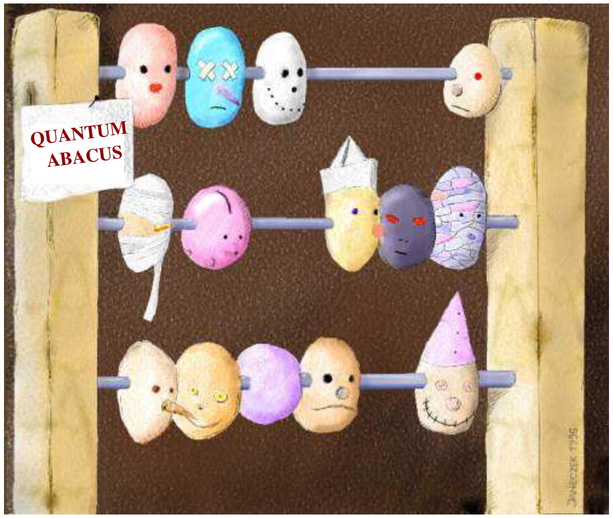
Quantum control:
Using measurements and dissipation



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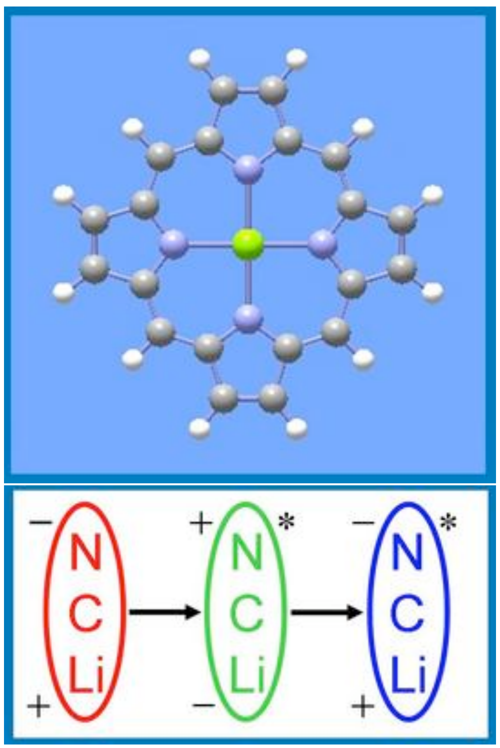
Kavli Institute Santa Barbara, April 2009

I

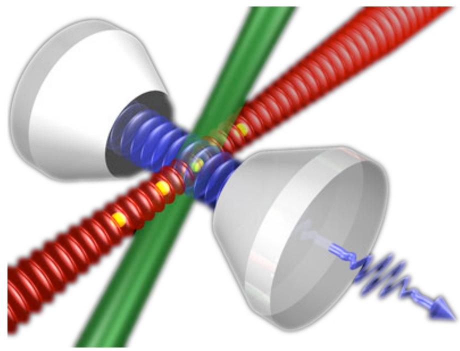
Quantum control: Systems and tasks

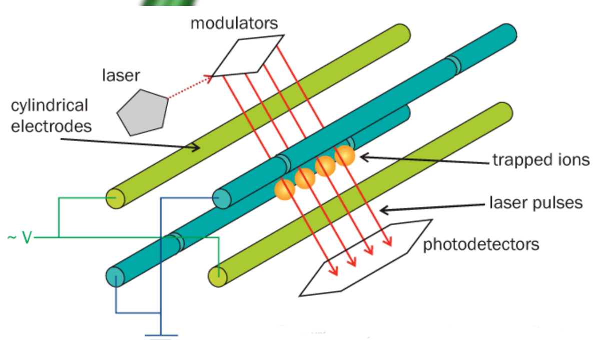
Physical systems

Chemistry

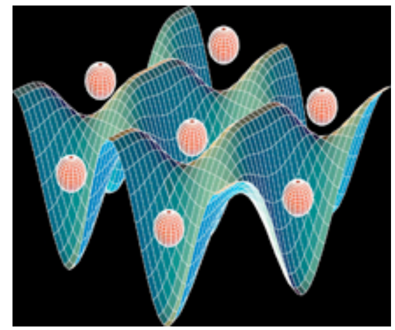


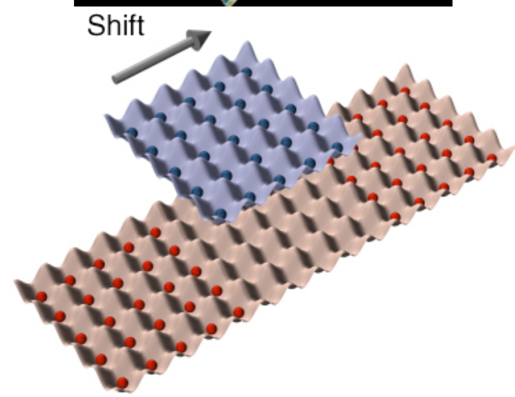
Quantum Optics





Condensed matter





Quantum Information

Quantum control from a quantum information perspective

Tasks:

- Implementation of unitary time evolutions (gate operations)
- Read out measurements
- Transfer initial state into a highly entangled state (one-way QC)
- Transfer of arbitrary states into a highly entangled state (cooling)
- State preparation of the unknown ground state of a Hamiltonian
- Quantum simulations

Related tasks:

- Accurate modelling of physical system
- Ground state cooling
- Transport of qubits

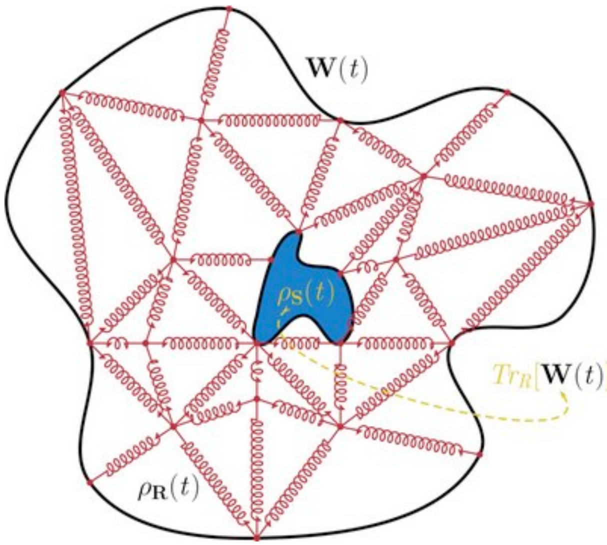
Decoherence

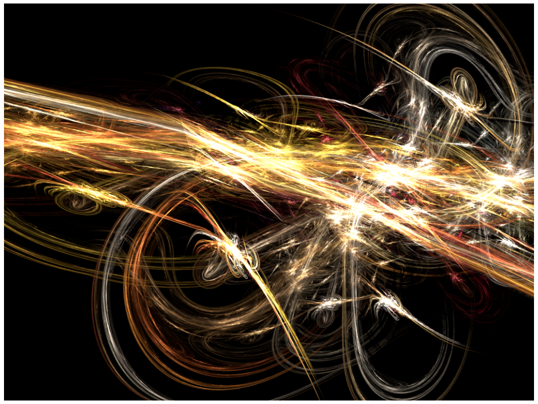
The position of the observer must be defined in order to determine the position of the rainbow. It is as if the act of observation is necessary to define the rainbow's position property, and hence its very existence:
no observer, no rainbow.

Decoherence

Decoherence and dissipation

Decoherence can be viewed as the loss of information from a system into the environment. (Wiki)





In physics, **dissipation** embodies the concept of a dynamical system where important mechanical modes, such as waves or oscillations, lose energy over time, typically due to the action of friction or turbulence. (Wiki)

Quantum control errors

Sources (classical and quantum):

- parameter fluctuations
- systematic errors
- finite level shifts
- phase fluctuations
- random spin flips
- emission of photons
- finite temperatures
- ...

Protection (active and passive):

- system optimisation
- optimal control
- quantum error correction
- decoherence-free states
- topological QC
- dynamical decoupling & bang-bang
- feedback
- using measurements
- using dissipation
- ...

Why use measurements and dissipation?

Unitary operations:

Measurements and dissipation:

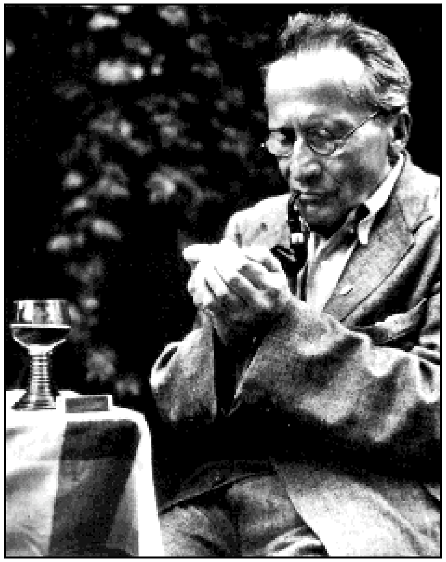
These can result in very robust and easy to implement **unitary operations**, as long as no information is revealed about the state of the qubits.

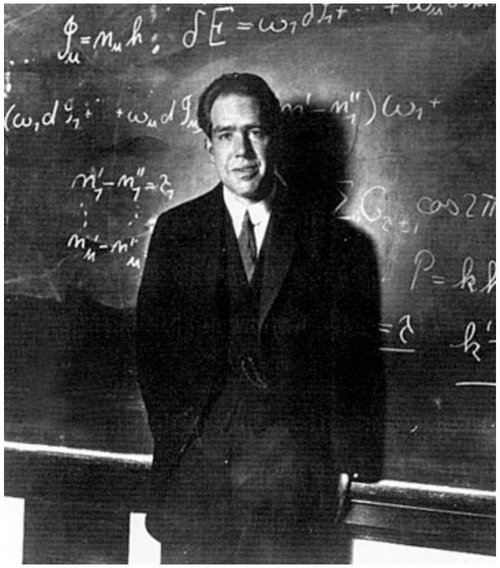
II

Dissipation in quantum optics: Macroscopic quantum jumps [1,2]

[1] Dehmelt, Bull. Am. Phys. Soc. **20**, 60 (1975).
[2] Blatt and Zoller, Eur. J. Phys. **9**, 250 (1988).

Historical debate on quantum jumps





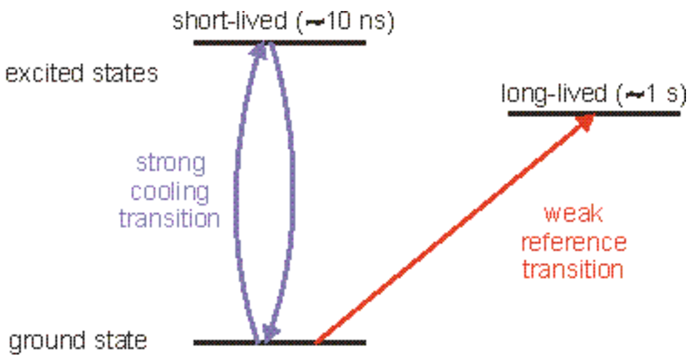
Schrödinger asserted that the application of QM to single systems would necessarily lead to nonsense such as quantum jumps. Bohr argued in response that the problem lay with the physics experiments of the time. [1,2]

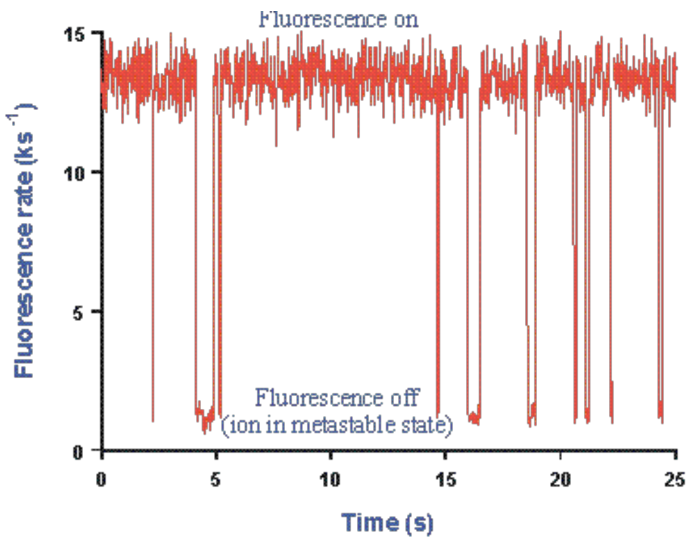
[1] Bohr, Philos. Mag. **26**, 476 (1913).

[2] Blatt and Zoller, Eur. J. Phys. **9**, 250 (1988).

Macroscopic quantum jumps

The existence of a random telegraph signal in the fluorescence of single ions, was predicted as early as 1975 by Dehmelt.[1,2]



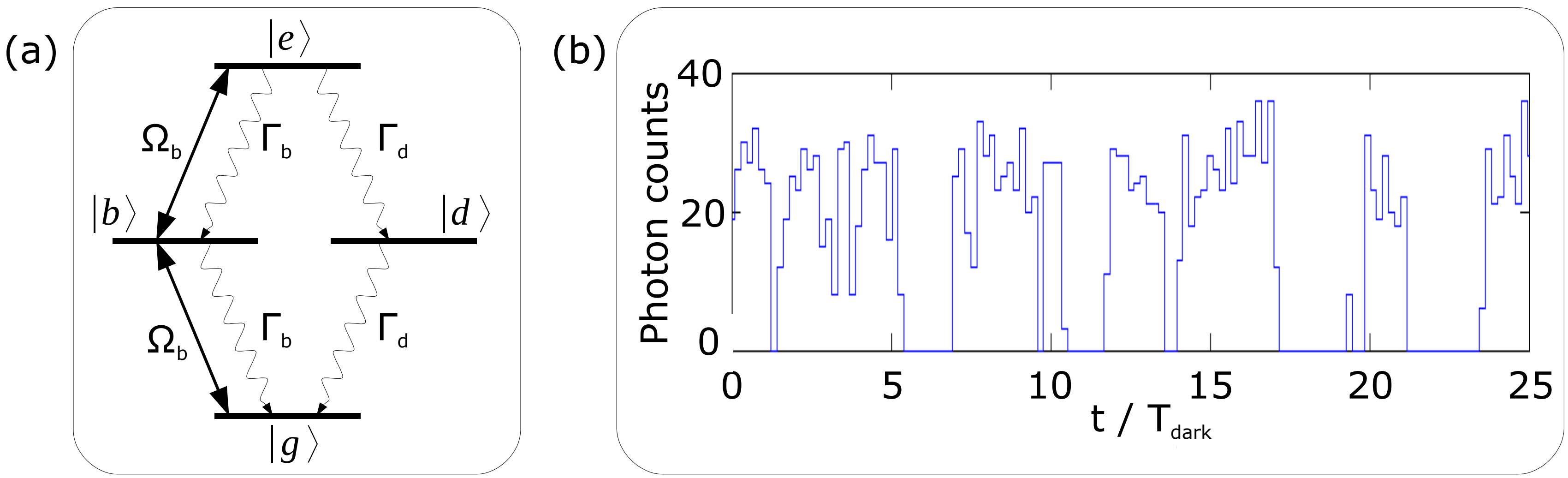


[1]Dehmelt, Bull. Am. Phys. Soc. **20**, 60 (1975).

[2]Nagourney *et al.*, PRL **56**, 2797 (1986); Sauter *et al.*, PRL **57**, 1696 (1986); Bergquist *et al.*, PRL **57**, 1699 (1986).

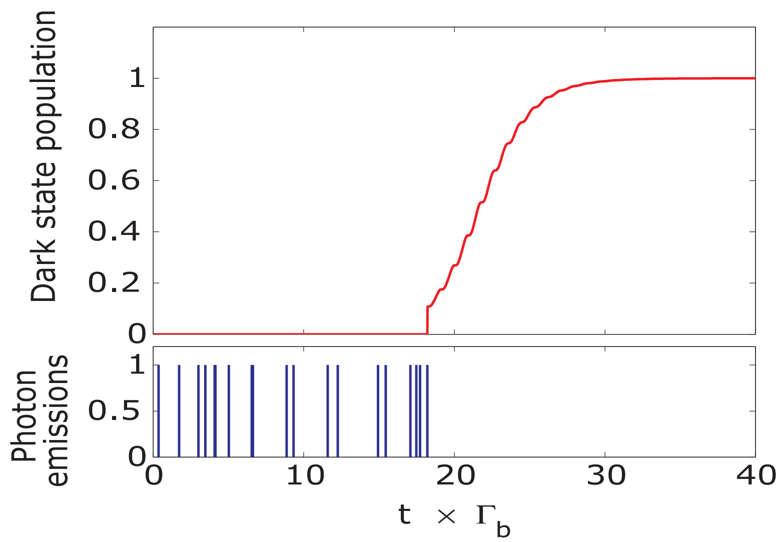
Another level scheme with quantum jumps [1]

We now look at a concrete example:



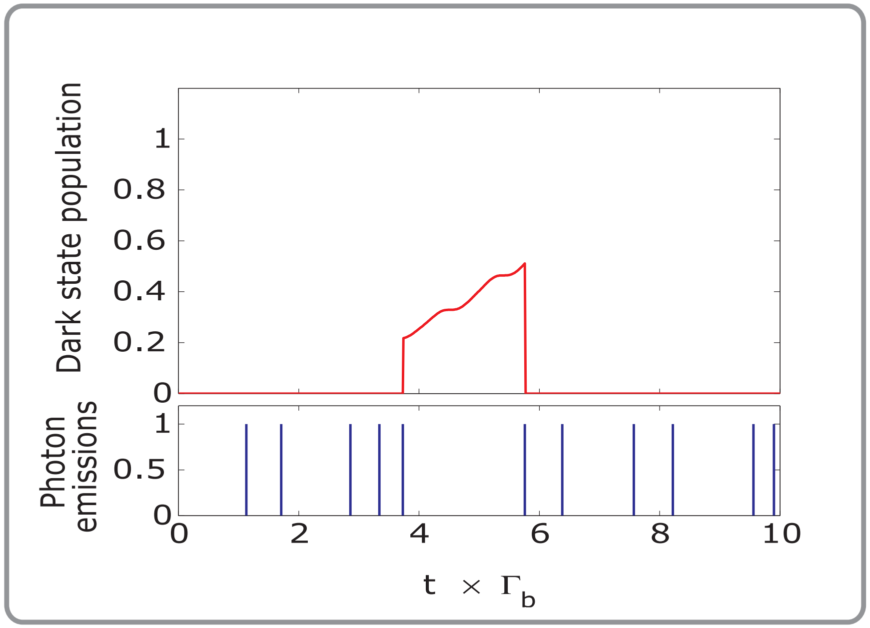
[1] Metz and Beige, PRA **76**, 022331 (2007).

Transition into a dark period



Possible trajectory of the four-level toy model for $\Omega_\mathrm{b} = \Gamma_\mathrm{b}$ and $\Gamma_\mathrm{d} = 10^{-2}\,\Gamma_\mathrm{b}$. The upper figure shows the population in the dark state $|b\rangle$; the vertical lines mark photon emissions. The population in $|b\rangle$ eventually reaches one.

Photon emissions within a light period



Again, the spontaneous emission of a photon results in the build up of population in $|b\rangle$. This time, another photon is emitted before the dark state population reaches one. The system remains in a macroscopic light period.

Quantum jump description

The no-photon evolution:

$$
\begin{aligned}
H_{\rm cond} &= \tfrac{1}{2}\hbar\Omega_{\rm b}\big[\,|b\rangle\langle e| + |g\rangle\langle b| + \text{H.c.}\big] \\
&\quad -\tfrac{\rm i}{2}\hbar\Gamma_{\rm d}\big[\,|b\rangle\langle b| + |d\rangle\langle d| + 2\,|e\rangle\langle e|\,\big] \\
&\quad -\tfrac{\rm i}{2}\hbar\Gamma_{\rm b}\big[\,|b\rangle\langle b| + |e\rangle\langle e|\,\big]
\end{aligned}
$$

Reset Operators:

$$
\begin{aligned}
R_{\rm d} &= |d\rangle\langle e| + |g\rangle\langle d| + |b\rangle\langle e| + |g\rangle\langle b| \\
R_{\rm b} &= |b\rangle\langle e| + |g\rangle\langle b|
\end{aligned}
$$

Characteristic time scales:

$$
T_{\rm dark} = \frac{1}{\Gamma_{\rm d}}\,, \quad T_{\rm light} = \frac{3 + 2\,x^2 + x^4}{\Gamma_{\rm d}}\,, \quad T_{\rm em} = \frac{3 + 2\,x^2 + x^4}{(2 + x^2)\Gamma_{\rm b}}
$$

with $x \equiv \Gamma_{\rm b}/\Omega_{\rm b}$ and for $\Gamma_{\rm d} \ll \Gamma_{\rm b}\,, \quad \Omega_{\rm b} \approx \Gamma_{\rm b}\,.$

Origin of the trajectories

Dynamics: The Hamiltonian entangles the system with the free radiation field.

Repeated photon measurements:

In case of an emission in the $\hat{\mathbf{k}}$-direction: [1]

$$|\psi\rangle \overset{\Delta t}{\longrightarrow} R_{\hat{\mathbf{k}}} \, |\psi\rangle / \| \cdot \|$$

with probability

$$\| \, R_{\hat{\mathbf{k}}} \, |\psi\rangle \, \|^2$$

$R_{\hat{\mathbf{k}}}$: reset operator

In case of no emission: [2]

$$|\psi\rangle \overset{\Delta t}{\longrightarrow} U_{\rm cond}(\Delta t, 0) \, |\psi\rangle / \| \cdot \|$$

with probability

$$\| \, U_{\rm cond}(\Delta t, 0) \, |\psi\rangle \, \|^2$$

$H_{\rm cond}$: non-Hermitian Hamiltonian

[1] Schön and Beige, Phys. Rev. A **64**, 023806 (2001).

[2] Hegerfeldt and Wilser, in *Classical and Quantum Systems*, Proceedings of the Second International Wigner Symposium, 1991 (World Scientific, Singapore, 1992), p. 104 and others

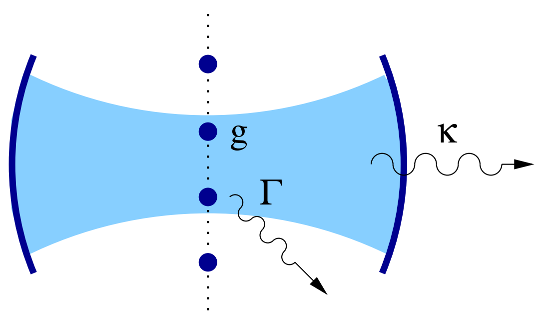
III

Using dissipation to manipulate decoherence-free states: The quantum Zeno effect [1–3]

[1] Beige, Braun, Tregenna, and Knight, PRL **85**, 1762 (2000).
[2] Marr, Beige, and Rempe, PRA **68**, 033817 (2003).
[3] Beige, PRA **67**, 020301(R) (2004).

Coupling atomic qubits via optical cavities

Atom-cavity setups possess all the necessary ingredients for quantum computing and other applications.



g: atom-cavity coupling constant

κ: spontaneous cavity decay rate

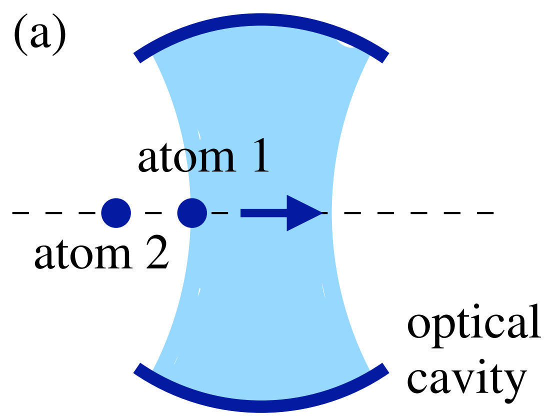
Γ: spontaneous atom decay rate

Main problems:

- dissipation due to two different decay channels
- inability to precisely control all experimental parameters

Dissipation-assisted adiabatic passage into an entangled state

Experimental setup:



Two two-level atoms can be prepared in a maximally entangled state by moving them slowly into an optical cavity.

The basic idea

If there is initially only one quanta of excitation in the system, then

$$H_{\rm int} \quad = \quad \hbar \left[\, g_1 |21;0\rangle + g_2 |12;0\rangle \,\right] \langle 11;1| + {\rm h.c.}$$

Adiabatic theorem:

The system remains constantly in a zero eigenstate.

Relevant eigenstate: $|\lambda_1\rangle = \left[\, g_1 |12;0\rangle - g_2 |21;0\rangle \,\right] / \| \cdot \|$

- when atom 2 enters the cavity: $|\lambda_1\rangle = |12;0\rangle$
- when both atoms see the same cavity coupling:

$$|\lambda_1\rangle = \left[\, |12;0\rangle - |21;0\rangle \,\right] / \sqrt{2}$$

Numerical results

Experimental parameters:

$$g_i(x_i) = g\ \exp\big(-(x_i/w_0)^2\big)$$

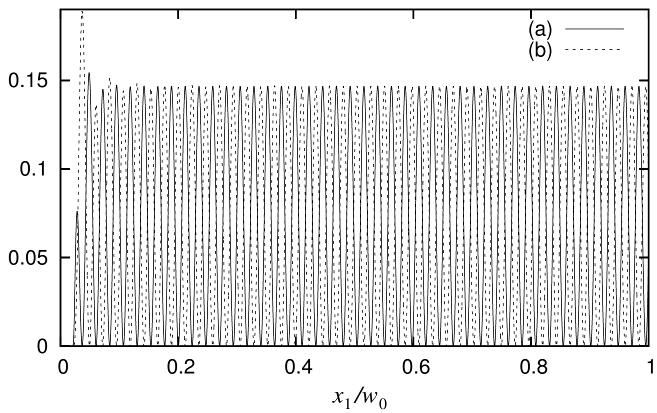
$$v = 5\,w_0 g\ \sin^2\big(\pi(x_1 + 4\,w_0)/5\,w_0\big)$$

$$w_0:\ \text{cavity waist},\ \Delta x = 2\,w_0$$

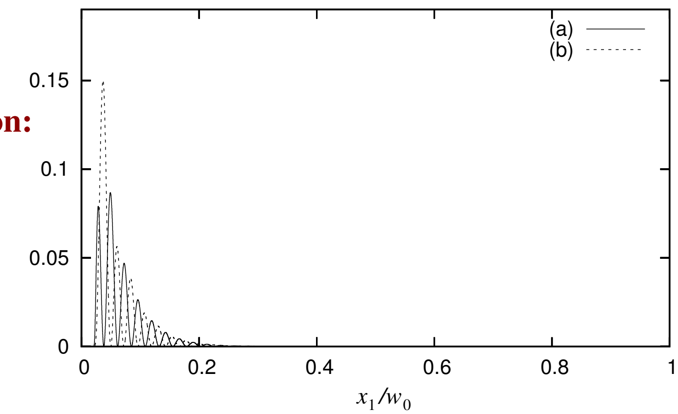
Population in the unwanted states: (a) $\big[\,g_2\,|12;0\rangle + g_1\,|21;0\rangle\,\big]/\|\cdot\|$

(b) $|11;1\rangle$

κ=0

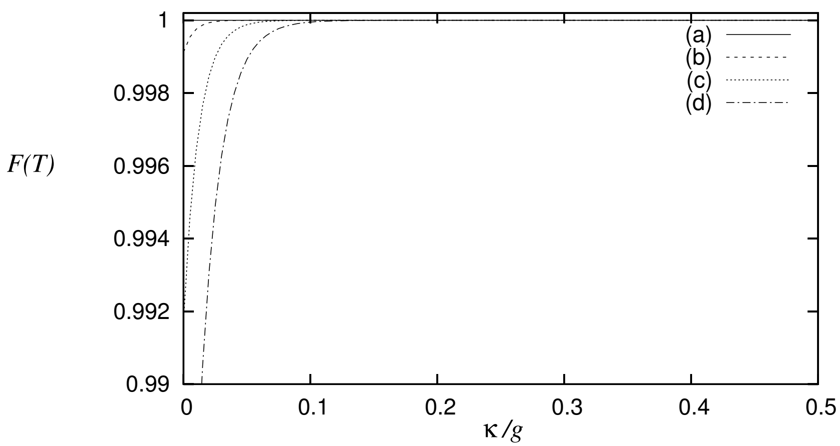


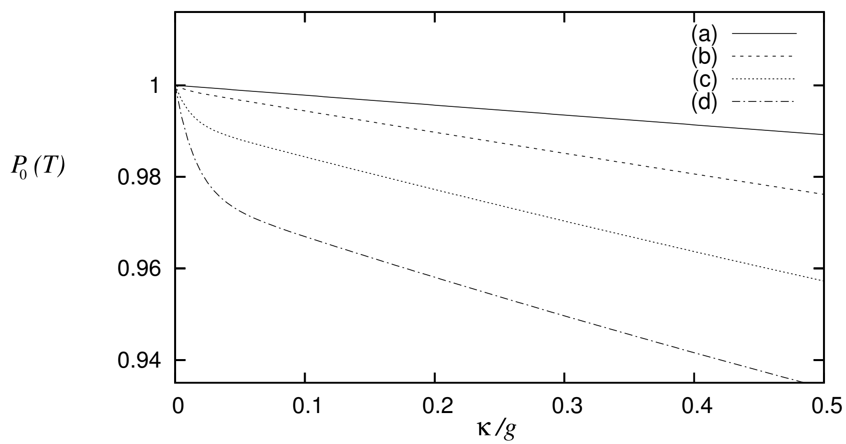
with dissipation: κ=0.2g



Fidelity and success rate $(\Gamma = 0)$

As a function of κ:





(a): $v_{\mathrm{max}} = 0.5\, w_0 g$, (b): $v_{\mathrm{max}} = w_0 g$, (c): $v_{\mathrm{max}} = 1.5\, w_0 g$, (d): $\kappa = 2\, w_0 g$ (d)

Interpretation via the quantum Zeno effect

The inverse quantum Zeno effect:

The time evolution of the system is an adiabatic passage (STIRAP) but the environment measures continuously, whether the system is indeed in the desired state:

$\Longrightarrow$ high fidelity of prepared state

The quantum Zeno effect:

Aanlogously, the quantum Zeno effect can be used to restrict the time evolution of a system onto a larger decoherence-free subspace:

$\Longrightarrow$ effective Hamiltonian $H_{\rm eff} = I\!\!P_{\rm DFS} \, H_{\rm Int} \, I\!\!P_{\rm DFS}$

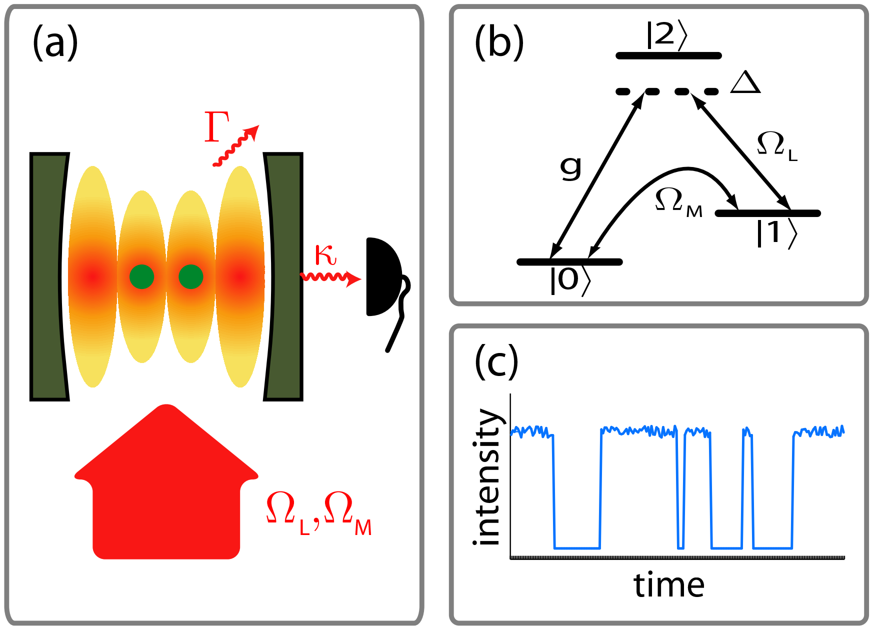
This Hamiltonian can be entangling even if $H_{\rm Int}$ isn't.

IV

Entangled state preparation using macroscopic quantum jumps [1–3]

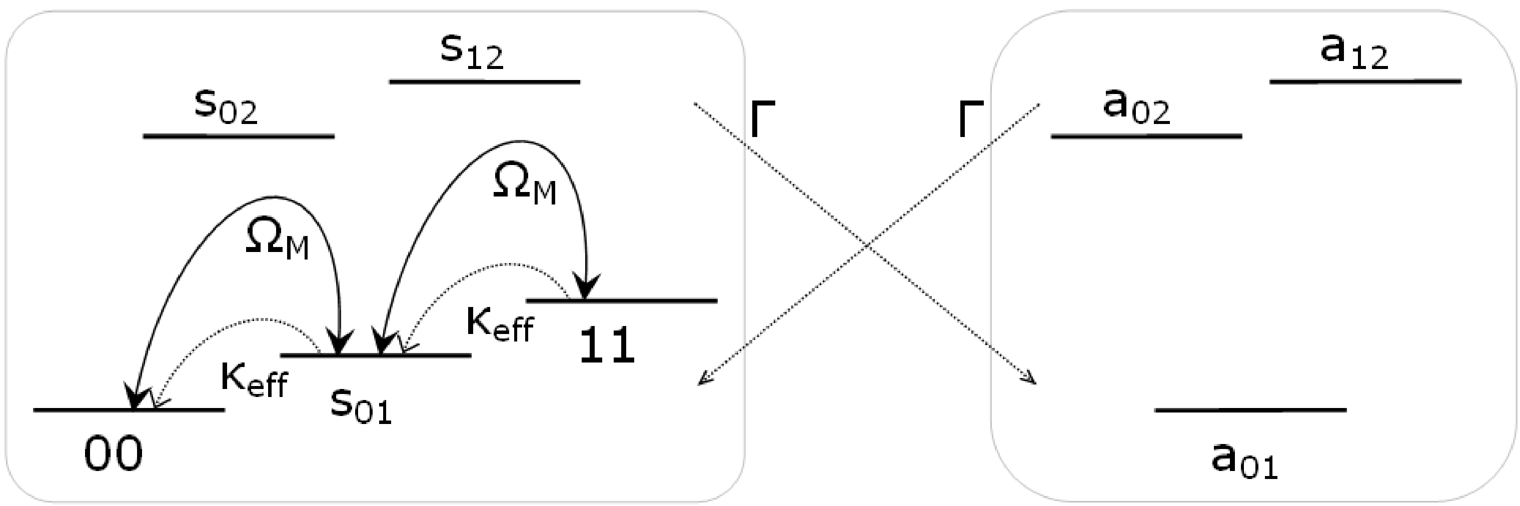
[1] Metz, Trupke, and Beige, PRL **97**, 040503 (2006).
[2] Metz and Beige, PRA **76**, 022331 (2007).
[3] Metz, Schön, and Beige, PRA **76**, 052307 (2007).

Experimental setup to entangle two atoms



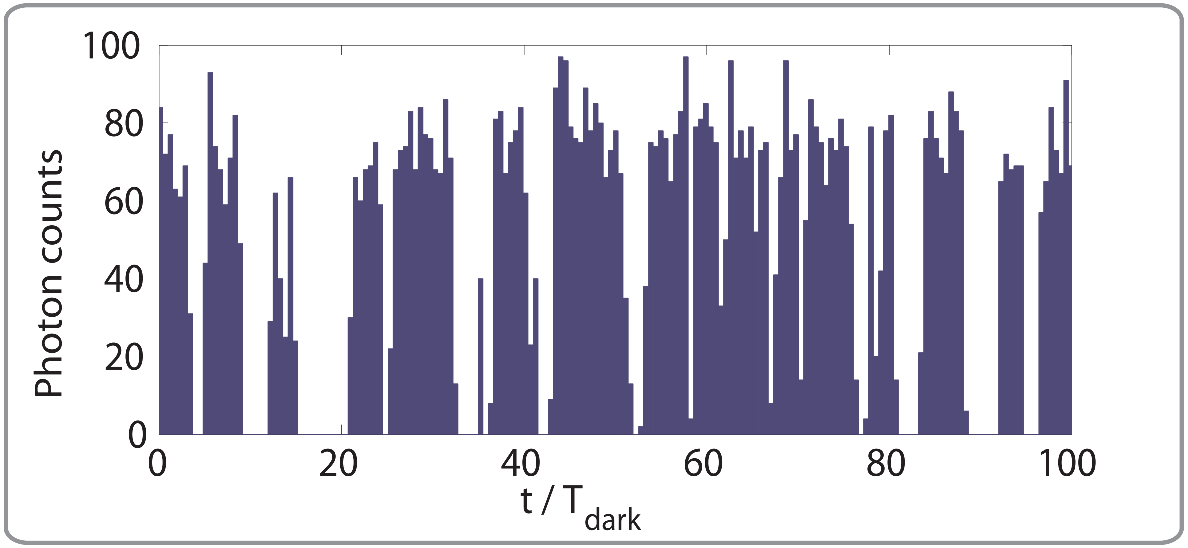
The successful generation of a maximally entangled atom pair is triggered on a macroscopic dark period. The laser should be turned off once the cavity emission stops.

Effective level scheme



An adiabatic elimination of the excited states due to a large detuning Δ shows that the atoms remain mainly in their ground states.

Macroscopic quantum jumps



Here: $\Delta = 50\,\kappa$, $\Gamma = 0.05\,\kappa$, $g = \Omega_{\rm L} = \kappa$, $\Omega_{\rm M} = 0.05\,\kappa$ and $\eta = 1$.

Achieving fidelities above 0.9 is possible even when using a relatively modest cavity with $C \equiv g^2/\kappa\Gamma$ is as low as 10 and when using a real-life single photon detector with an efficiency as low as $\eta = 0.2$.

Entanglement growth using parity measurements

Two entangled qubit pairs:

$$\begin{aligned} |\psi\rangle &= (|01\rangle - |10\rangle) \otimes (|01\rangle - |10\rangle)/2 \\ &= (|0101\rangle - |1001\rangle - |0110\rangle + |1010\rangle)/2 \end{aligned}$$

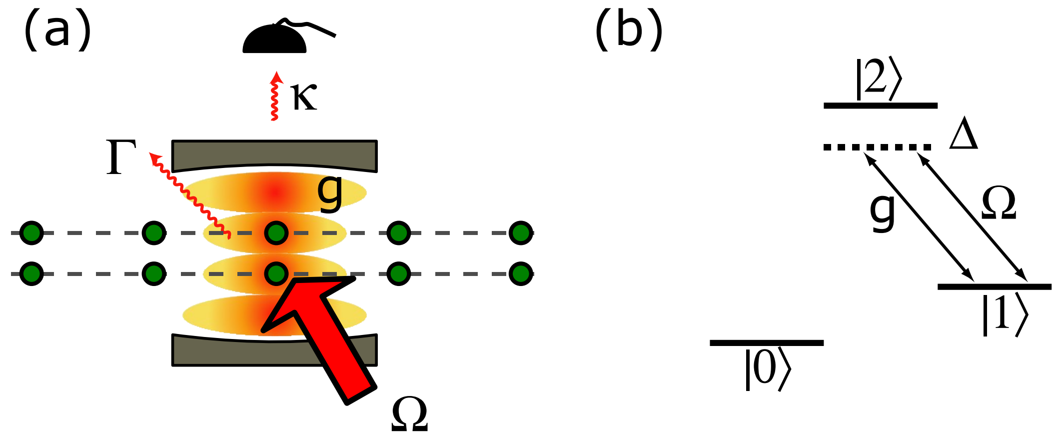
Projection of atom 2 and 3 onto $|01\rangle$, $|10\rangle$ subspace:

$$|\psi\rangle \rightarrow (|0101\rangle + |1010\rangle)/\sqrt{2}$$

GHZ-state!

An incomplete parity measurement

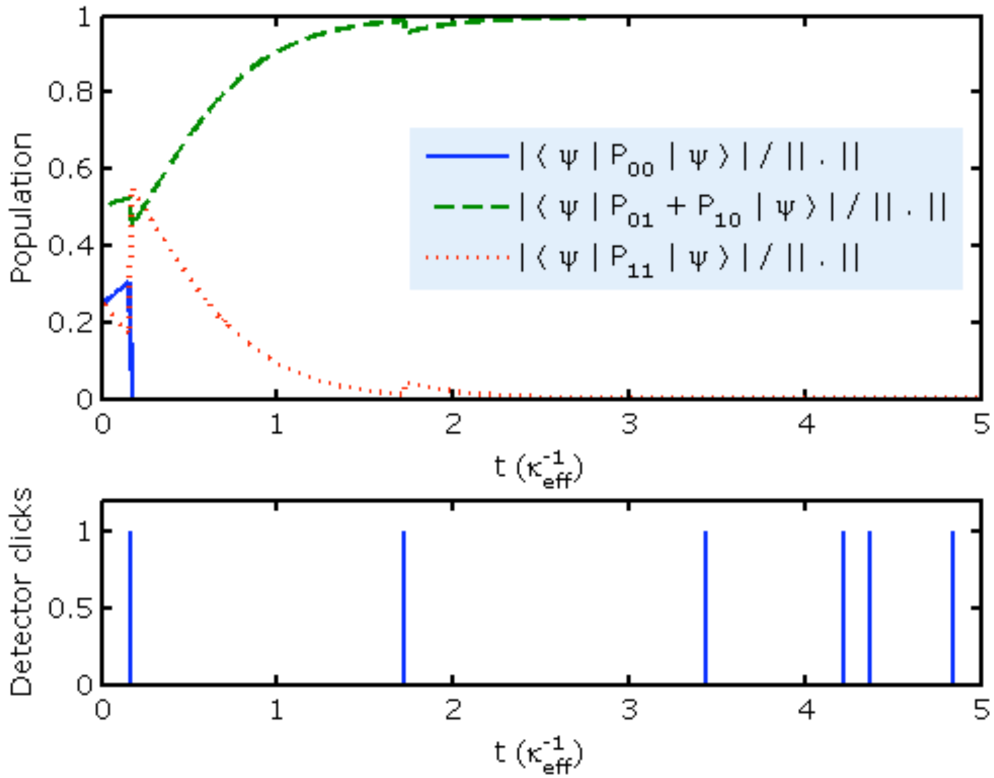
Experimental setup:



The successful completion of the projection onto the $\{|01\rangle, |10\rangle\}$ subspace is heralded by the emission of photons as if there is only one emitting atom inside the resonator.

$\Longrightarrow$ "electron shelving"

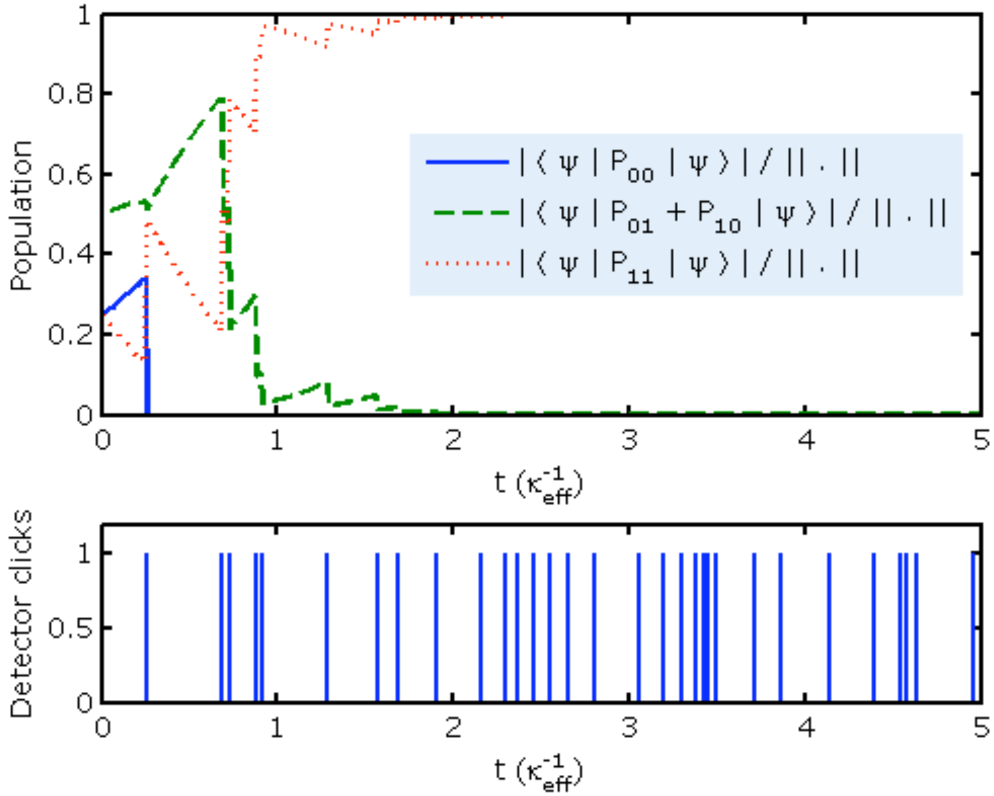
Relatively low emission rate



Parameters: $\Gamma = 0.1\,\kappa$, $g = \kappa$, $\Delta = 50\,\kappa$, $\Omega = \kappa$ $(C = 10)$

Initial state: $|\psi\rangle = (|00\rangle + |01\rangle + |10\rangle + |11\rangle)/2$

Maximum emission rate



Parameters: $\Gamma = 0.1\,\kappa$, $g = \kappa$, $\Delta = 50\,\kappa$, $\Omega = \kappa$ $(C = 10)$

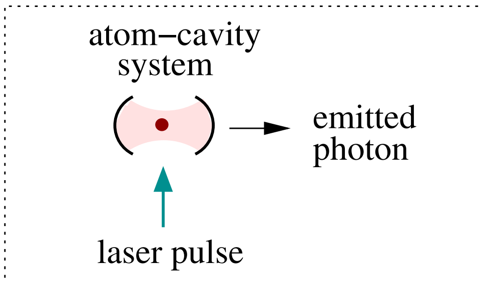
Initial state: $|\psi\rangle = (|00\rangle + |01\rangle + |10\rangle + |11\rangle)/2$

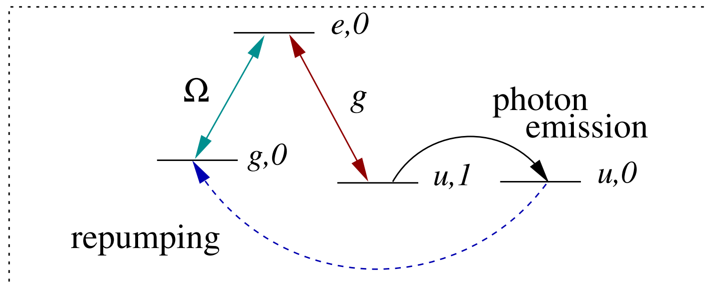
V

Linking distant qubits via photon measurements [1–4]

[1] Lim, Kwek, and Beige, PRL **95**, 030505 (2005).
[2] Barrett and Kok, PRA **71**, 060310(R) (2005)
[3] Lim, Barrett, Beige, Kok, and Kwek, PRA **73**, 012304 (2006).
[4] Busch, Kyoseva, Trupke, and Beige, PRA **78**, 040301(R) (2008).

Generation of a single photon on demand [1,2]





Reliable single photon source:

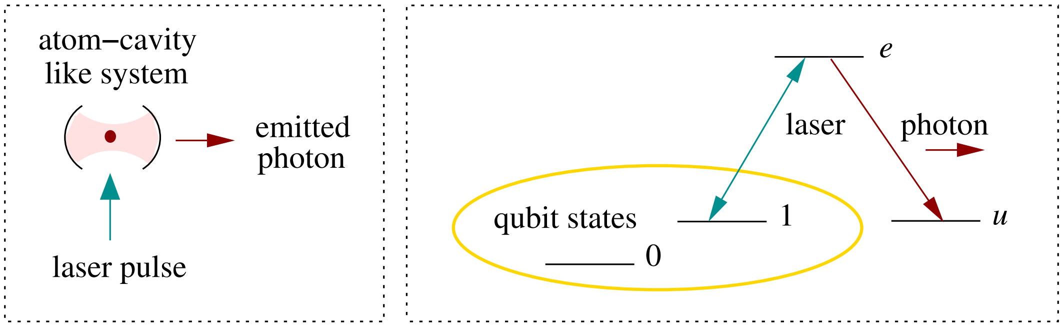
- STIRAP process places **one photon** in the cavity mode
- leakage of photon through cavity mirror yields

$$|g\rangle \quad \longrightarrow \quad |g;1\rangle$$

[1] Law and Kimble, J. Mod. Opt. **44**, 2027 (1997); Kuhn, Hennrich, Bondo, and Rempe, Appl. Phys. B **69**, 373 (1999).

[2] Kuhn, Hennrich, and Rempe, PRL **89**, 067901 (2002).

Generation of an encoded flying qubit [1]



Generation of an additional time-bin encoded qubit:

- information is stored in stationary qubits like $\alpha\,|0\rangle + \beta\,|1\rangle$
- generation of a single photon on demand such that

$$\alpha\,|0\rangle + \beta\,|1\rangle \quad \longrightarrow \quad \alpha\,|0;\mathsf{E}\rangle + \beta\,|1;\mathsf{L}\rangle$$

$|\mathsf{E}\rangle$ and $|\mathsf{L}\rangle$ denote a single photon created at an early and a late time, respectively.

[1] Lim, Beige, and Kwek, PRL **95**, 030305 (2005).

Photon pair absorption without erasing qubits

For two photons:

- **arbitrary two-qubit state:** $|\psi_{\mathrm{in}}\rangle = \alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|10\rangle + \delta\,|11\rangle$
- **state after creation of two photons:**
$$|\psi_{\mathrm{enc}}\rangle = \alpha\,|00;\mathsf{EE}\rangle + \beta\,|01;\mathsf{EL}\rangle + \gamma\,|10;\mathsf{LE}\rangle + \delta\,|11;\mathsf{LL}\rangle$$
- **measurement outcome:** $|\mathsf{EE}\rangle + \mathrm{e}^{\mathrm{i}\varphi_1}\,|\mathsf{EL}\rangle + \mathrm{e}^{\mathrm{i}\varphi_2}\,|\mathsf{LE}\rangle + \mathrm{e}^{\mathrm{i}\varphi_3}\,|\mathsf{LL}\rangle$
- **final state:** $|\psi_{\mathrm{fin}}\rangle = \alpha\,|00\rangle + \mathrm{e}^{-\mathrm{i}\varphi_1}\,\beta\,|01\rangle + \mathrm{e}^{-\mathrm{i}\varphi_2}\,\gamma\,|10\rangle + \mathrm{e}^{-\mathrm{i}\varphi_3}\,\delta\,|11\rangle$

A photon pair measurement in a mutually unbiased basis always results in a two-qubit phase gate.

A Repeat-Until-Success (RUS) quantum gate

Encoded two-qubit state using the mutually unbiased basis:

$$|\psi_{\rm enc}\rangle = \tfrac{1}{2}\sum_{i=1}^{4} |\psi_i\rangle|\Phi_i\rangle$$

$$\text{with} \quad |\psi_1\rangle = {\rm e}^{-{\rm i}\pi/4}\, Z_1\big(\tfrac{1}{2}\pi\big)\, Z_2\big(-\tfrac{1}{2}\pi\big)\, U_{CZ}\, |\psi_{\rm in}\rangle\,,$$

$$|\psi_2\rangle = -{\rm e}^{{\rm i}\pi/4}\, Z_1\big(-\tfrac{1}{2}\pi\big)\, Z_2\big(\tfrac{1}{2}\pi\big)\, U_{CZ}\, |\psi_{\rm in}\rangle\,,$$

$$|\psi_3\rangle = |\psi_{\rm in}\rangle\,, \quad |\psi_4\rangle = -{\rm i}\, Z_1(\pi)\, Z_2(\pi)\, |\psi_{\rm in}\rangle$$

$$Z_i(\varphi) = {\rm diag}\,(0, {\rm e}^{-{\rm i}\varphi})\,, \quad U_{\rm CZ} = {\rm diag}\,(1,1,1,-1)$$

A measurement of $|\Phi_{1,2}\rangle$ results in a universal phase gate, while a measurement of $|\Phi_{3,4}\rangle$ yields the initial qubits up to local operations.

On average, the whole process has to be repeated twice.

VI

Cooling atoms into entangled states [1–4]

[1]Kraus, Büchler, Diehl, Kantian, Micheli, and Zoller, PRA **78**, 042307 (2008).

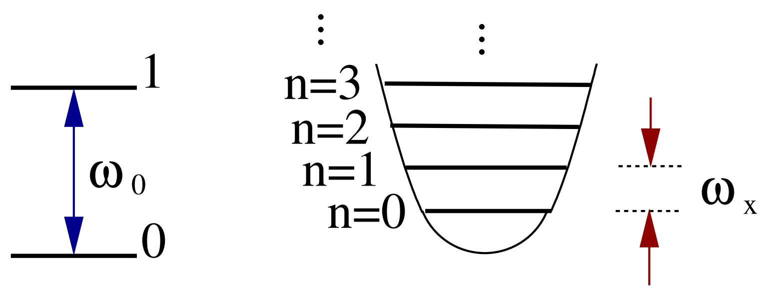
[2]Verstraete, Wolf, and Cirac, arXive:0803.0613.

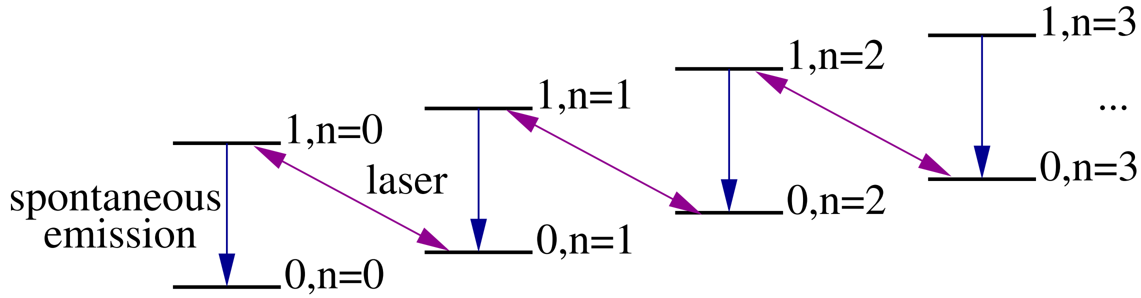
[3]Ticozzi and Viola, arXive:0809.0613.

[4]Vacanti and Beige, NJP (submitted); arXive:0901.3909.

Sideband cooling of a single particle

A single two-level atom can be cooled very efficiently using a laser with frequency $\omega_0 - \omega_x$ and spontaneous emission. [1]





[1]Wineland and Dehmelt, Bull. Am. Phys. Soc. **20**, 637 (1975).

Setup for entanglement generation

- **The qubits:** degenerate ground states $|g_0\rangle \equiv |0\rangle$ and $|g_1\rangle \equiv |1\rangle$
  with interaction $H_{\rm Int} = \sum_{n=0}^{2^N-1} E_n \, |\lambda_n\rangle\langle\lambda_n|$

- **The cooling device:** excited atomic states $|e_0\rangle$ and $|e_1\rangle$
  with laser driving of g_0–e_0 and g_1–e_1 transitions

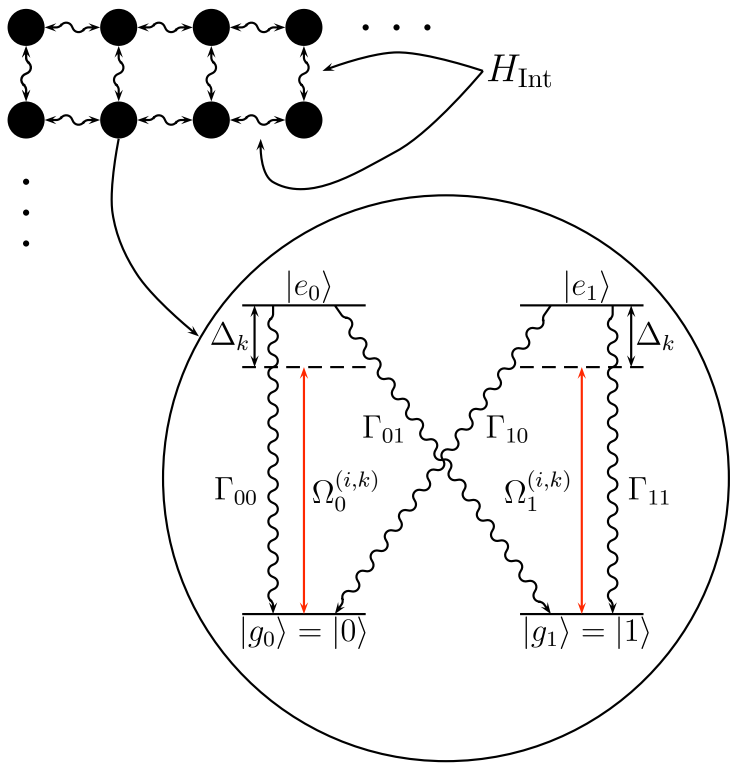
- **The total Hamiltonian:**

$$H_{\rm I} \;=\; \sum_{n=0}^{4^N-1} E_n \, |\lambda_n\rangle\langle\lambda_n| + \sum_{n=0}^{4^N-1} \sum_{m\neq n} \chi_{nm} \, |\lambda_n\rangle\langle\lambda_m| + {\rm H.c.}$$

- **Aim:** preparation of the qubit ground state $|\lambda_0\rangle$

Level scheme of a single atom

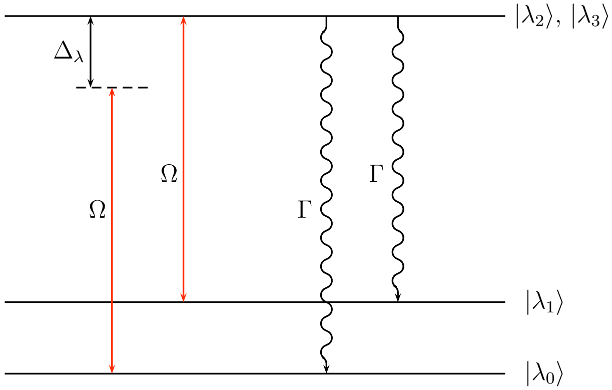
We consider a system of strongly interacting atomic qubits which is driven by K laser fields to auxiliary excited states $|e_0\rangle$ and $|e_1\rangle$:



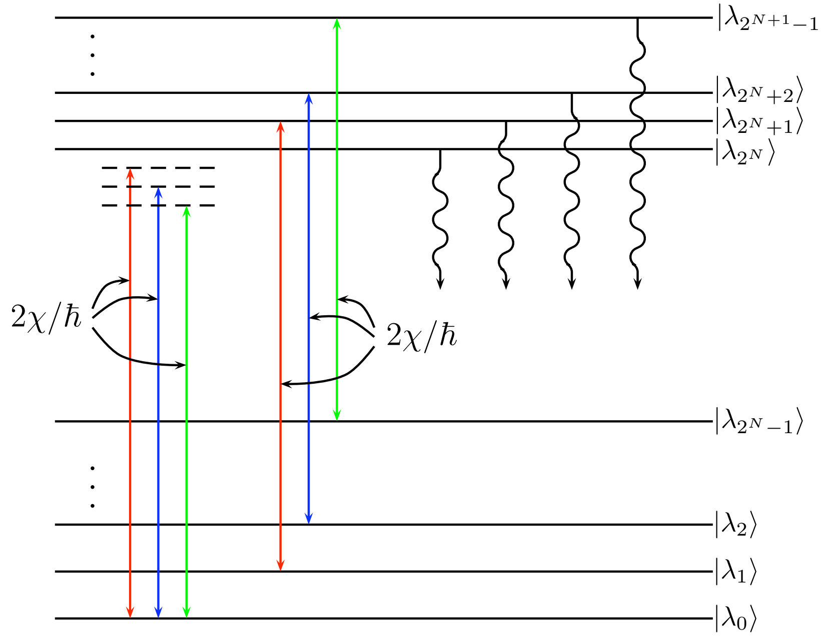
Level scheme of the combined system

The detuning we need to cool the system into $|\lambda_0\rangle$ comes exactly from the fact that $|\lambda_0\rangle$ is the ground state of the system:

The one-qubit case:

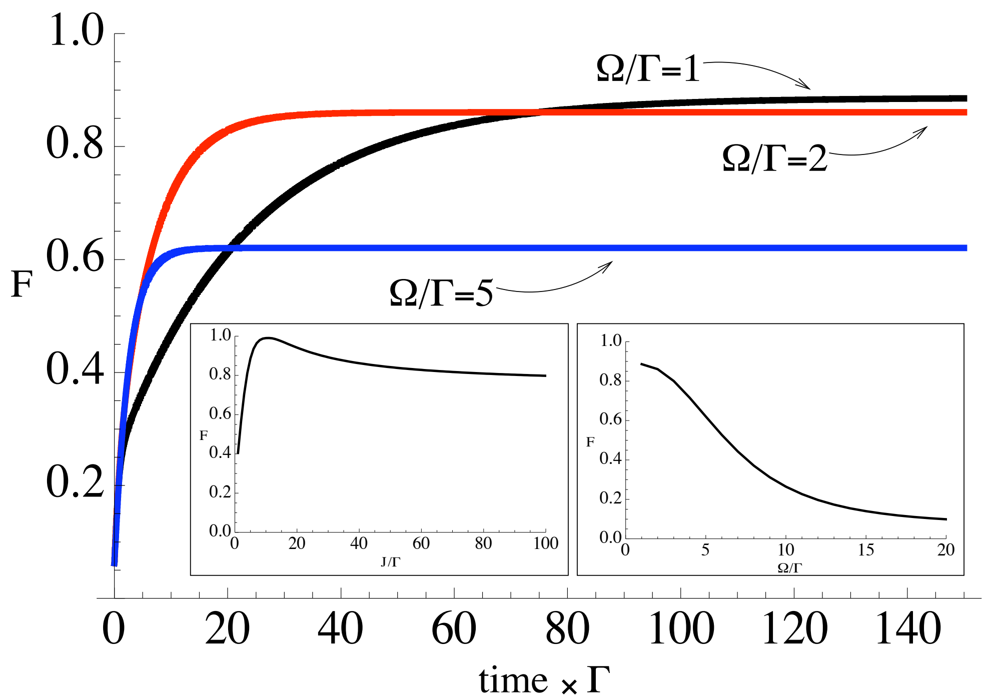


The many-qubit case:



A two-qubit example

Here are numerical results for the spin-spin Heisenberg Hamiltonian
$H = \hbar J \vec{\sigma}_1 \cdot \vec{\sigma}_2 = -3\hbar J\,|\lambda_0\rangle\langle\lambda_0| + \sum_{n=1}^{3} \hbar J\,|\lambda_n\rangle\langle\lambda_n|$
with $|\lambda_0\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$:



VII

Conclusions

Final remarks

Measurements and dissipation provide a very useful tool for the coherent control of open quantum systems:

- state preparation and gate operations via no-photon measurements
- state preparation and gate operations via photon detection
- state preparation via the observation of quantum jumps
- state preparation via cooling
- active feedback
- ...

Motivation for using dissipation is to obtain simple and feasible entangling schemes which are robust against parameter fluctuations.

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