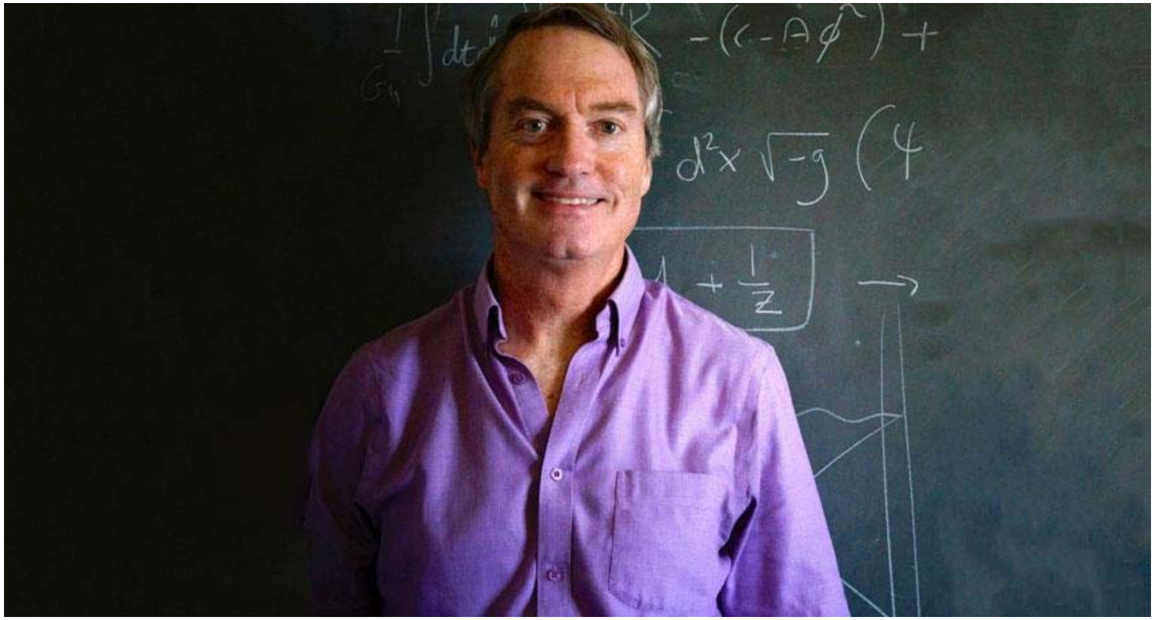
Joe Polchinski Science Symposium



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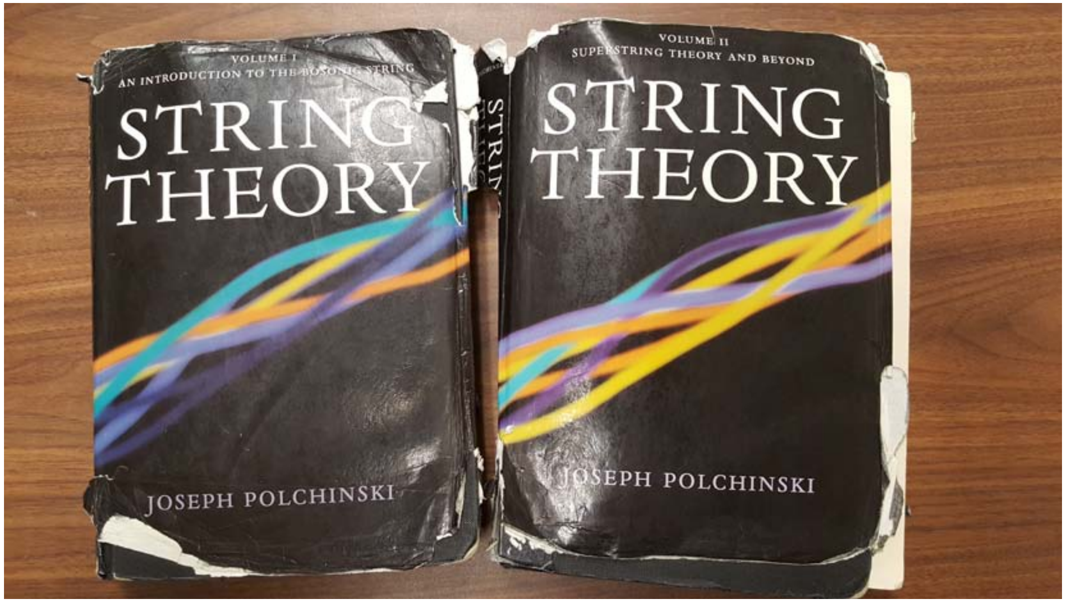
(McMaster University / Perimeter Institute)

My first encounter with Joe



After my PhD in condensed matter physics, I served my military duty as a civil servant in Korea. During my spare time, I read Joe's `big books' on string theory from cover to cover with joy. I became to admire Joe for his deep understanding of physics and clear explanation.

Interactions with Joe

- I first met Joe in person when I came to KITP as a postdoc in 2006. During my stay at KITP, I occasionally had discussions with Joe on gravity. Joe was always generous with his time to listen to even wrong ideas. Once I was confused about passive and active diffeomorphisms. As Joe gently corrected me, he added “Einstein was once confused about that too, so you have a good company”. Joe was a generous and caring person.

- More recently, I have been working on two subjects in which Joe left big footprints. The first is field theories of metals, and the other is renormalization group.

Effective theories of metals

- Joe’s 1992 TASI lecture on “Effective field theory and the Fermi surface” put Landau Fermi liquid theory on a firm theoretical ground along with the work of Shankar

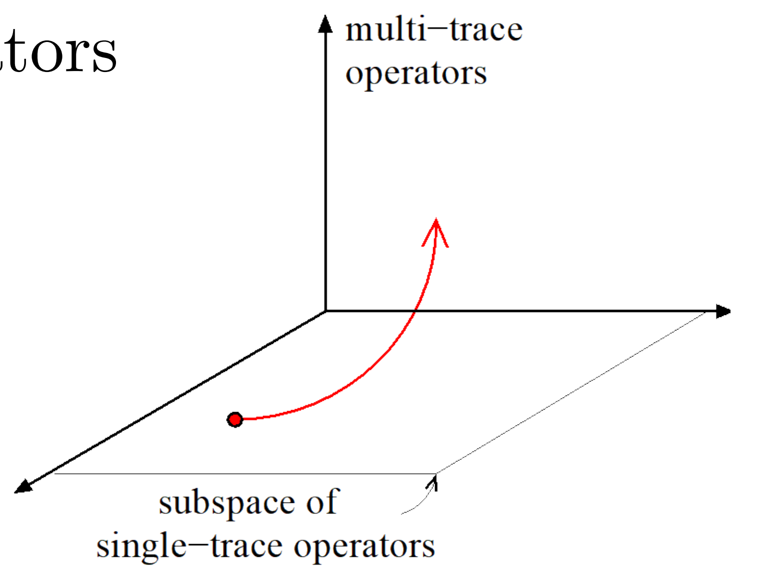
- The 1994 paper on “Low-energy dynamics of the spinon-gauge system” was one of the first papers that studied field theories of non-Fermi liquids (NFL) systematically. The paper provided the way to think about the problem from the perspective of effective field theory. It also taught me that NFL is a rich research area, which partly led me to work on this problem. Many of recent developments have been built on Joe’s work.

Exact RG and quantum RG

- Joe's exact RG equation can be written as a differential operator acting on Boltzmann weights. According to the Wilson's picture, the renormalized Boltzmann weight defines the RG flow in the space of all couplings

$$e^{-S[\phi;J_1',J_2',J_3',..]} = e^{-dzH[\delta/\delta\phi,\phi]}\; e^{-S[\phi;J_1,J_2,J_3,..]}$$

J_n : sources for n-th trace operators



Exact RG and quantum RG

- If one views the coarse graining operator as a quantum operator acting on states whose wavefunctions are given by Boltzmann weights, it is natural to invoke the superposition principle to represent a Boltzmann weight with general couplings as a linear superposition of Boltzmann weights that include only a subset of couplings. The subset is chosen to span the full theory space as a vector space.

$$e^{-S[\phi;J_1,J_2,J_3,..]} = \int Dj_1 \ \Psi(j_1) \ e^{-S[\phi;j_1,0,0,..]}$$

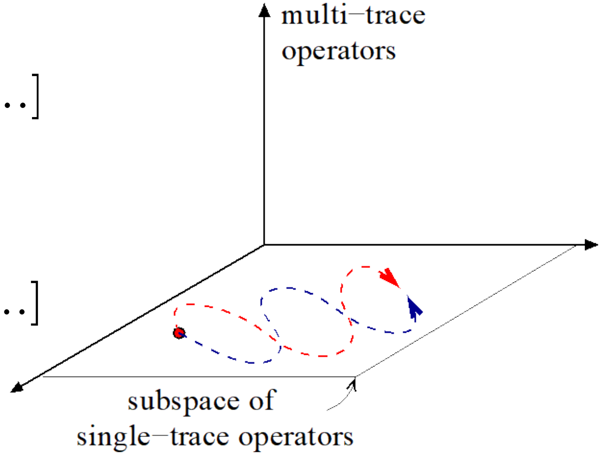
Exact RG and quantum RG

- In this picture, one can interpret the RG flow as a quantum evolution of wavefunctions defined in the subset of couplings. The coarse graining operator then gives an action for dynamical couplings, which includes metric. This may provide a way to understand the emergence of dynamical gravity in the AdS/CFT correspondence.

$$e^{-S[\phi;J_1,J_2,J_3,..]} = \int Dj_1 \ \Psi(j_1) \ e^{-S[\phi;j_1,0,0,..]}$$

$$\Downarrow$$

$$e^{-S[\phi;J'_1,J'_2,J'_3,..]} = \int Dj_1 \ \Psi'(j_1) \ e^{-S[\phi;j_1,0,0,..]}$$



Exact RG and quantum RG

- I am still struggling with questions that Joe had raised over the years :

“It seems to me that the key question of locality still has to be addressed. In your paper this would be the question, how big are the higher derivative terms represented by …”

State dependent spread of entanglement in relatively local Hamiltonians

1811.07241

Locality

- A key principle in quantum field theory
- However, locality can not be a part of the fundamental theory that includes dynamical gravity
  - If quantum fluctuations of geometry is strong, there can not be a well defined notion of what is near and what is far

Emergent locality

- Still, the theory that includes dynamical gravity can not be an arbitrary non-local theory
  - Local effective field theories should emerge if quantum fluctuation of geometry is weak

- In the emergent local theory, the notion of locality is determined by saddle-point geometry

- Geometry is determined by state, and so is the locality

Relatively local Hamiltonian

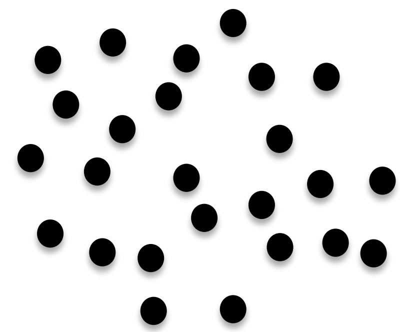
- A class of non-local theories from which local theories emerge within a subset of states that exhibit local structures of entanglement

Example

- Hilbert space : N real fields on each of L sites

$$\hat{\phi}_i^a \left|\phi\right\rangle = \phi_i^a \left|\phi\right\rangle$$

$$i = 1, 2, .., L \quad a = 1, 2, .., N$$



- A sub-Hilbert space with symmetry $S_N \ltimes Z_2^N \subset O(N)$

$$S_N : \phi_i^a \quad \rightarrow \quad \phi_i^{P_a}$$

$$Z_2^N : \phi_i^a \quad \rightarrow \quad (-1)^{\delta_{ab}} \phi_i^a$$

Symmetric sub-Hilbert space

- General symmetric operators can be written as polynomials of single-trace operators

$$O_{i_1 i_2 .. i_{2n}} \equiv \frac{1}{N} \sum_a \phi^a_{i_1} \phi^a_{i_2} ... \phi^a_{i_{2n}} .$$

- States whose wavefunctions involve only single-trace operators form a complete basis of the symmetric sub-Hilbert space

$$\left| \mathcal{T} \right\rangle = \int D\phi \; e^{-N \sum_n T_{i_1 i_2 .. i_{2n}} O_{i_1 i_2 .. i_{2n}}} \left| \phi \right\rangle$$

- General states in the symmetric Hilbert space can be written as

$$\left| \chi \right\rangle = \int D\mathcal{T} \; \left| \mathcal{T} \right\rangle \chi(\mathcal{T})$$

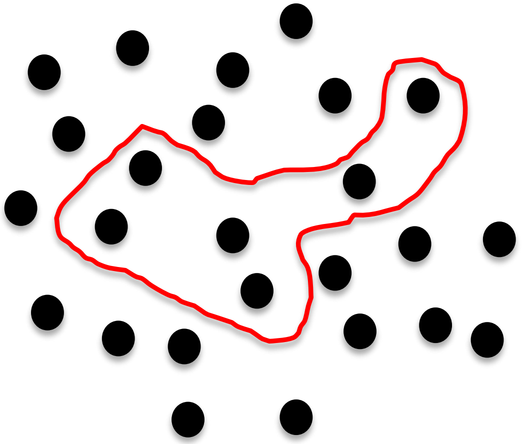
* $D\mathcal{T}$ should be defined along the imaginary axis

States with local structures

- A state is defined to have a local structure if the entanglement entropy of any sub-region is proportional to the `volume’ of its boundary associated with a lattice which provides a well defined notion of distance

- General states do not have local structure

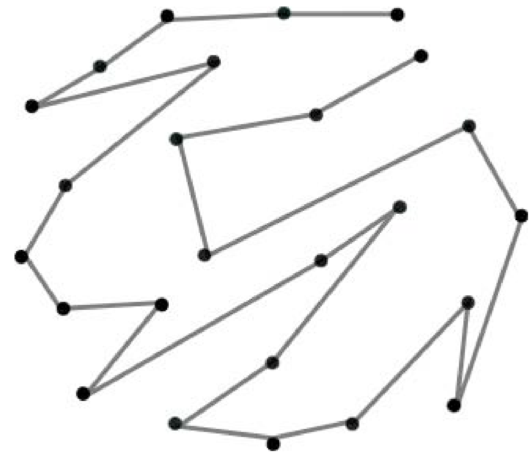
- The dimension, topology and geometry of the lattice, if there is a local structure, are properties of states



States with/without local structures

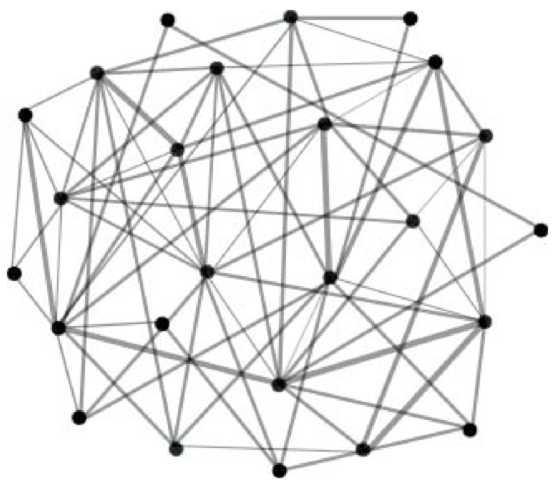
$$|T\rangle = \int D\phi_i^a \; e^{-T_{ij}\phi_i^a\phi_j^a} \;\; |\phi\rangle$$

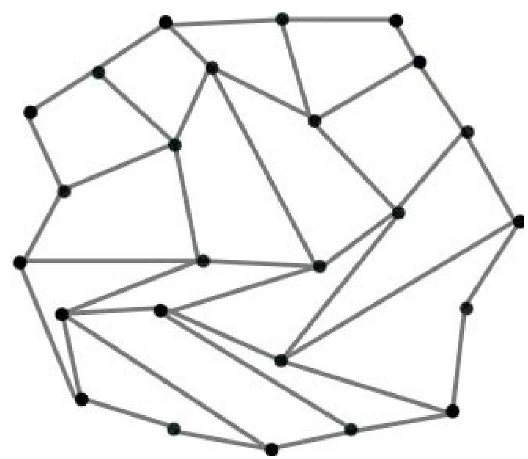
$$I_{ij} \propto |T_{ij}|^2$$



1D local structure

No local structure





2D local structure

Wanted :
Hamiltonian whose locality is determined by those of states

Relatively local Hamiltonian

$$\hat{H} = -R\sum_{i,j}\sum_{a}\hat{T}^a_{ij}(\hat{\phi}^a_i\hat{\phi}^a_j) + U\sum_i\sum_a\hat{\pi}^a_i\hat{\pi}^a_i + \frac{\lambda}{N}\sum_i\sum_{a,b}(\hat{\phi}^a_i\hat{\phi}^a_i)(\hat{\phi}^b_i\hat{\phi}^b_i)$$

sum over all
pairs of sites

standard ultra-local terms

quantum operator that measures the
pre-existing mutual information
between site i and j

$$\hat{T}^a_{ij} = \sum_{b\neq a}\frac{\hat{\pi}^b_i\hat{\pi}^b_j}{N-1}$$

$$\frac{\langle T|\hat{T}_{ij}|T\rangle}{\langle T|T\rangle} = 2(T^{*-1}+T^{-1})^{-1}_{ij}$$

H has the full lattice permutation symmetry

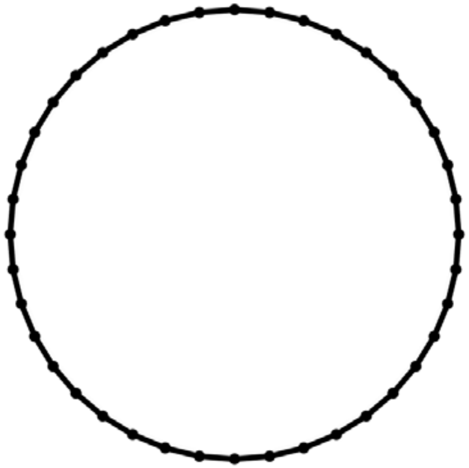
Induced Hamiltonian for the collective variables

$$\begin{aligned}\mathcal{H} &= -R\sum_n\Big[2n(2n-1)T_{ijj_1j_2..j_{2n-2}}P_{ij}P_{j_1j_2..j_{2n-2}} - 4n^2T_{jj_1j_2..j_{2n-1}}T_{ii_1i_2..i_{2n-1}}P_{ij}P_{j_1j_2..j_{2n-1}i_1i_2..i_{2n-1}}\Big]\\ &+U\sum_n\Big[2n(2n-1)T_{iij_1j_2..j_{2n-2}}P_{j_1j_2..j_{2n-2}} - 4n^2T_{ij_1j_2..j_{2n-1}}T_{ii_1i_2..j_{2n-1}}P_{j_1j_2..j_{2n-1}i_1i_2..i_{2n-1}}\Big] + \lambda P_{ii}^2\\ &+\frac{R}{N}\sum_n\Big[2n(2n-1)T_{ijj_1j_2..j_{2n-2}}P_{ijj_1j_2..j_{2n-2}} - 4n^2T_{jj_1j_2..j_{2n-1}}T_{ii_1i_2..i_{2n-1}}P_{ijj_1j_2..j_{2n-1}i_1i_2..i_{2n-1}}\Big]\Big\}\end{aligned}$$

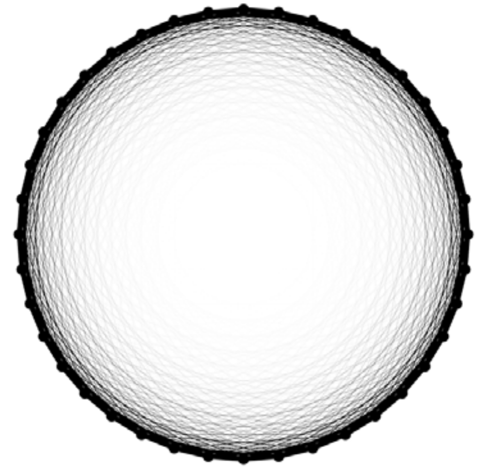
$$[T_{j_1,j_2,\ldots,j_n}, P_{i_1,i_2,\ldots,j_m}] = -\frac{1}{N}\delta_{n,m}\delta_{i_1,(j_1}\delta_{i_2,j_2}\ldots\delta_{i_n,j_n)}$$

- In the large N limit, semi-classical approximation is applicable
- When only bi-local fields are turned on at t=0, time evolution generates only bi-local fields to the leading order in 1/N

Time evolution of entanglement bonds



(a)



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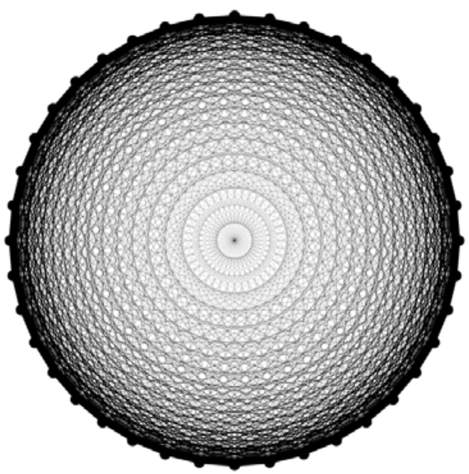

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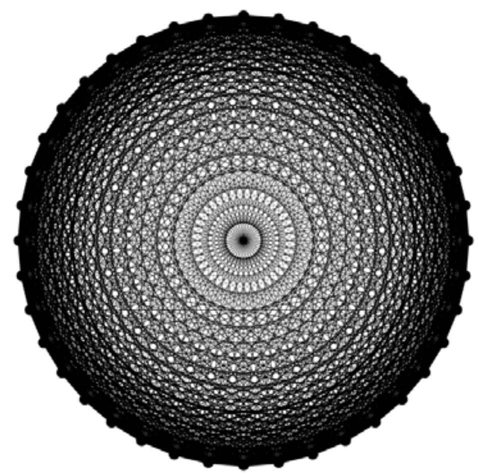
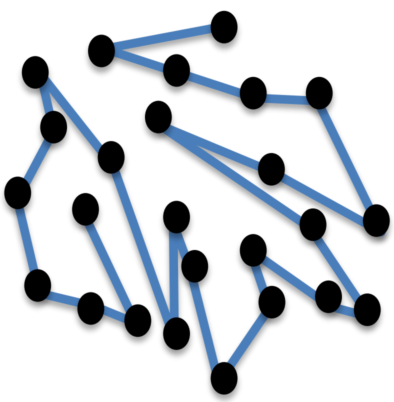
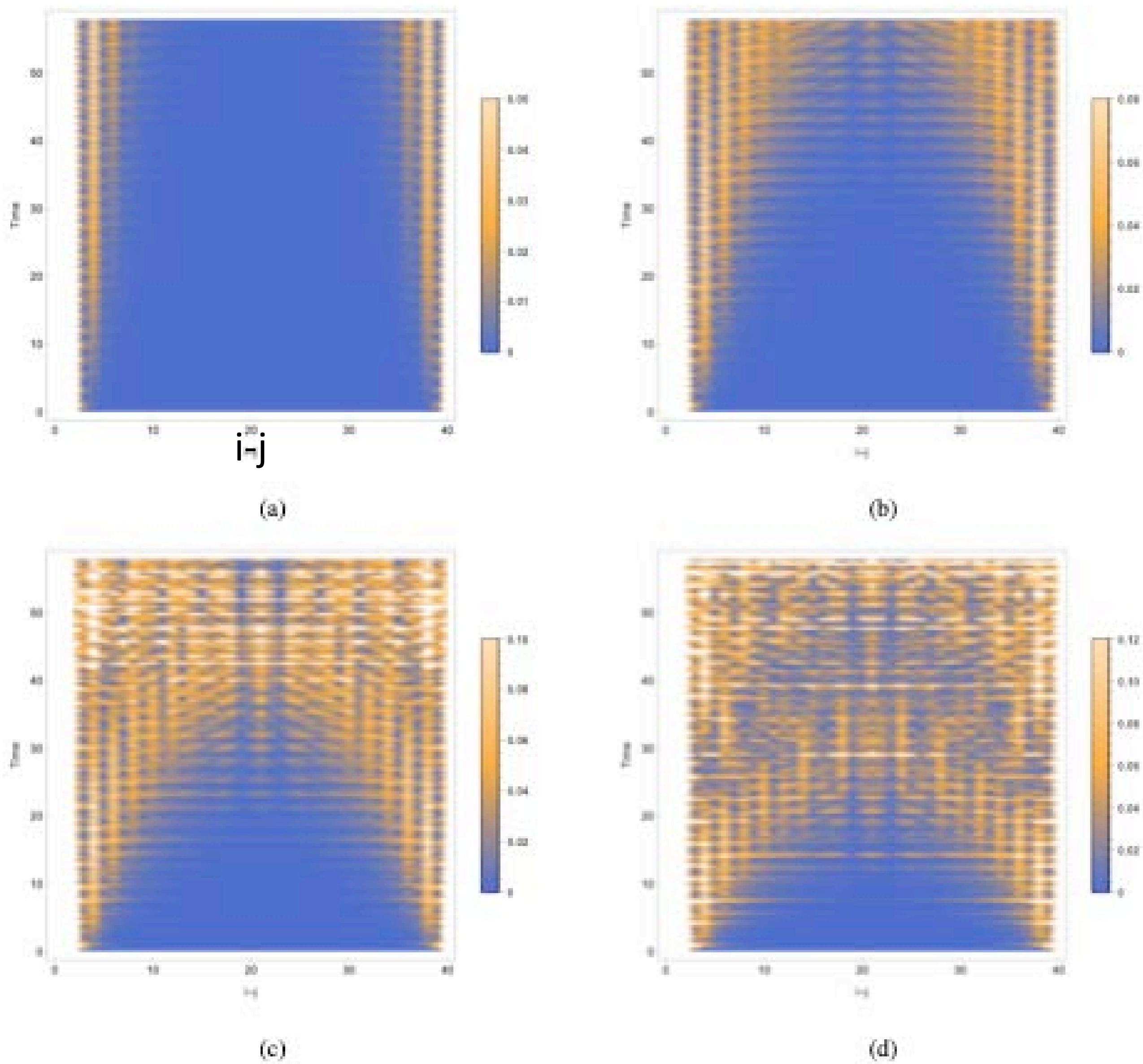


(d)

State dependent spread of entanglement

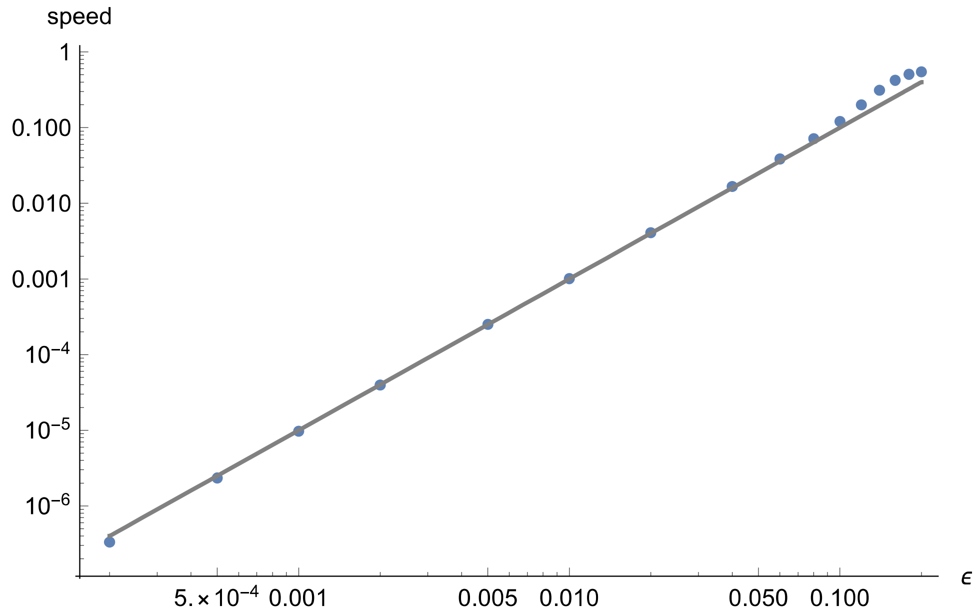
$T_{ij}(t)$





State dependent coordinate speed of entanglement spread

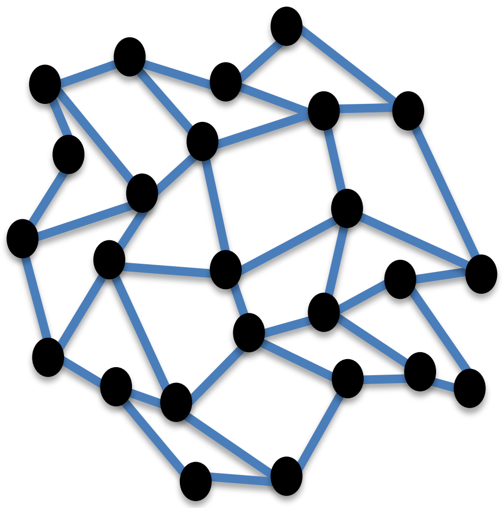
Coordinate Speed at which the range of entanglement bonds grow

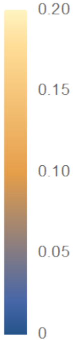


Entanglement entropy of the initial state

- Coordinate speed strongly depends on the initial state because geometry that determines proper distance is state dependent
- No Lieb-Robinson type bound for the coordinate speed of the entanglement spread

State dependent dimensionality

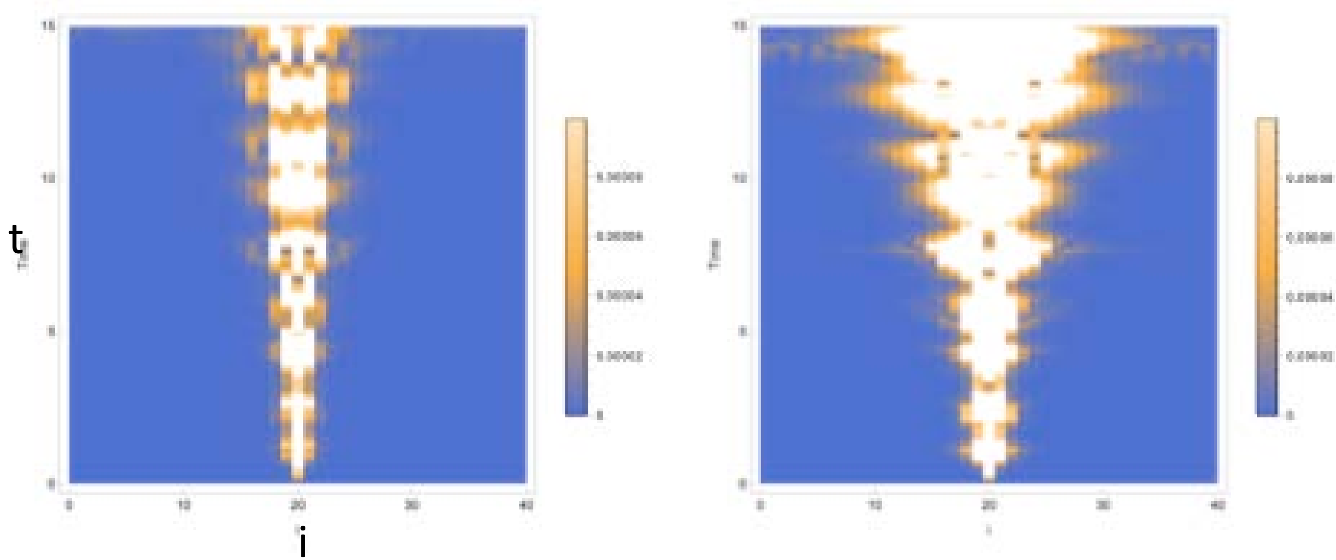




Fluctuations around the saddle point

- A small local perturbation added in the initial state propagates on top of the background geometry that is dynamically set by condensate of multi-local collective variables

$$T_{ij} = \bar{T}_{ij} + \delta T_{ij}$$

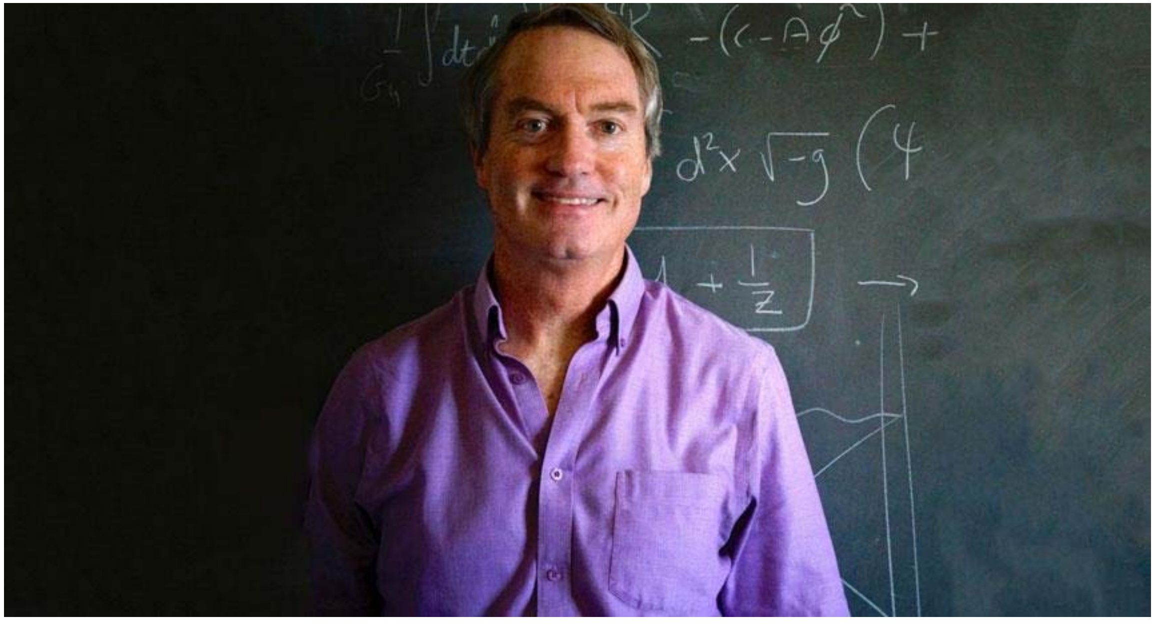


A few remarks

- The dimension, topology and geometry of the emergent local theory are determined by the initial state on which the Hamiltonian acts

- If one insists that the Wheeler De-Witt Hamiltonian arises from matter fields in a similar way, the underlying Hamiltonian for the the matter fields must be relatively local [1803.00556]

- Relatively local Hamiltonian is one way to realize ER=EPR [Maldacena, Susskind]



A giant in physics

who was kind and generous to others