THE LARGE N HARMONIC (AND ANHARMONIC) OSCILLATOR AS A STRING THEORY

N. ITZHAKI

BASED (MOSTLY) ON HEP-TH/0408180 WITH J. MCGREEVY.

INTRODUCTION

THE GAUGE/STRING THEORY DUALITY IS USEFUL TO

1- LEARN ABOUT GAUGE THEORIES FROM STRING THEORY. (THE GOAL OF THIS PROGRAM)

2- LEARN ABOUT THE S.T FROM THE G.T

* I'LL DESCRIBE ANOTHER EXAMPLE OF THE GAUGE/STRING DUALITY THAT FOR THE MOST PART IS USELESS FOR 1 BUT QUITE INTERESTING FOR 2.

OUR STARTING POINT:

$$S=\frac{1}{2}\int dt\, Tr\left((D_0 x)^2 - x^2\right)$$ (BERENSTEIN)

AS USUAL IN GAUGED Q.M

$A_0=0 \Rightarrow$ PROJECTS ON SINGLET

||

N FREE FERMIONS

IN

THE ENERGY LEVELS ARE

$$E_j = j+\frac{1}{2}$$

SO THE GROUND STATE ENERGY

IS $$E_0^{tot}=\frac{1}{2}+\frac{3}{2}+\dots+\frac{(2N-1)}{2}=\frac{N^2}{2}+O$$

* NORMALLY (AdS/CFT, USUAL $c=1$) THE WAY THE GAUGE/STRING DUALITY WORK IS:

STRINGS IN D+1 ⟷ GAUGE THEORY IN D-DIM.

EXTRA (LIOUVILLE) DIRECTION SPACE-LIKE

* HERE

1- NO BOUNDARY.

2- THE LIOUVILLE DIR. IS TIME-LIKE

FIRST CONCRETE EXAMPLE OF EMERGENT TIME.

PLAN OF THE TALK

- THE MODEL AND ITS SYMMETRIES

  HARMONIC OSCILLATOR $\Longleftrightarrow$ CHIRAL BOSON

- THE DUAL STRING THEORY

- COMPARISON:

  1- CORRELATION FUNCTION

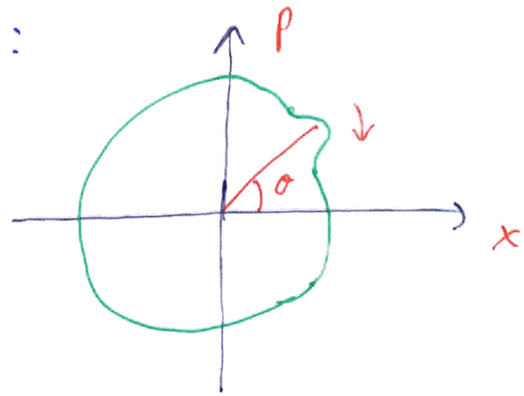
  2- GROUND STATE ENERGY (INCLUDING LOOPS)

- STRING THEORY FOR:

  1- NON TRIVIAL EXPECTATION VALUES.

  2- ANHARMONIC OSCILLATOR.

SEMI-CLASS:



LOOKS LIKE A CHIRAL BOSON.

WE CAN MAKE THIS PRECISE:

$$Z = \operatorname{tr} q^H = q^{\frac{N}{2}} \sum_{k_1=0}^{\infty} \sum_{k_2=k_1+1}^{\infty} \cdots \sum_{k_N=k_{N-1}+1}^{\infty} q^{\sum_1^N k_n}$$

DO THE SUMS WE GET

$$Z = q^{\frac{N^2}{2}} \prod_{n=1}^{N} \frac{1}{1-q^n}$$

$$\Rightarrow \quad H = \frac{\alpha_0^2}{2} + \sum_{n=1}^{N} \alpha_{-n}\alpha_n \,,$$

$$\alpha_0 = N \,, \qquad [\alpha_n, \alpha_{-n}] = n\,\delta_{n+m}$$

IN TERMS OF THE Q.M:

$Tr((a^\dagger)^k) \Longleftrightarrow \alpha_k$

$\left(a = \frac{1}{\sqrt{2}}(x+ip), \quad a^\dagger = \frac{1}{\sqrt{2}}(x-ip) \right)$

* HILBERT SPACE LIKE IN QCD_2

HAMILTONIAN IS DIFF.

(# OF BOXES IN A TABLEAU NOT $C_2(R)$)

SYMMETRIES

SEMI-CLASSICALLY:

$u = \frac{1}{\sqrt{2}}(x+ip), \quad v = \frac{1}{\sqrt{2}}(x-ip)$

$\{u,v\}_{P.B} = i$

HAMILTONIAN IS $H = uv$

$\Rightarrow u(t) = e^{-it}u(0), \quad v(t) = e^{it}v(0)$

SO THESE ARE ALL CONSERVED CHARGES:

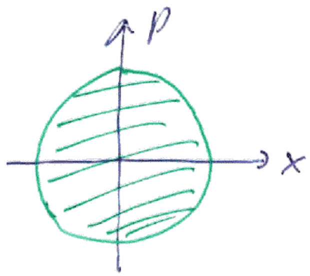
$Q_{n,m} = e^{i(m-n)t} u^n v^m$

THAT FORM THE W_∞ ALGEBRA

$\{Q_{nm}, Q_{n'm'}\} = i(nm' - mn') Q_{n+n'-1, m+m'-1}$

THE GROUND STATE

IS $uv=N$



EXCITATIONS WE GET BY

AREA-PRESERVING TRANSFORMATIONS

$$h_{nm}=u^n v^m$$

WHEN ACTING ON THE GROUND

STATE ONLY $|n-m|$ MATTERS.

THE SAME AT THE Q.M

LEVEL:

$$Tr(a^\dagger)|0\rangle \simeq Tr(a a^{\dagger 2})|0\rangle$$

BUT

$$Tr(a^\dagger) \neq Tr(a a^{\dagger 2})$$

A LITTLE PUZZLE:

TOO MANY QUANTUM OPERATORS

PER S.C DEFORMATION.

EXAMPLE:

$Tr(a^2 a^{\dagger 2})$ AND $Tr(a a^\dagger a a^\dagger)$

ARE DIFFERENT

BUT CORRESPOND TO THE

SAME S.C DEFORMATION

$$h_{2,2}$$

SOLUTION:

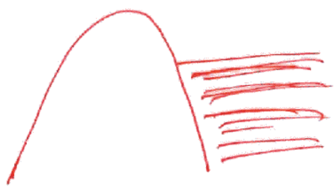
IS GAUGE INVARIANCE

$$j=[x, p_0 x]=i[a,a^\dagger]=0$$

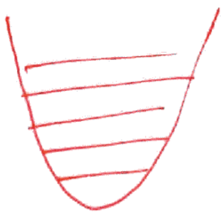
DUAL STRING THEORY

WE KNOW THAT THE USUAL $c=1$ IS DUAL TO

$$u(x) = \frac{1}{2\alpha'} x^2$$



$\Rightarrow$ TO GET



WE NEED TO TAKE

$$\alpha' \to -\alpha'$$

SO THE STRING HAS A NEGATIVE TENSION!!!

MIGHT BE OK IN 2D.

A BETTER WAY TO THINK ABOUT THIS IS TO KEEP $\alpha' > 0$ AND WICK ROTATE ALL DIRECTIONS.

SO WE HAVE $D-1$ TIME-LIKE DIRECTIONS AND ONLY ONE SPATIAL DIR.

AGAIN TROUBLES IN $D>2$ BUT FOR $D=2$ JUST MEANS THAT

SPATIAL $\Longleftrightarrow$ TIME

SO WE START WITH

$$T(z)=\frac{1}{\alpha'}\left(\partial x\partial x-\partial\varphi\partial\varphi\right)+\frac{Q}{\sqrt{\alpha'}}\partial^2\varphi,\quad Q=b+\frac{1}{b}=2$$

AND

$$S=\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}\left(-\partial_\alpha x\partial^\alpha x+\partial_\alpha\varphi\partial^\alpha\varphi+\sqrt{\alpha'}R^{(2)}Q\varphi+\mu_0 e^{2b\varphi/\sqrt{\alpha'}}\right)$$

NOW WE CAN TAKE

$\alpha'\to-\alpha'$ OR $x\to ix$, $\varphi\to i\varphi$

EITHER WAY WE GET

$$T(z)=\frac{1}{\alpha'}\left(\partial\varphi\partial\varphi-\partial x\partial x\right)+i\frac{Q}{\sqrt{\alpha'}}\partial^2\varphi$$

AND

$$S=\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}\left(-\partial_\alpha\varphi\partial^\alpha\varphi+\partial_\alpha x\partial^\alpha x+i\sqrt{\alpha'}R^{(2)}Q\varphi+\mu_0 e^{i2b\varphi/\sqrt{\alpha'}}\right)$$

-10-

FROM THE HARMONIC OSCILLATOR POV MOST CONVENIENT TO WORK WITH $\alpha'=1$

$$\left(u(x)=-\frac{1}{2\alpha'}x^2\iff u(x)=-\frac{x^2}{2}\right)$$

SO THAT'S WHAT WE DO FROM NOW ON.

NOW THE STRING COUPLING CONSTANT IS:

$$g_s=e^{i2\varphi}$$

DIFFERS FROM THE USUAL CASE:

1- NO WEAKLY AND STRONGLY COUPLED REGION $|g_s|=1$

$\Rightarrow$ NO GOOD EXPANSION PARAMETER ?

-11-

2- WE CAN COMPACTIFY THE LIOUVILLE DIR. φ.

$$R = \frac{m}{2}, \quad m = 1, 2, 3, \ldots$$

FOR THE PER. CLOSED STRING. BUT OPEN STRING PHYSICS IMPLIES

$$m = 2, 4, 6, \ldots$$

OUR PROPOSED STRING DUAL HAS $R = 1$ SO

$$\varphi \sim \varphi + 2\pi$$

A GOOD WAY TO THINK ABOUT THIS:

X IS THE Q.M TIME

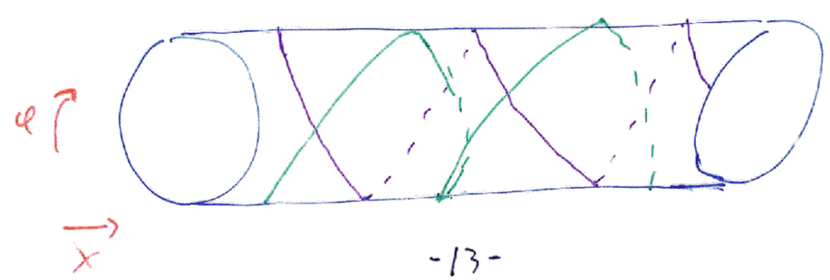
φ $\widetilde{\text{IS}}$ θ (WHICH HAS THE SAME PERIODICITY)

INDEED THE SAME VELOCITY

$$\frac{d\varphi}{dx} = 1 = \frac{d\theta}{dt}$$

BUT

BOTH CHIRALITIES IN THE BULK AND ONLY ONE IN THE Q.M



LET'S START TO COMPARE:

ON THE Q.M SIDE WE CAN CALCULATE

$$\mathcal{O}(bra; ket) = \langle bra | ket \rangle$$

(SINCE TIME DEP. IS TRIVIAL THESE CONTAIN ALL G.I INFO.)

SIMPLEST EXAMPLE:

$$\left.\begin{array}{l} \langle bra| = \langle 0|\, tr(a^{k_1}) \\ |ket\rangle = tr((a^\dagger)^{k_2})\,|0\rangle \end{array}\right\} \Rightarrow \mathcal{O}(k_1;k_2) \sim \delta_{k_1,k_2} N^{k_1}$$

AS USUAL THERE ARE $1/N^2$ CORR.

$$\mathcal{O}(k_1;k_2) = \delta_{k_1,k_2} \sum_{l=0}^{[k_1/2]} C_l(k_1)\, N^{k_1-2l}$$

WE CAN ALSO CALCULATE THE ANALOG OF A 3-POINT FUNCTION

$\langle BRA| = \langle 0|\, Tr(a^k)$, $\quad |KET\rangle = Tr(a^\dagger)^{k_1}\, Tr(a^\dagger)^{k_2}\,|0\rangle$

GIVES

$$\mathcal{O}(k;k_1,k_2) \sim \delta_{k,k_1+k_2}\, N^{k-1}$$

ON THE S.T SIDE WE HAVE

$$T_k^{\pm} = c\bar{c}\, \underbrace{e^{-ik(x\pm\varphi)}}_{\substack{\text{TACHYON} \\ \text{W.F}}}\; \underbrace{e^{i2\varphi}}_{\substack{\text{STRING} \\ \text{C.C}}}$$

WE CAN CALCULATE S-MATRIX AMPLITUDES:

$$A(\underbrace{k_1, k_2, \dots, k_{n^+}}_{n^+ - \text{POSITIVE CHIRALITY}};\ \underbrace{k_{n^+ +1}, \dots, k_{n^+ + n^-}}_{n^- - \text{NEGATIVE CHIRALITY}}) =$$

$$= \int Dx D\varphi \, e^{-S} \prod_{i=1}^{n_+ + n_-} V_i$$

LIKE IN SINE-GORDON WE DO

$$= \int Dx D\varphi \, e^{-S_0} \sum_{n=0}^{\infty} \frac{\mu_0^n}{n!} \Big(\underbrace{\int d^2\sigma \sqrt{g}\, e^{i2\varphi}}_{\text{SCREENING OPERATOR}} \Big)^n \prod_{1}^{n_+ + n_-} V_i$$

* MUCH BETTER BECAUSE OF THE S_0.

* EVEN BETTER WHEN WE INT. THE ZERO MODE:

$\varphi = \varphi^0 + \tilde{\varphi}$, $x = x^0 + \tilde{x}$

INTEGRATE x^0: $K^+_{tot} + K^-_{tot} = 0$

INTEGRATE φ^0: $2-2g-(n+n_+ + n_-) + \frac{1}{2}(K^+_{tot} - K^-_{tot}) = 0$

$\Downarrow$

$$\boxed{K^-_{tot} = -K^+_{tot} = 2-2g-(n+n_+ + n_-)}$$

* ONE IMPORTANT CONCLUSION IS THAT μ_0 IS INDEED THE GENUS EXPANSION PARAMETER.

$\Downarrow$

$$\mu_0 \sim N$$

* $1 \to 1$ AMPLITUDE:

$A(k;-k) \sim \mu_0^k \sim N^k$ $V(T^+_{k>0} \Leftrightarrow Tr((a^+)^k)$

$T^-_{k<0} \Leftrightarrow Tr(a^k))$

* CHIRALITY WORKS FINE: NO $T^+_{k<0}$ AND $T^-_{k>0}$.

* JUST LIKE IN QM $g \le [\frac{k}{2}]$

* $1 \to 2$: $A(k;-k_1,-k_2) \sim \delta_{k,k_1+k_2}\, \mu_0^{k-1}$ ✓

A CLOSER LOOK

ON THE Q.M SIDE THE $1 \to 1$ IS

$$O(k;k) = k N^{k}$$

AND THE $1 \to m$ GIVES

$$O(k;k_1,k_2,\dots,k_m) = k(k-1)\cdots(k-m+2)\,k_1 k_2 \cdots k_m \times N^{k-m+1}$$

ON THE S.T SIDE

1- FOR THE SCREENING OPERATORS

$$\mu = \pi \frac{\mu_0\,\Gamma(b^2)}{\Gamma(1-b^2)} = \text{FIXED} = N \qquad b \to 1 \qquad \mu_0 \to \infty$$

2- IF ONE OF THE $k_i > 0$ WE GET ZERO.

3- WITH LEG FACTORS f_i

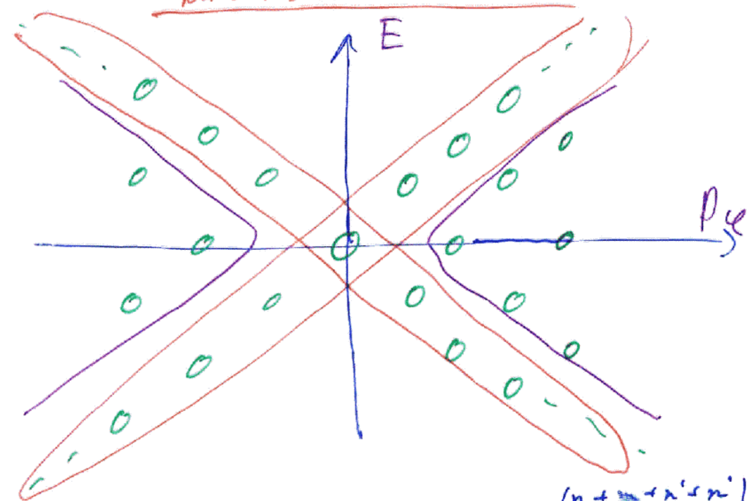
$$O = \Pi f_i\, A$$

THERE OTHER SINGLE TRACE OPERATORS

$$Tr\left(a^m (a^\dagger)^n\right)$$

THESE SHOULD CORRESPOND TO OTHER CLOSED STRING MODES IN THE BULK.

DISCRETE STATES



$$A \sim \Gamma\left((n-m)-(n'-m')\right) N^{(n+m+n'+m')}$$

BEYOND TREE LEVEL

FOR THE VACUUM ENERGY WE HAVE

$$n = 2 - 2g$$

SO CONTRIBUTIONS FROM

1- $g=0 \Rightarrow n=2 \Rightarrow E_S \sim N^2$

2- $g=1 \Rightarrow n=0 \Rightarrow E_T \sim N^0$

BUT ON THE Q.M. SIDE

$$E = \frac{N^2}{2} + 0$$

$$\Downarrow$$

$$E_T = 0$$

A SIMPLER WAY TO CALCULATE THE VACUUM ENERGY IS TAKE $X \sim X + \beta$ AND LOOK AT

$$\ln Z = -\beta E_0 \quad \beta \to \infty$$

-21-

A RELATIVELY SIMPLE CALCULATION BECAUSE THERE IS NO BACKGROUND CHARGE ($R_T^{(2)} = 0$, NO INSERTIONS).

$$\Downarrow$$

IN THAT CAL. X AND φ ARE TWO FREE BOSONS.

IN THE USUAL C=1 CASE THIS WAS DONE: (BERSHADSKY, KLEBANOV)

$$X \sim X + 2\pi R \Rightarrow \frac{Z}{V_{Lio}} = c\left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right)$$

HERE WE TAKE $X \longleftrightarrow \varphi$

$$\Downarrow$$

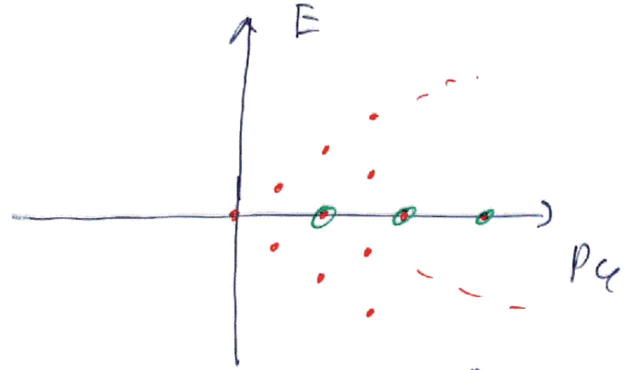
$$V_L \to V_X = \beta, \quad \frac{Z}{\beta} = ic\left(\frac{R}{\sqrt{\alpha'}} - \frac{\sqrt{\alpha'}}{R}\right)$$

SO FOR $R = 1 = \sqrt{\alpha'}$

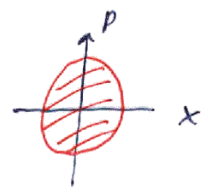
$$E_T = 0 \qquad 0 = 1 + 12\sum n$$

-22-

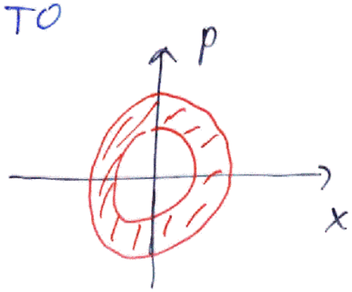
WHAT ABOUT THE OTHER MODES ?

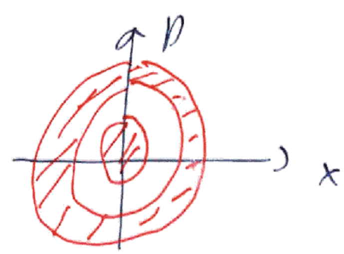


THE ⊙ TAKE



TO

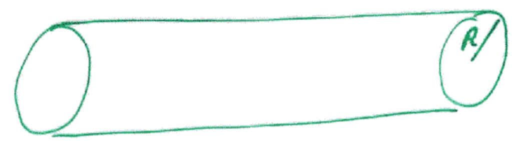




ETC.

CONCLUSIONS

A NEW EXAMPLE OF GAUGE/STRING DUALITY, IN WHICH THE BULK TIME IS AN EMERGENT DIRECTION.



SHOULD BE INTERESTING TO TAKE R TO BE LARGE

$R=1 \Rightarrow R=k$

* INTERESTING ALSO BECAUSE APPEARS TO BE RELATED TO THE FQHE.
* CAN THIS BE GENERALIZED TO HIGHER DIM? FOR EXAMPLE IMAGINARY NS-5-BRANES

CONCLUSIONS

* A NEW EXAMPLE OF GAUGE/STRING DUALITY.

  IT IS INTERESTING BECAUSE IT SEEMS TO WORK DIFFERENTLY:

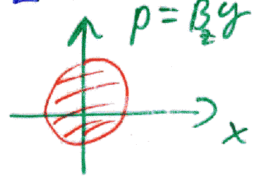
  1- THE EXTRA BULK DIRECTION IS TIME-LIKE.

  2- MORE LIKE COS.

  3- DIFF. BULK OBSERVABLES.

* WOULD BE VERY INTERESTING IF COULD BE GEN. TO HIGHER DIM.

* RELATION WITH QHE:



DO OTHER R'S (2, 3, ...) GIVE THE FQHE?