QUANTUM CONTROL LANDSCAPES
and
CONTROL WITH ENGINEERED RESERVOIRS

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Program "Control of Complex Quantum Systems"
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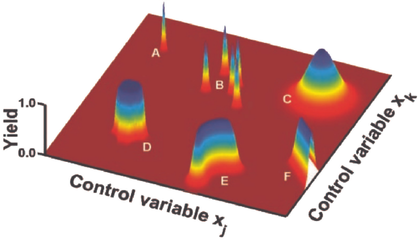
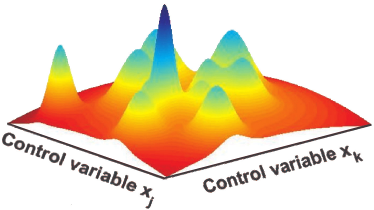
QUANTUM CONTROL LANDSCAPES





PRL **106,** 120402 (2011) PHYSICAL REVIEW LETTERS week ending 25 MARCH 2011

Are there Traps in Quantum Control Landscapes?

Alexander N. Pechen* and David J. Tannor

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OUTLINE

- H. Rabitz, M. Hsieh, C. Rosenthal, Science **303**, 1998 (2004): Conjecture: No traps at all (under controllability assumption only).
- P. de Fouquieres and S.G. Schirmer, arXiv:1004.3492: Examples of traps.
- A. Pechen and D.J. Tannor, PRL **106**, 120402 (2011): Discovery of a general trapping behavior.
- A. Pechen and N. Il'in, PRA **86**, 052117 (2012): Absence of traps for the LZ system.

BASIC DEFINITIONS

Control problem:

$$i\dot{U}_t^\varepsilon = (H_0 + V\varepsilon(t))U_t^\varepsilon; \qquad J(\varepsilon) = \widehat{J}(U_T^\varepsilon) \to \max$$

- Optimal controls: Global maxima of J.
- Traps: Local maxima of J.
- Second-order traps: $\delta J/\delta\varepsilon = 0$, $H = \frac{\delta^2 J}{\delta\varepsilon^2} \leq 0$, $J(\varepsilon) < J_{\max}$.
- Regular controls: Jacobian $\delta U_T^\varepsilon/\delta\varepsilon$ has full rank.

$$\frac{\delta J}{\delta\varepsilon} = \frac{\delta\widehat{J}}{\delta U_T^\varepsilon} \cdot \frac{\delta U_T^\varepsilon}{\delta\varepsilon}$$

OBJECTIVES

State-to-state transfer:

$$J_{\mathrm{i}\to\mathrm{f}}(\varepsilon) = |\langle \mathrm{f}|U_T^\varepsilon|\mathrm{i}\rangle|^2$$

Maximizing mean value of an observable O:

$$J_O(\varepsilon) = \mathrm{Tr}[U_T^\varepsilon \rho_0 U_T^{\varepsilon\dagger} O]$$

Generation of a unitary process W:

$$J_W(\varepsilon) = \frac{1}{4}|\mathrm{Tr}(U_T^\varepsilon W^\dagger)|^2$$

REGULAR CONTROLS: NO TRAPS

Theorem: Typical $\widehat{J}(U)$ have exactly one maximum and one minimum value (hence have no traps).[1][2] Similarly for open systems.[3]

Theorem: $\widehat{J}(U)$ has no traps implies that regular controls are not traps.[3]

Conjecture: No traps at all for typical quantum control problems.

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[1] H. Rabitz, M. Hsieh, C. Rosenthal, *Science'04* and subsequent works.

[2] Related works: J. von Neumann (1937); R. Brockett (1989); S.J. Glaser, T. Schulte-Herbrüggen, M. Sieveking, O. Schedletzky, N.C. Nielsen, O.W. Sørensen, C. Griesinger (1998).

[3] R. Wu, A. Pechen, H. Rabitz, M. Hsieh, B. Tsou, *J. Math. Phys.* **49**, 022108 (2008).

NON-REGULAR CONTROLS

Non-regular controls exist. Can they be traps?

P. de Fouquieres and S.G. Schirmer[4]:

- Existence of critical points for any $0 < J_{\mathrm{i}\to\mathrm{f}} < 1$.
- Example of a second-order trap.
- Example of a full trap.

[4] P. de Fouquieres, S.G. Schirmer, arXiv:1004.3492.

NON-REGULAR CONTROLS: SECOND-ORDER TRAPS

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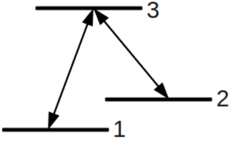
General result: If at least one $V_{ij} = 0$ in the basis $|i\rangle$ of H_0, then there exist ρ_0 and O for which $\varepsilon(t) \equiv 0$ is a second-order trap.

Example: Λ-atom. Control $\varepsilon(t) = 0$ is a second-order trap.
(However, not a full trap.)



ESTIMATING ATTRACTING BASIN

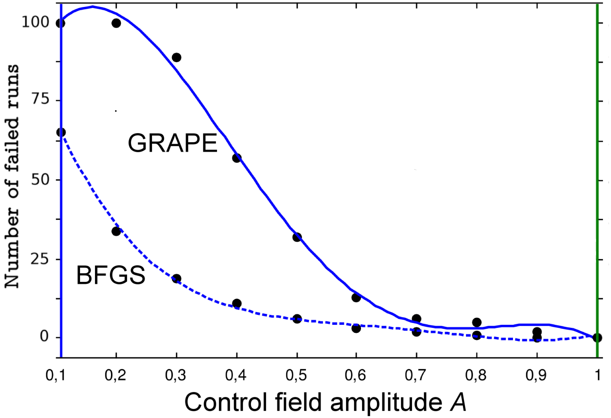


Figure: Number of failed runs vs. amplitude of the initial control.

Remark: For some realistic system parameters, $A = 0.1$ corresponds to the field intensity $I \approx 10^{12}$ W/cm^2.[5]

[5] A.N. Pechen, D.J. Tannor, *Isr. J. Chem.* **52**, 467 (2012).

LANDAU-ZENER SYSTEM: NO TRAPS

PHYSICAL REVIEW A **86**, 052117 (2012)

Trap-free manipulation in the Landau-Zener system

Alexander Pechen[1,2,*] and Nikolay Il'in[2]

Landau-Zener system:

$$i\dot{U}_t^\varepsilon = (\Delta\sigma_x + \varepsilon(t)\sigma_z)U_t^\varepsilon$$

Theorem: No traps for all typical $J(\varepsilon)$.

Noise in the optimal control: $\mathbb{E}(J) = J(\varepsilon_0) - \mathcal{D} + o(\sigma^2)$

$$\mathcal{D}_{\mathrm{AWN}}^{\mathrm{i}\to\mathrm{f}} \leq \sigma^2 T; \qquad \mathcal{D}_{\mathrm{MWN}}^{\mathrm{i}\to\mathrm{f}} \leq \sigma^2 E$$
$$\mathcal{D}_{\mathrm{AWN}}^{W} = \sigma^2 T; \qquad \mathcal{D}_{\mathrm{MWN}}^{W} = \sigma^2 E$$

Noise: see K. Moore, C. Brif, M. Grace, A. Donovan, D. Hocker, T. Ho, R. Wu, H. Rabitz, *PRA* **86**, 062309 (2012).

CONTROL WITH ENGINEERED RESERVOIRS

ENVIRONMENT AS A CONTROL

- C. Altafini (PRA'04): Markovian open quantum systems with coherent control are uncontrollable.

  The lack of controllability "*naturally calls for richer classes of control fields than just unitary ones to be studied.*"

- A.P. and H. Rabitz: Engineer state of the environment and use it as incoherent control.[6]

- A.P.: Markovian open quantum systems with coherent and incoherent controls are (approximately) controllable.[7]

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[6] A. Pechen, H. Rabitz, "Teaching the environment to control quantum systems", *PRA* **73**, 062102 (2006).

[7] A. Pechen, "Engineering arbitrary pure and mixed quantum states", *PRA* **84**, 042106 (2011).

ENVIRONMENT AS A CONTROL

PHYSICAL REVIEW A **73**, 062102 (2006)

Teaching the environment to control quantum systems

Alexander Pechen* and Herschel Rabitz†

Open quantum systems: control by tailoring Hamiltonian and non-Hamiltonian aspects of the evolution

$$\frac{d\rho_t}{dt} = -i[H_0 + V\varepsilon(t), \rho_t] + \mathcal{L}_{n_\omega(t)}(\rho_t)$$

- Coherent control: $\varepsilon(t)$.
- Incoherent control: $n_\omega(t)$ — density of environmental particles in state $|\omega\rangle$.

$$\mathcal{L}_{n_\omega(t)}(\rho) = \sum_j \left(2L_j\rho L_j^\dagger - L_j^\dagger L_j\rho - \rho L_j^\dagger L_j\right), \quad L_j = L_j(n_\omega(t))$$

PHYSICAL EXAMPLE: INCOHERENT PHOTONS

- Environment: Incoherent photons (or phonons).
- Control: Spectral density n_ω of incoherent photons.
- Evolution: $\mathcal{L}_{n_\omega} = \sum_{i<j} A_{ij}[(n_{\omega_{ij}} + 1)L_{Q_{ij}} + n_{\omega_{ij}} L_{Q_{ji}}]$

$$\left(L_Q(\rho) = 2Q\rho Q^\dagger - Q^\dagger Q\rho - \rho Q^\dagger Q, \qquad Q_{ij} = |i\rangle\langle j|\right)$$

ENGINEERING ARBITRARY QUANTUM STATES

PHYSICAL REVIEW A **84**, 042106 (2011)

Engineering arbitrary pure and mixed quantum states

Alexander Pechen

$$\rho_{\mathrm{i}} \xrightarrow{\varepsilon, n_\omega} \rho_{\mathrm{f}} = \sum p_i |\phi_i\rangle\langle\phi_i|$$

Incoherent control $n_{\omega_{ij}} = p_j/(p_i - p_j)$:

$$\rho_{\mathrm{i}} \xrightarrow{n_{\omega_{ij}}} \tilde{\rho} = \sum p_i |i\rangle\langle i|$$

Coherent control $|i\rangle \xrightarrow{\varepsilon} |\phi_i\rangle$ (assume fast unitary controllability[8]):

$$\tilde{\rho} \xrightarrow{\varepsilon} U_T^{\varepsilon} \tilde{\rho} U_T^{\varepsilon\dagger} = \rho_{\mathrm{f}}$$

Result (complete controllability): Approximate steering of any ρ_{i} into any ρ_{f} with coherent and incoherent photons.

[8] S.E. Sklarz, D.J. Tannor, N. Khaneja, *PRA* **69**, 053408 (2004).

ENGINEERING ALL-TO-ONE KRAUS MAPS

J. Phys. A: Math. Theor. **40** (2007) 5681–5693 JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

Controllability of open quantum systems with Kraus-map dynamics

Rong Wu, Alexander Pechen, Constantin Brif and Herschel Rabitz

Department of Chemistry, Princeton University, Princeton, NJ 08544, USA

All-to-one Kraus map: $\forall \rho_{\rm i} \xrightarrow{\Phi_{\rho_{\rm f}}} \rho_{\rm f}$

Theorem: For any $\rho_{\rm f}$ the corresponding all-to-one Kraus map $\Phi_{\rho_{\rm f}}$ (mathematically) exists.

Theorem: Control scheme of PRA'11 gives a physical prescription for producing arbitrary all-to-one Kraus maps.

APPLICATIONS OF ENGINEERED ENVIRONMENTS

- Preparing entangled states; S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Buchler, P. Zoller, *Nat.Phys.'08*; H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, H.P. Büchler, A Rydberg quantum simulator, *Nat.Phys.'10*
- Improving quantum computation; F. Verstraete, M. Wolf, J. Cirac, *Nat.Phys.'09*.
- Inducing multiparticle entanglement dynamics; J.T. Barreiro, P. Schindler, O. Ghne, T. Monz, M. Chwalla, C.F. Roos, M. Hennrich, R. Blatt, *Nat.Phys.'10*.
- Quantum metrology of open dynamical systems: Precision limits through environment control (talk by L. Davidovich).

Quantum computing with mixed states and non-unitary circuits:

- D. Aharonov, A. Kitaev, N. Nisan, quant-ph/9806029.
- V.E. Tarasov, J. Phys. A 35, 5207 (2002).

SOME RECENT RELATED WORKS

- A. Grigoriu, H. Rabitz, G. Turinici, "Controllability analysis of quantum systems immersed within an engineered environment", Preprint hal-00696546 (2012).
- V. Bergholm, T. Schulte-Herbrueggen, How to Transfer between Arbitrary *n*-Qubit Quantum States by Coherent Control and Simplest Switchable Noise on a Single Qubit, `arXiv:1206.4945`
- G. Baggio, F. Ticozzi, L. Viola, Quantum State Preparation by Controlled Dissipation in Finite Time: From Classical to Quantum Controllers, `arXiv:1209.5568`

CONCLUSIONS

QUANTUM CONTROL LANDSCAPES

- Analysis of traps is important for determining the efficiency of local algorithms.
- Several important results have been obtained:
  - Regular controls: no traps.
  - Non-regular controls: some are second-order traps (for $n \geq 3$).
  - LZ system: no traps at all.

CONTROL WITH ENGINEERED ENVIRONMENTS

- Physical model: incoherent photons.
- Creation of arbitrary quantum states.
- Generation of all-to-one Kraus maps.

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MARIE CURIE ACTIONS

Thank you!