Driven Double-Quantum Dots

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Outline

- Introduction to charge qubits and detectors
- Microwave driven double dot qubit
- Electron-phonon coupling
- Breakdown of RWA
- Conclusions

Steady state current

- at T=0, system relaxes (phonon emission) to ground state $\rho(\infty) = |g\rangle\langle g|$
- Detector current is proportional to ground state occupation of nearest $I_{ss} \propto \langle l | \rho(\infty) | l \rangle$

Experiments (cont.)

• DiCarlo et al. PRL 92 226801 (2004)

Microwave driven charge qubit

• Bare qubit Hamiltonian can be described in terms of *localized* states $|1\rangle$ and $|2\rangle$

• Add time dependent driving field at frequency: $\omega_{rf} = \phi + \eta$ $H_{\text{driv}} = \frac{e\vec{E}(t)\cdot\vec{a}}{2}(|1\rangle\langle 1| - |2\rangle\langle 2|), \quad \vec{E}(t) = \vec{E}_0 \cos[(\phi + \eta)t]$

Microwave driven charge qubit *|e e|* = sin ⌅ ⌥*^x* + cos ⌅ ⌥*z*, *{|g , |e }* = *{*sin(⌅*/*2)*|r* + and ⁼ ↵⇥² ⁺ ². We transform *^H^q* to ^a frame rotat-

 $\frac{1}{2}$ rotation from a and make rotating $\frac{1}{2}$, $\frac{1}{2}$ tansion in to rotating mark and make rotating wave • Transform to rotating frame and make rotating wave approximation

$$
\tilde{H}_{\text{RWA}} = -(\eta \,\sigma_z^e + \Omega \,\sigma_x^e)/2. \qquad \eta = \omega - \phi
$$

• RWA Hamiltonian is diagonal in 'dressed' basis & has energy
salitting given by the effective Babi frequency splitting given by the effective Rabi frequency

$$
\Omega' = \sqrt{\Omega^2 + \eta^2}
$$
\n
$$
\Omega = e\vec{E}_0 \cdot \vec{a} \sin \theta, \qquad \theta = \tan^{-1} \left(\frac{\Delta}{\epsilon}\right)
$$
\n
$$
\sin(\theta) \text{ is just the dipole moment } \langle e|\hat{z}|q \rangle
$$

Hamiltonian is *H*ph = ^q ⌦q*a†* ^q*a*q. The electron–phonon $\sin(\theta)$ is just the dipole moment $\langle e|\hat{z}|g\rangle$

Phonon Environment we work in units where the unit of the uni We include the international terms of phonons, for which the bare \mathbf{r} with a strongly driven, or driven, \mathbf{D} dissipative two-level systems (2LSs) is reviewed in [13]. , and ˜ denotes the rotating basis. This Hamiltonian is diagonalised in the dressed basis *{| , |*+ *}* =

• Phonon bath is major source of relaxation *{*cos(*/*2)*|g* sin(*/*2)*|e ,*sin(*/*2)*|g* + cos(*/*2)*|e }*: *d* α *//*2 where α

$$
{e-p}=\sum{\mathbf{q}}i_{\mathbf{q}}\hat{\varrho}(\mathbf{q})(\mathbf{q}-\mathbf{q}),\qquad \qquad (a
$$

 \int_{0}^{1}

^q*a*q. The electron–phonon

 Γ_{\cdot}

^q)*,* (2)

*^j c^j*⁰ is the

 $\overline{\mathcal{L}}$

 $\ddot{}$

listic that phonons contains the position of t η can be piezoelectric of deformation potential • M_q can be piezoelectric or deformation potential Fourier transform of the electron density operator and this and the ever-present phonon induced relaxation re- Γ

proportional to I just perturbs phonon energies, so the • Restrict electronic states in electron density operator $\hat{\varrho}$ and get a spin boson model *i* Postrict electronic states in electron density operator $\hat{\epsilon}$ and set **•** Restrict electronic states in electron density operator $\hat{\varrho}$ and get the volume of the lattice and for periodicity and for periodicity where the second version of the second version

$$
H_{\rm e-p} \approx \sigma_z \sum_{\mathbf{q}} g_{\mathbf{q}} \left(a_{\mathbf{q}}^{\dagger} + a_{\mathbf{q}} \right),
$$

Steady state current

With driving, get resonant peaks when $\omega = \phi$.

Past experimental results st expe

 in the yello n.
D as *V*6 is increase -1.09 (pA) I D

2*=U* [18]. We demon-

... and in isolated quantum dots: -10 -5 0 5 pe $\overline{\mathsf{P}}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ do
Co

ance mea-

d. & denotes

*S*2*=dV*6 as a

the upper (lower) dot.

function of

Petta et al, PRL 93 186802 2004

FIG. 3 (color). (a) *ID* as a function of *V*² and *V*⁶ with *Vsd* ! Peaks should remain below 0.5

Recent experimental results

Reilly et al, unpublished 2012

 \triangledown

without driving

with driving

Surprise! n
E to the one use of the one use \blacksquare double dot. Q $\overline{}$ gize *S*2*=dV*6 as a *N*%, where $\ddot{}$ the upper (lower) doesn't do

The peaks go over 0.5! i.e. *population inversion*: the systems spends longer in the excited state

Master equation. The dynamics of the dynamic waster equation for the master equation trix, $\mathcal{L}_{\mathcal{A}}$ we integrate the von Neumann $\mathcal{L}_{\mathcal{A}}$ we integrate the von Neumann $\mathcal{L}_{\mathcal{A}}$ trace over phonon modes, resulting in the modes of \mathbf{M} as the can induce population inraising and lowering rates. version. For the purpose of discussion we take *T* ! 0 and

• Derive master equation from dynamics of joint density matrix R: • Derive master equation from dynamics of joint density matrix R: and replacing *WI*"*t* 0 \$! *WI*"*t*\$, which is valid for weak **P** Derive master equation from dynamics of ioint density matrix R.

$$
\dot{\rho}_I(t) = -\int_0^t dt' \operatorname{Tr}_B[H_I(t), [H_I(t'), R(t')]].
$$

• Two versions of secular / rotating wave approximation a) in dressed basis $\dot{\rho_I} = \sum J(\omega')$ \mathcal{L} appearing terms in the *H*_I (*t*), and the *approximation* •Two versions of secular / rotating wave approximation $S_{\mathcal{L}}$ *^I* !^X • Two versions of secular / rotating wave approximation $\frac{1}{i}$ $\frac{1}{i}$ $\frac{1}{i}$ *a***) in dressed basis_,** \blacksquare $\dot{\rho}_I = \sum J(\omega') \left[(N(\omega') + 1) \mathcal{D}[P_{\omega'}] \right]$ [|][−] 〉 ′ from the versions of sociality from the dependent of the decay time scale. This precludes making and $\phi = \begin{pmatrix} \omega_0 & \omega_1 \\ \vdots & \vdots & \vdots \\ \omega_n & \omega_n & \omega_n \end{pmatrix}$ $\hat{\rho}_I = \sum J(\omega') \left[(N(\omega') + 1) \mathcal{D}[P_{\omega'}]\rho_I + N(\omega') \mathcal{D}[P_{\omega'}^\dagger]\rho_I \right]$

$$
\dot{\rho_I} = \sum_{\omega'} J(\omega') \left[(N(\omega') + 1) \mathcal{D}[P_{\omega'}]\rho_I + N(\omega')\mathcal{D}[P_{\omega'}^\intercal]\rho_I \right]
$$
\n
$$
\sum_{\text{Spectral density}} J(\omega) = 2\pi \sum_{\mathbf{q}} |g_{\mathbf{q}}|^2 \delta(\omega - \omega_{\mathbf{q}})
$$

where
$$
P_0 = \cos \theta \cos \varphi \sigma_z^d/2
$$
, $P_{\Omega'} = -\cos \theta \sin \varphi \sigma_z^d$,
\n $P_{\omega_0 \pm \Omega'} = \mp \sin \theta (1 \pm \cos \varphi) \sigma_{\mp}^d/2$, $P_{\omega_0} = -\sin \theta \sin \varphi \sigma_z^d/2$.
\n $N(\omega) = (e^{\omega/k_B T} - 1)^{-1}$
\n $\mathcal{D}[A]\rho \equiv A\rho A^{\dagger} - (A^{\dagger} A\rho + \rho A^{\dagger} A)/2$ **photons** phonons

 $\overline{\mathbb{L}}$

System, then the Master equation ment, !⁰ *SY,Z*(!*n*). We also assume that the driving nelling is allowed. *H*leads is the free Hamiltonian of the ment, !⁰ *SY,Z*(!*n*). We also assume that the driving field is such that *Ster equation* i.e. that the noise spectra are slowly varying over free spectra are slowly varying over free spectra are slowly varying over free spectra are slowly varying to the slowly varying over free spectra are slowly varying to th

• Derive master equation from dynamics of joint density matrix R: • Derive master equation from aynamics of joint deni • Derive master equation from dynamics of joint density matrix R: boson coupling to a generic bath of bosons [?], where *bⁱ*

$$
\dot{\rho}_I(t) = -\int_0^t dt' \, \text{Tr}_B[H_I(t), [H_I(t'), R(t')]].
$$

• Two versions of secular / rotating wave approximation a) in dressed basis $\dot{\rho_I} = \sum$ ω' $J(\omega')$ \mathcal{L} $(N(\omega') + 1) \mathcal{D}[P_{\omega'}]\rho_I + N(\omega')\mathcal{D}[P_{\omega'}^{\dagger}]\rho_I$ $\overline{\mathbb{L}}$ temperature. However, in the set of the set o $\rho_I = \sum J(\omega') \left[(N(\omega')+1)\,\mathcal{D}[P_{\omega'}]\rho_I + N(\omega')\mathcal{D}[P_{\omega'}^\dagger] \right]$ $\overline{\omega'}$ **c** *H*phon, and make our first rotating wave approximation • Iwo versions of secular / rotating wave approximation a) in dressed basis detuning the mass \mathbf{S} $\rho_I = \sum J(\omega')$ temperature. However, in this Letter, we restrict our $\mathcal{L}_{\mathcal{L}}$ (1) $\mathcal{D}[D_{\perp}]$ $\mathcal{L}_{\mathcal{L}}$ (*N*(\mathcal{L}^{\prime}) $\mathcal{D}[D^{\dagger}]$) $c(r)+1/D[P_{\omega'}]\rho_I+N(\omega')D[P_{\omega'}']\rho_I]$ **• Two versions of secular / rotating wave approximation** from the versions of secarary frequency that approximation.

a) in dressed basis $\hat{\rho}_I = \sum J(\omega') \left[(N(\omega') + 1) \mathcal{D}[P_{\omega'}]\rho_I + N(\omega') \mathcal{D}[P_{\omega'}^\dagger]\rho_I \right].$

b) in bare energy basis where a
In hare energy hasis sity, $\frac{1}{2}$ and $\frac{1}{2}$ is the occupation of phonon of p b) in bare energy basis *x* α *I* (*t*) in bare energy basis in the double commutator. Namely, the terms α Instead, I will go mad with Laplace transforms directly on Eq. (3). To demonstrate the approach, I will focus on

$$
\dot{\rho}_I(t) = -i[-\frac{\eta}{2}\sigma_z^{(e)} - \frac{\Omega}{2}\sigma_x^{(e)}, \rho_I(t)] + \Gamma_{\varphi}\mathcal{D}[\sigma_z^{(e)}]\rho_I(t) + \Gamma_r\mathcal{D}[|g\rangle\langle e|]\rho_I(t)
$$

Solve Bloch vector in steady state steady state properties, so the RWA is not restrictive. Solve Bloch vector in steady state steady state properties, so the RWA is not restrictive.

$$
\rho(t) = (\mathbb{I} + x_d(t)\sigma_x^d + y_d(t)\sigma_y^d + z_d(t)\sigma_z^d)/2
$$

 p opulations: $\alpha = \alpha + \beta$ the equations of motions of motions of motion for the components of the B lochi \mathcal{A} populations:

$$
\dot{z}_d = (\Gamma_- - \Gamma_+) - (\Gamma_- + \Gamma_+) z_d
$$

RWA in

 $\overline{P}(W \mid W)$ in $\overline{P}(W \mid W)$ and $\overline{P}(W \mid W)$ + *J*(⁰)sin² ⌅ sin² ↵(*N*(⁰) + 1*/*2))*/*2*, ^z* = (*J*(0) cos² ⌅ cos² ↵(*N*(0) + 1*/*2)*/*2 a) in dressed basis (Rabi >> decay) + *J*(⁰)sin² ⌅ sin² ↵(*N*(⁰) + 1*/*2))*/*2*,* (PRL 95 106901, 2005)

Saturated on resonance between dress between the terms appearance of the terms and the terms appearance of the terms and the terms appearance of the terms appearance of the terms appearance of the terms appearance of the t Asymmetric peak and the department of the

200 b) in bare energy basis (Rabi << decay) (PRL 96 017405, 2006)

Asymmetric peak Mosammetric (Lorentzian) peak Unsaturated

Problem...

strong driving approx \Rightarrow

saturation on resonance | ansa + asymmetric peaks **(c)** $\overline{}$

'HWXQLQJI
'HWXQLQJI

'HWXQLQJI
'HWXQLQJI

 $\mathbf{1}$

 \mathbf{L}

 \mathbf{I}

Dynamical master equation !0 *,*!00*,q,q*0 $\overline{}$ **IC** ²*P*!0⇢(*t*)*P*!00*e*(*si*(! 0 +! 00))*t*0 ✓ 1 **PACS 19** I. MASTER EQUATION CONTINUES. To solve the dynamics of the qubit, we develop a master equation for the qubit density matrix, ⇢ = Tr*^B R* where

 $\bullet\,$ von Neuman equation *R* van Neuman aquation

\n- \n von Neuman equation\n
$$
\dot{\rho}_I(t) = -\int_0^t dt' \operatorname{Tr}_B[H_I(t), [H_I(t'), R(t')]]
$$
\n
$$
H_I(t) = \sigma_z(t) \sum_q g_q^* a_q^\dagger e^{i\omega_q t} + g_q a_q e^{-i\omega_q t},
$$
\n
\n- \n Transform\n
$$
\sigma_z(t) = \sum_{\omega' \in \mathcal{W}} P_{\omega'} e^{i\omega' t}.
$$
\n
$$
s\bar{\rho}_s - \rho_0 = \sum_{\nu, \nu} \left(i \tilde{J}(\omega' + is) - i \tilde{J}(-\omega'' - is) \right) P_{\nu} \bar{\rho}_{s-i(-\nu' - is)P_{\nu'}} P_{\nu}.
$$
\n
$$
- \left(i \tilde{J}(\omega' + is) \bar{\rho}_{s-i(-\nu' - is)P_{\nu'}} P_{\nu} - i \tilde{J}(-\omega'' - is) P_{\nu'} P_{\nu} \bar{\rho}_{s-i(-\nu' - is)P_{\nu'}} P_{\nu'} \bar{\rho}_{s-i(-\nu' - is)P_{\nu'}} P_{\nu'}.
$$
\n
\n

• Solve by expansion $\bar{\rho}_s = \sum_{\vec{s}} \frac{\rho_\nu}{\sqrt{s}}.$ $E_{\rm c} = 1$ far from zero. Therefore, we really do care about dynamics at this frequency. That suggests we take a guess that ¯⇢*^s* $\nu \in {\pm \Omega', 0}$ $\bar{\rho}_{\nu}$ $\begin{array}{cc} \nu \in \{\pm \Omega', 0\} \end{array} s - i \nu \end{array}.$ *.* (14) ⇢¯*^s* = over *q*, this turns into a branch cut along the negative real axis. In what follows, we only ever evaluate *J*˜ with a real Also, if more frequencies (from a discrete set of Floquet eqigenfrequencies) are included in *z*(*t*) then it is suitable to • Solve by expansion $\tilde{J}(x) = \sum \frac{|g_q|^2}{n}$ $\sum_{\zeta \in \{1, \zeta \leqslant \zeta\}} s - i\nu$ $\nu \in \{\pm \Omega^\prime, 0\}$ correction and annihilation operators (recalling the cyclic property of the trace). Writing the trace of this correction of the trace of the tr

$$
\tilde{J}(x) = \sum_{q} \frac{|g_q|^2}{\omega_q + x}
$$

Dynamical master equation !0 Substituting this into Eq. (10) gets a little messy, but conceptually it is not too hard to see what will happen. The LHS will have poles at *s* = *±i*⌦⁰ , and the RHS will have poles at *s* = *i*(!⁰ + !00) and *s* = *i*(!⁰ + !⁰⁰ *±* ⌦⁰

• Keeping only DC pole in ρ and matching residues at s=0 gives traditional Lindblad master eqn... *iJ*˜(!⁰ ⁺ *is*) *iJ*˜(! *is*) !⁰ !0*P*! *iJ*˜(!⁰ *is*)*P†* g only DC pole in ρ and matching residues $\frac{1}{2}$ $\mathbf g$ ives traditional Eniquiad $\mathbf g$ sampe as what happened in the case of the case of the case of the case of the RWA is the RWA of the RWA is the RWA \bullet interpting only but the find ρ and matering residues at s=0 gives traditional Lindblad master eqn... at s v S ives craditional Emporad master equ

(!⁰⁰ *is*)*P†*

 $\overline{}$

Im(*s*)

, where Re

!00*P*!⇢¯*^si*(!0!00)

 $\overline{\Omega}$

Re(*s*)

$$
0 = \sum_{\omega'} J(\omega') \mathcal{D}[P'_{\omega}]\rho_{ss} + iF(\omega')[P^{\dagger}_{\omega'}P_{\omega'}, \rho_{ss}] \qquad \qquad \int_{0+i\Omega'}^{\text{im(s)}}
$$

 \tilde{C} course, this is not too hard to solve. One way to do so is to treat Eq. (13) as a matrix equation for the vector \tilde{C}

 $\bigcup_{i=1}^{n}$ is well by $\bigcup_{i=1}^{n}$

• Include dynamical poles... where *D*[*A*]⇢ ⌘ *A*⇢*A†* (*A†A*⇢ + ⇢*A†A*)*/*2. So this is just what we would get if we had made the standard RWA in *J*ˆ ⁺(!⁰ + *is*)¯⇢*^si*(!0!00)*P†* !00*P*! ⁺ *^J*^ˆ Now we proceed to substitute Eq. (14) into Eq. (15), and match the residues at *s* = *i*⌫⁰

$$
i\nu'\bar{\rho}_{\nu'} = \sum_{\substack{\nu, \omega' \\ \omega' + \nu - \nu' \in W}} \left(\hat{J}_+(\omega' - \nu') + \hat{J}_-(\omega' + \nu) \right) P_{\omega'}\bar{\rho}_{\nu} P^{\dagger}_{\omega' + \nu - \nu'} - i\Omega'
$$

There are several useful things to say about this system of equations. The RHS is traceless (using the cyclic property) \pm

$$
\bar{\rho}_{s} = \sum_{\nu, \nu} \left(i \tilde{J}(\omega' + is) - i \tilde{J}(-\omega'' - is) \right) P_{\nu} \bar{\rho}_{s-i}(\nu' + \nu) P_{\nu} \qquad \qquad \bar{\rho}_{s} = \sum_{\nu \in \{\pm \Omega', 0\}} \frac{\bar{\rho}_{\nu}}{s - i \nu}.
$$

$$
- \left(i \tilde{J}(\omega' + is) \bar{\rho}_{s-i}(\nu' + \nu) P_{\nu} P_{\nu} - i \tilde{J}(-\omega'' - is) P_{\nu} P_{\bar{\rho}_{s-i}(\nu' + \nu')} \right)
$$

No Problem...

 $\overline{1}$ 0 $\overline{1}$ 1 ϵ/ω_0 \mathcal{C} \mathbf{r} M $\left| \mathbf{\left(\mathbf{a}\right) }\right|$ $\overline{}$ \mathcal{F}

strong driving approx \Rightarrow

saturation on resonance + asymmetric peaks

1.0

strong decay approx \Rightarrow no inversion + Lorentzian peaks $\overline{3}$ dB $\overline{$ L Orantzian Das

neither approx \Rightarrow asymmetric + unsaturated

Preliminary **Results**

note the wiggles...