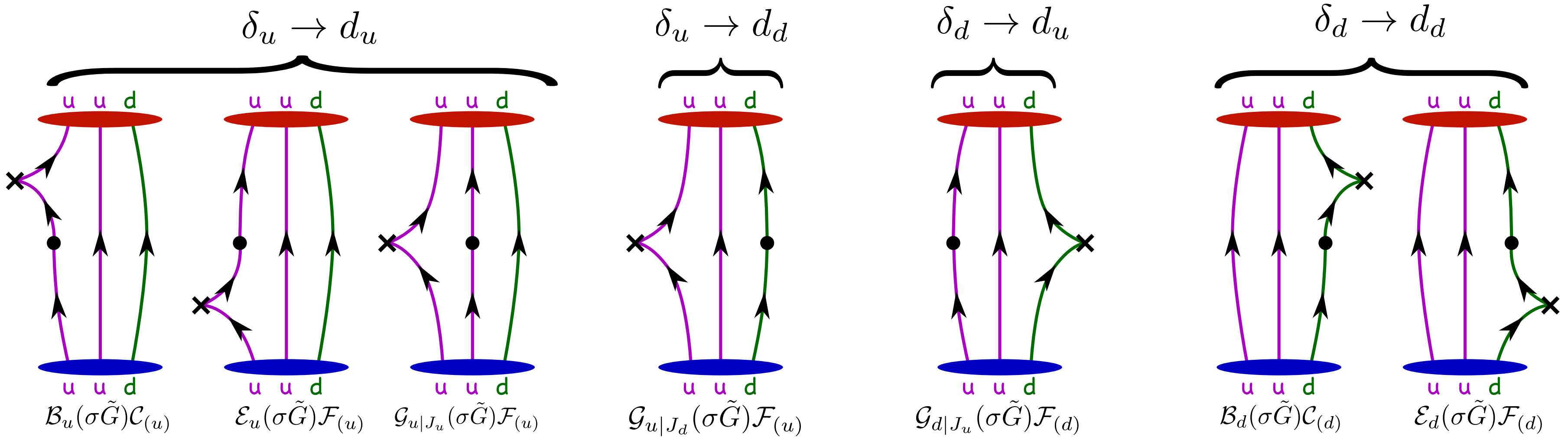
- Now: Only quark-connected insertions



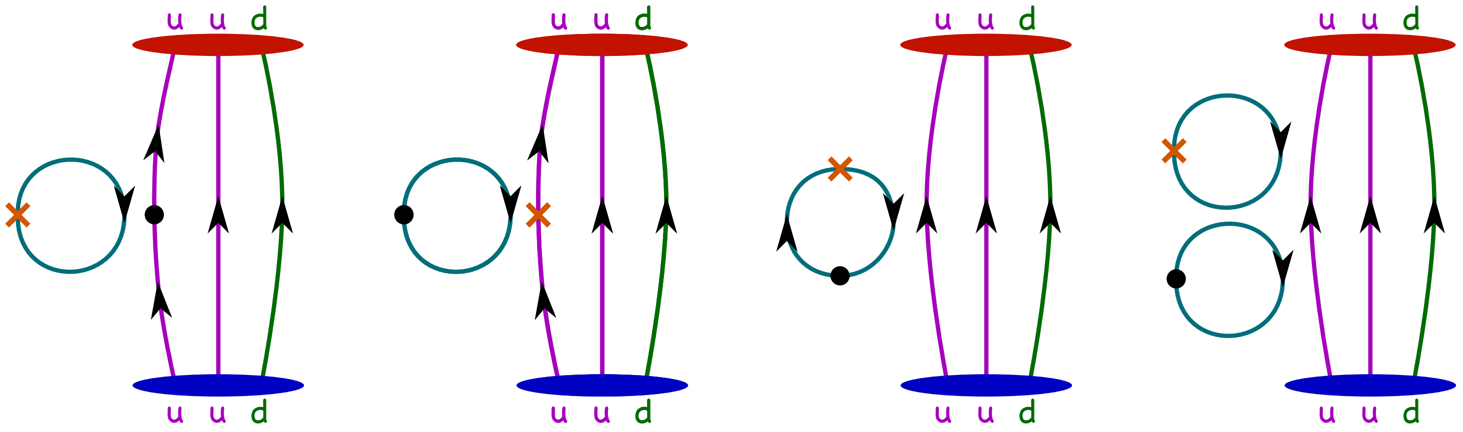
Quark-Gluon EDM: Insertions of dim-5 Operators

$$\mathcal{L}^{(5)} = \sum_q \tilde{d}_q\, \bar{q}(G\cdot\sigma)\gamma_5 q \quad \begin{array}{l} \nearrow\ \langle N(y)\,\bar{N}(0) \int d^4x(\tilde{G}\cdot\sigma)\rangle \\ \searrow\ \langle N(y)\,[\bar{q}\gamma^\mu q](z)\,\bar{N}(0) \int d^4x(\tilde{G}\cdot\sigma)\rangle \end{array}$$

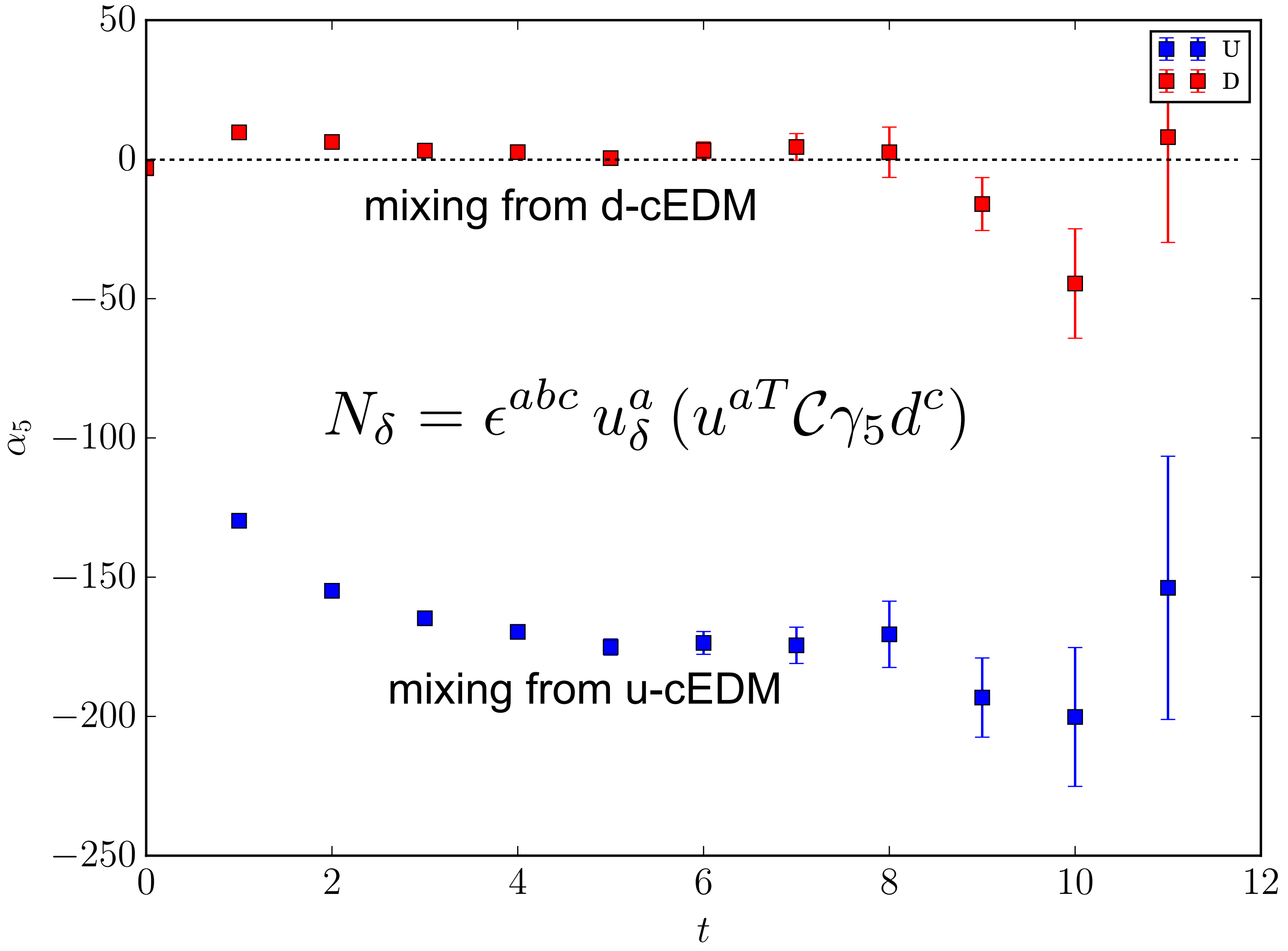
- Now: Only quark-connected insertions



- Some day: Single- and double-disconnected diagrams (contribute to isosinglet cEDM, mix with θ-term)



Parity Mixing (Proton)

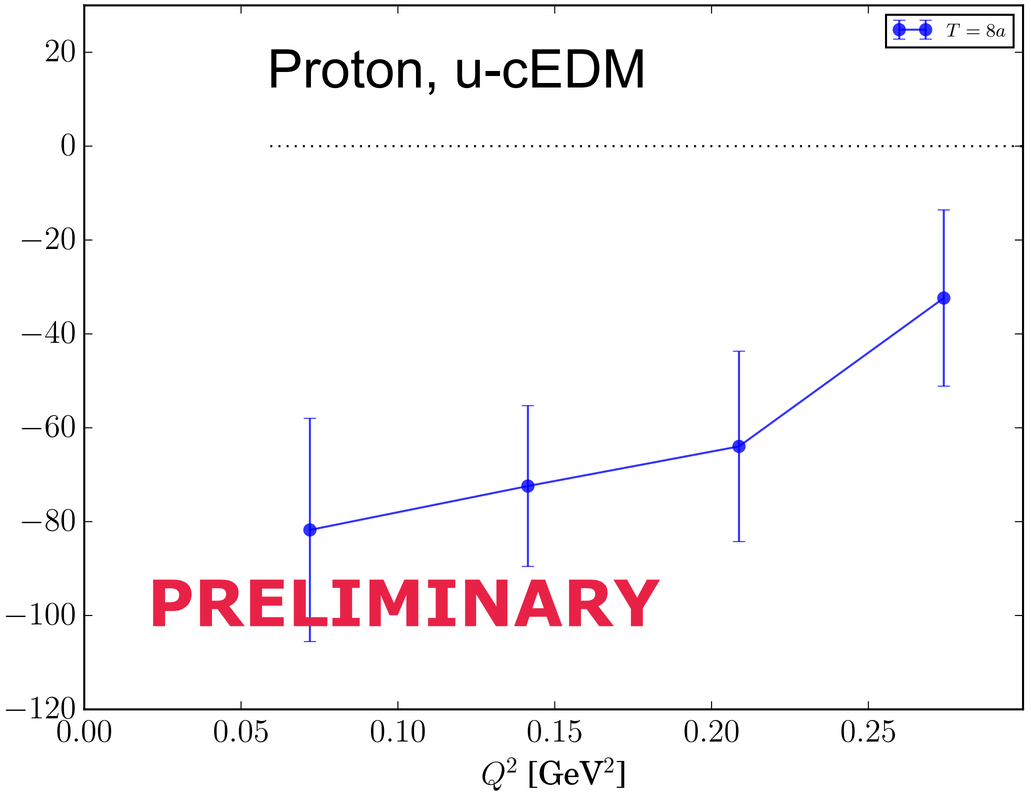


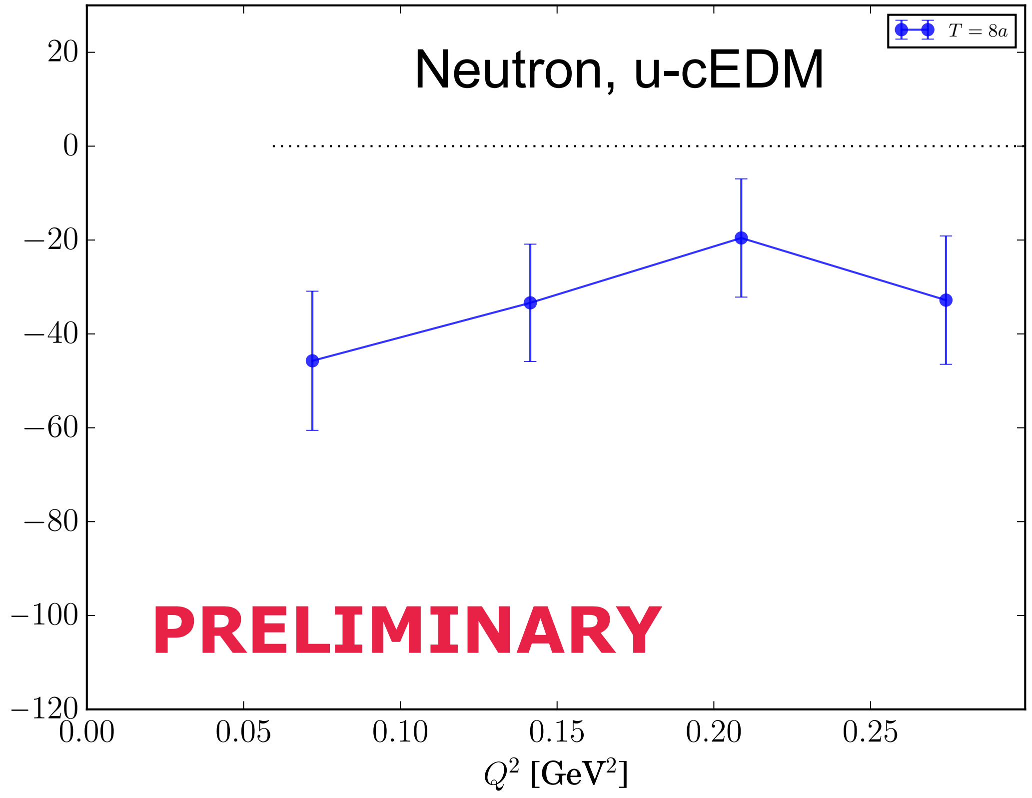
$$N_\delta = \epsilon^{abc}\, u_\delta^a \left(u^{aT}\mathcal{C}\gamma_5 d^c\right)$$

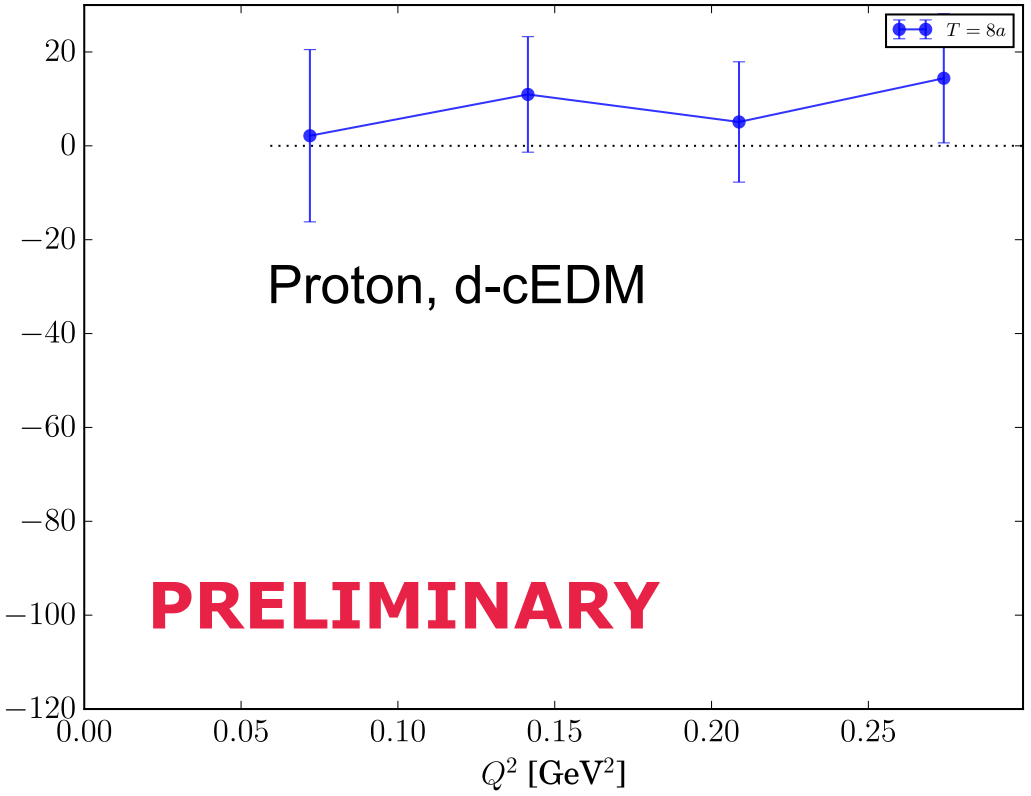
$$\langle N(t)\bar{N}(0)\rangle_{\cancel{CP}} = e^{i\alpha_5\gamma_5}\frac{-i\not{p} + m_N}{2m_N} e^{2i\alpha_5\gamma_5} e^{-E_N t}$$

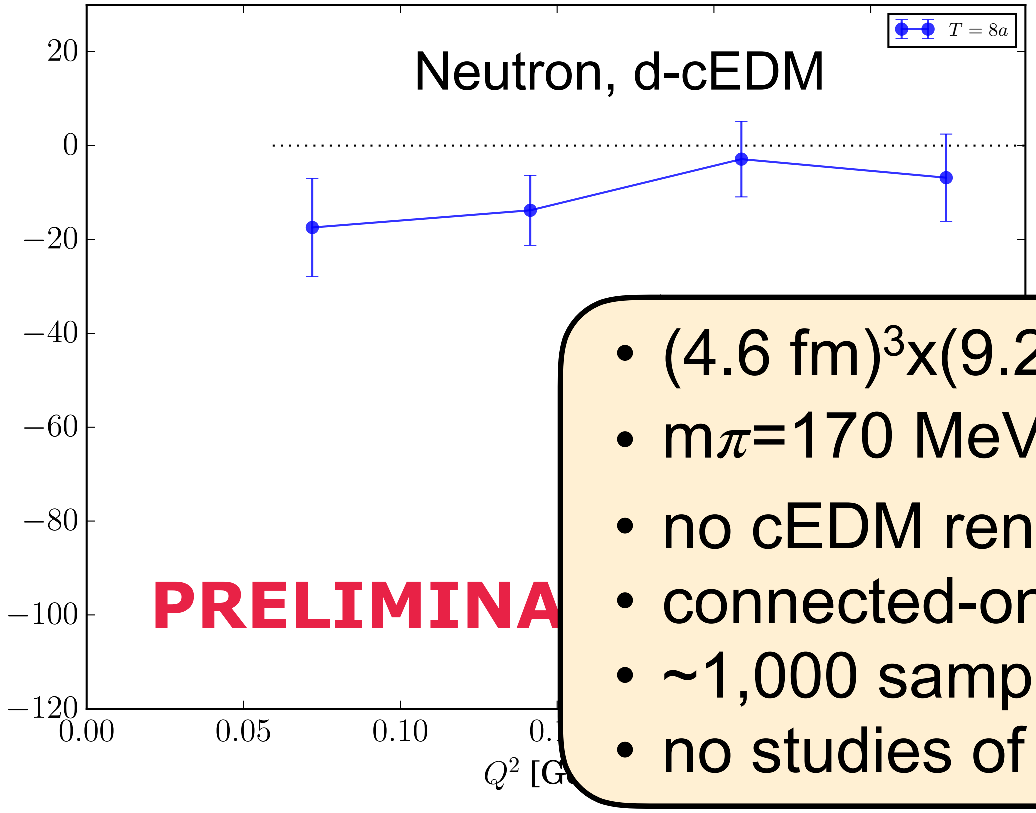
$$\hat{\alpha}_5 = \frac{\alpha_5}{\tilde{d}} = -\frac{\mathrm{ReTr}\left[T^+\gamma_5\cdot C_{2pt}^{\overline{CP}}(t)\right]}{\mathrm{ReTr}\left[T^+\cdot C_{2pt}^{CP}(t)\right]}, \quad t\to\infty$$

Proton & Neutron EDFF Form Factors


Proton, u-cEDM

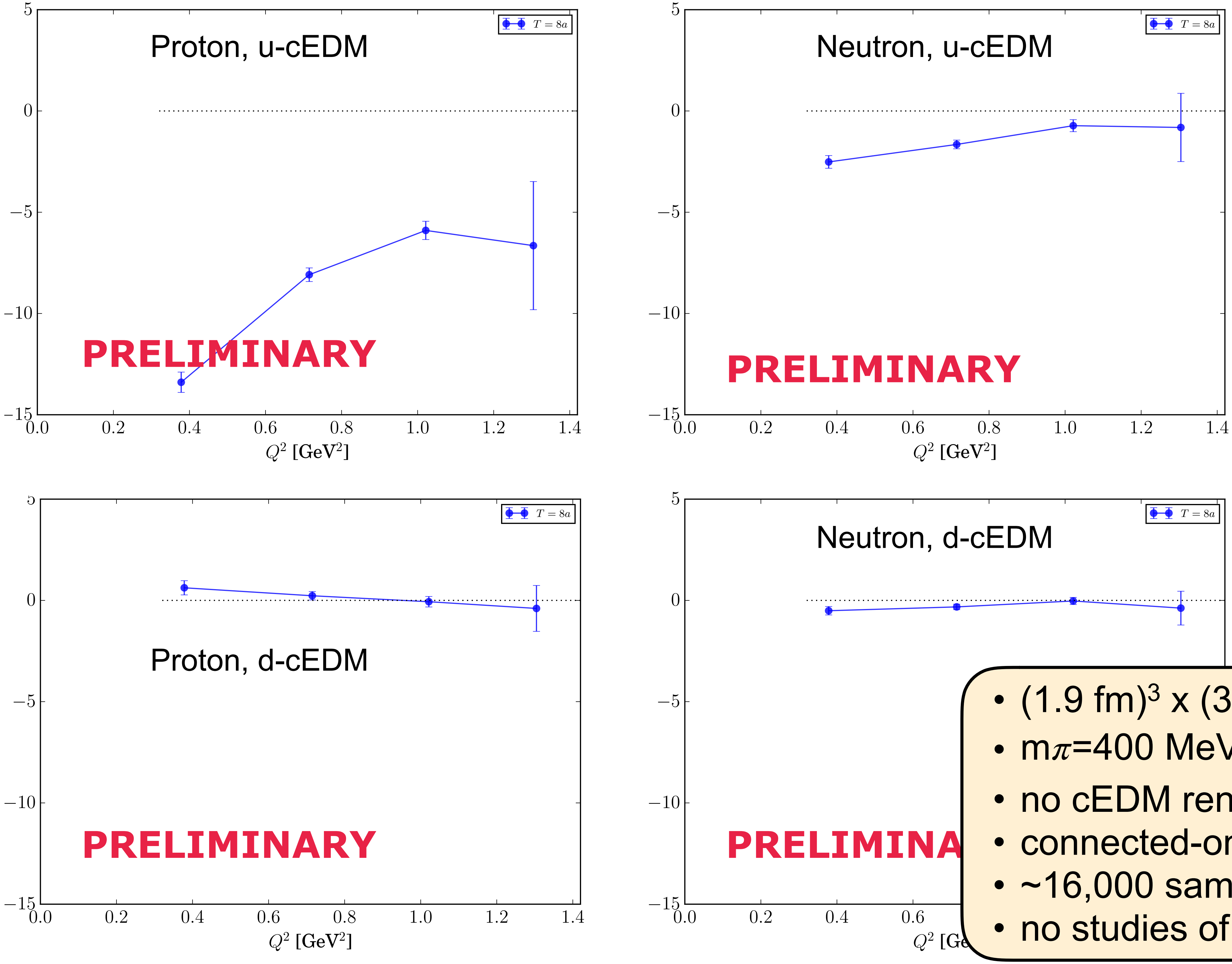

Neutron, u-cEDM


Proton, d-cEDM


Neutron, d-cEDM

- (4.6 fm)^{3}x(9.2 fm) box
- mπ=170 MeV
- no cEDM renormalization
- connected-only
- ~1,000 samples
- no studies of sys.errors yet

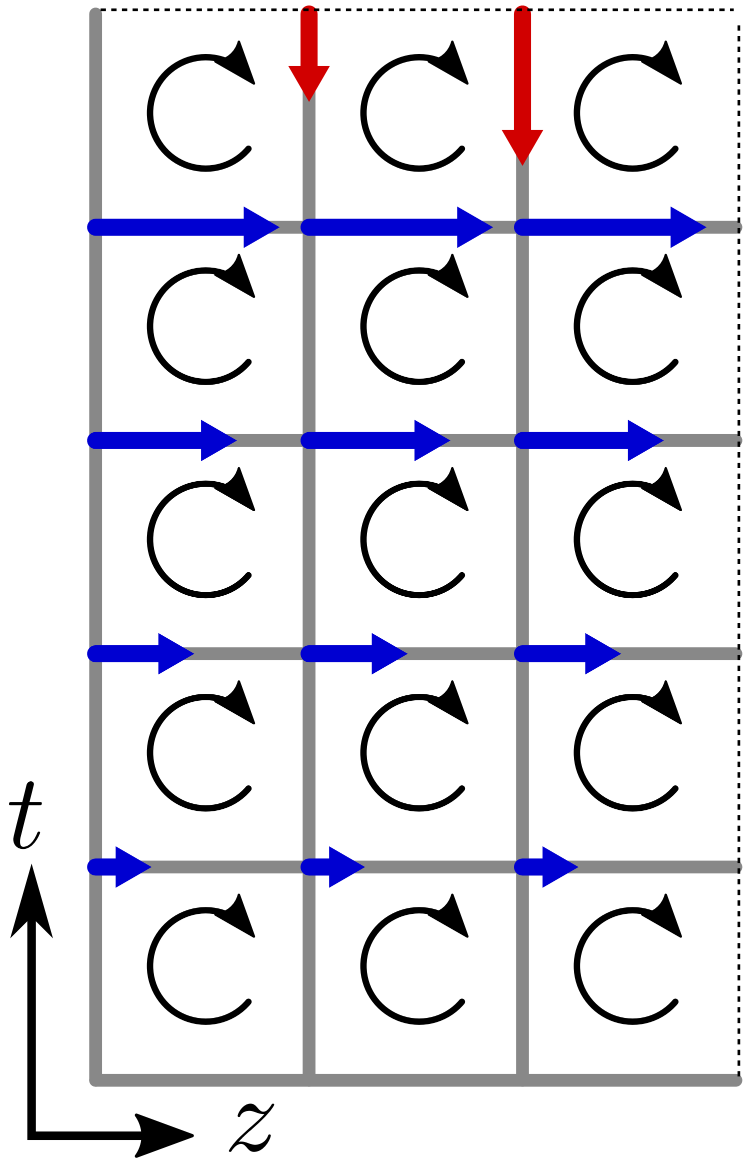
Proton & Neutron EDFF Form Factors



- $(1.9\text{ fm})^3$ x (3.8 fm)
- mπ=400 MeV
- no cEDM renormalization
- connected-only
- ~16,000 samples
- no studies of sys.errors yet

Background Electric Field

Accessing magnetic and electric moments at $Q^2=0$
Imag.Minkowski/Real Euc. electric field on a lattice
[W.Detmold et al (2009)] : calculation of hadron polarizabilities



Full flux through the "side" of the periodic box $= q\Phi = 2\pi$

Constant Electric field has to be quantized, $\mathcal{E}_{\min} = \frac{1}{|q_d|}\frac{2\pi}{L_x L_t}$

$$U_\mu \to e^{iqA_\mu} U_\mu$$

$$A_z(z,t) = n\,\mathcal{E}_{\min}\cdot t$$

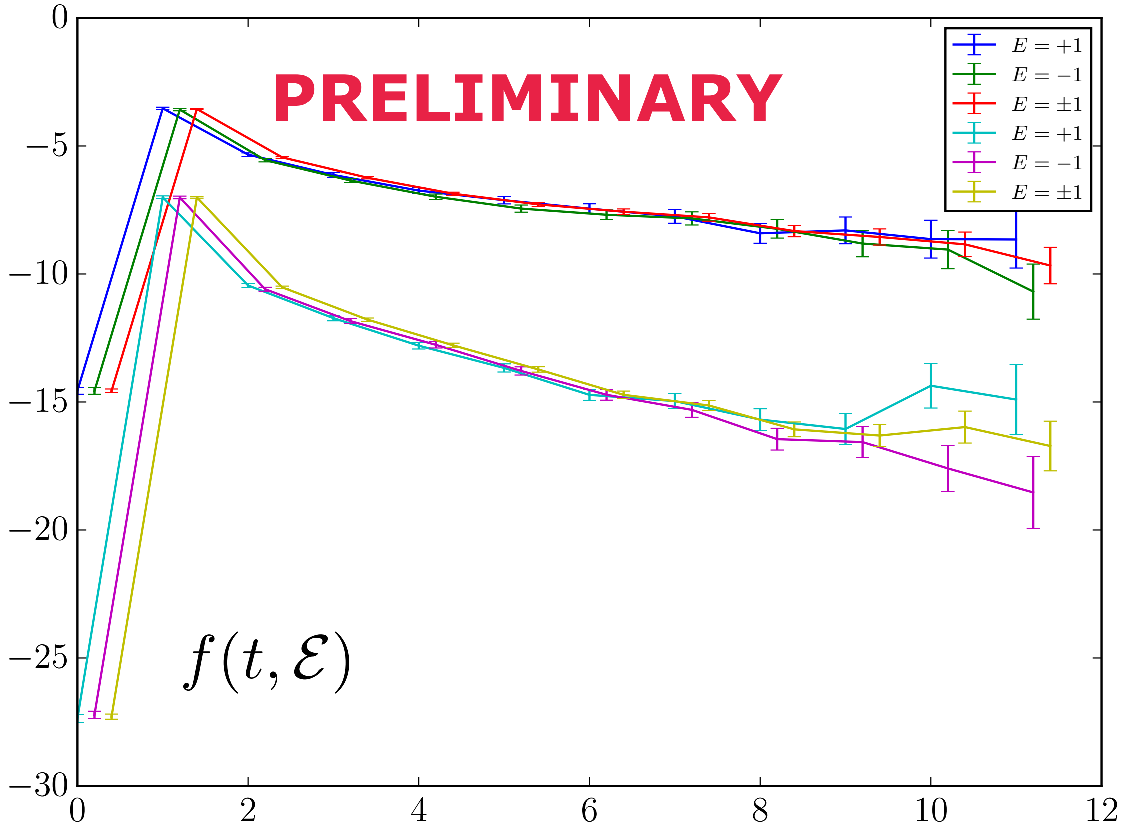
$$A_t(z,t=L_t-1) = -n\,\mathcal{E}_{\min}\cdot L_t z$$

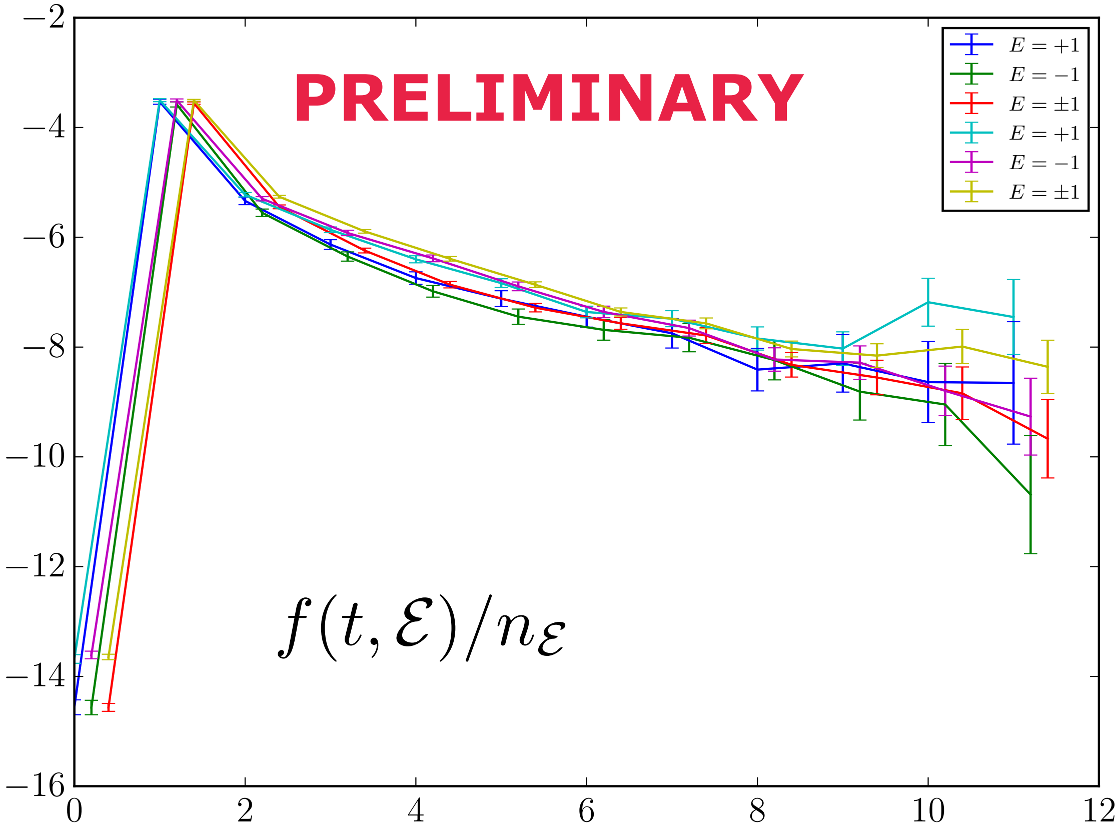
CP-odd Neutron Energy Shift

$$\langle N(t)\bar{N}(0)\,\mathcal{O}_{\overline{CP}}\rangle_{\mathcal{E}} \sim e^{-E_N t}\left[A - d_N \mathcal{E}_z \Sigma_z t\right]$$

$$f(t,\mathcal{E}) = \frac{\mathrm{ReTr}\left[\Sigma_z \cdot \langle N(t)\bar{N}(0)\,\mathcal{O}_{\overline{CP}}\rangle_{\mathcal{E}}\right]}{\mathrm{ReTr}\left[\langle N(t)\bar{N}(0)\rangle_{\mathcal{E}}\right]} \sim A + d_N \mathcal{E} t$$

- $(1.9\text{ fm})^3$ x (3.8 fm)
- m_π=400 MeV
- $\mathcal{E}_{min}$=0.0966 GeV^2=490 MeV/fm





- Linearity in $\tilde{d}_q/d_N$, t , and $\mathcal{E}$
- No renormalization yet

Electric field
on 16^3x32 lattice

$$\mathcal{E} = \frac{6\pi}{L_x L_t} \approx 0.1\ \mathrm{GeV}^2$$

$$\approx 500\ \mathrm{M(e)V/fm}$$

Summary

- Calculations of θ-induced NEDM are very noisy close to the physical pion mass

- Additional techniques may be necessary

    local sampling of topology?

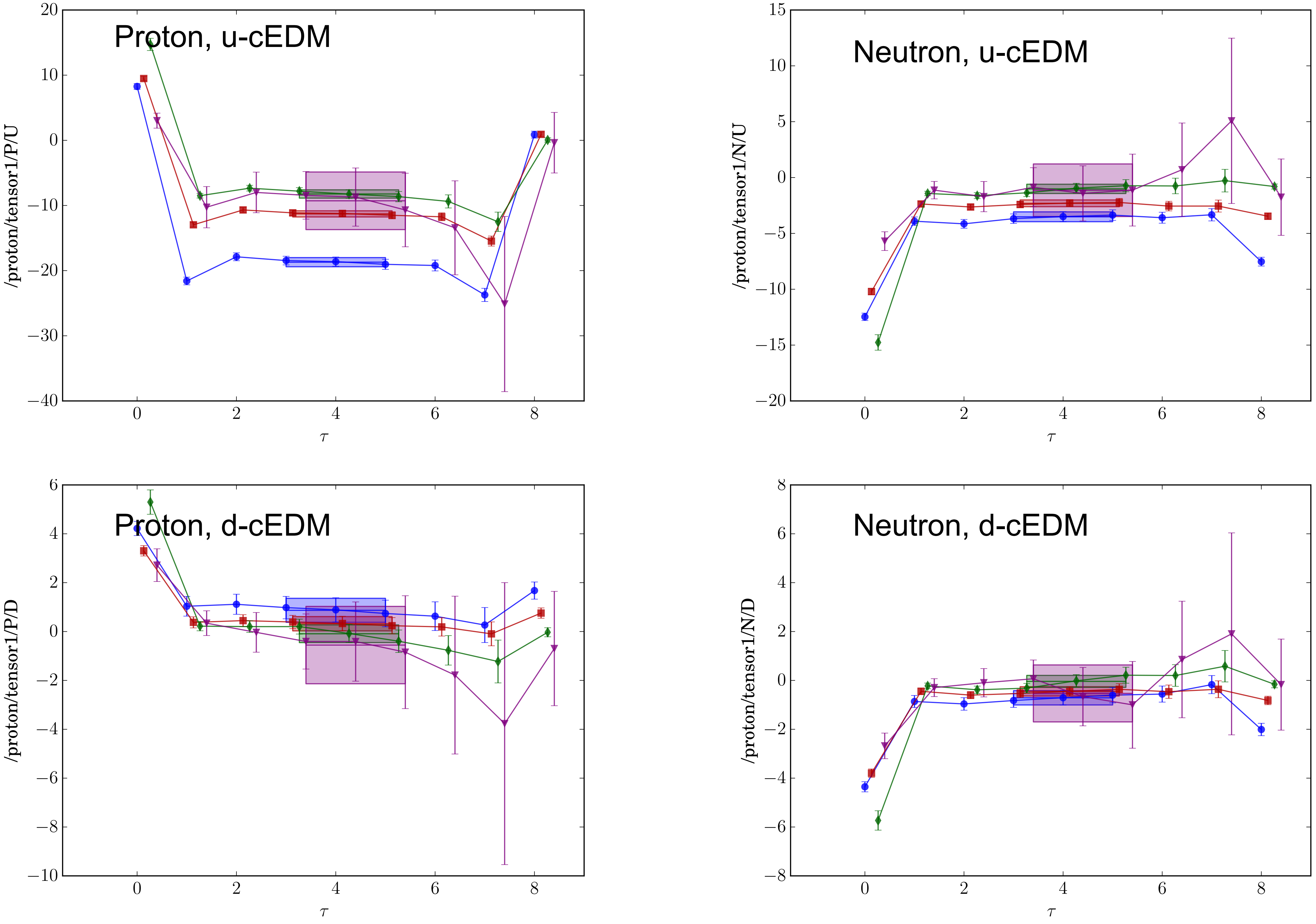
- Initial results for *quark-connected* cEDM-induced EDFF look promising

- Preliminary study with background field methods shows expected qualitative behavior

- Challenges in computing NEDM from cEDM

    subtraction of lower-dimension operators
    disconnected diagrams
    mixing with θ-term in the isoscalar channel

F3 Plateaus (16c32, 400 MeV)



F3 Plateaus (32c64, 170 MeV)