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Perturbative Saturation -
The long and the short of it.

Why is high energy QCD interesting?

Experimentally DESY, LHC, RHIC

Theoretically: QCD has two facets

a) short distance (perturbative) physics
(high Q^2 DIS, Drell-Yan, jets)

b) long distance (nonperturbative) physics
(masses, form factors ...)

High energy scattering nontrivially involves both:

Maybe through one we can learn about the other ?!

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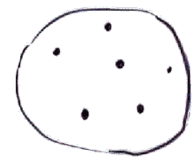
Why hard ?

Hadron en face



There are fast quantum fluctuations around "partons" - virtual gluons
They are not seen if too fast.

Higher energy - Lorentz boost the hadron.
Time gets Lorentz dilated.
Fast fluctuations freeze - the disk "blackens".

Hit it with a small probe - it will scatter.
Intrinsic transverse momentum:

$$Q^2 \sim \frac{1}{S_\perp} \quad \text{grows with blackness}$$

Expect Q^2 to dominate final states -

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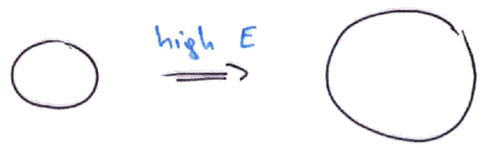
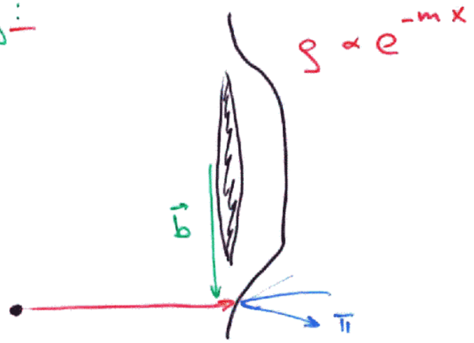
Why soft?

Transverse size effectively grows with energy.



Heisenberg:



For inelastic scattering – energy in the overlap > m_π

$$\sigma_{tot} = 2\pi b_{max}^2$$

$$\frac{s}{m} e^{-m_\pi b_{max}} = m_\pi$$

$$b_{max} \simeq \frac{1}{m_\pi} \ln \frac{s}{m_\pi^2}$$

$$\boxed{\sigma_{tot} < \frac{2\pi}{m^2} \ln^2 \frac{s}{m_\pi^2}}$$ Froissart bound

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So at high energy:

* global aspects of scattering are determined by peripheral long distance physics (σ_{tot}, $\sigma_{elastic}$ ...)
* the final state multiplicities are dominated by hard perturbative component ("minijets")

Perturbative QCD saturation approach

Can we make sense of it all perturbatively?

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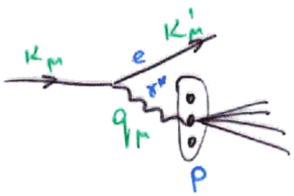
Scattering in QCD - parton model.

Deeply Inelastic Scattering



$\} X$

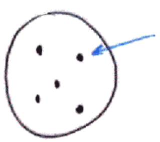
$ep \to eX$

$Q^2 = -q^2 \gg m_H$

$x = \dfrac{Q^2}{2P\cdot q}$ $\quad \left[W^2 = (p+q)^2 = Q^2\left(\frac{1}{x}-1\right) \right]$

W: energy of the $\gamma^* P$ system

Parton model:



partons $\equiv$ quarks & gluons

x - fraction of the proton momentum carried by the struck parton

γ^* - "partonometer" of size $\sim \frac{1}{Q}$

$$\boxed{\sigma_{DIS} \simeq \frac{\alpha_{em}}{Q^2} \sum_i e_i^2 N^i_{parton}(x)}$$

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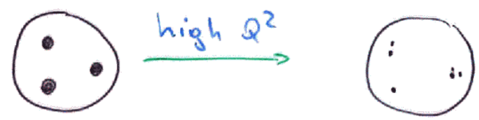
Pert QCD - $N_i(x, Q^2)$ depends on resolution (Q^2)



DGLAP evolution

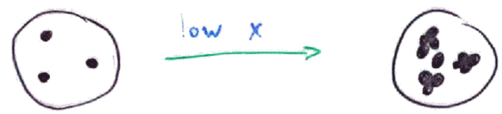
$$\frac{d\, N_i(x\, Q^2)}{d \ln Q^2} = \alpha_s \int_y P_{ij}(x,y)\, N_j(y, Q^2)$$

$$\delta N = \left| {}^{x}_{y} \ \ {}_{y-x} \right|^2 \otimes N(y)$$

DGLAP fits work well

But what happens at lower x ?

Partons split, but mainly on the same transverse scale - partonic density grows.



"Gluon cloud" cools down - extra gluons "materialize".

The protons become blacker

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Low x linear BFKL evolution:

$$\frac{d\varphi(\bar{k})}{d\ln x} = \alpha_s \int_{\bar{p}} K(\bar{k},\bar{p})\,\varphi(\bar{p})$$

[$\varphi(Q) = \frac{\partial G(Q^2)}{\partial \ln Q^2}$ - gluon density at fixed transverse momentum]

BFKL is a linear equation:

# of emitted gluons is proportional to the number of existing gluons -

no interaction between gluons!

But at low enough x gluons overlap $\Rightarrow$ must interact.

Evolution must become nonlinear.

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BFKL itself tells us it is wrong:

$$\varphi(k) \sim \exp\left\{\omega_0 \ln\frac{1}{x} - \frac{\ln^2(k^2/k_0^2)}{a^2 \ln 1/x}\right\}$$

$\omega_0 = 4\ln 2\,\frac{\alpha_s N}{\pi}$; $a^2 \simeq \#\,\alpha_s$

Cross section:

$$\sigma \simeq \int \frac{dk^2}{k^2}\varphi(k) \propto e^{\omega_0 \ln 1/x} = s^{\omega_0}$$

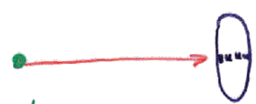
Froissart bound is violated !!

Worse than that: at fixed impact par.:

$$T_{Y=\ln 1/x}(r,R) = \alpha^2\left(\frac{r}{R}\right)^2 \exp\{\omega_0 Y - \dots\} >> 1$$

Unitarity is violated at fixed b.

The same problem of nonlinearity:



Many gluons at same impact parameter: scattering probability is NOT proportional to the number of gluons.

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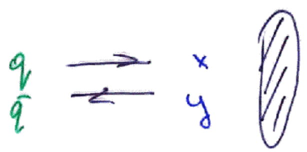
Multiple scatterings - nonlinear QCD evolution

Change frame & probe: energy in the probe, probe is a color dipole



$N(xy)$ - dipole scattering probability

Boost the dipole - more gluons appear in its wave function

Probability to find one extra gluon at coordinate z is the intensity of "e-m" field

$$P(z) = \int \frac{dk^+}{2\pi}\left[\mathcal{A}(k^+,z)\right]^2 = \frac{\alpha_s N_c}{2\pi}\ln\frac{1}{x}\,\frac{(x-y)^2}{(x-z)^2(y-z)^2}$$

So under boost:

$$\rightleftarrows_{y}^{x} \Longrightarrow \left[1-\int d^2z\, P(z)\right]^{1/2} \rightleftarrows_{y}^{x} + \int d^2z\, P(z)^{1/2} \rightleftarrows_{y}^{z\,x}$$

10^4

Names & acronyms

Gribov - Levin - Ryskin

Mueller - Qiu

$$\frac{dN}{dY} \sim N - N^2 \quad \text{- Kovchegov}$$

$$\left.\begin{aligned}\frac{dN_1}{dY} &\sim N_1 - N_2\\ \frac{dN_2}{dY} &\sim N_2 - N_4\\ &\vdots\end{aligned}\right\}\ \text{Balitsky - JIMWLK}$$

Color Glass Condensate

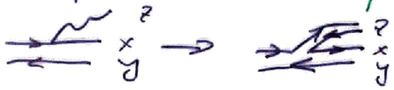
$\boxed{\text{CGC}}$

Cold Gluon Cloud

At large N_c

a ∿∿ b equivalent to $\alpha \longrightarrow \alpha'$, $\beta \longleftarrow \beta'$

So:



$$|xy\rangle \xrightarrow{\text{boost}} \Big[1-\int_z P(z)\Big]^{1/2}|xy\rangle + \int_z P^{1/2}(z)\,|(xz),(yz)\rangle$$

So for the scattering probability:

$$N(xz,yz) = N(xz)+N(yz) - \underbrace{P(xz,yz)}_{\text{probability that both dipoles scatter}}$$

"Formal" large N_c: $P(xz,yz) = N(xz)\,N(yz)$

All said and done:

$$\frac{dN(xy)}{d\ln 1/x} = \frac{\alpha_s N_c}{2\pi}\int d^2z\,\frac{(x-y)^2}{(x-z)^2(y-z)^2}\Big[\underbrace{N(xz)+N(yz)-N(yx)}_{\text{BFKL}} - \underbrace{N(xz)\,N(yz)}_{\text{multiple scattering correction}}\Big]$$

Amplitude is unitary:

$$\frac{dN}{d\ln 1/x}\Big|_{N=1} = 0$$

Qualitative behavior:

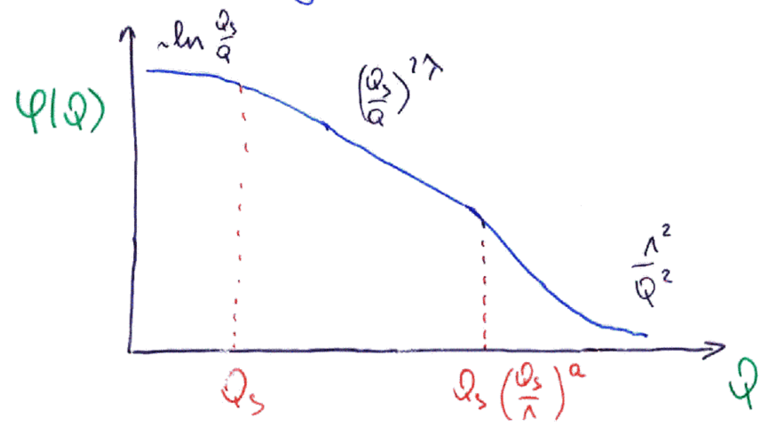
$$N(x-y) = \begin{cases} \left[Q_s^2(x-y)^2\right]^{\lambda} & (x-y) < Q_s^{-1} \\ 1 & |x-y| > Q_s^{-1} \end{cases}$$

$\lambda \leftarrow$ anomalous dimension

$$Q_s^2 \propto \exp\Big\{\frac{4\alpha_s N}{\pi}\#\,\ln\frac{1}{x}\Big\}$$

$$\lambda \simeq 0.64$$

For intrinsic gluon distribution:



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Qualitatively very striking:

$$\varphi(k) = \left[\frac{Q_s^2(y)}{Q^2}\right]^{\lambda} \quad ; \quad \boxed{\lambda < 1}$$

Very strong shadowing:

$Q_s^2(y) \propto A^{1/3}$ – atomic number

Thus $\varphi_A(k) \propto [A^{1/3}]^{\lambda} \ll A^{1/3}$

Pre(post)dict: hadronic multiplicities strongly suppressed in nuclear collisions relative to scaled p-p multiplicities.

Predict: Cronin enhancement in d-A (or p-A) quickly disappears as energy increases.

[Perhaps RHIC data - but needs a lot of "courage" to make such statement.]

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What about soft physics?

$$\sigma = \int d^2b \, N(\vec{r}, \vec{b}) \qquad \left[\vec{r} = \vec{x} - \vec{y}, \ \vec{b} = \frac{\vec{x}+\vec{y}}{2}\right]$$

Impact parameter

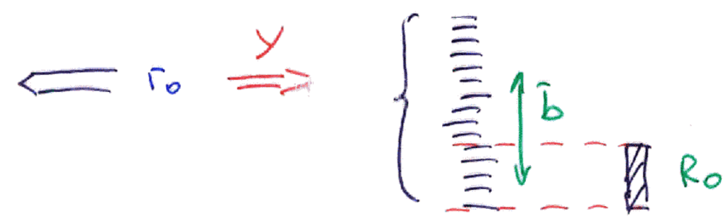
We can understand $\vec{b}$ - dependence:

The number of dipoles in the projectile grows with energy $n(r, b; \underbrace{r_0, b_0}_{\text{parent dipole}})$

n - satisfies BFKL equation (even though N does not)

$$n_y(r, b; r_0) = \frac{1}{r^2} \exp\left\{\omega_0 y - \ln\frac{16 b^2}{r r_0} - \frac{\ln^2 \frac{16 b^2}{r r_0}}{a^2 y}\right\}$$

The projectile swells in space



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Suppose target is characterised by some Q_s

The scattering probability is close to 1 if there is at least one dipole of size Q_s^{-1} in the overlap area:

Equate $n(r = Q_s^{-1}, b) \cdot R^2$ to 1:

$$b_{max}^2 = \frac{1}{16} r_0 Q_s^{-1} \exp\left\{ \frac{\alpha_s N_c}{\pi} \varepsilon Y \right\}$$

$$\varepsilon \approx 7\zeta(3)\left[-1 + \sqrt{1 + \frac{8 \ln 2}{7\zeta(3)}} \right]$$

$$\frac{\alpha_s N_c}{\pi} \frac{\varepsilon}{\omega_0} \simeq 0.87$$

Nonlinear evolution still violates the Froissart bound.

$$\sigma \propto b_{max}^2 \propto s^{\frac{\alpha_s N_c \varepsilon}{\pi}}$$

(But the physics is completely different from BFKL problem)

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Physics: perturbative massless gluons

$\Longrightarrow$ long range Coulomb fields

if $\rho(b) \sim b^{-\lambda}$

b_{max} : $s\,\rho(b) \simeq m_\pi$

$$b_{max} \propto s^{1/\lambda}$$

No cure for soft maladies from nonlinearities...

Really need confinement

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Have we exhausted perturbative approach?

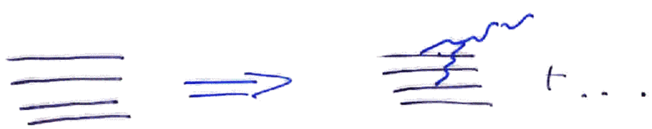
Certainly not.

1. Higher order in α_s correction - may have strong effect on λ, ω etc.

2. "Pomeron loops"

We had:

But at high energy the projectile wave function becomes dense:



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Is understanding of soft physics hopeless?

Maybe not. Maybe "soft pomeron" is perturbative $[\sigma_{tot} \sim s^{0.08}]$

Iff scattering cross section is dominated by black but small constituents.

Constituent quarks?

$\sigma_{tot} \Rightarrow$ diameter of $q \sim 0.3 fm$

There is still room for Coulomb tails (tales)

Need to understand corrections to the leading order...

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Pomeron loops become important when

$$\alpha^2 x^2 n_p(x^2, y) \sim 1$$

Is there a window for multiple scattering to be dominant nonlinearity:

$$\alpha^2 x^2 n_p(x^2, y) \quad \text{vs} \quad N(x)$$

When target is small (dipole etc...),

$N(x) \simeq \alpha^2$ - no window.

If target "dense" to begin with

$N(x, Y_0) \gg \alpha^2$ - then there is window of rapidities.

But eventually both corrections are equally important.