Oscillatory particle banding in a rotating fluid

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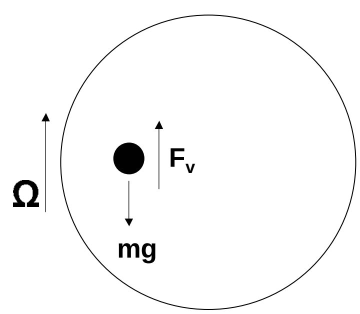
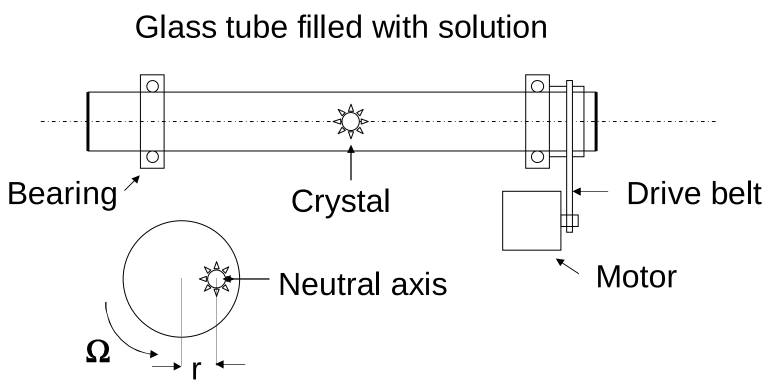
Particle levitation

A particle in a fluid can be "levitated" at an off-axis point if the fluid rotates

$F_v = 6\pi\eta\ a\ r\Omega$

For a spherical particle (Stokes)



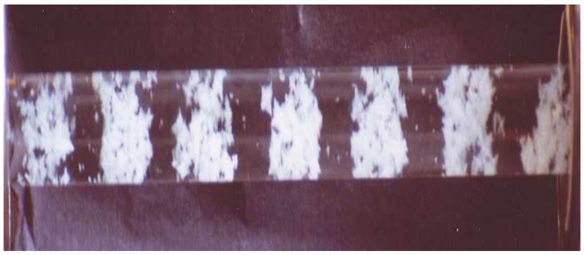


Experimental Apparatus for growing crystals by levitation

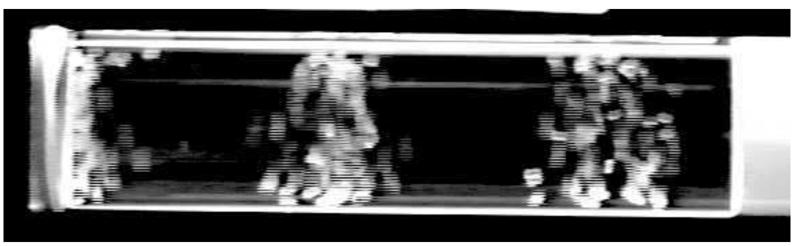
- Experimentally, this position is stable
- Crystals were grown in 1991-2 from seeds in super-saturated solution using dynamic levitation.

When there are many particles, a new phenomenon occurs:

The particles accumulate in bands, with separation proportional to the tube diameter (Λ/R~2)



Banding in a tube with *l/R* =8.72



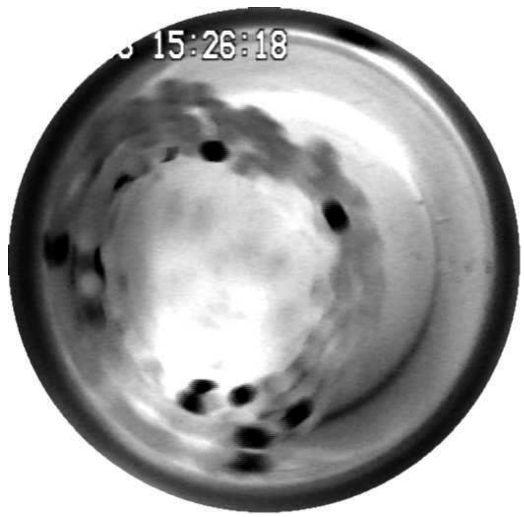
Band at the tube end

No band at the tube end

Ω=9.4rad/s, 220 plexiglass cylinders; Λ/R~4 in this case

Looked at from the end of the tube:

Ω



Dependence on parameters:

- Not dependent on type of particle, size, shape
- Not dependent on Ω if it is in range between settling and centrifuging
- Not dependent on viscosity up to 10X water
- Dependent on buoyancy: bubbles interleave particles.

Hydrodynamic numbers for fluid=water

- Reynolds # for particles ~ 100-300
- Eckman # (viscous /Coriolis) ~ 10^{-3}
- Rossby # (non-linear/linear) ~0.5

These show that the problem is not so simple, and will involve non-linear and viscous effects in the long run

Previous work on suspensions in horizontally rotating fluids:

Observation of banding of 100μm SiO_2 particles in glycerol solutions
W. R. Matson, B. J. Ackerson, and P. Tong, Phys. Rev. E 67, 50301 (2003)

Theory to explain banding period observed by Matson *et al* by means of averaging hydrodynamic interactions between particles
J. Lee and A. J. C. Ladd, Phys. Rev. Lett., 89, 104301, (2002)

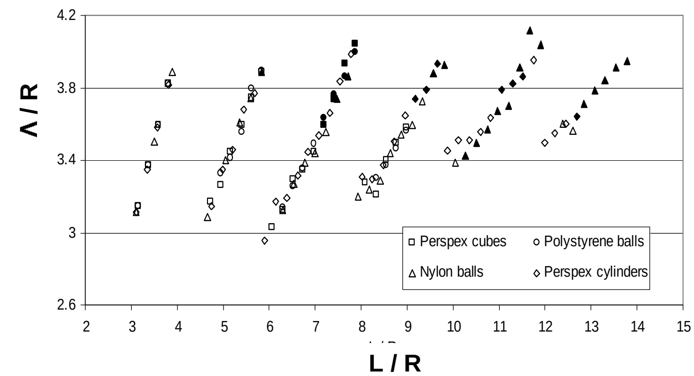
Observation of banding of sand particles in water: work concentrated on orbits of single particles
A. P. J. Breu, C. A. Kreulle and I. Rehberg, Europhys. Lett. 62, 491 (2003)

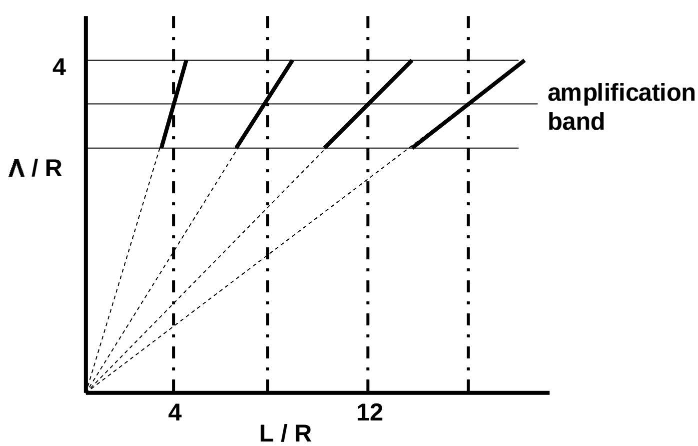
Observation of banding in crystal growth from supersaturated solutions: attributed possibly to growth dynamics
S. G. Lipson, J. Phys.: Condens. Matter 13 5001 (2001)

Observation of banding of different types of particles and bubbles, suggestion of a wave interaction mechanism
G. Seiden and S. G. Lipson, Physica A 314, 272 (2002)

Observation of banding of neutrally-buoyant particles in *partially-filled* rotating tubes- the free surface makes it a different problem
M. Tirumkudulu, A. Tripathi and A. Acrivos, Physics of Fluids 11, 507-9 (1999).

Dependence on the tube length



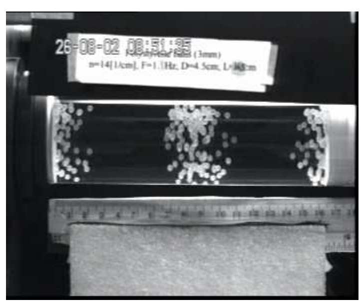


A model for a resonator with a finite amplification band and wavelength determined by boundary conditions at the ends (e.g. a laser resonator)

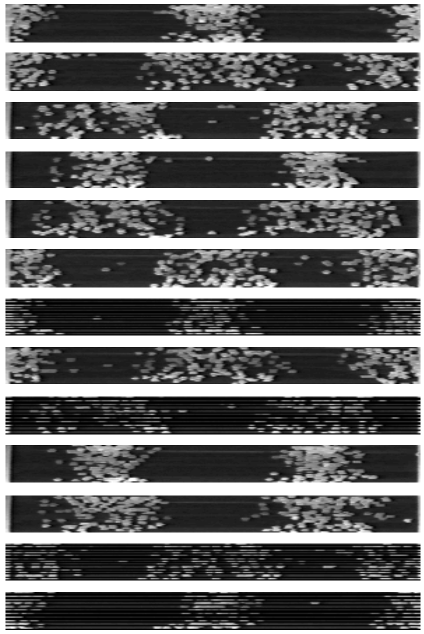
Oscillations

- There are two equivalent patterns for each tube length:
- If the tube has length nΛ, there can be particle at both ends, or neither
- If the tube has length (n+ ½) Λ, there is a band at one end and none at the other
- Oscillations between the two possibilities slowly take place.

Short film of the oscillations



oscillations.mpg



Oscillations

Bands at intervals of 3.5 sec. in a tube of length 2Λ (20 sec oscillation period)

Inertial waves

- In the atmosphere they are related to Rossby waves, which are created when an easterly air-stream crosses an obstacle.
- Coriolis force is important (restoring force)
- Solution of Navier-Stokes in a rotating frame of reference.

D. Tritton, ``Physical Fluid Dynamics'', Oxford : Clarendon Press, (1988)
H. P. Greenspan ``Theory of Rotating Fluids'' Cambridge: Cambridge University Press, (1969)

Navier-Stokes with Coriolis force

$$\frac{\partial(\rho v)}{\partial t} + v.\nabla(\rho v) + \Omega\times(\rho v) + 2\Omega\times(\Omega\times\rho r)$$
$$= -\eta\nabla p + \rho g + F$$

Includes non-linear term, centrifugal force, gravity, viscosity and an external force F

Note: g is oscillatory at Ω in this frame!

Now simplify by ignoring non-linear term and viscosity, and replacing *p* by an effective pressure:

$$p^* = p - \tfrac{1}{2}\rho(\Omega\times r)^2 - \rho g \cdot r$$

Whence:

$$\frac{\partial(\rho v)}{\partial t} + 2\Omega\times\rho v = -\nabla p^* + F$$

Now with an oscillatory dependence of *p** on time:

$$p^* \propto \exp(i\omega t)$$

we get a wave equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p^*}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 p^*}{\partial\theta^2}+\left(1-\frac{4\Omega^2}{\omega^2}\right)\frac{\partial^2 p^*}{\partial z^2}=f(F)$$

which we solve like the waveguide equation in electro-magnetic theory

If we assume the particles to be driving the fluid at ω=Ω (which is reasonable, but has to be inspected), the homogeneous solution is p_m*(r, θ, z, t):

$$p^* = p_0 J_m(\gamma r)\cos(m\theta-\Omega t)\cos(kz)$$

Where $k=\frac{\gamma}{\sqrt{3}}$

The velocity components for the m=−1 mode are

$$v_r = V_0\left[J_1'(\gamma r) + \frac{2J_1(\gamma r)}{\gamma r}\right]\sin(\theta+\Omega t)\cos(kz);$$

$$v_\theta = V_0\left[2J_1'(\gamma r) + \frac{J_1(\gamma r)}{\gamma r}\right]\cos(\theta+\Omega t)\cos(kz);$$

$$v_z = \frac{\gamma}{k}V_0 J_1(\gamma r)\sin(\theta+\Omega t)\sin(kz).$$

Why m=-1? Because it is stationary in the lab frame!

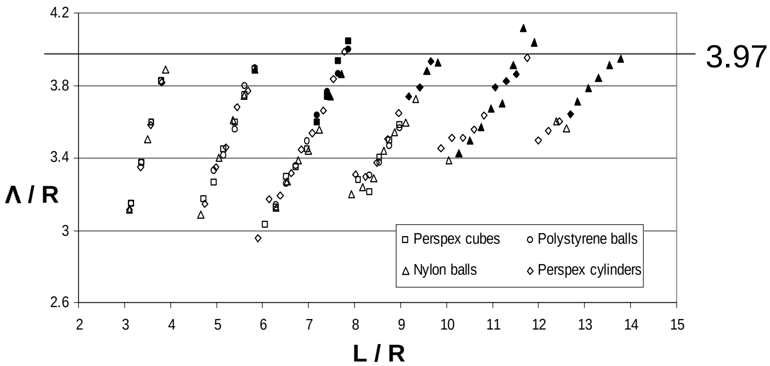
Boundary conditions: v_r=0 on the curved walls (r=R) and v_z=0 at the ends

lead to an eigenvalue equation:

$$J_1'(\gamma r) + \frac{2J_1(\gamma r)}{\gamma r} = 0$$

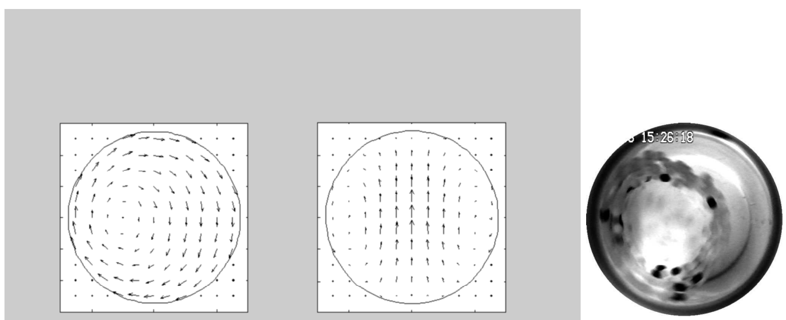
With a first solution γR=2.74 and wavelength λ=3.97R.

Compare with experimental results….



But note that this means λ=Λ, i.e. the bands are spaced by one *wavelength, and not a* half *wavelength as you might expect. Why?*

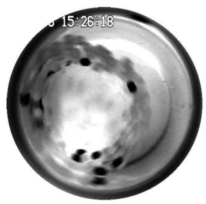
Fluid velocity in the plane $kz=2n\pi$

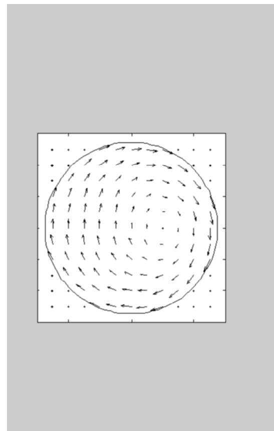


(a) (b) (c)

(a) shows v in the lab frame, and (b) in the rotating frame:
(a) matches the particle motion in the rotating fluid, (c).

Fluid velocity in the plane $kz=(2n+1)\pi$

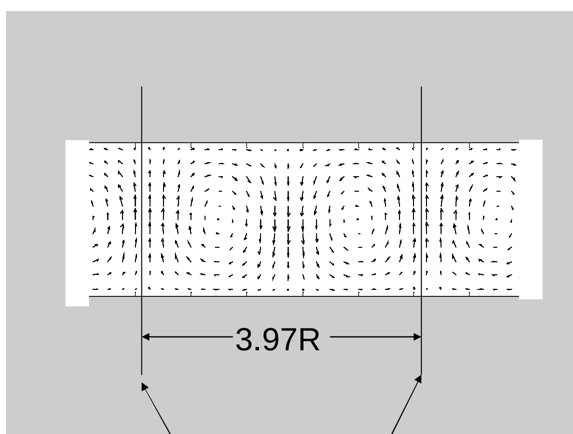




A bubble plane

Doesn't match- so particles don't accumulate here; but bubbles do match.

In the center plane x=0



Ends of tube could be here: $v_z=0$

The oscillations

- In alternate nodal planes the particle accumulate. But any nodal plane satisfies the end boundary conditions.
- If ω is not exactly equal to Ω, there can be beats between the two patterns : hence the oscillations?
- We have not yet dealt with this satisfactorily.

Viscosity

- Effect of viscosity: what is the viscous damping length of inertial waves?
- As the viscosity is increased, the 3-D particle motion becomes very clear, but the banding gets more complicated. No clear results yet.
- The interaction between the particles and the fluid will have to be introduced as a perturbation.
- At some stage particles are trapped in both types of node (remember the crystal bands, Λ/R~2, not 4)

- How does viscosity affect the resonance condition?
- If we put the interaction force $F=-\alpha\rho v$, we find

$$\frac{\gamma^2}{k^2} = \frac{4\Omega^2}{\Omega^2 + \alpha^2} - 1$$

instead of 3, which increases ω and decreases Λ, as seen in the experiments

Conclusions

- Banding was observed when several particles are suspended in a rotating inviscid fluid.
- The banding period Λ is determined approximately by the radius of the tube and exactly by its length.
- Interpreted as excitation of inertial waves in the rotating fluid.
- Particles accumulate in alternate nodes of the standing wave, and bubbles in the interleaving planes.
- Oscillations occur between two degenerate possibilities, the origin of which is not yet clear.