Evgeny Epelbaum, RUB, Bochum, Germany

Frontiers in nuclear physics, KITP, Santa Barbara
August 22 - November 4, 2016

Quark mass dependence of the nuclear force

(in connection with anthropic considerations)

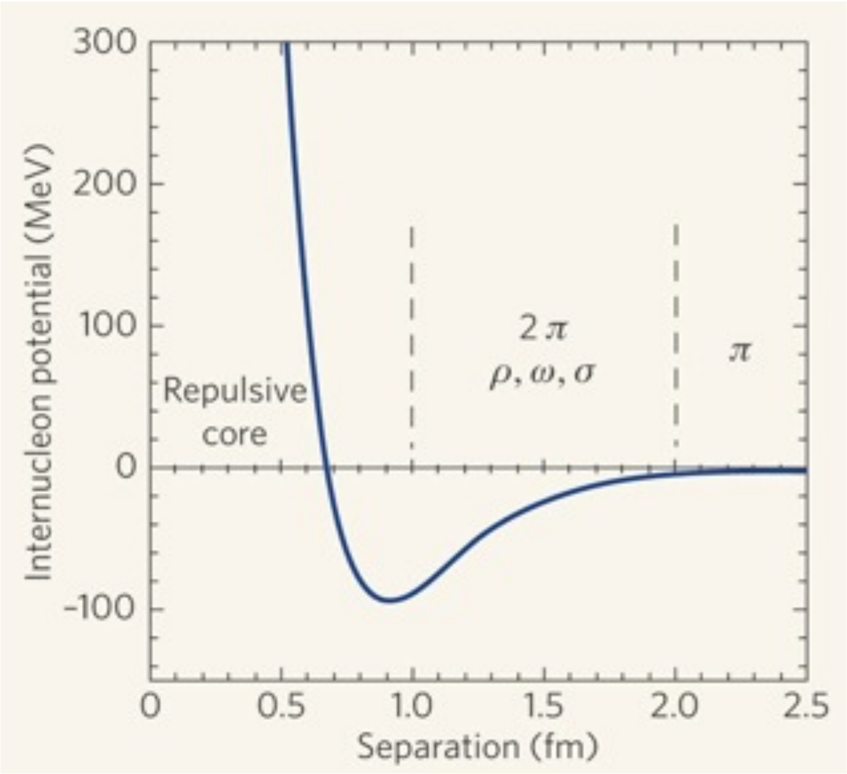
Part I

- Motivation
- Quark mass dependence of the nuclear force (near the physical point)

Part II: application to the Hoyle state (by Dean Lee)

Motivation

How does the nucleon force depend on the value of the quark masses?



— interesting on its own

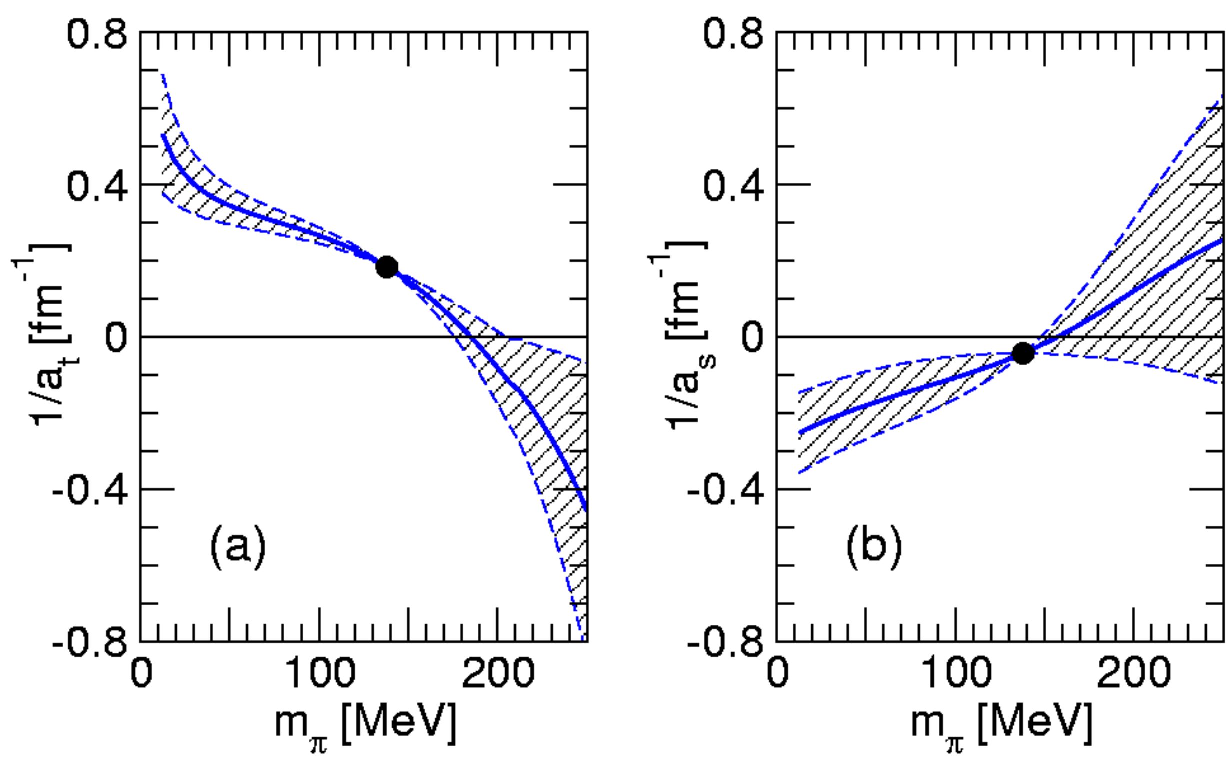
— nuclear physics could be dramatically different for $M_\pi \neq M_\pi^{phys}$ (e.g. P-wave bound states)

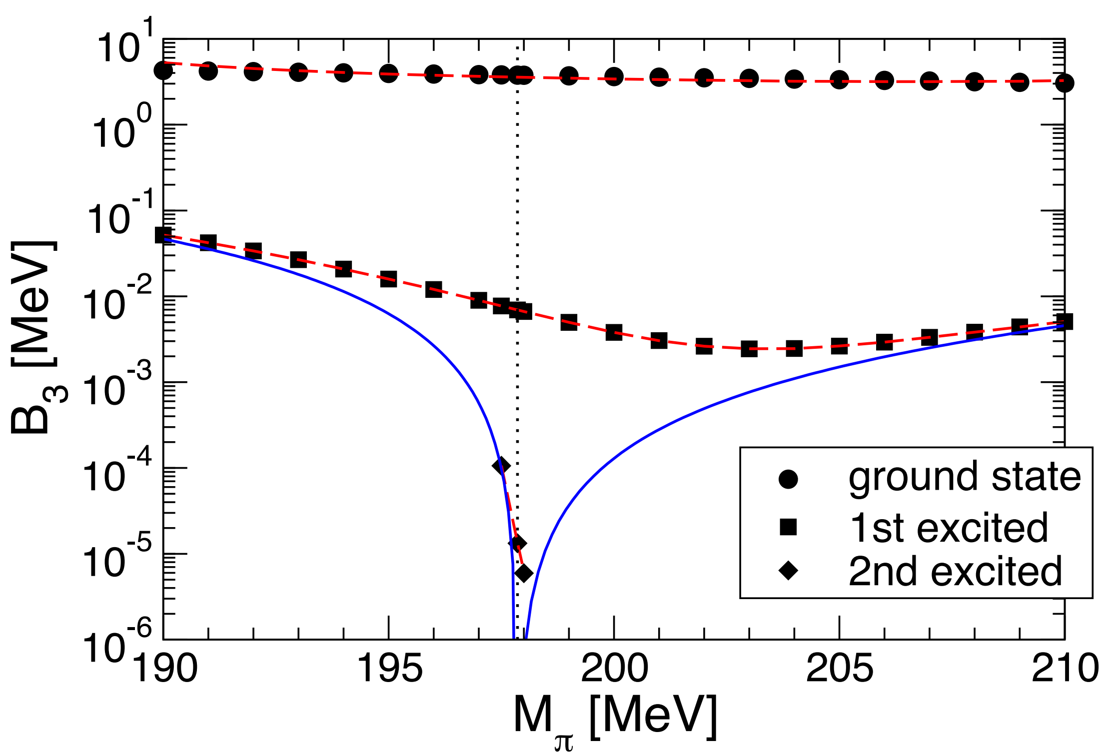
Bulgac, Miller, Strickman '97

NPLQCD; π-less EFT (Barnea et al)

— is QCD close to the infrared RG limit cycle?

Braaten, Hammer '03; EE, Hammer, Meißner, Nogga '06





Motivation

— constraining possible time variation of the SM parameters: m_q-dependence of the nuclear force + nuclear physics + theory of BBN + abundancies of light elements

Bedaque, Luu, Platter '11, Berengut, EE, Flambaum, Hanhart, Meißner, Nebreda, Pelaez '13

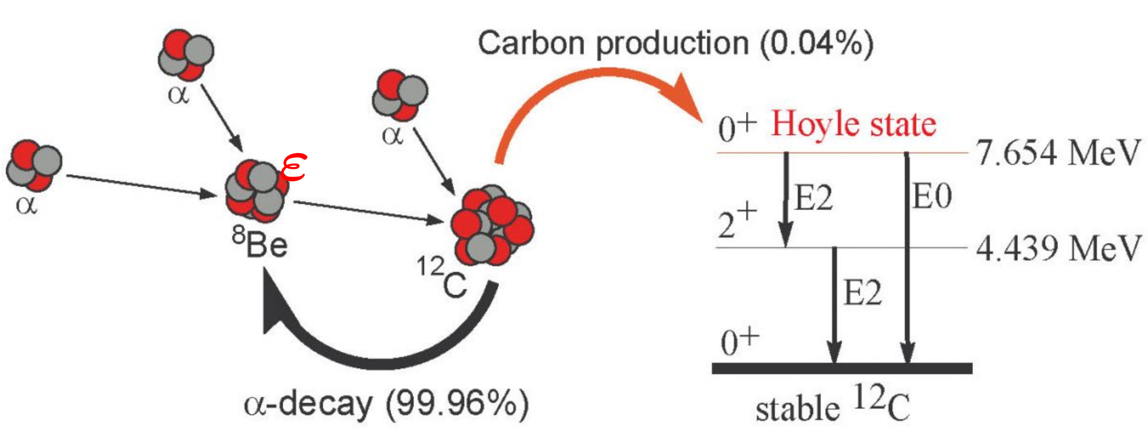
— „anthropic considerations" in connection with the Hoyle state EE, Krebs, Lähde, Lee, Meißner '13

early 1953: Fred Hoyle predicts a resonant state in ^{12}C about 7.7 MeV above the ground state to explain carbon production in stars
Hoyle, Astrophys. J. Suppl. 1 (1954) 121

summer 1953: resonant state at 7.68 ± 0.03 MeV measured at the Kellogg Radiation Lab
Dunbar, Pixley, Wenzel, Whaling, Phys. Rev. 92 (1953) 649

For a critical discussion of a possible anthropic significance of Hoyle's discovery see:
Kragh, „An anthropic myth: Fred Hoyle's carbon-12 resonance level", Arch. Hist. Exact Sci. 64 (210) 721

Reaction rate for the triple alpha process: $$r_{3\alpha} \simeq 3^{\frac{3}{2}} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_{\rm B} T} \right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_{\rm B} T} \right)$$



where $\varepsilon \equiv E_{12}^\star - 3E_4 = 379.47(18)\ \mathrm{keV}$

Changing ε by ~100 keV destroys production of either ^{12}C or ^{16}O Livio et al.'89; Oberhummer, et al.'00

How robust is ε with respect to variations of the light quark mass?

Nuclear chiral EFT

The long-standing challenge of ab-initio calculation of the Hoyle state (as a 12-nucleon system) has been solved recently EE, Krebs, Lee, Meißner, PRL 106 (2011) 192501

This opens the way for studying the (linear) response of ε to small variations of the light quark mass around the physical value EE, Krebs, Lähde, Lee, Meißner, PRL 110 (13) 112502; EPJA 49 (13) 82

The framework: chiral EFT $$\sum \left[\frac{-\vec{\nabla}_i^2}{2\, m_N} + V_{ij}^{2N} + V_{ijk}^{3N} \right] |\Psi\rangle = E\, |\Psi\rangle$$

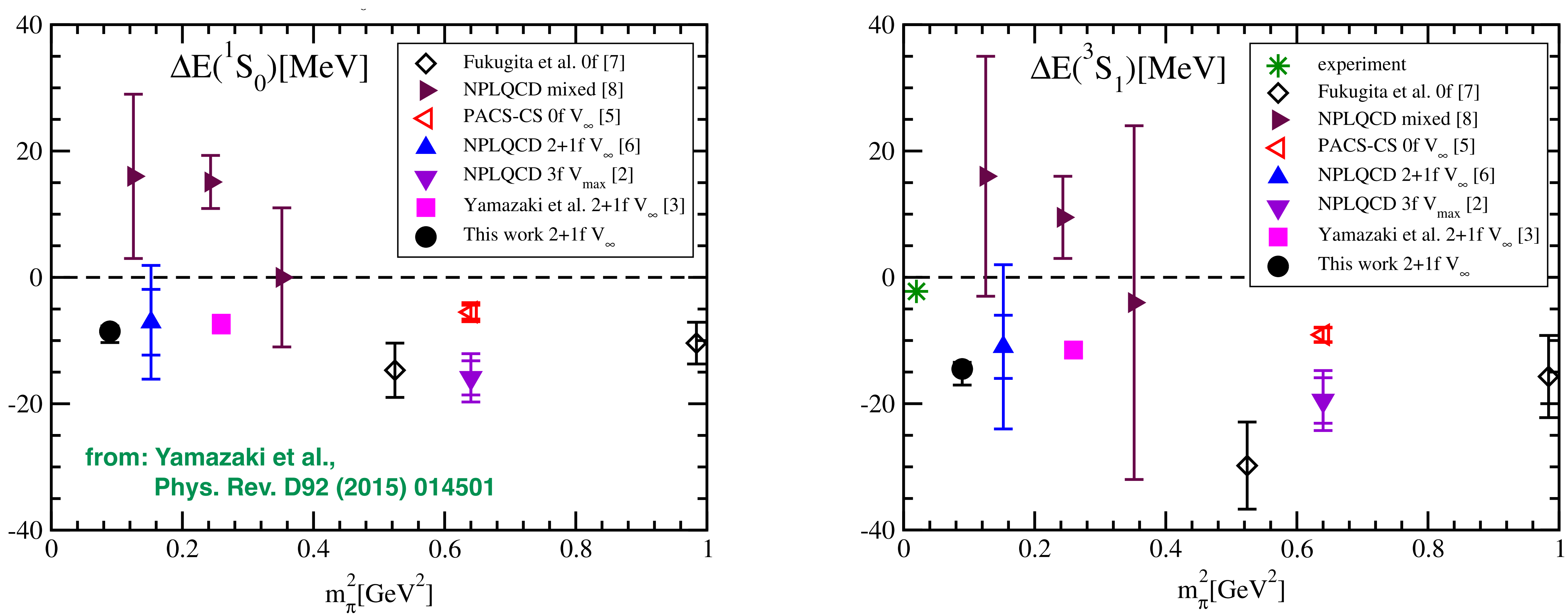
Here and in the following, we work in the limit of exact isospin symmetry and express our results in terms of K-factors: $K_X^q \equiv \frac{m_q}{X} \frac{\partial X}{\partial m_q}\Big|_{m_q^{\rm phys}}$

Sources of the quark mass dependence:

- ✔ **- nucleon mass:** $K_{m_N}^q = 0.048^{+0.002}_{-0.006}$
- ✔ **- long-range force:** explicit and implicit (g_A, F_π) quark mass dependence
- ✘ **- short-range NN force:** M_π-dependence poorly known, parametrize (up to NLO) via:

spin-singlet ($^1S_0$): $\bar{A}_s \equiv \frac{\partial a_s^{-1}}{\partial M_\pi}\Big|_{M_\pi^{\rm phys}}$ spin-triplet ($^3S_1$): $\bar{A}_t \equiv \frac{\partial a_t^{-1}}{\partial M_\pi}\Big|_{M_\pi^{\rm phys}}$

Lattice-QCD results for NN scattering observables

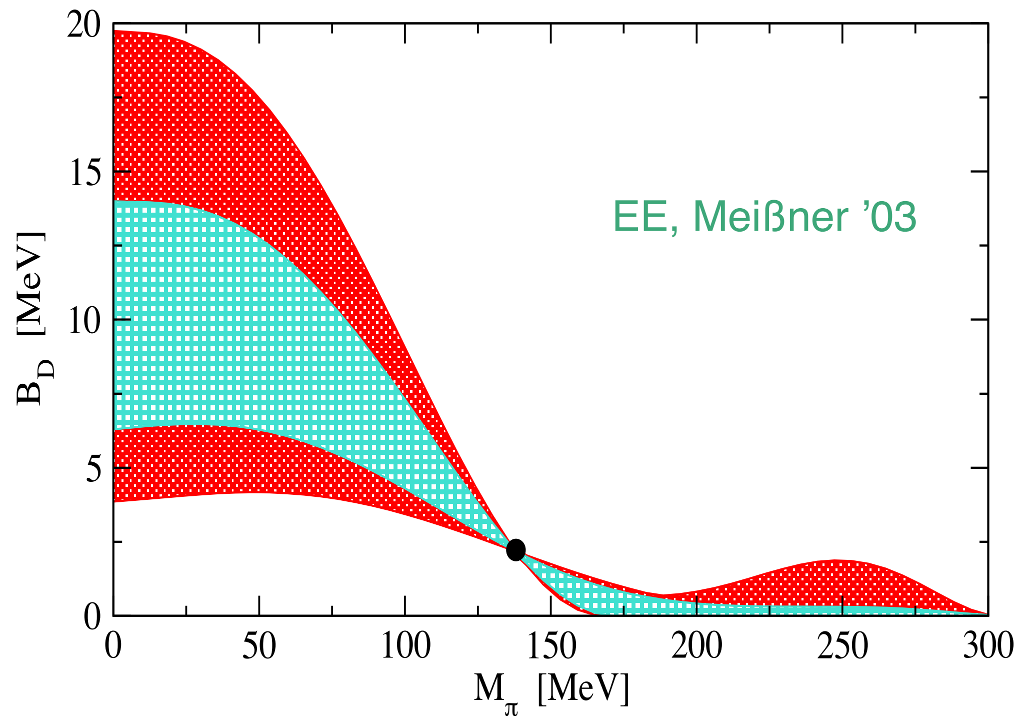


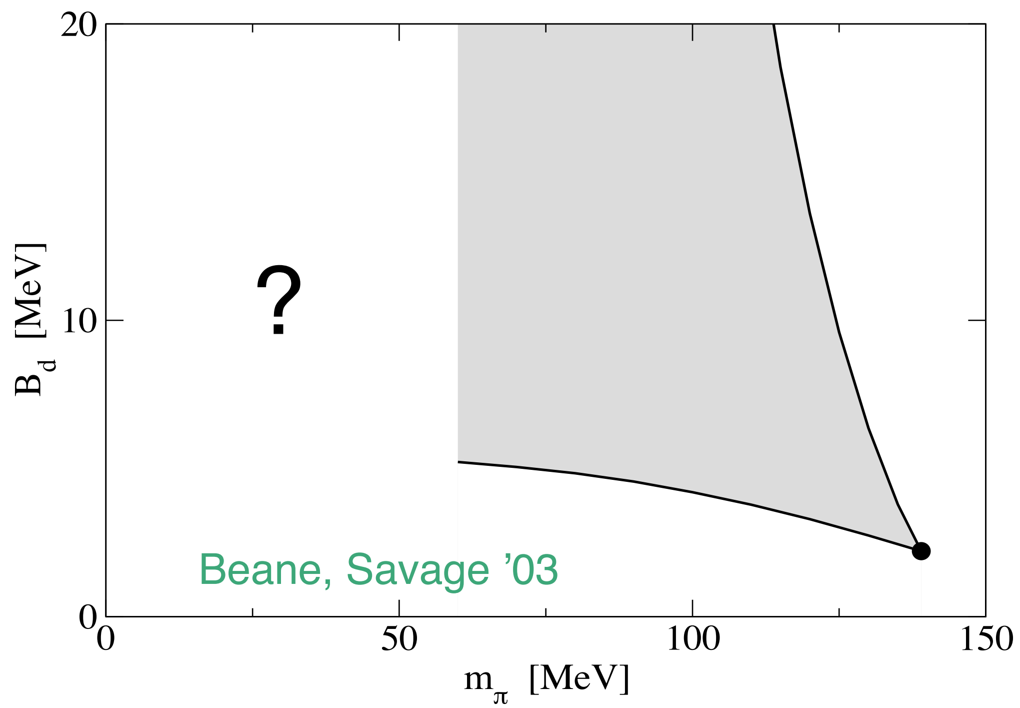
Further, the HAL QCD Collaboration claims [by first generating the NN potential] weaker attraction in both $^1S_0$ and $^3S_1$-$^3D_1$ channels and no bound states for $M_\pi > 411$ MeV Ishii et al.'12

Estimations based on chiral EFT ??

Lattice-QCD results for NN scattering observables

Pion mass dependence of the deuteron BE at NLO





Estimations based on chiral EFT ??

(large uncertainty mainly due to the lack of knowledge of m_q-dependent short-range LECs…)

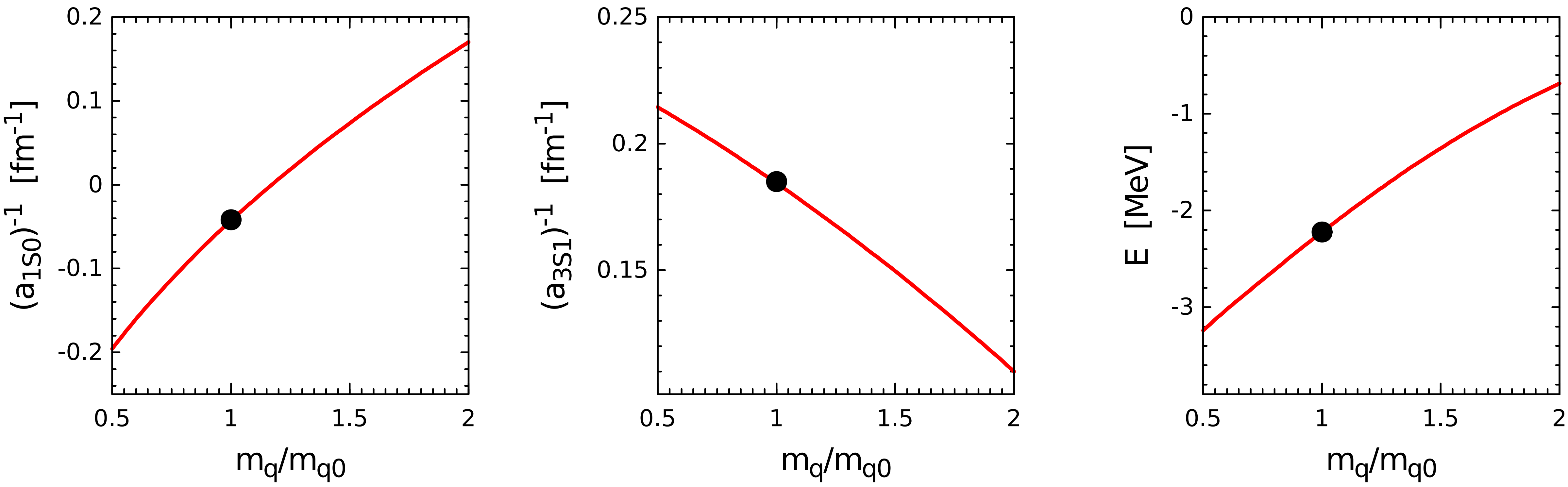
S-wave χ extrapolations in the renormalizable approach

EE, Gegelia,'12,'13

At LO, M_π-dependence of the amplitude is due to pion propagator in the OPEP
→ parameter-free prediction!

$$\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+M_\pi^2}\,\tau_1\cdot\tau_2 + C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2$$

$$T(\vec{p}\,',\vec{p})=V_{2N}^{(0)}(\vec{p}\,',\vec{p}) + \frac{m_N^2}{2}\int\frac{d^3k}{(2\,\pi)^3}\frac{V_{2N}^{(0)}(\vec{p}\,',\vec{k})\,T(\vec{k},\vec{p})}{(k^2+m_N^2)\,(E-\sqrt{k^2+m_N^2}+i\,\epsilon)}$$



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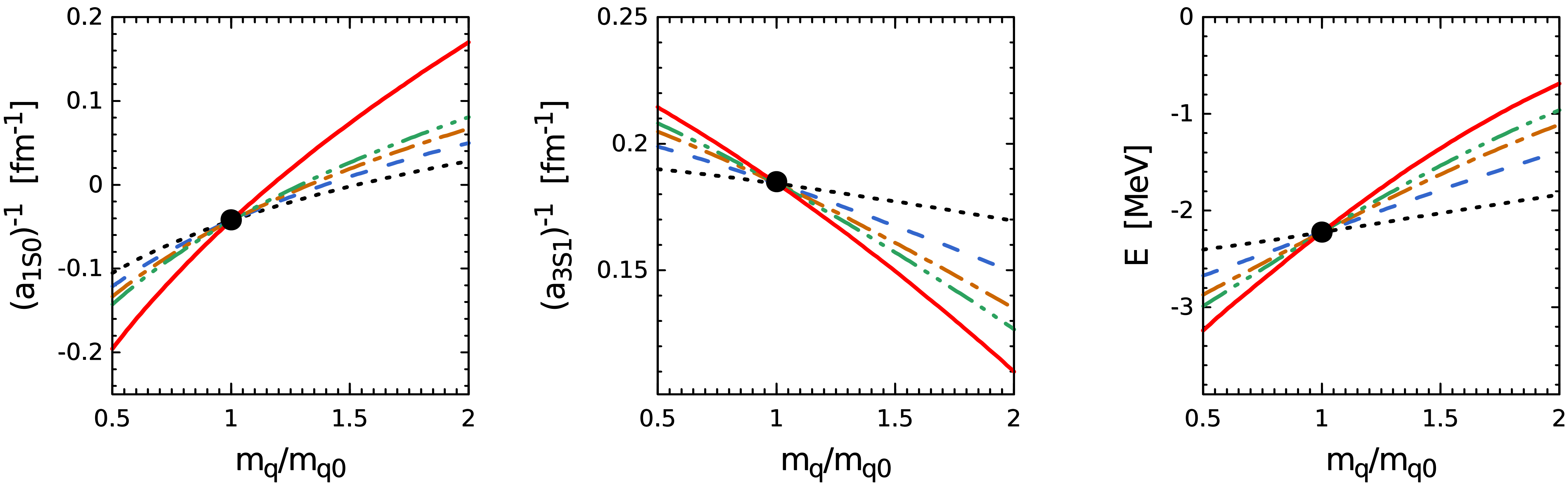
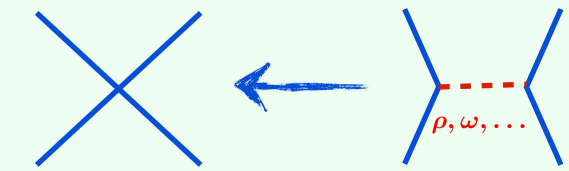
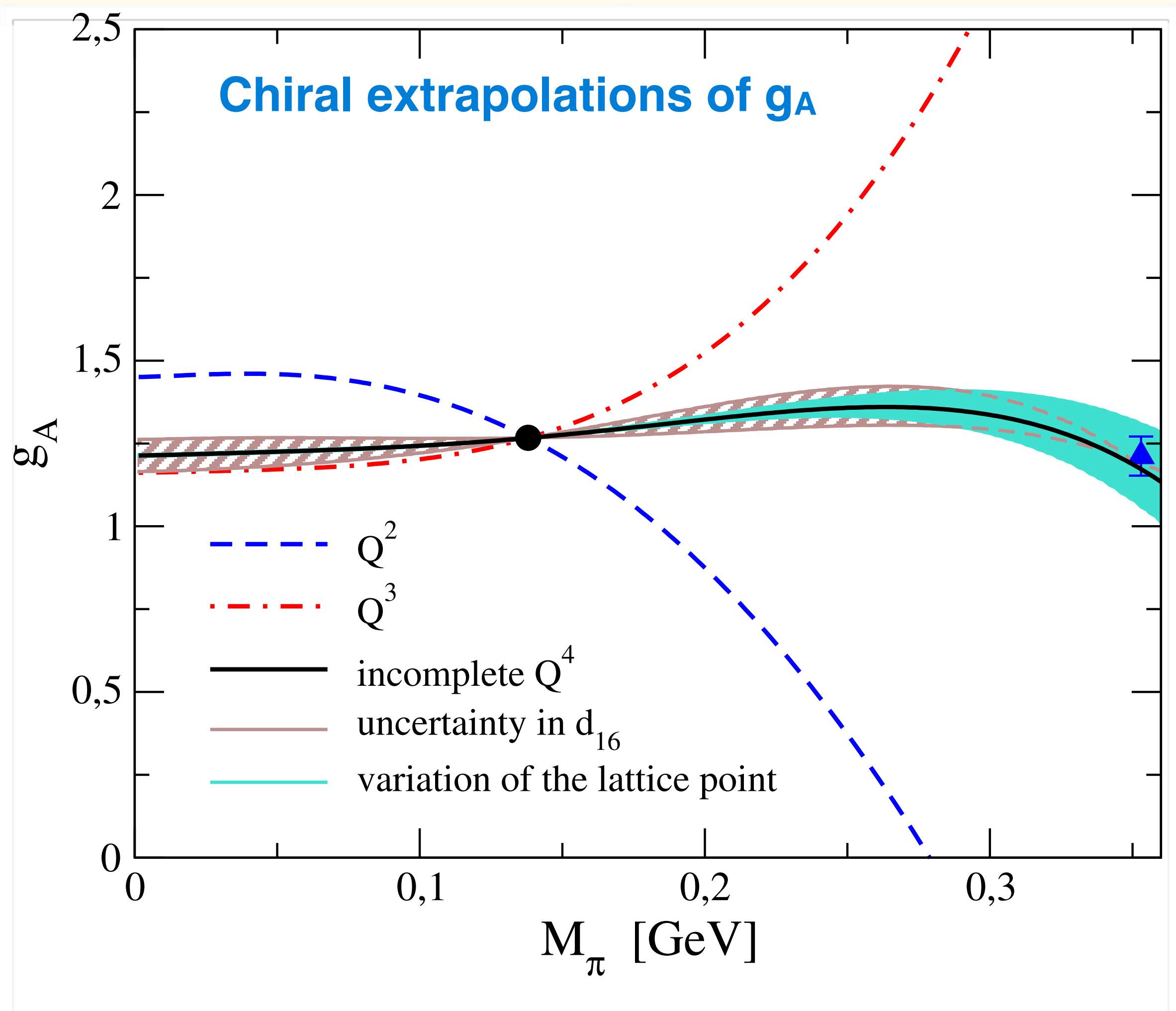


Quark mass dependence of the NN force

Berengut, EE, Flambaum, Hanhart, Meißner, Nebreda, Pelaez '13

- Use ChPT combined with lattice-QCD data to constrain the M_π-dependence of the nucleon mass and long-range part of the force
- M_π-dependence of contact interactions from resonance saturation [EE, Meißner, Glöckle, Elster '02] + unitarized ChPT + lattice-QCD





Resonance saturation of the various LECs based on the Bonn B potential

LEC	$\mathrm{N^2LO}$ fits	$\sigma + \rho + \omega$
$\tilde{C}^{\rm res}_{1S0}$	$-(0.12 \ldots 0.16)$	-0.12
$C^{\rm res}_{1S0}$	$(1.16 \ldots 1.37)$	1.28
$\tilde{C}^{\rm res}_{3S1}$	$-(0.13 \ldots 0.16)$	-0.10
$C^{\rm res}_{3S1}$	$(0.42 \ldots 0.72)$	0.66
$C^{\rm res}_{\epsilon 1}$	$-(0.36 \ldots 0.47)$	-0.41

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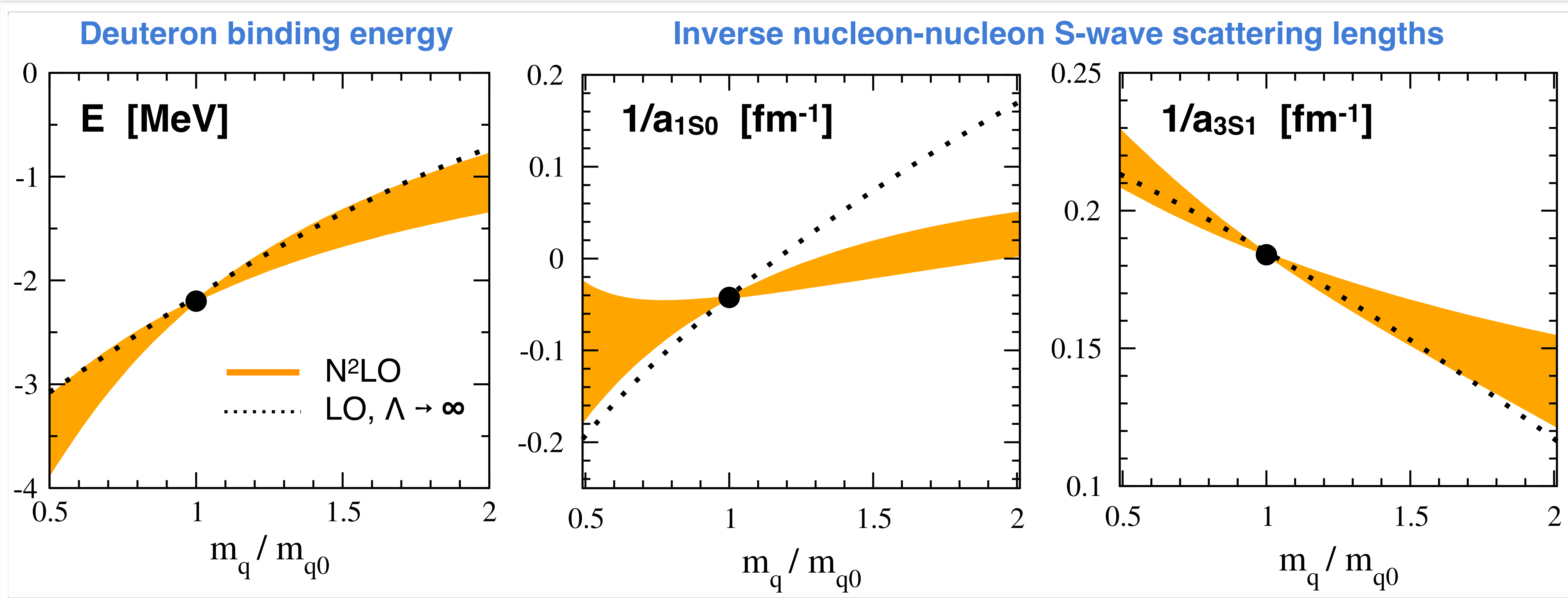

In terms of K-factors $K_X^q \equiv \frac{m_q}{X}\frac{\partial X}{\partial m_q}\Big|_{m_q^{\rm phys}}$ we find: $K^q_{a_s} = 2.3^{+1.9}_{-1.8}, \quad K^q_{a_t} = 0.32^{+0.17}_{-0.18}$

to be compared with earlier calculations:

$K^q_{a_s} = 5 \pm 5, \quad K^q_{a_t} = 1.1 \pm 0.9$ (W, NLO) EE et al. '03

$K^q_{a_s} = 2.4 \pm 3.0, \quad K^q_{a_t} = 3.0 \pm 3.5$ (KSW, NLO) Beane, Savage '03

Impact on BBN: limits on m_q variation at the time of BBN: $\delta m_q / m_q = 0.02 \pm 0.04$

Quark mass dependence of the NN force

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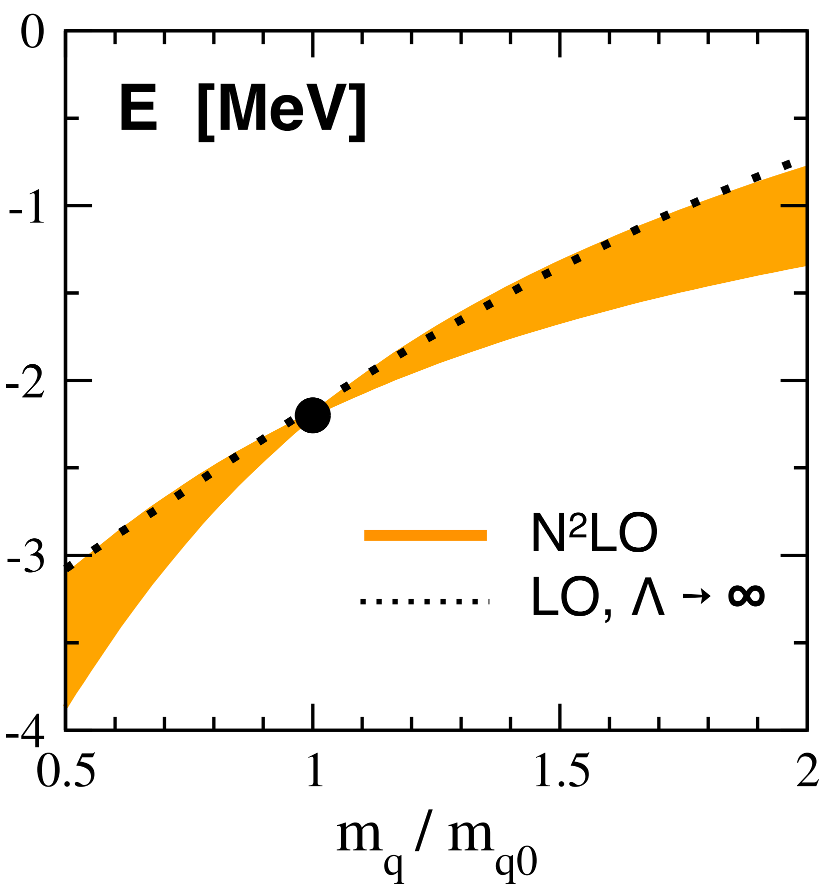
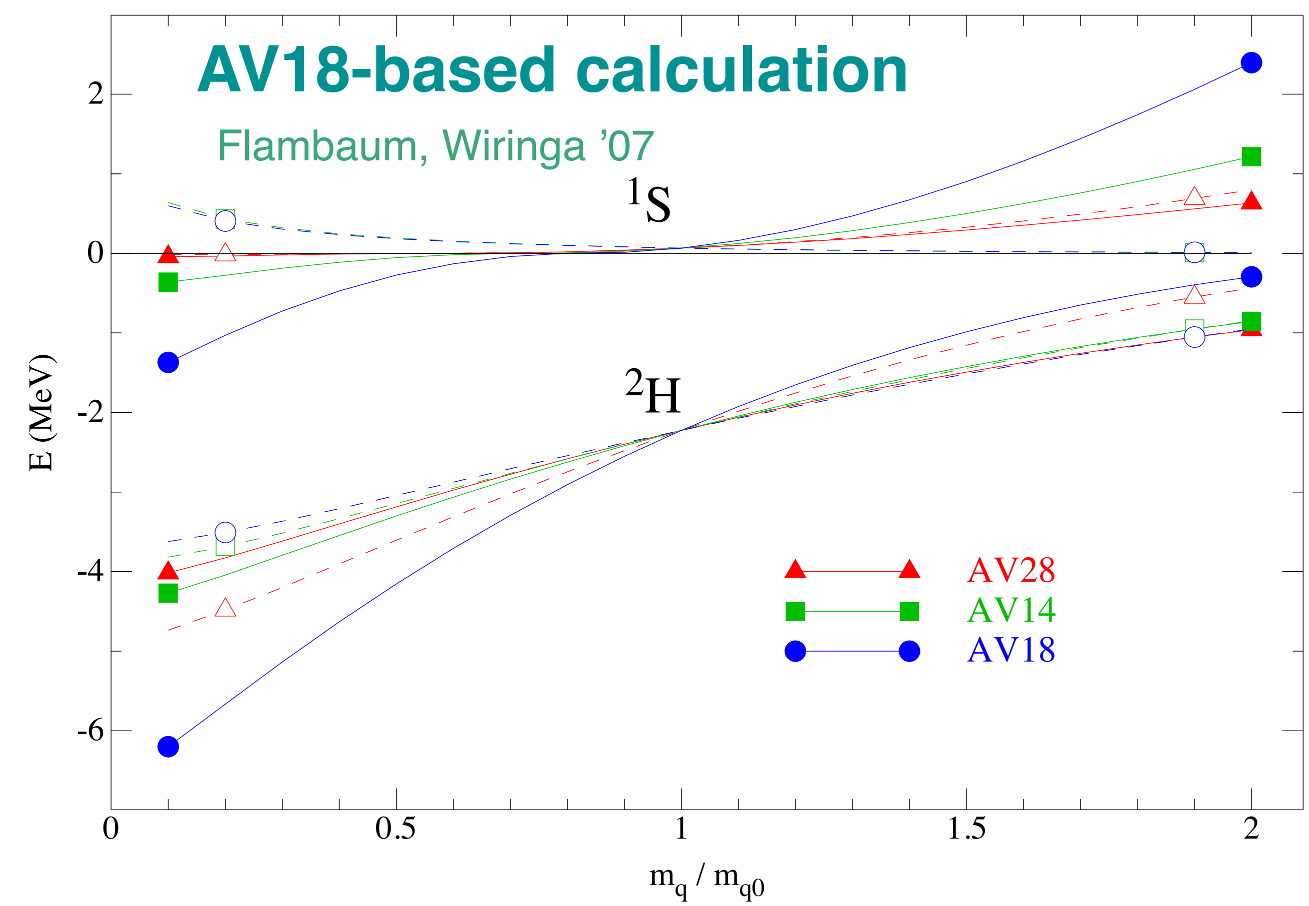
Deuteron binding energy





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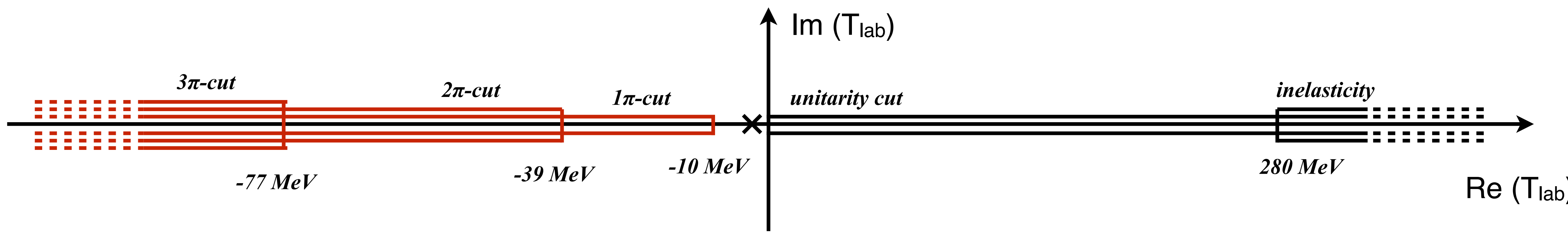
Low-energy theorems for NN scattering

Baru, EE, Filin, Gegelia, PRC 92 (15) 014001; Baru, EE, Filin, PRC 94 (16) 014001

The long-range interaction (1π) governs the low-energy behavior of the amplitude and implies correlations between coefficients in the ERE which may be regarded as Low Energy Theorems



— very good / fair predictive power in the $^3S_1$ / $^1S_0$ channel at physical M_π

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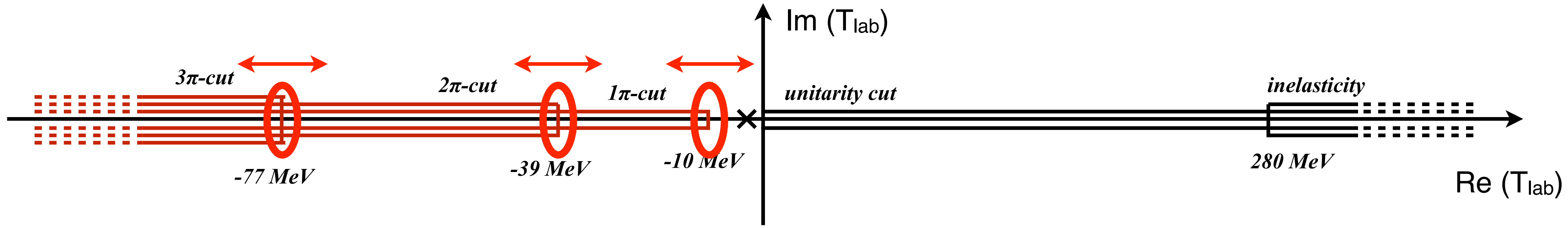

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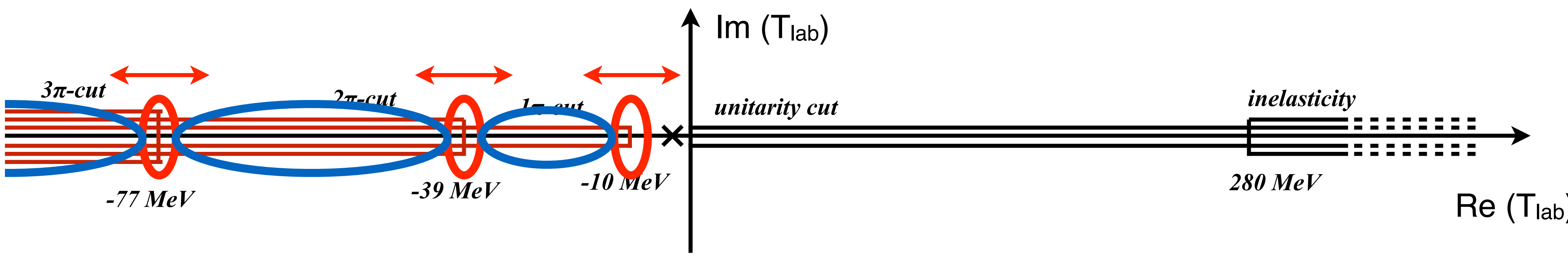

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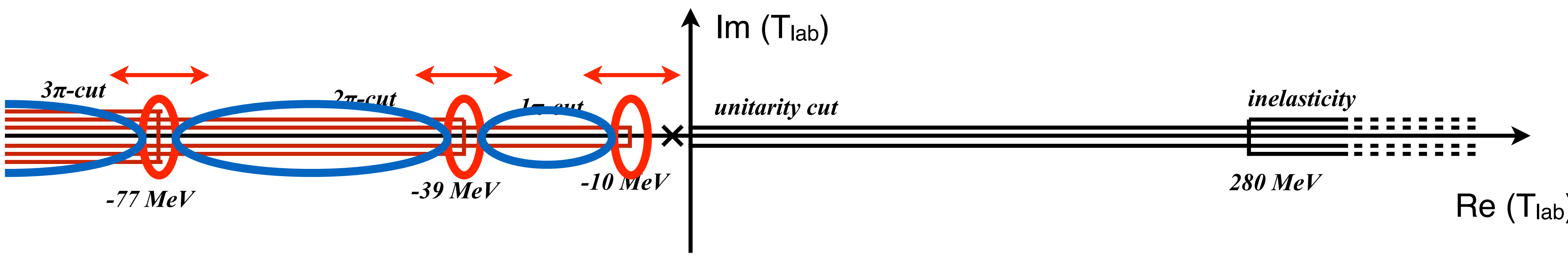

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— need a single lattice data (e.g. BE) as input at a given M_π to reconstruct the amplitude…

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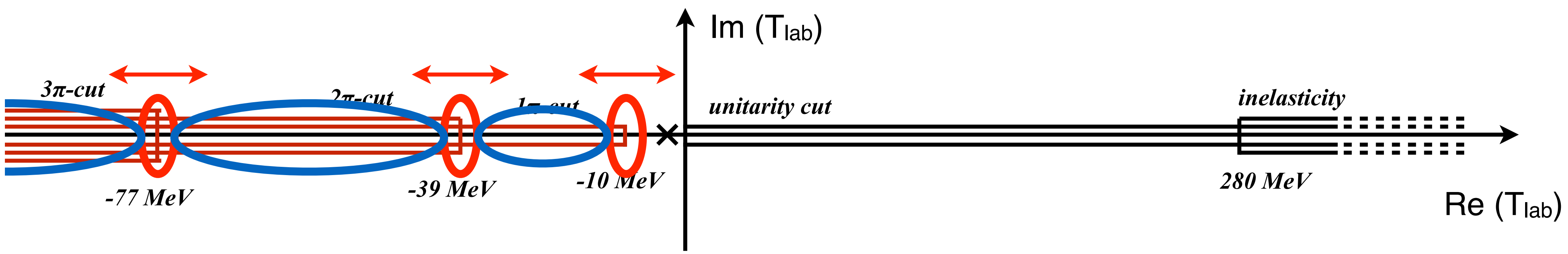

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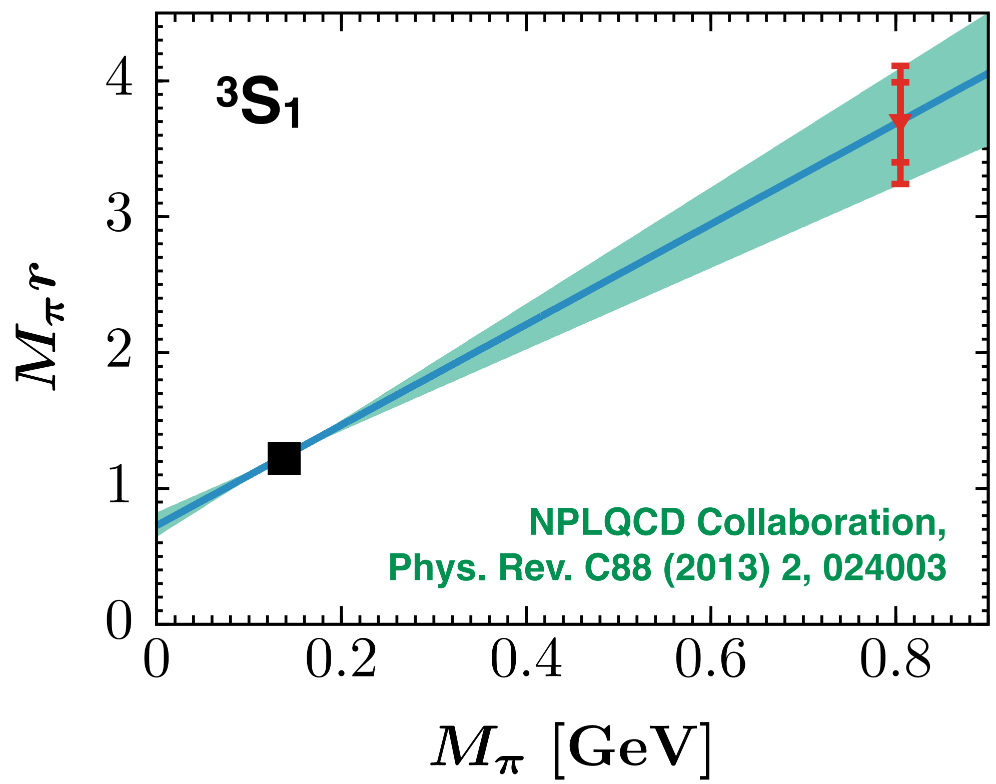
— need a single lattice data (e.g. BE) as input at a given M_π to reconstruct the amplitude…

⟶ can be used to extrapolate the scattering amplitude in energy at fixed M_π.
No reliance on the chiral expansion: $M_\pi \to \infty$ limit well defined!

Low-energy theorems for NN scattering

Use the conjectured linear M_π-behavior of $M_\pi\, r^{(3S1)}$ as input Baru, EE, Filin, Gegelia '15

$$M_\pi r \cong C^{(^3S_1)} + D^{(^3S_1)} M_\pi^2 \qquad \text{where} \qquad C^{(^3S_1)} = 1.149^{+0.009\,+0.011}_{-0.009\,-0.009}, \quad D^{(^3S_1)} = 3.95^{+0.45\,+0.45}_{-0.49\,-0.55}\ \text{GeV}^{-2}$$



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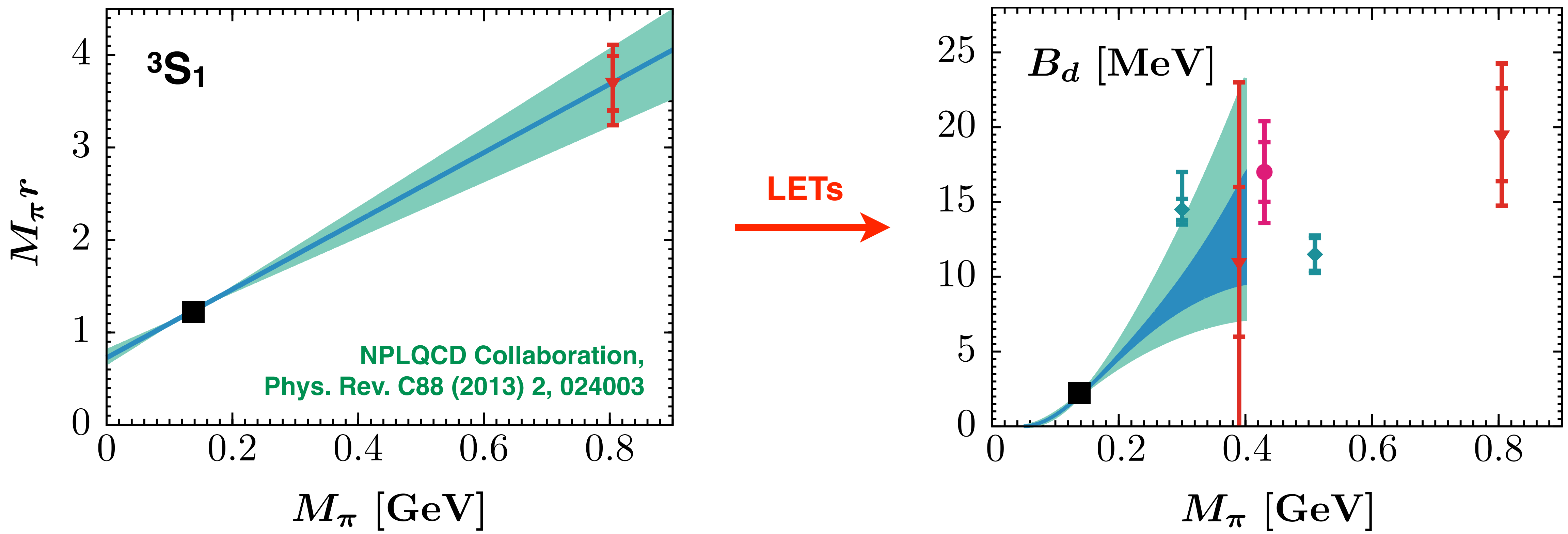

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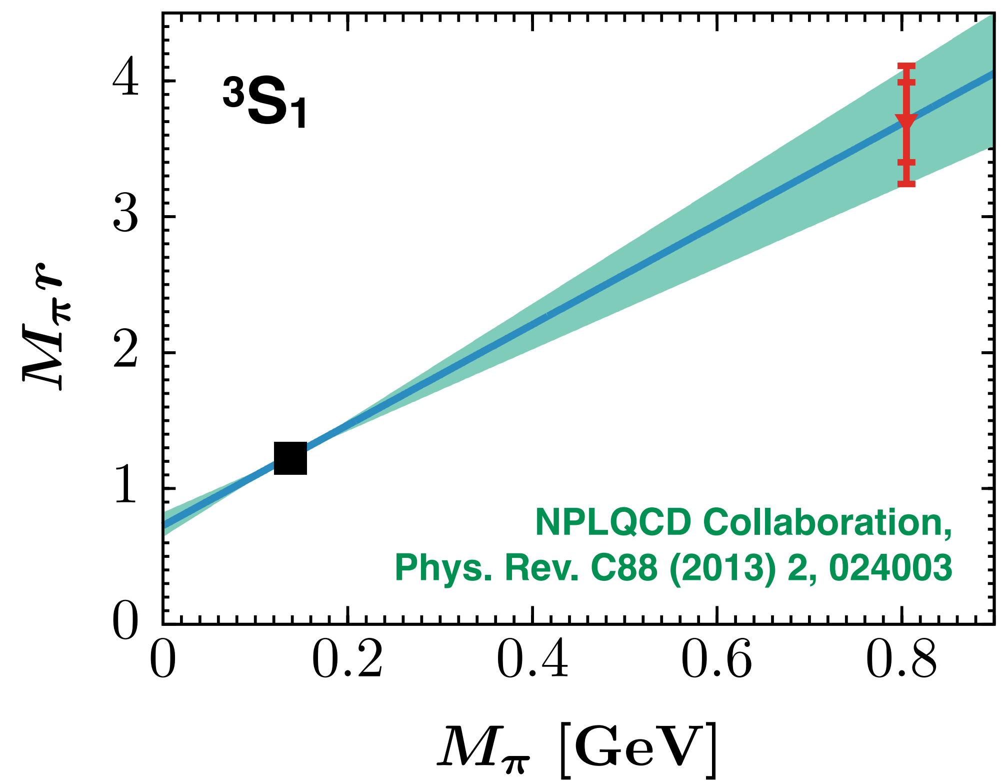

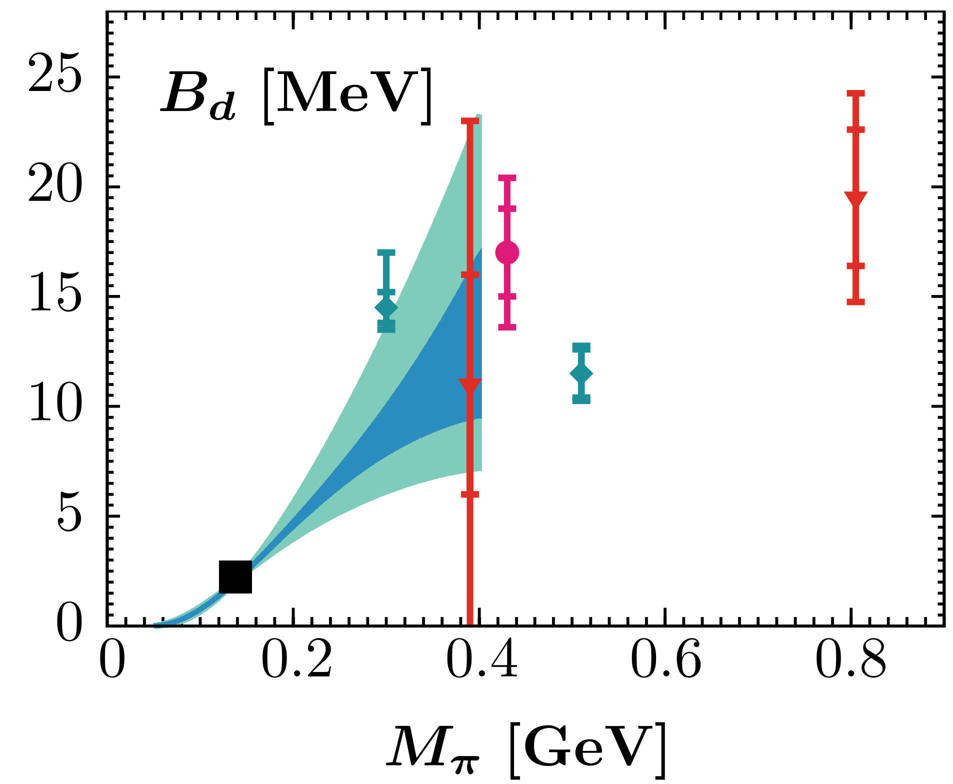

This leads to $K^q_{a_t} = -0.6 \pm 0.1$ (the error includes the theoretical uncertainty of the LETs and lattice results, but NOT the systematic uncertainty of the assumed linear extrapolation of the effective range).

Summary

- The lack of information about quark mass dependence of the NN contact interactions leads to large uncertainties in χ extrapolations of nuclear observables. It can be parametrized by

spin-singlet ($^1S_0$): $\bar{A}_s \equiv \left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$ spin-triplet ($^3S_1$): $\bar{A}_t \equiv \left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$

- Employing resonance saturation (combined with unitized ChPT + lattice-QCD), one finds at N^2LO:

$\bar{A}_s \simeq 0.29^{+0.25}_{-0.23}$ $\bar{A}_t \simeq -0.18^{+0.10}_{-0.10}$ (the uncertainty due to resonance saturation is not included!)

These results are compatible with the LO chiral EFT predictions (large uncertainty) & with the phenomenological analysis by Flambaum, Wiringa (no uncertainty estimate provided).

- Using LETs in combination with the conjectured linear dependence of $M_\pi\, r^{(^3S_1)}$ seems to reproduce the lattice-QCD trend for the ^2H BE and leads to $\bar{A}_t \sim 0.3$

⟶ need more precise lattice-QCD calculations near the physical point