Driven Double-Quantum Dots

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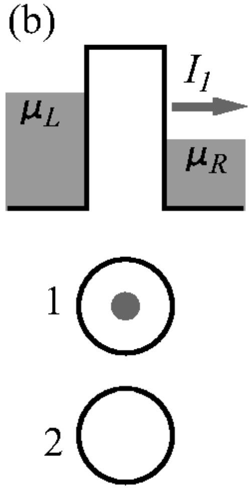
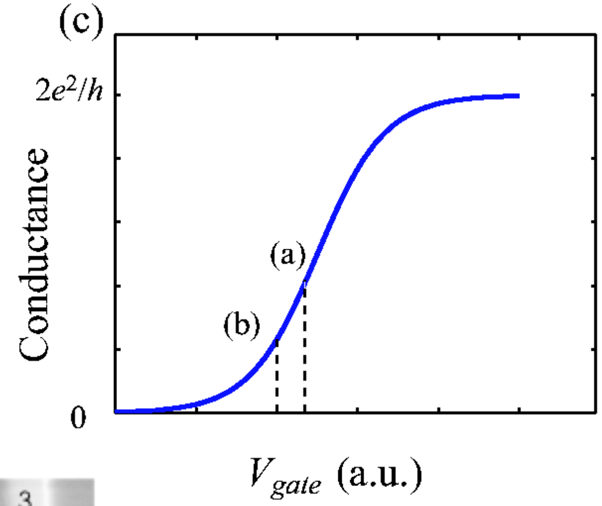
ARC CENTRE OF EXCELLENCE FOR
ENGINEERED QUANTUM SYSTEMS

Outline

- Introduction to charge qubits and detectors
- Microwave driven double dot qubit
- Electron-phonon coupling
- Breakdown of RWA
- Conclusions

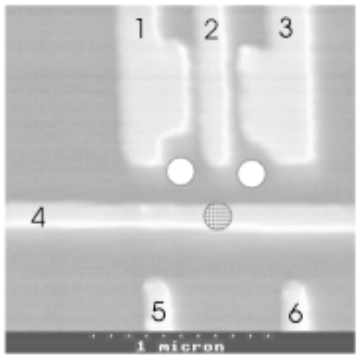
Continuous measurement of a single qubit



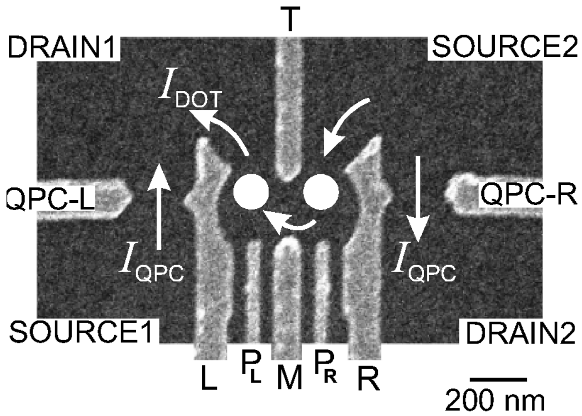


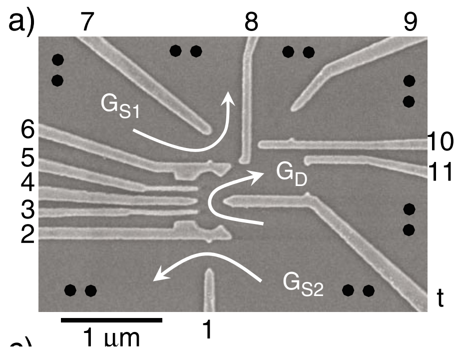
- Experiments:

Gardelis et al. Phys. Rev. B67 073302, (2003),



Elzerman et al. Phys. Rev.B67 161308, (2003)

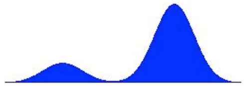




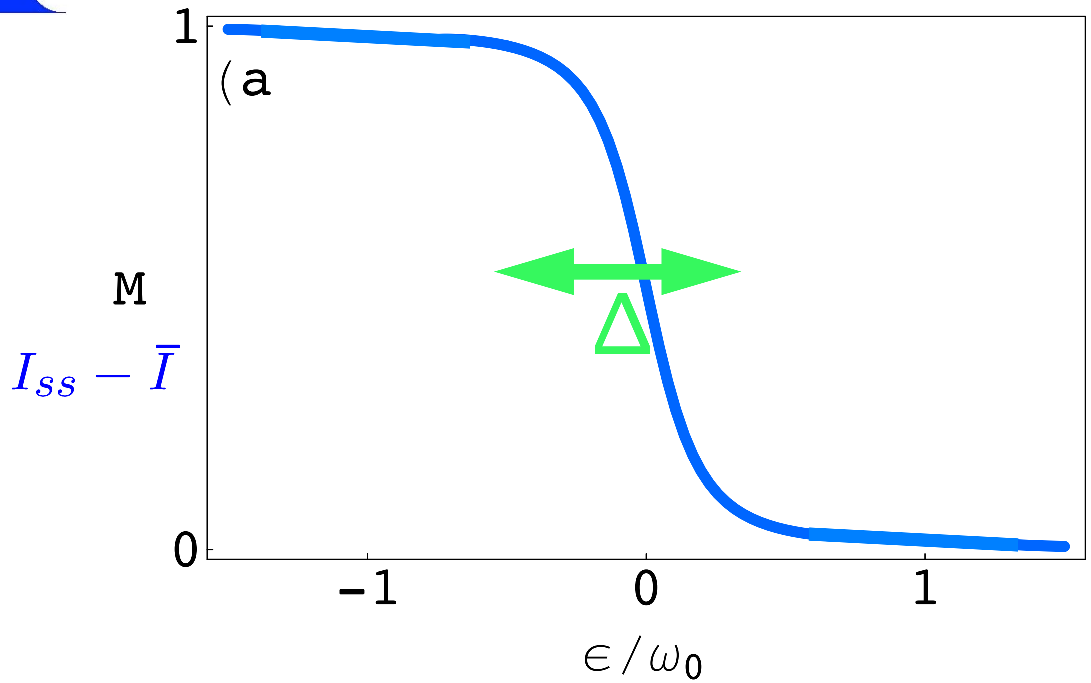
Petta et al, PRL 93 186802 2004

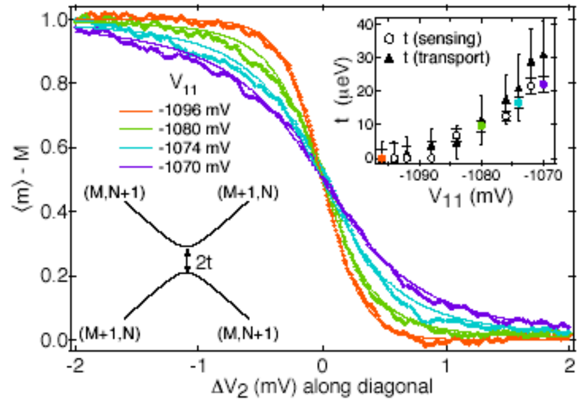
Steady state current

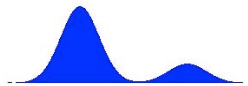
- at T=0, system relaxes (phonon emission) to ground state $\rho(\infty) = |g\rangle\langle g|$
- Detector current is proportional to ground state occupation of nearest well $I_{ss} \propto \langle l|\rho(\infty)|l\rangle$



$\Omega = 0$

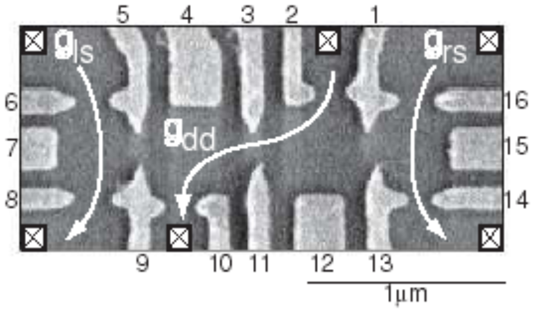


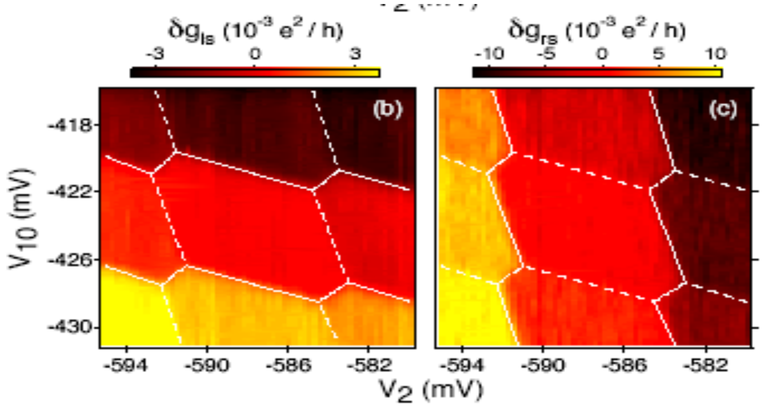


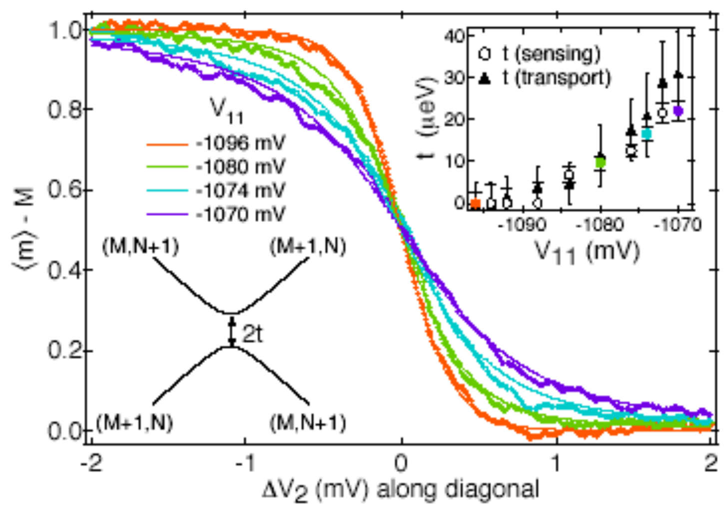


Experiments (cont.)

- *DiCarlo et al. PRL 92 226801 (2004)*





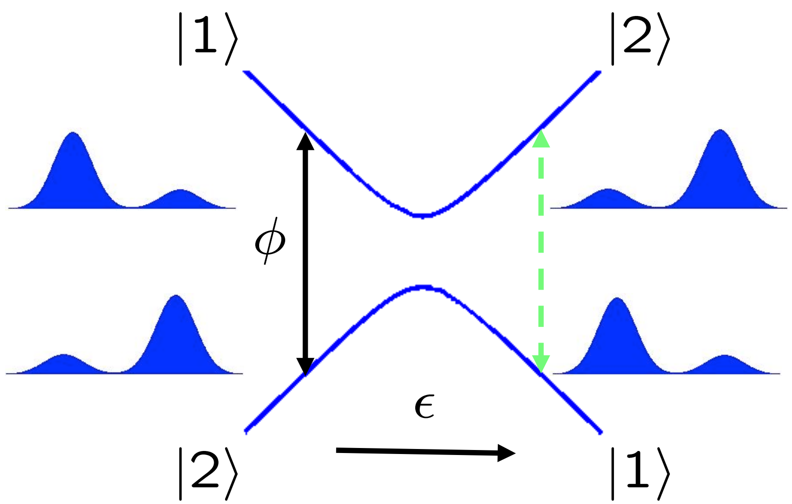


Microwave driven charge qubit

- Bare qubit Hamiltonian can be described in terms of *localized* states $|1\rangle$ and $|2\rangle$

$$H_{\mathrm{qb}} = -\frac{\Delta}{2}\left(|1\rangle\langle 2| + |2\rangle\langle 1|\right) - \frac{\epsilon}{2}\left(|1\rangle\langle 1| - |2\rangle\langle 2|\right)$$

$$\phi = \sqrt{\Delta^2 + \epsilon^2}$$



- Add time dependent driving field at frequency: $\omega_{rf} = \phi + \eta$

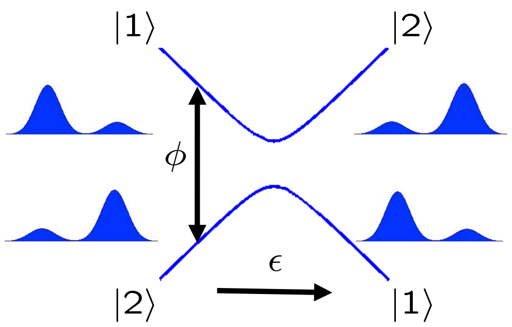
$$H_{\mathrm{driv}} = \frac{e\vec{E}(t)\cdot\vec{a}}{2}\left(|1\rangle\langle 1| - |2\rangle\langle 2|\right), \quad \vec{E}(t) = \vec{E}_0 \cos\left[(\phi+\eta)t\right]$$

Microwave driven charge qubit

- Transform to rotating frame and make rotating wave approximation

$$\tilde{H}_{\rm RWA} = -(\eta\,\sigma_z^e + \Omega\,\sigma_x^e)/2. \qquad \eta = \omega - \phi$$

- RWA Hamiltonian is diagonal in 'dressed' basis & has energy splitting given by the effective Rabi frequency



$$\Omega' = \sqrt{\Omega^2 + \eta^2}$$

$$\Omega = e\vec{E}_0\cdot\vec{a}\,\sin\theta, \qquad \theta = \tan^{-1}\left(\frac{\Delta}{\epsilon}\right)$$

sin(θ) is just the dipole moment $\langle e|\hat{z}|g\rangle$

Phonon Environment

- Phonon bath is major source of relaxation

$$ {}_{\mathrm{e-p}} = \sum_{\mathbf{q}} i \quad {}_{\mathbf{q}} \hat{\varrho}(\mathbf{q}) (\quad {}_{\mathbf{q}} - \quad {}^{\dagger}_{-\mathbf{q}}), $$

- $M_{\mathbf{q}}$ can be piezoelectric or deformation potential

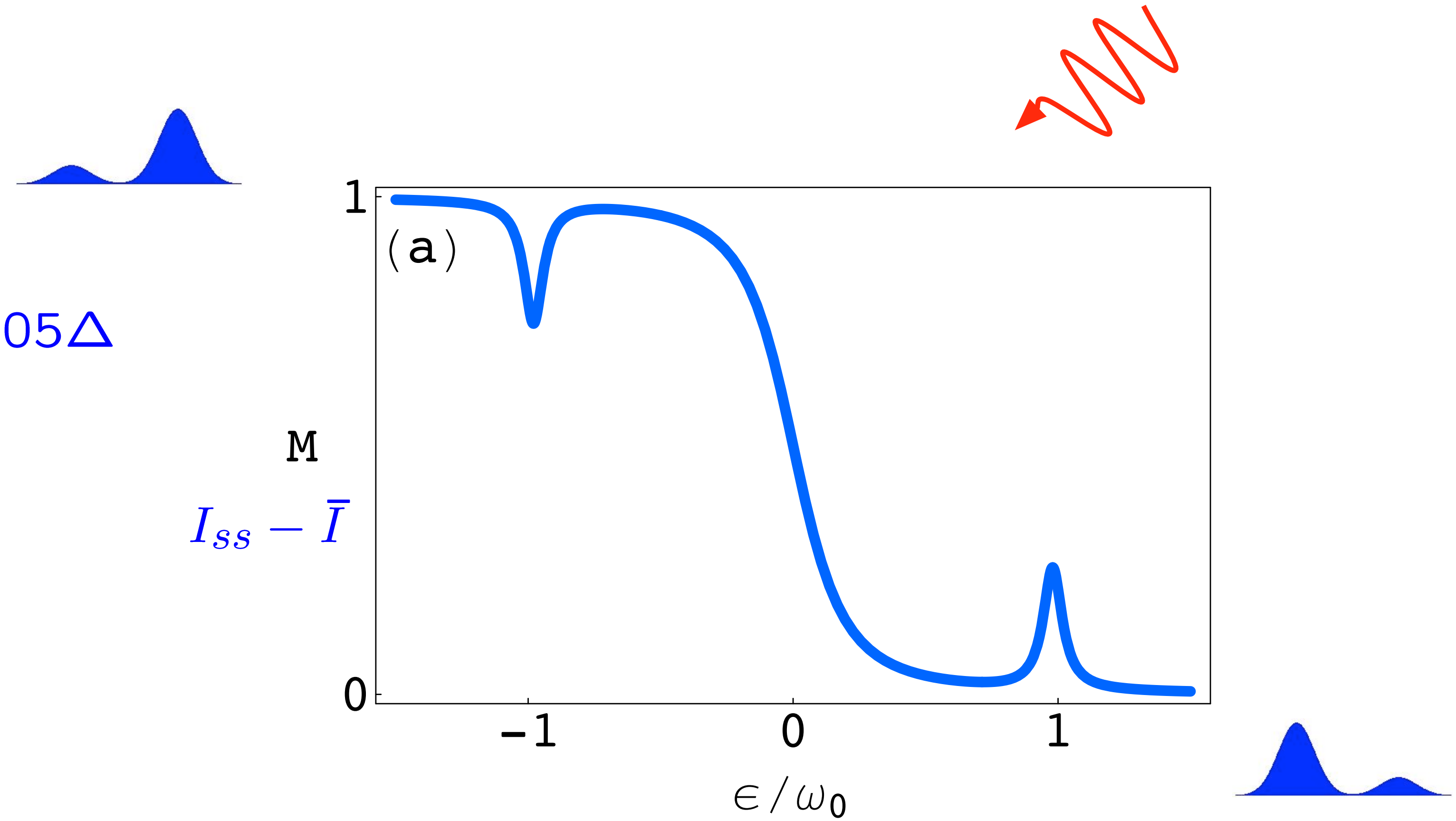
- Restrict electronic states in electron density operator $\hat{\varrho}$ and get a spin boson model

$$ H_{\mathrm{e-p}} \approx \sigma_z \sum_{\mathbf{q}} g_{\mathbf{q}} \left(a^{\dagger}_{\mathbf{q}} + a_{\mathbf{q}} \right) , $$

Steady state current

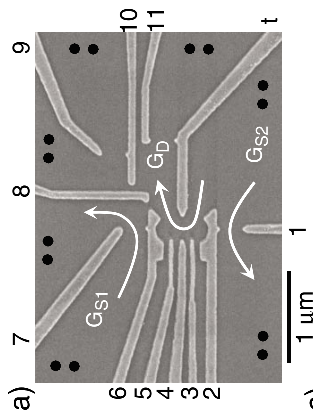
With driving, get resonant peaks when ω=ϕ.

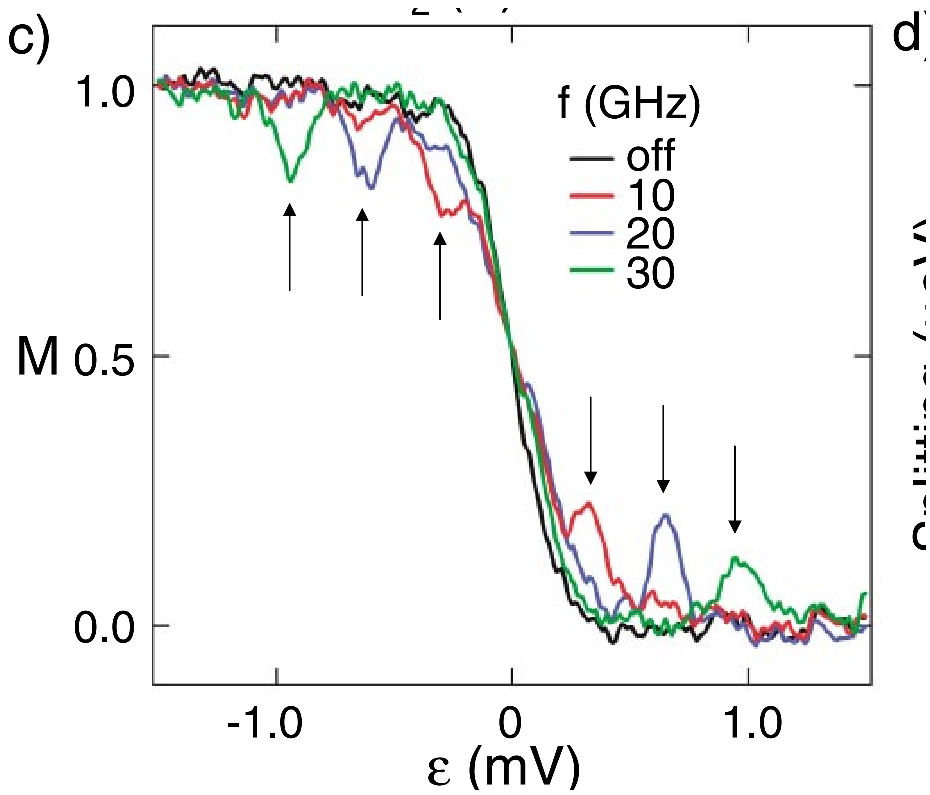
$\Omega = 0.05\Delta$

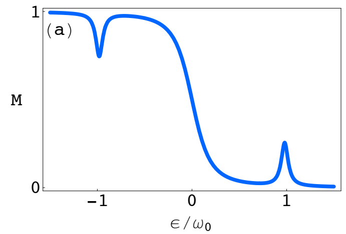


Past experimental results

… and in isolated quantum dots:





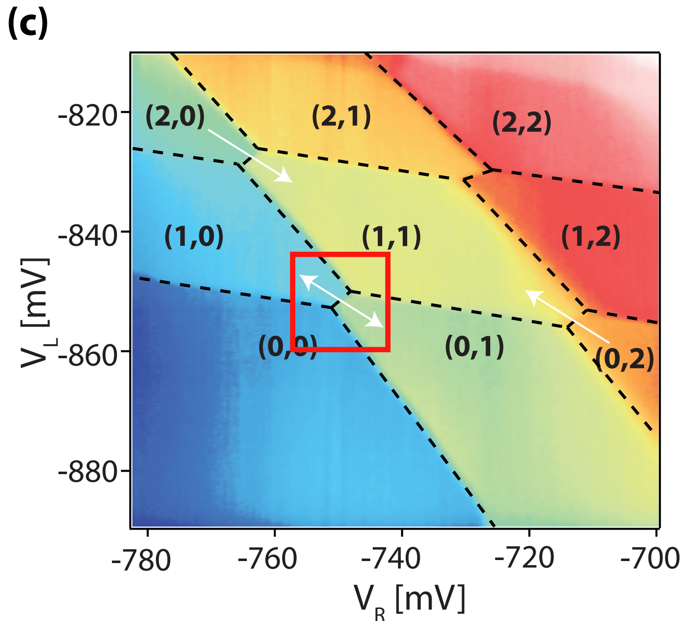


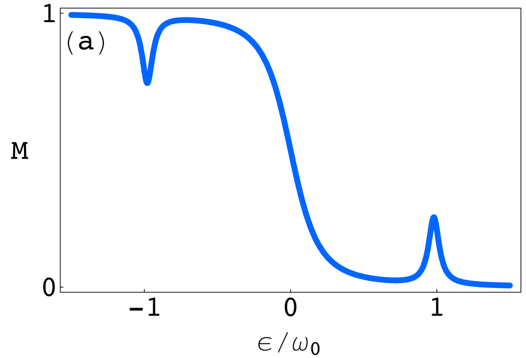
Petta et al, PRL 93 186802 2004

Peaks should remain below 0.5

Recent experimental results

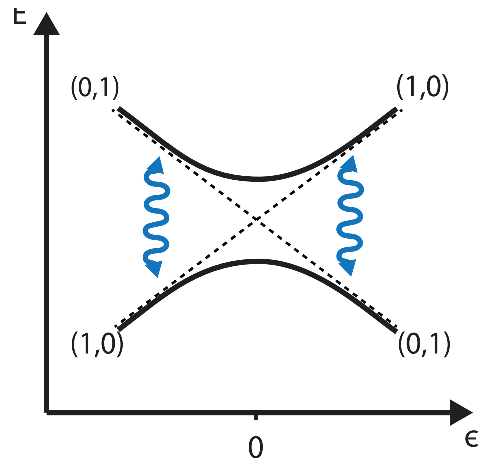
Reilly et al, unpublished 2012

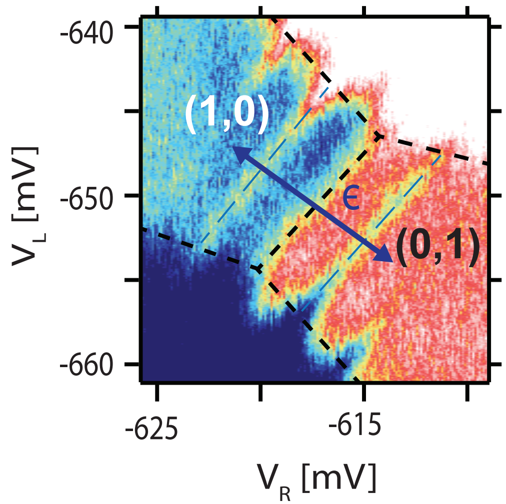




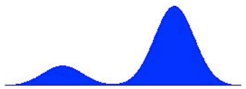
without driving

with driving

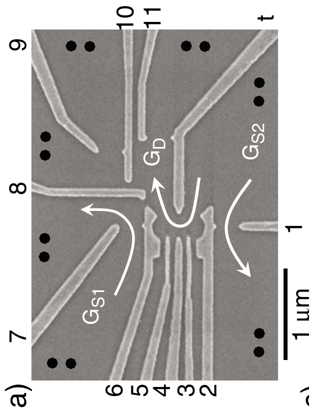


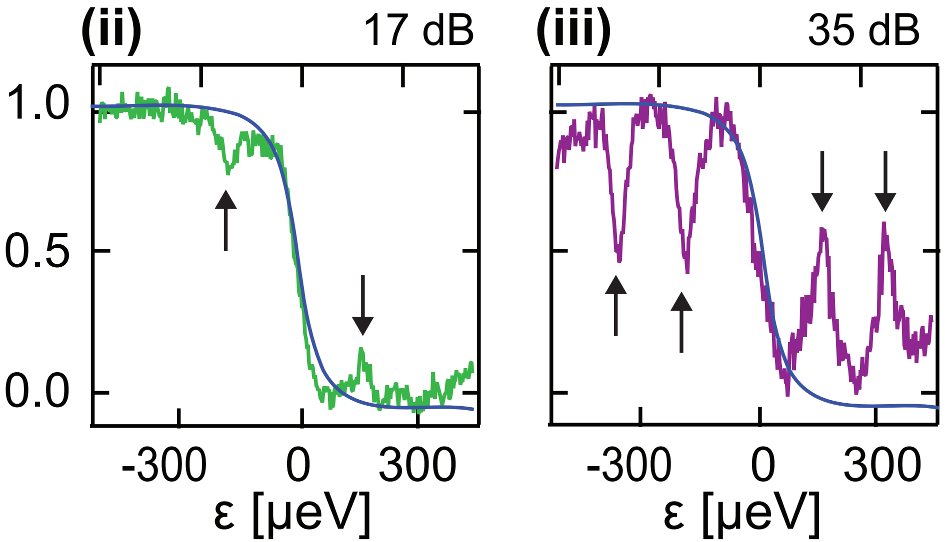


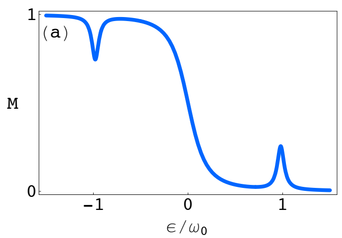
Surprise!

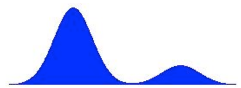


Reilly et al, unpublished, 2012









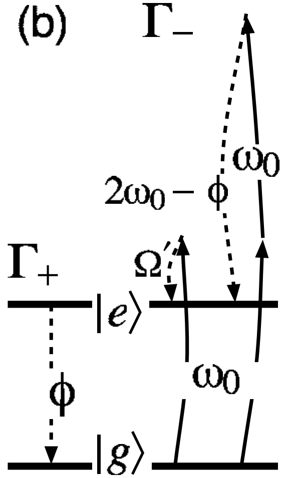
The peaks go over 0.5!

i.e. *population inversion*: the systems spends longer in the excited state

Master equation

- Derive master equation from dynamics of joint density matrix *R*:

$$\dot{\rho}_I(t) = -\int_0^t dt' \, \mathrm{Tr}_B[H_I(t), [H_I(t'), R(t')]].$$

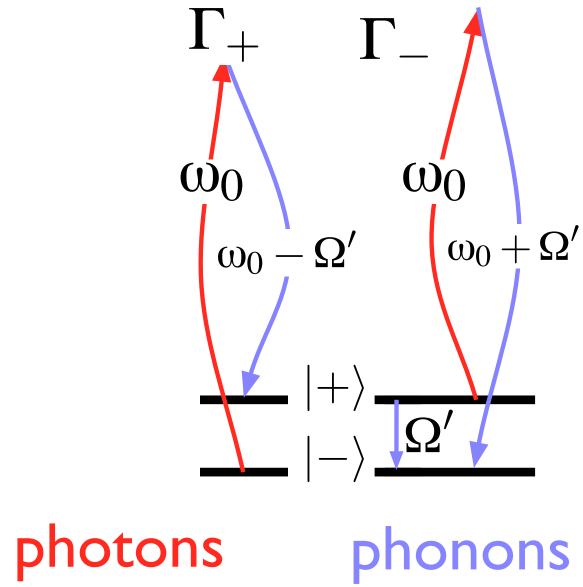


- Two versions of secular / rotating wave approximation

a) in dressed basis

$$\dot{\rho}_I = \sum_{\omega'} J(\omega') \left[(N(\omega')+1)\, \mathcal{D}[P_{\omega'}]\rho_I + N(\omega')\mathcal{D}[P^\dagger_{\omega'}]\rho_I \right]$$

Spectral density $J(\omega) = 2\pi \sum_{\mathbf{q}} |g_{\mathbf{q}}|^2 \delta(\omega - \omega_{\mathbf{q}})$



where $P_0 = \cos\theta\cos\varphi\,\sigma_z^d/2$, $P_{\Omega'} = -\cos\theta\sin\varphi\,\sigma_-^d$,
$P_{\omega_0 \pm \Omega'} = \mp\sin\theta\,(1 \pm \cos\varphi)\,\sigma_\mp^d/2$, $P_{\omega_0} = -\sin\theta\sin\varphi\,\sigma_z^d/2$.

$N(\omega) = (e^{\omega/k_B T} - 1)^{-1}$

$\mathcal{D}[A]\rho \equiv A\rho A^\dagger - (A^\dagger A\rho + \rho A^\dagger A)/2$

Master equation

• Derive master equation from dynamics of joint density matrix R:

$$\dot{\rho}_I(t) = -\int_0^t dt' \, \mathrm{Tr}_B[H_I(t), [H_I(t'), R(t')]].$$

•Two versions of secular / rotating wave approximation

a) in dressed basis

$$\dot{\rho}_I = \sum_{\omega'} J(\omega') \left[(N(\omega') + 1) \, \mathcal{D}[P_{\omega'}]\rho_I + N(\omega')\mathcal{D}[P^{\dagger}_{\omega'}]\rho_I \right]$$

b) in bare energy basis

$$\dot{\rho}_I(t) = -\, i[-\frac{\eta}{2}\sigma_z^{(e)} - \frac{\Omega}{2}\sigma_x^{(e)}, \rho_I(t)] + \Gamma_\varphi \mathcal{D}[\sigma_z^{(e)}]\rho_I(t) + \Gamma_r \mathcal{D}[|g\rangle\langle e|]\rho_I(t)$$

Solve Bloch vector in steady state

$$\rho(t) = (\mathbb{I} + x_d(t)\sigma_x^d + y_d(t)\sigma_u^d + z_d(t)\sigma_z^d)/2$$

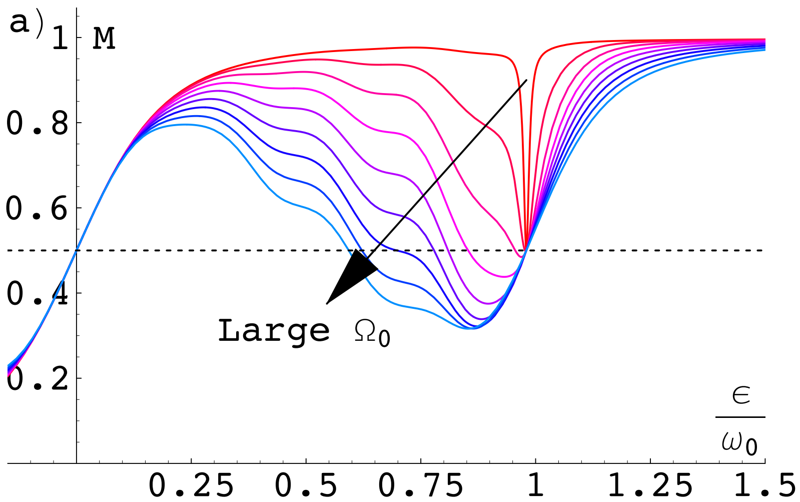
populations:

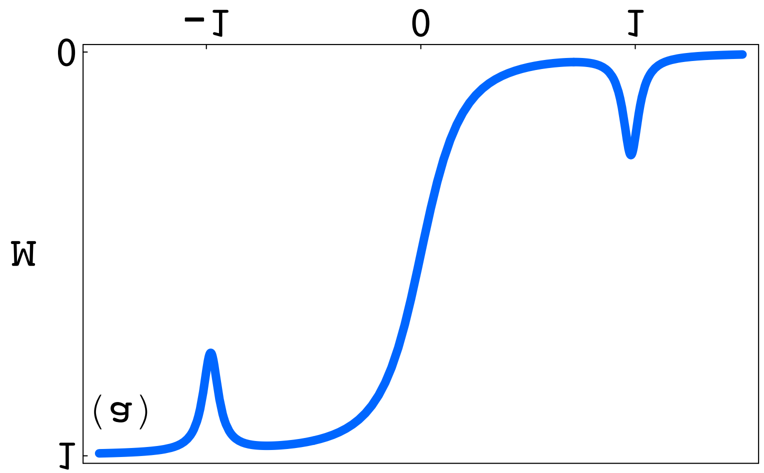
$$\dot{z}_d = (\Gamma_- - \Gamma_+) - (\Gamma_- + \Gamma_+) z_d$$

RWA in

a) in dressed basis (Rabi >> decay)
(PRL 95 106901, 2005)

b) in bare energy basis (Rabi << decay)
(PRL 96 017405, 2006)

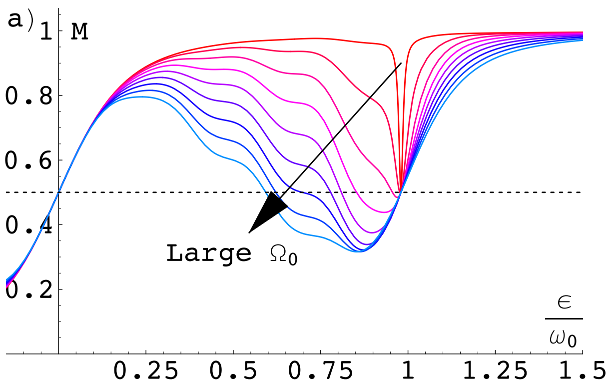


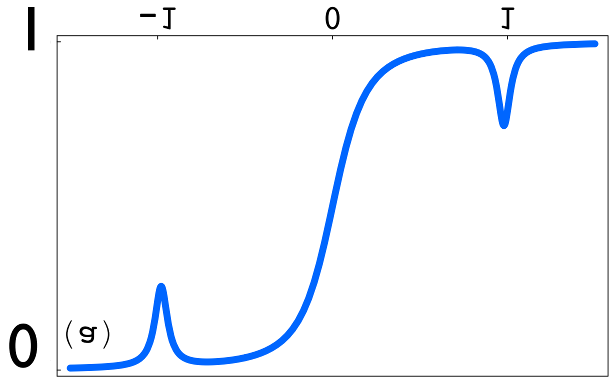


Asymmetric peak
Saturated on resonance

Symmetric (Lorentzian) peak
Unsaturated

Problem...





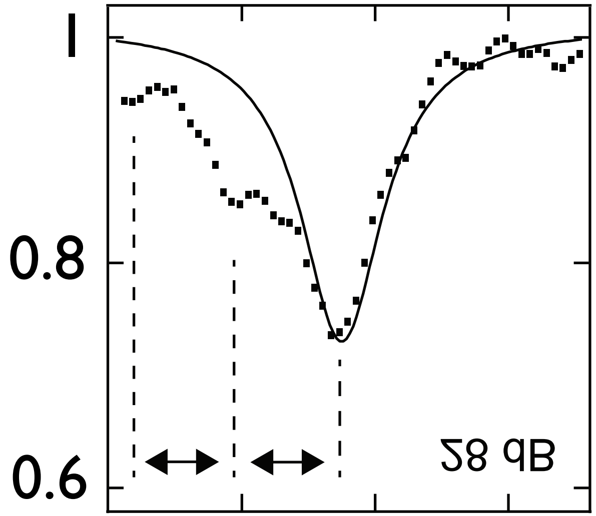
strong driving approx ⇒

saturation on resonance
+ asymmetric peaks

strong decay approx ⇒

unsaturated +
Lorentzian peaks

Data:
unsaturated
asymmetric



Dynamical master equation

- von Neuman equation

$$\dot{\rho}_I(t) = -\int_0^t dt' \, \mathrm{Tr}_B[H_I(t), [H_I(t'), R(t')]].$$

$$H_I(t) = \sigma_z(t) \sum_{\mathbf{q}} g_{\mathbf{q}}^* a_{\mathbf{q}}^\dagger e^{i\omega_{\mathbf{q}} t} + g_{\mathbf{q}} a_{\mathbf{q}} e^{-i\omega_{\mathbf{q}} t},$$

- Transform

$$\sigma_z(t) = \sum_{\omega' \in \mathcal{W}} P_{\omega'} e^{i\omega' t}.$$

$$s\bar{\rho}_s - \rho_0 = \sum_{\,',\,''} \left(i\tilde{J}(\omega' + is) - i\tilde{J}(-\omega'' - is) \right) P_{\,'} \bar{\rho}_{s-i(\,'+\,'')} P_{\,''}$$

$$- \left(i\tilde{J}(\omega' + is) \bar{\rho}_{s-i(\,'+\,'')} P_{\,''} P_{\,} - i\tilde{J}(-\omega'' - is) P_{\,''} P_{\,} \bar{\rho}_{s-i(\,'+\,'')} \right)$$

- Solve by expansion

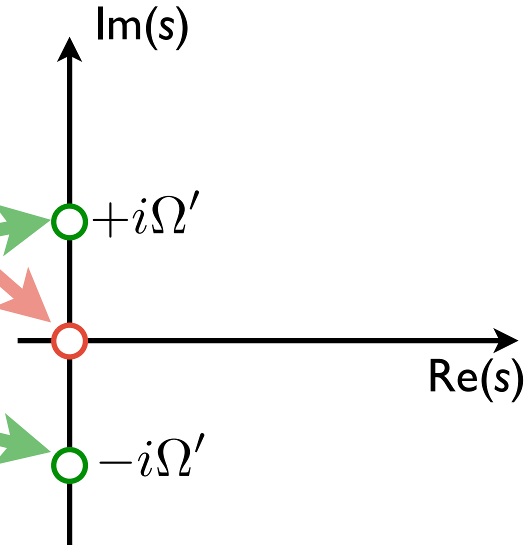
$$\tilde{J}(x) = \sum_q \frac{|g_q|^2}{\omega_q + x}$$

$$\bar{\rho}_s = \sum_{\nu \in \{\pm\Omega', 0\}} \frac{\bar{\rho}_\nu}{s - i\nu}.$$

Dynamical master equation

- Keeping only DC pole in ρ and matching residues at s=0 gives traditional Lindblad master eqn…

$$0 = \sum_{\omega'} J(\omega')\mathcal{D}[P'_{\omega}]\rho_{ss} + iF(\omega')[P^{\dagger}_{\omega'}P_{\omega'}, \rho_{ss}]$$



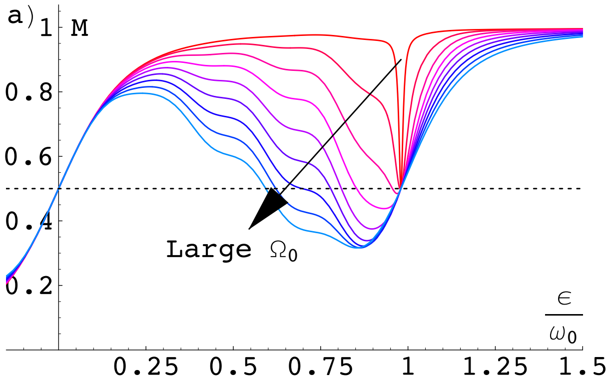
- Include dynamical poles...

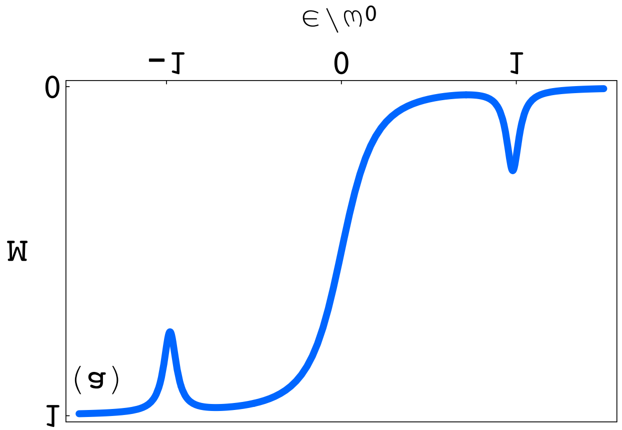
$$\begin{aligned} i\nu'\bar{\rho}_{\nu'} \;=\; & \sum_{\substack{\nu,\,\omega'\,:\\ \omega'+\nu-\nu'\in\mathcal{W}}} \left(\hat{J}_{+}(\omega'-\nu') + \hat{J}_{-}(\omega'+\nu)\right) P_{\omega'}\bar{\rho}_{\nu}P^{\dagger}_{\omega'+\nu-\nu'} \\ & - \left(\hat{J}_{+}(\omega'-\nu')\bar{\rho}_{\nu}P^{\dagger}_{\omega'+\nu-\nu'}P_{\omega'} + \hat{J}_{-}(\omega'+\nu)P^{\dagger}_{\omega'+\nu-\nu'}P_{\omega'}\bar{\rho}_{\nu}\right). \end{aligned}$$

$$\bar{\rho}_s = \sum_{\nu\in\{\pm\Omega',0\}} \frac{\bar{\rho}_{\nu}}{s-i\nu}.$$

$$\begin{aligned} s\bar{\rho}_s - \rho_0 \;=\; & \sum_{',\,''} \left(i\tilde{J}(\omega'+is) - i\tilde{J}(-\omega''-is)\right) P_{'}\bar{\rho}_{s-i('+'')}P_{''} \\ & - \left(i\tilde{J}(\omega'+is)\bar{\rho}_{s-i('+'')}P_{''}P \;\; - i\tilde{J}(-\omega''-is)P_{''}P\;\bar{\rho}_{s-i('+'')}\right) \end{aligned}$$

No Problem...





strong driving approx ⇒

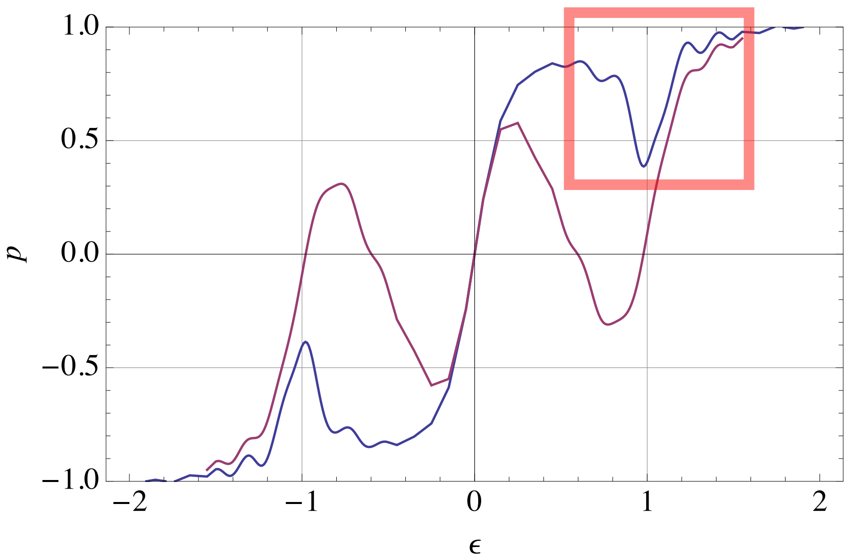
saturation on resonance + asymmetric peaks

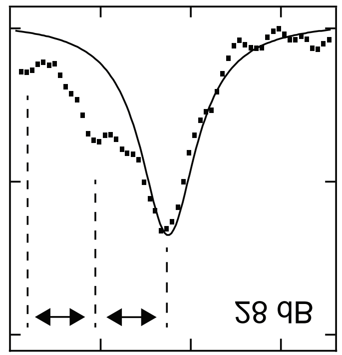
strong decay approx ⇒

no inversion + Lorentzian peaks

neither approx ⇒

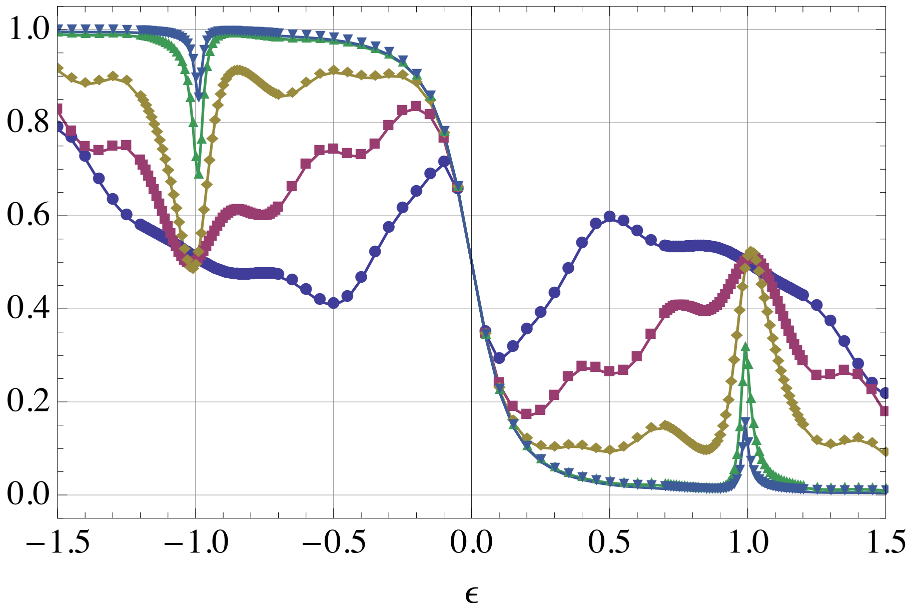
asymmetric + unsaturated





Preliminary Results

Theory



note the wiggles...