(Opto)-Mechanical Quantum Interface: Noise Resilient Operations via Control Techniques

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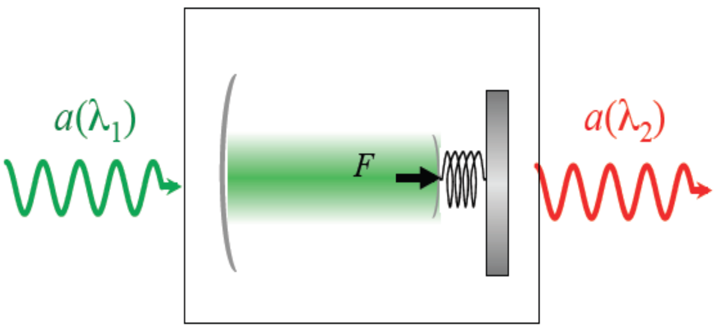
Supported by

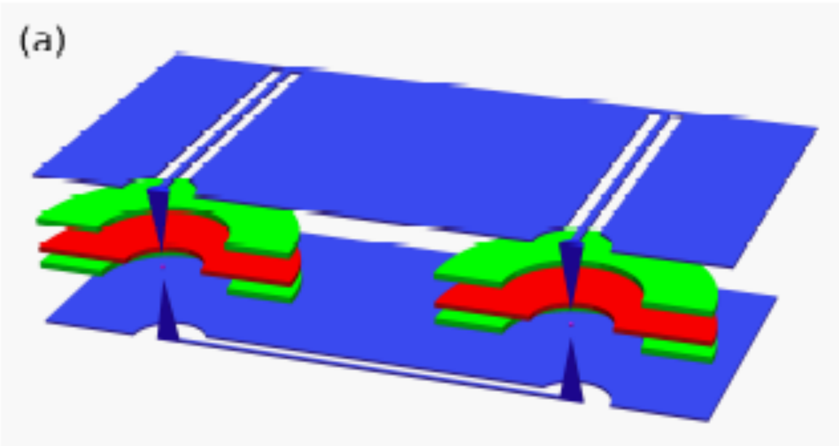
Outline

1. Quantum "mechanical" systems

2. Optomechanical quantum interface
   High fidelity state transfer
   Robust photon entanglement
   (via dark mode)



3. Parametric coupling of trapped particle motion with superconducting circuits



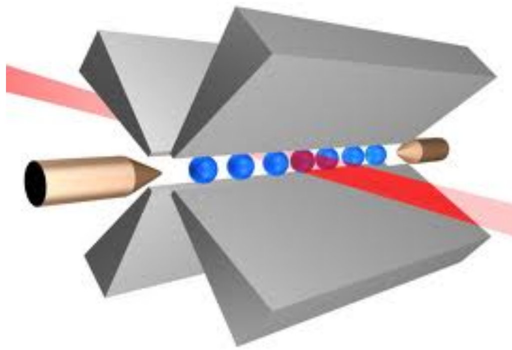
4. Conclusions

Mechanical Systems in the Quantum Limit

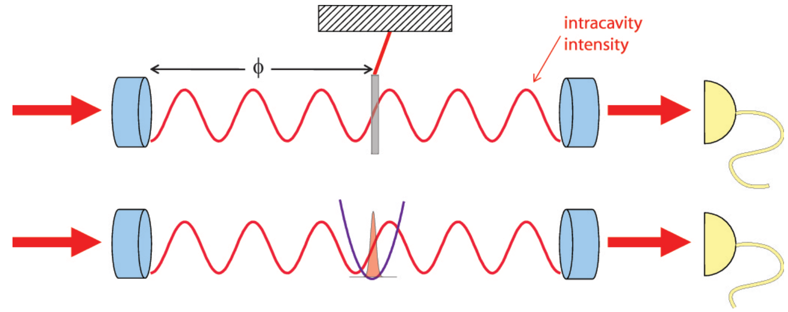
Classical system
- acoustic frequency
- Room temperature
- thermal excitations

Quantum limit
- high frequency
- relatively high Q-factor
- strong coupling with other nanoscale devices

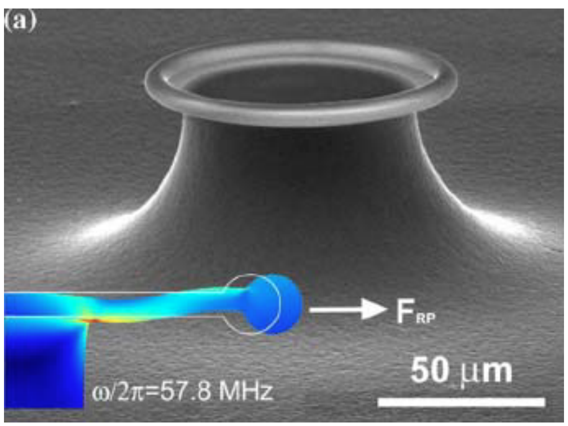
Mechanical Systems in the Quantum Limit



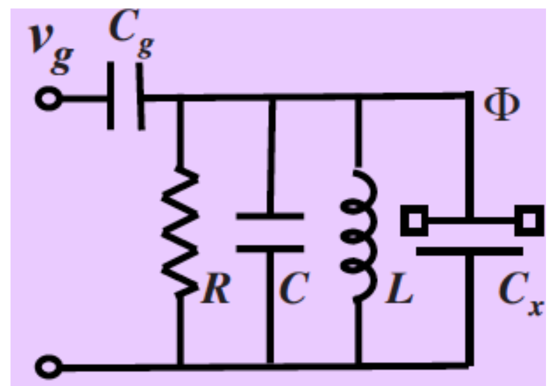
Harmonic motion of trapped ions
(Brown et al, Nature 2011)



Atomic cloud in optical cavity
(Brooks et al, Nature 2012)



Optomechanical systems
(Kippenberg,Vahala, Science 2008, review)



Nanoelectromechanical systems
(O'Connell et al, Nature 2010
Teufel et al, Nature 2011)

Mechanical Systems in the Quantum Limit

Recent progresses

- Strong coupling between light and mechanical modes
  microwave: Teufel et al Nature (2011)
  $\omega_m/2\pi, 10\,\mathrm{MHz}$ $\kappa/2\pi, 100\,\mathrm{kHz}$ $g/2\pi, 1\,\mathrm{MHz}$
  optical experiment: Verhagen et al Nature (2012)
  $\omega_m/2\pi, 100\,\mathrm{MHz}$ $\kappa/2\pi, 7\,\mathrm{MHz}$ $g/2\pi, 10\,\mathrm{MHz}$

- Mechanical modes reach quantum ground state – cavity cooling or high resonator frequency – reported in several recent experiments

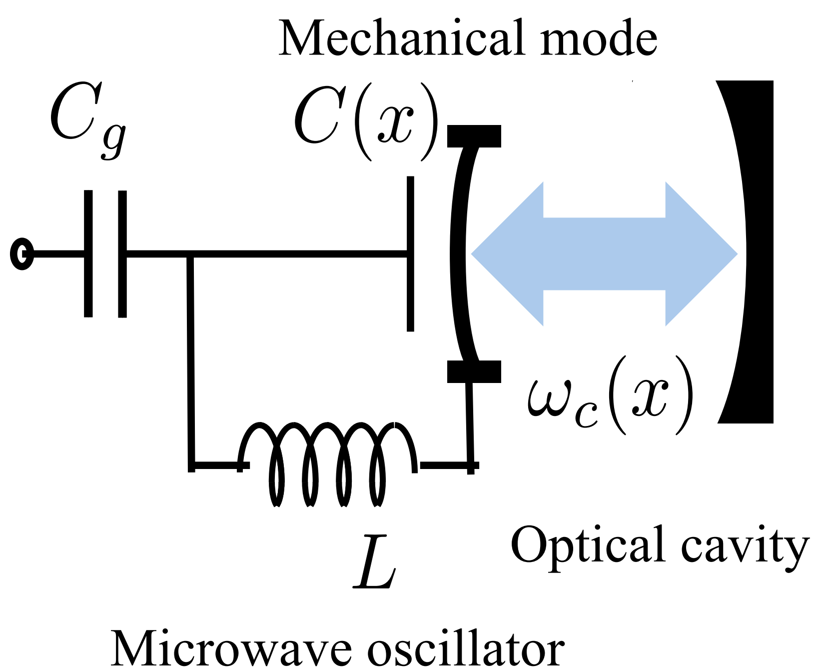
- Optomechanically induced transparency, mechanical dark mode
  Weis et al, Science (2010), Teufel et al, Nature (2011), Safavi-Naeini et al Nature (2011), C. Dong et al Science (2012)

Recent review: Aspelmeyer, Meystre, Schwab, Phys. Today (2012)

Optomechanical Quantum Interface

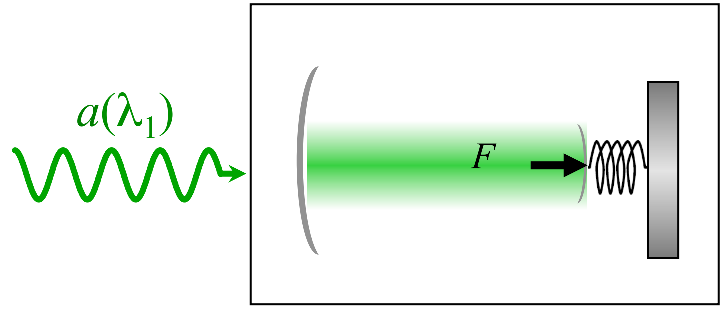
- Strong/controllable light-matter coupling
- Can connect very different systems – hybrid system
- Connect different parts of a quantum network

Cirac, Zoller, Kimble, Mabuchi, PRL (1997).

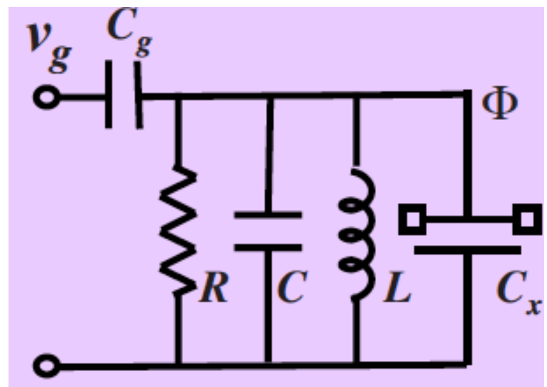


Mechanical Effects of Light

Radiation pressure force on the mirror – cavity backaction



Optical cavity + movable mirror
Photon scattered by mirror
Forces on mirror ~ photon number



Superconducting resonator - NEMS
Mechanical motion changes capacitance
Forces on NEMS ~ photon number

Various effects studied:
Cooling to quantum limit
Strong coupling regime
…

$$H_{int} = G_0 a^\dagger a \hat{x} = F \cdot \hat{x} = \hbar \Delta\omega \cdot a^\dagger a$$

e.g. C.K. Law, PRA (1995)

Optomechanical Quantum Interface

Radiation pressure force and effective linear coupling

Cavity-mechanical mode coupling: mechanical shift of cavity resonance

$$H_G = -G_i a_i^\dagger a_i q$$

Pumping on cavity mode – steady state amplitude, Δ_i: laser detuning

$$a_{is} = \frac{-iE_i}{\kappa_i/2 - i(\Delta_i + G_i q_s)}$$

Red sideband driving – effective linear coupling
(all terms relative to steady state)

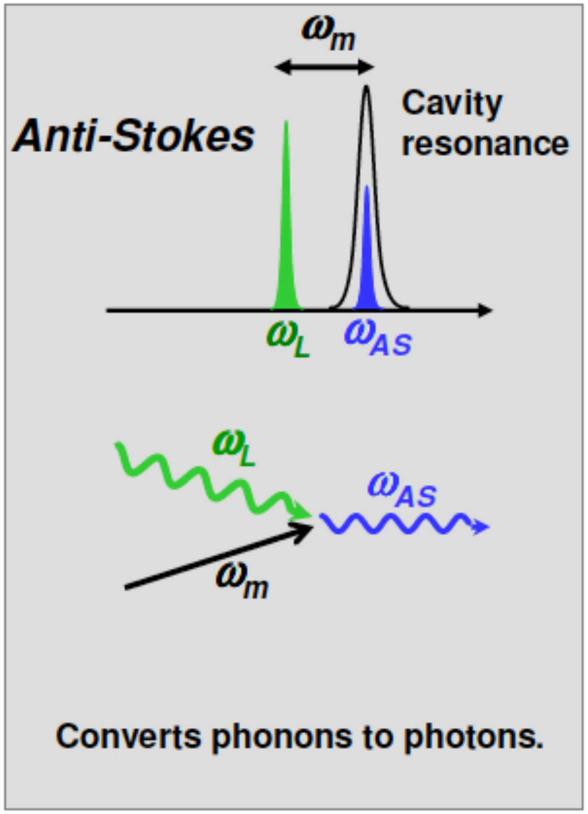
$$H_{eff} = \epsilon_i a_i^\dagger b_m + \epsilon_i^\star b_m^\dagger a_i$$

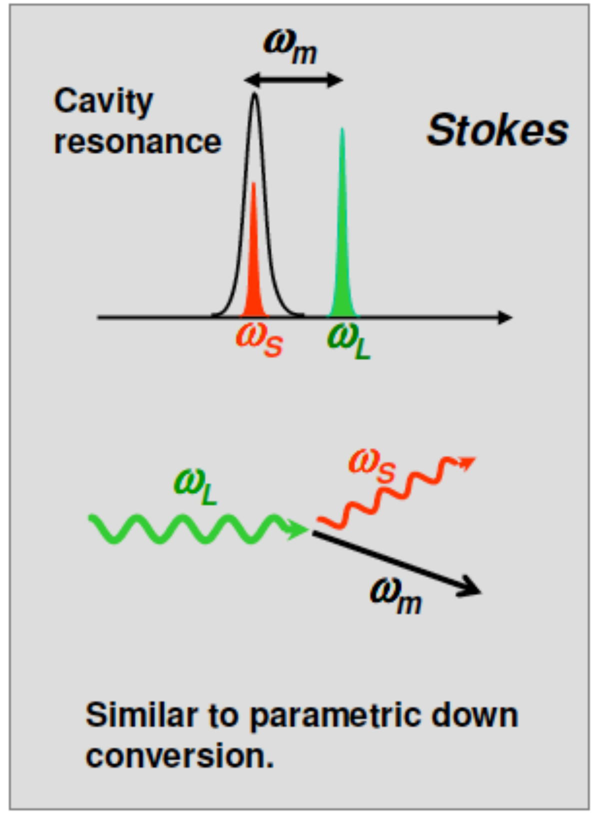
Blue sideband driving – effective linear coupling (instability etc…)

$$H_{eff} = i\epsilon_i \left(a_i^\dagger b_m^\dagger - b_m a_i\right)$$

Optomechanical Quantum Interface

Radiation pressure force and effective linear coupling





Red detuned driving

Blue detuned driving

Optomechanical Quantum Interface

Radiation pressure force and effective linear coupling

Red sideband driving – beam-splitter operation

$$H_{eff} = \epsilon_i a_i^\dagger b_m + \epsilon_i^\star b_m^\dagger a_i$$

Generate transformation - Swapping of modes with for π/2 pulse

$$a_i(t) = \cos(\epsilon_i t) a_i(0) + i \sin(\epsilon_i t) b_m(0)$$
$$b_m(t) = \cos(\epsilon_i t) b_m(0) + i \sin(\epsilon_i t) a_i(0)$$

Blue sideband driving – parametric amplifier

$$H_{eff} = i\epsilon_i \left(a_i^\dagger b_m^\dagger - b_m a_i \right)$$

Generate two-mode squeezing – Gaussian EPR pairs and entanglement
Combined with beam-splitter – squeezing of each mode

$$a_i(t) = \cosh(\epsilon_i t) a_i(0) + i \sinh(\epsilon_i t) b_m^\dagger(0)$$
$$b_m(t) = \cosh(\epsilon_i t) b_m(0) + i \sinh(\epsilon_i t) a_i^\dagger(0)$$

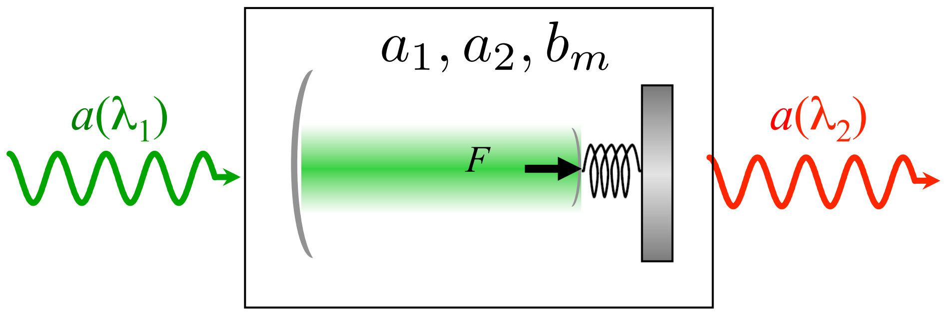
Optomechanical Quantum Interface

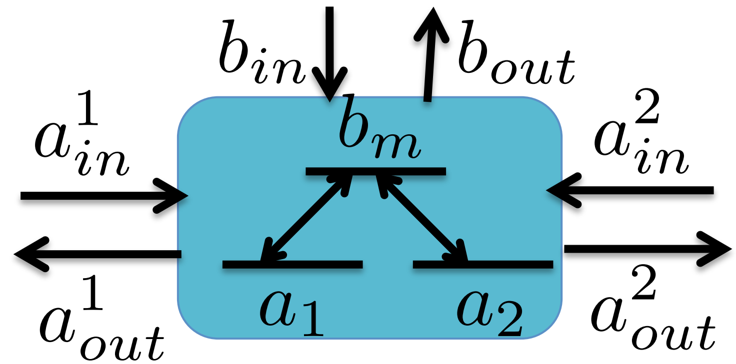
Outstanding questions for this talk:

- Quantum manipulations of light modes via mechanical mode
- Effect of mechanical noise
- Can we suppress effect of noise?
- Answer: via dark mode – control the pumping

Optomechanical Quantum Interface

Two cavity modes (quantum channels) and a mechanical mode (interface)



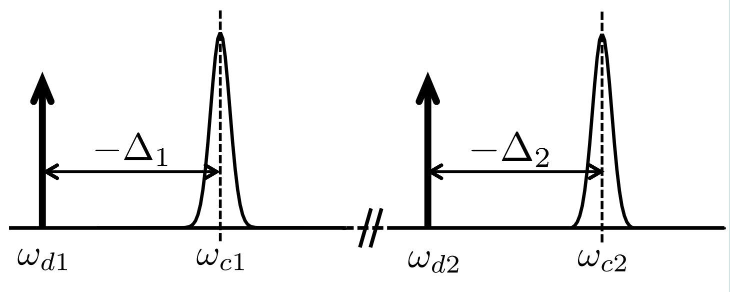


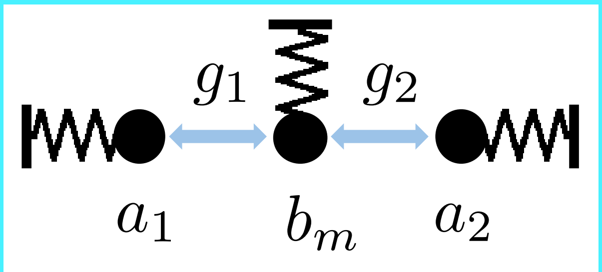
1. Cavity modes can have distinct frequency/system – microwave, optical … (hybrid quantum system)
2. Input, output channels for all three modes – mechanical thermal noise
3. Thermal noise can degrade fidelity/robustness of quantum schemes
4. Extended models: multi-modes/coupling configurations

Optomechanical Quantum Interface

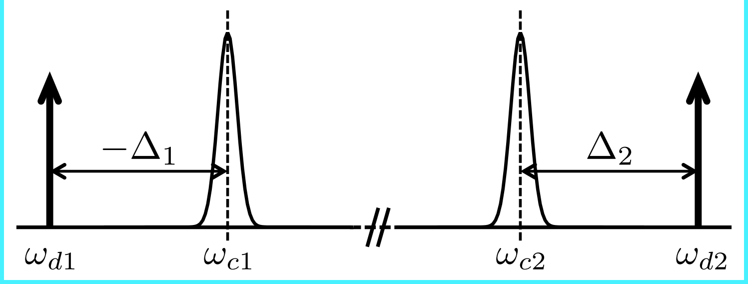
- **System: two cavity modes (microwave, optical), mechanical mode**
- **Linearization under strongly pumped cavity modes and RWA**

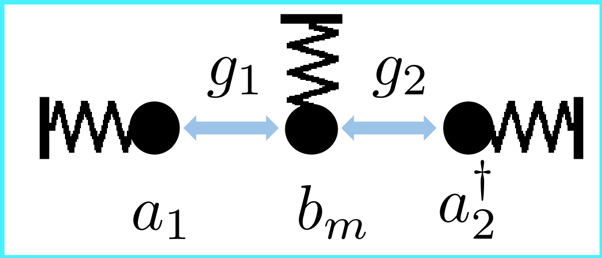
Red-detuned – Red-detuned
- quantum wavelength conversion
- discrete state entanglement





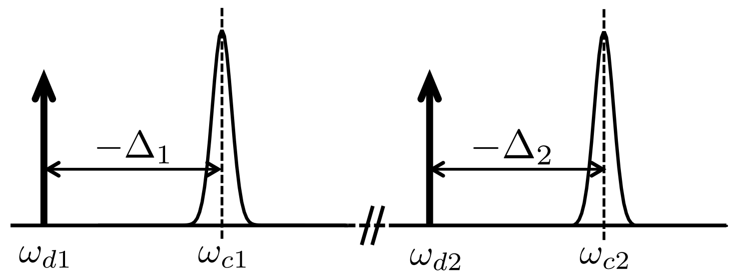
Red-detuned – Blue-detuned
- continuous variable entanglement





Quantum Wavelength Conversion

Beam-splitter operations



Cavity mode a1 – red-detuned drive $-\Delta_1 = \omega_m$
Cavity mode a2 – red-detuned drive $-\Delta_2 = \omega_m$
Both coupling with mechanical mode bm

System Hamiltonian in the strong coupling regime with RWA

$$H_I = \sum_{i=1,2} \hbar g_i (a_i^\dagger b_m + b_m^\dagger a_i) + H_{I,diss}$$

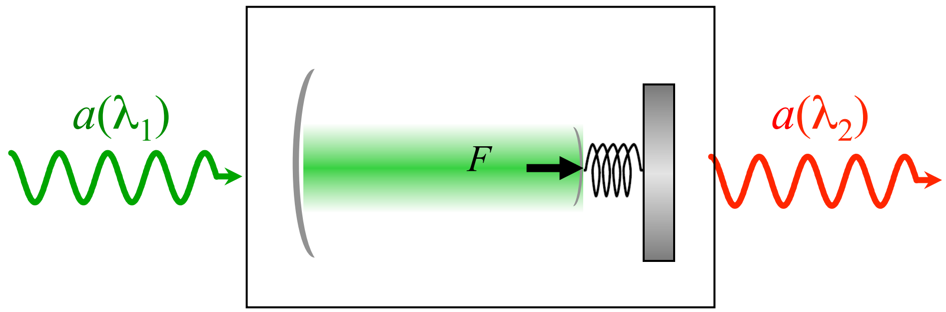
Hamiltonian used for quantum state transfer

Quantum Wavelength Conversion

Goals

Two cavity modes and
a mechanical mode:

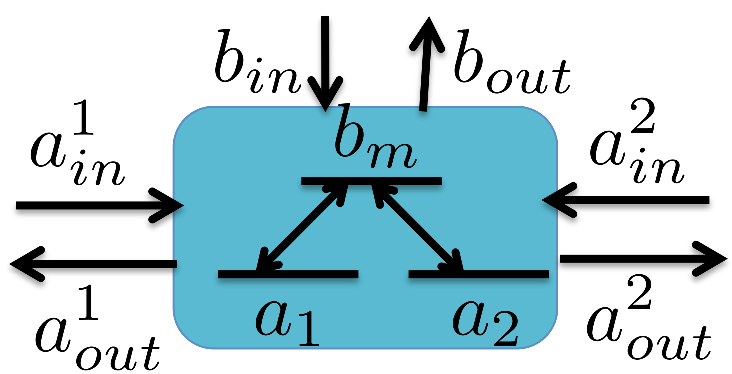
$$a_1, a_2, b_m$$



1. Conversion of pre-prepared quantum state in one cavity to the other. Cavity modes have distinct frequency

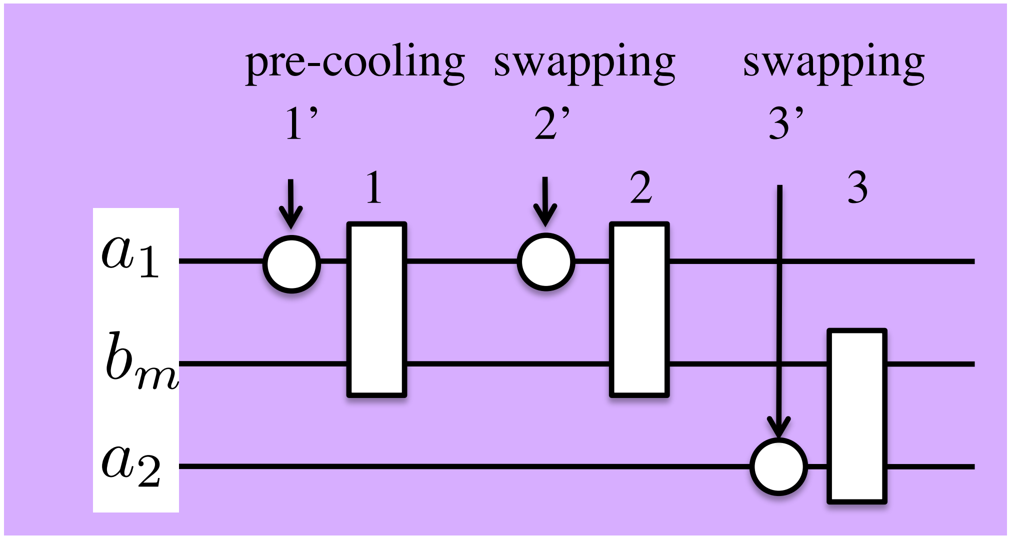
2. Transmission of pulse from one input port to another output port at different frequency

$$a_{in}^1 \rightarrow a_{out}^2$$



Quantum Wavelength Conversion

Transfer of quantum state : transient scheme by "2" swap pulses



Double-swap scheme:

1. Swap modes a_1 and b_m
   - initial state to b_m
2. Swap modes b_m and a_2
   - initial state to a_2
3. Solving quantum Langevin equation

- Swapping via mechanical mode, thermal noise degrades conversion fidelity
- Cavity damping degrades conversion fidelity
- Fidelity for gaussian states reduces as: $-\gamma_m T(2n_{th}+1)\cosh(2r)/4$
  T = time of operation, n_{th}=thermal number $-\kappa_i T(\cosh(2r)-1)/2$
- Pre-cooling pulse '1': swap a_1 and b_m – transient cooling to partially remove thermal noise of mechanical mode

Tian, Wang, PRA 82, 053806 (2010)

Quantum Wavelength Conversion

Transfer of quantum state : transient scheme by "2" swap pulses

PRL **107,** 133601 (2011) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2011

Storing Optical Information as a Mechanical Excitation in a Silica Optomechanical Resonator

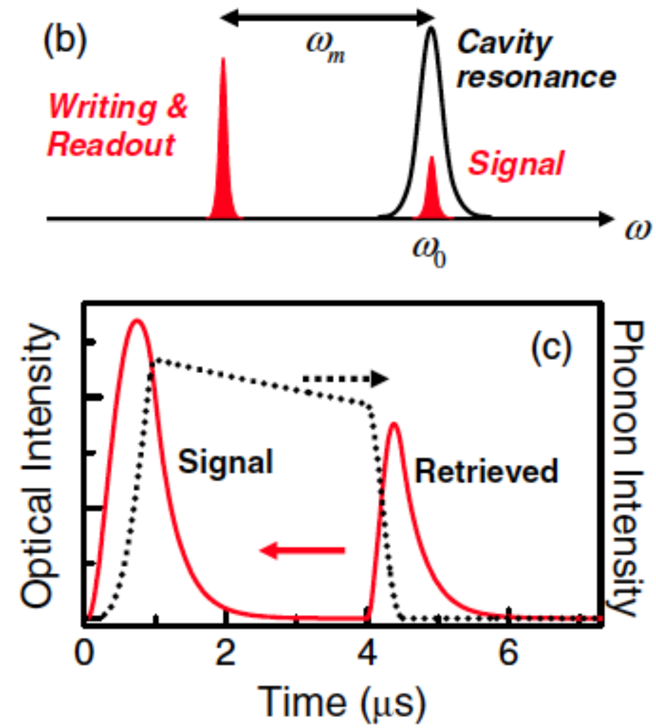
Victor Fiore,[1] Yong Yang,[1] Mark C. Kuzyk,[1] Russell Barbour,[1] Lin Tian,[2] and Hailin Wang[1]

1. Matter of principle demonstration of optomechanical swap operation

2. Optical and mechanical signal swap

3. Signal swap back to cavity after some free time

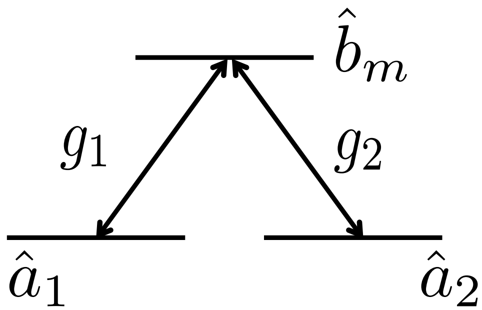
4. Signal retrieved by another swap



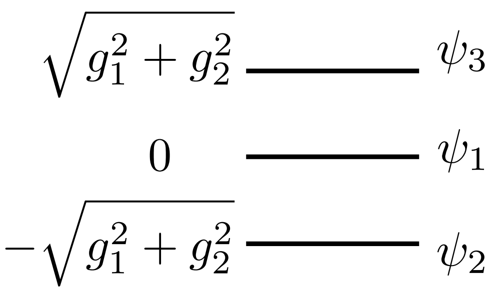
Quantum Wavelength Conversion

Adiabatic scheme via mechanical dark mode

$$H = \sum_{i=1,2} -\hbar\Delta_i a_i^\dagger a_i + \boxed{\hbar g_i (a_i^\dagger b_m + b_m^\dagger a_i)} + \hbar\omega_m b_m^\dagger b_m$$



Eigenmodes at $-\Delta_i = \omega_m$



<u>**No damping:**</u> mechanical dark mode

$$\psi_1 = (-g_2 a_1 + g_1 a_2)/g_0$$

Dark mode energy separated from other modes $g_0 = \sqrt{g_1^2 + g_2^2}$

$$\lambda_1 = 0,\ \lambda_{2,3} = \pm\sqrt{g_1^2 + g_2^2}$$

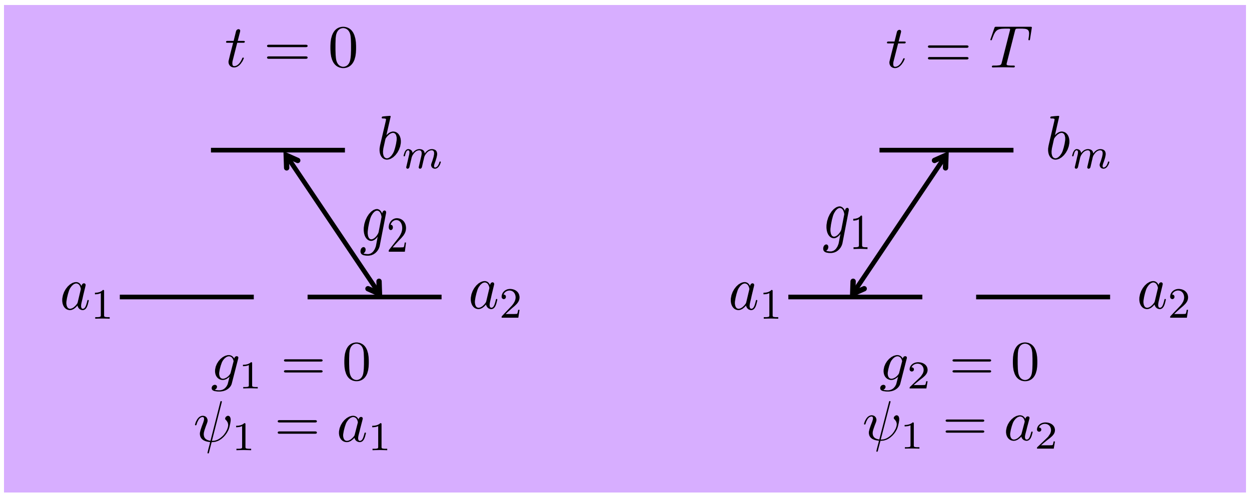
Remains in dark mode when adjusting coupling $g_{1,2}$ adiabatically (Landau-Zener condition)

$$|dg_i/dt|/g_0 \ll g_0$$

Quantum Wavelength Conversion

Adiabatic scheme via mechanical dark mode

$$\psi_1 = (-g_2 a_1 + g_1 a_2)/g_0$$



At time t=0, g_1=0, g_2=-g_0, dark mode $a_1(0)$
at time t=T, g_1=g_0, g_2=0, dark mode $a_2(T)$
Initial state in mode a_1 is transferred to mode a_2

$$a_2(T) = a_1(0)$$

Two-way state swapping scheme, S. Huang and L. Tian, in preparation (2013)

Quantum Wavelength Conversion

Adiabatic scheme via mechanical dark mode

Langevin eq. in interaction picture

$$i d\vec{v}(t)/dt = M(t)\vec{v}(t) + i\sqrt{K}\vec{v}_{in}(t)$$

$$\vec{v}(t) = [a_1, b_m, a_2]^{\mathrm{T}}$$

$$M(t) = \begin{pmatrix} -i\frac{\kappa_1}{2} & g_1(t) & 0 \\ g_1(t) & -i\frac{\gamma_m}{2} & g_2(t) \\ 0 & g_2(t) & -i\frac{\kappa_2}{2} \end{pmatrix}$$

<u>Finite damping:</u> we treat damping terms in M(t) as perturbation terms
Dark mode contains small contribution from mechanical mode

$$\psi_1 = \left(-\frac{g_2}{g_0}a_1 \boxed{-\frac{i(\kappa_1-\kappa_2)g_1 g_2}{2g_0^3}b_m} + \frac{g_1}{g_0}a_2\right)$$

<u>Not totally dark!</u>

Eigenenergy is modified – causes damping

$$\lambda_1 = -i\left(\frac{g_1^2}{2g_0^2}\kappa_2 + \frac{g_2^2}{2g_0^2}\kappa_1\right)$$

Hence, adiabatic conversion can be affected by mechanical noise
How to characterize these effects?

Quantum Wavelength Conversion

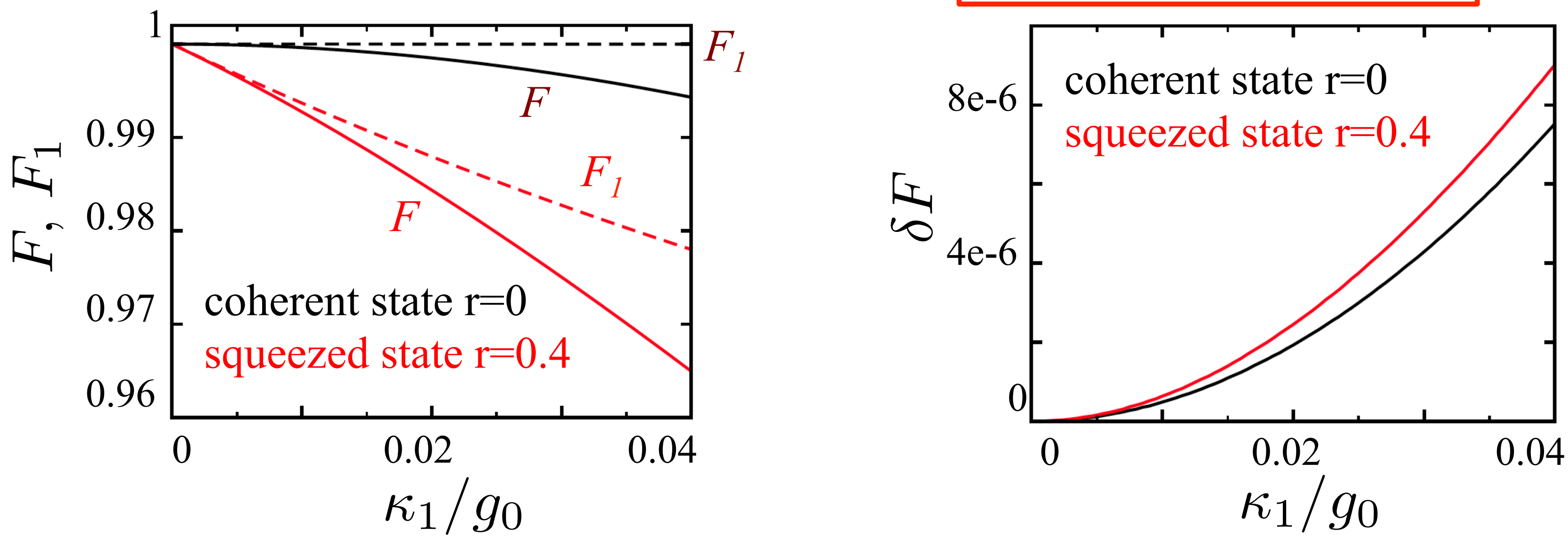
Adiabatic scheme via mechanical dark mode

Fidelity for gaussian states at time T: $F = F_1 F_2$

$$F_1 \approx 1 - f(0,T)(\cosh(2r) - 1) - f_s \cosh(2r) \qquad f(0,T) \sim (\kappa_1 + \kappa_2)T/4$$

$$F_2 \approx 1 - f^2(0,T) y(\alpha)/2. \qquad f_s \lesssim \gamma_m (2n_{th} + 1) T ((\kappa_1 - \kappa_2)/4g_0)^2$$

2-swap pulse scheme:- $\kappa_i T(\cosh(2r) - 1)/2 \; - \gamma_m T(2n_{th} + 1)\cosh(2r)/4$



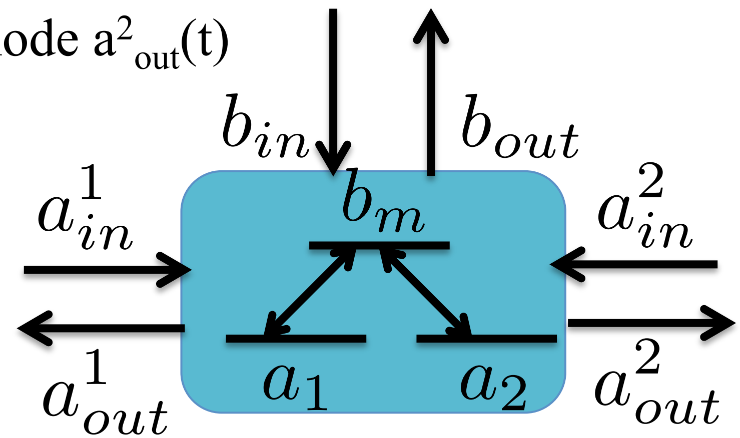
Fidelity plotted for κ_2=0, F_1, linear vs κ_1, F_2, quadratic vs κ_1

$\delta F = F(0) - F(\gamma_m)$ describes contribution from mechanical noise

Quantum Wavelength Conversion

Pulse transmission at impedance matching and constant coupling

Input mode $a^1_{in}(t)$ to be transferred to output mode $a^2_{out}(t)$
Noise operators $a^2_{in}(t)$ and $b_{in}(t)$



Using Langevin equation at constant M and input-output relation

$$\vec{v}_{out}(t) = \vec{v}_{in}(t) - \sqrt{K}\vec{v}(t)$$

Transmission matrix – unitary operator

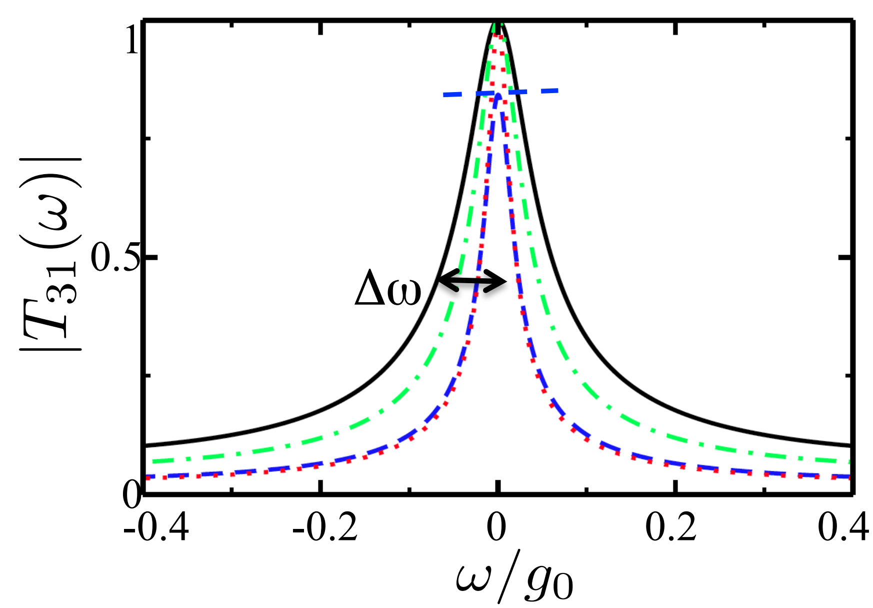
$$\vec{v}_{out}(\omega) = \widehat{T}(\omega)\vec{v}_{in}(\omega)$$

Output operator $$a^2_{out}(\omega) = \widehat{T}_{31}(\omega)a^1_{in}(\omega) + \widehat{T}_{32}(\omega)b_{in}(\omega) + \widehat{T}_{33}(\omega)a^2_{in}(\omega)$$

Condition for high fidelity $\widehat{T}_{31}(\omega) \to 1 \quad \widehat{T}_{32}(\omega), \widehat{T}_{33}(\omega) \to 0$

Quantum Wavelength Conversion

Pulse transmission at impedance matching and constant coupling



- **optimal transmission condition** $g_1^2\kappa_2 = g_2^2\kappa_1$ $\widehat{T}_{31}(\omega) \to 1$
  Blue-dashed curve shows non-optimal transmission
- Half width derived $\Delta\omega \sim \kappa_i$ (~ 0.2 – 0.4 in plots)
  Fidelity drops with input pulse spectral width σ_ω
  High fidelity for $\sigma_\omega \ll \Delta\omega$

L. Tian, PRL 108, 153604 (2012). See also
Y. D. Wang & A. Clerk, PRL 108, 153603 (2012)

Quantum Wavelength Conversion

Adiabatic scheme via mechanical dark mode

Sciencexpress Reports

Optomechanical Dark Mode

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*To whom correspondence should be addressed. E-mail: hailin@uoregon.edu

Thermal mechanical motion hinders the use of a mechanical system in applications such as quantum information processing. While the thermal motion can be overcome by cooling a mechanical oscillator to its motional ground state, an alternative approach is to exploit the use of a mechanically-dark mode that can protect the system from mechanical dissipation. We have realized such a dark mode by coupling two optical modes in a silica resonator to one of its mechanical breathing modes in the regime of weak optomechanical coupling. The dark mode,

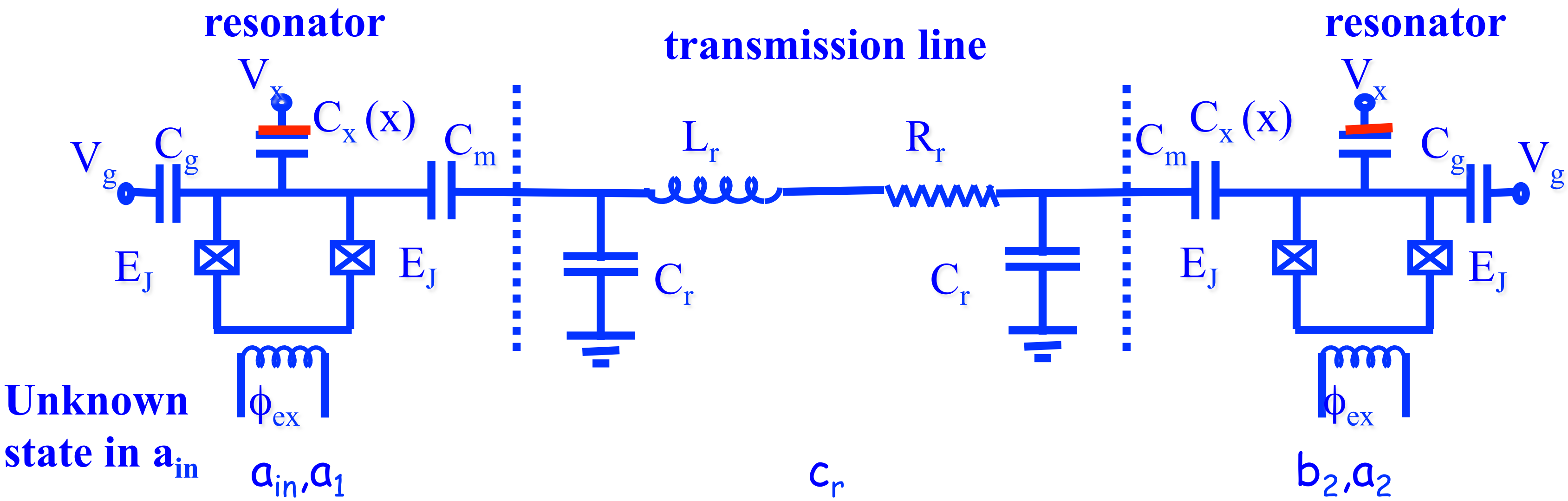
other, effectively mediating an optomechanical coupling between the two optical modes. This type of mechanically-mediated coupling can be immune to thermal mechanical motion, providing a promising mechanism for interfacing hybrid quantum systems (*9*, *14*, *15*).

To introduce the optomechanical dark mode, we consider an optomechanical system, in which two optical modes couple to a mechanical oscillator with optomechanical coupling rates G_1 and G_2, respectively (see Fig. 1B). As illustrated in Fig. 1C, the optomechanical coupling is driven by

Entanglement in Optomechanical Systems

Various approaches in optomechanics: (photons, photon-phonon)

- Stationary state schemes – e.g. Wipf et al, NJP (2008), Vitali et al, PRL (2007), Paternostro et al, PRL (2007)
- Pulsed scheme – L. Tian, S.M. Carr, PRB (2006), S. G. Hofer et al, PRA (2011), Vanner et al, PNAS (2011)



Potential issues:

- Couplings/entanglement constrained by stability conditions
- Thermal noise in mechanical mode

Entanglement in Optomechanical Systems

Current work: L. Tian, preprint arXiv:1301.5376

- Motivated by recent experimental progress in the strong coupling regime
- Gives clear picture of the physics of cv entanglement generation in both cavity state and cavity output

Strength of this system:

- Stability conditions less constrain on couplings – strong entanglement
- Strong and robust entanglement in both cavity states and cavity output

(via Bogoliubov dark mode and quantum interference)

- Can be applied to hybrid systems bridging microwave to optical regime

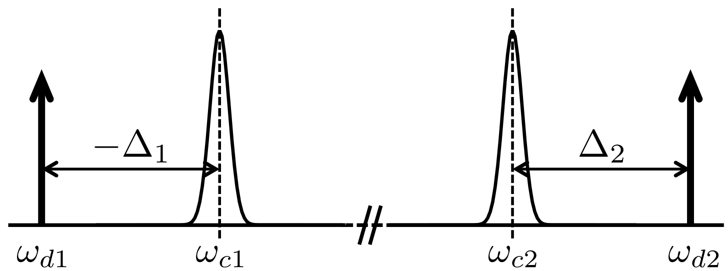
Related work:

- Stationary state scheme: Barzanjeh et al, PRL (2012)
- Measurement based ideas in atomic systems: Muschik et al PRA (2011)
- Other recent work:

Y.D. Wang, A.A. Clerk, arXiv (2013),

H. Tan, G. Li, and P. Meystre, arXiv (2013)

Robust Entanglement Generation



Cavity mode a1 – red-detuned drive $-\Delta_1 = \omega_m$
Cavity mode a2 – blue-detuned drive $\Delta_2 = \omega_m$
Both coupling with mechanical mode bm

System Hamiltonian in the strong coupling regime under RWA

$$H_I = \hbar g_1 (a_1^\dagger b_m + b_m^\dagger a_1) + i\hbar g_2 (a_2^\dagger b_m^\dagger - a_2 b_m) + H_{I,diss}$$

Stability conditions in strong coupling regime: $\dfrac{g_1^2}{g_2^2} > \max\left\{\dfrac{\kappa_2}{\kappa_1}, \dfrac{\kappa_1}{\kappa_2}\right\}$
Which indicates $g_1 > g_2$

$$g_1 = g_0 \cosh(r) \quad g_2 = g_0 \sinh(r) \quad g_0 = \sqrt{g_1^2 - g_2^2}$$

Continuous Variable Entanglement

Two modes under parametric amplifier coupling

$$H_s = -g_s \left(a_1 a_2 + a_1^\dagger a_2^\dagger \right)$$

System operators evolve in terms of Bogoliubov modes

$$a_1(t) = \beta_1(r) = \cosh(r)\, a_1 + i \sinh(r)\, a_2^\dagger$$
$$a_2(t) = \beta_2(r) = \cosh(r)\, a_2 + i \sinh(r)\, a_1^\dagger$$

Entanglement – two-mode squeezed vacuum state (a Gaussian state)
Covariance matrix

$$V = \frac{1}{2} \begin{pmatrix} \cosh(2r) & 0 & \sinh(2r) & 0 \\ 0 & \cosh(2r) & 0 & -\sinh(2r) \\ \sinh(2r) & 0 & \cosh(2r) & 0 \\ 0 & -\sinh(2r) & 0 & \cosh(2r) \end{pmatrix}$$

Logarithmic negativity, ref. e.g. Vidal and Werner, PRA (2002)

$$E_N = 2r \log_2 e$$

Ref: Braunstein, van Loock, RMP (2005)

Robust Entanglement Generation

Bogoliubov dark mode and two brights modes

1. "dark" mode, λ_1=0 – one of Bogoliubov modes in two-mode squeezing

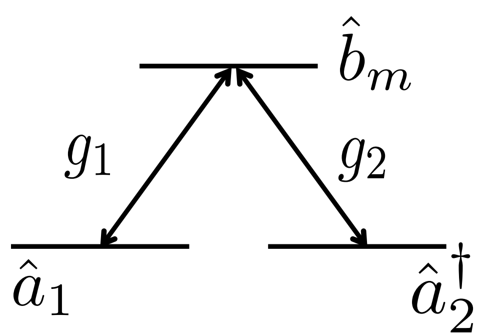
$$\alpha_1 = -i\sinh(r)a_1 + \cosh(r)a_2^\dagger$$

2. Two other modes and eigenenergies – bright modes $\lambda_{2,3} = \pm g_0$

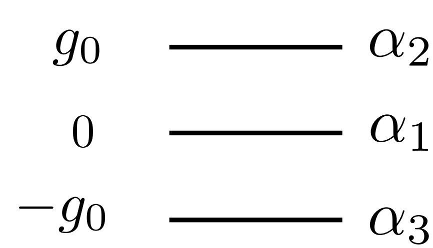
$$\alpha_{2,3} = \frac{1}{\sqrt{2}}\left(\cosh(r)a_1 \pm b_m + i\sinh(r)a_2^\dagger\right)$$

3. Relations to Bogoliubov modes: $\alpha_1 = \beta_2^\dagger;\ (\alpha_2 + \alpha_3)/\sqrt{2} = \beta_1$

4. Coupling diagram, energy spectrum, and symmetry



Eigenmodes



Robust Entanglement Generation

Bogoliubov dark mode and two brights modes

Finite damping rates: Langevin equation for system operators and perturbation

1. Eigenmodes – first order corrections $x_i \sim \kappa_i/g_0, \gamma_m/g_0$
2. Relations to Bogoliubov modes:

$$\alpha_1 = \beta_2^\dagger + x_1 b_m; \; (\alpha_2 + \alpha_3)/\sqrt{2} = \beta_1 - \sqrt{2} x_3 b_m$$

3. Eigenvalues

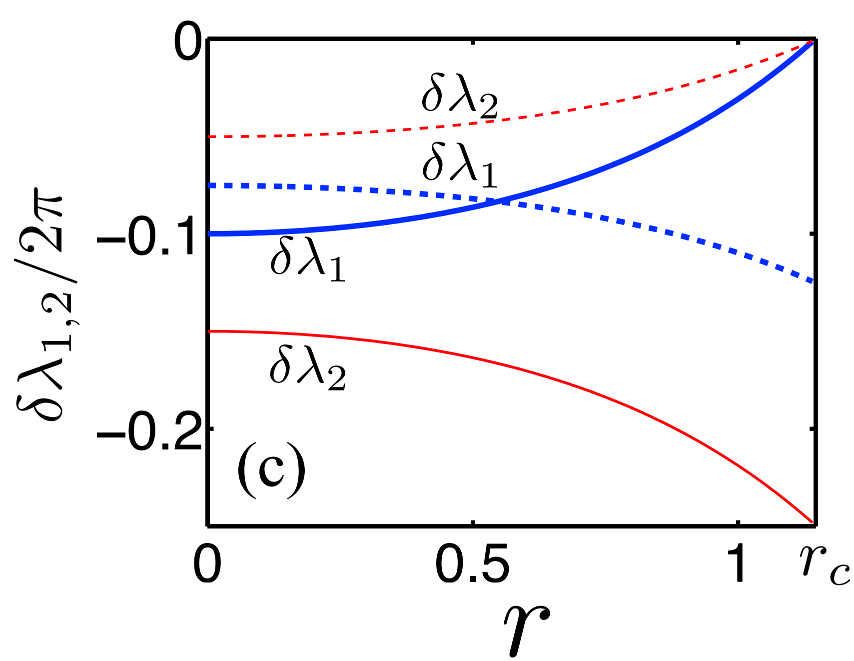
$$\lambda_1 = i\delta\lambda_1 \text{ and } \lambda_{2,3} = \pm g_0 + i\delta\lambda_2$$

4. Stability conditions == $\delta\lambda_i < 0$

3. Dependence on damping rates (interesting effect on entanglement)

(κ1, κ2)=(0.3, 0.2) – solid

(κ1, κ2)=(0.2, 0.3) – dashed



Robust Entanglement Generation

Central idea

- Entanglement generated via mechanical mode – effect of noise
- Excitation of dark mode doesn't involve mechanical mode => $\beta_2(r)$
- Excitation of bright modes mix cavity and mechanical modes
- Quantum interference cancels mechanical modes => $\beta_1(r)$
- Cavity/cavity output operators have forms of Bogoliubov operators to leading order with mechanical noise suppressed

$$\beta_1(r) = \cosh(r)a_1 + i\sinh(r)a_2^\dagger$$
$$\beta_2(r) = \cosh(r)a_2 + i\sinh(r)a_1^\dagger$$

Robust Entanglement Generation

Entanglement of cavity states – time domain

Solving Langevin equation in time domain for operator evolution
Zero damping rates:

$$\alpha_1(t) = \alpha_1(0);\ \alpha_{2,3}(t) = \exp(\mp i\varphi(t))\alpha_{2,3}(0)$$

Bogoliubov modes at time t
- Dark mode $\beta_2(t) = \beta_2(0)$
- Bright modes mixing $\beta_1(t) = \beta_1(0)\cos\varphi(t) - ib_m(0)\sin\varphi(t)$

At time t_n with $\varphi(t_n) = n\pi$, Bogoliubov modes are free of mechanical mode
r=squeezing parameter at t; r_0=squeezing parameter at t_0

$$\begin{pmatrix} a_1(t) \\ a_2^\dagger(t) \end{pmatrix} = \begin{pmatrix} \cosh(r) & -i\sinh(r) \\ i\sinh(r) & \cosh(r) \end{pmatrix} \begin{pmatrix} \cosh(r_0)(-1)^n & i\sinh(r_0)(-1)^n \\ -i\sinh(r_0) & \cosh(r_0) \end{pmatrix} \begin{pmatrix} a_1(0) \\ a_2^\dagger(0) \end{pmatrix}$$

Robust Entanglement Generation

Entanglement of cavity states – time domain

Finite damping rates – solving Langevin equation in eigenbasis

$$a_1(t_n) = \left[(-1)^n \cosh(r) a_1(0) - i \sinh(r) a_2^\dagger(0)\right] + f_1(a_1(0), a_2^\dagger(0)) + y_1 b_m(0) + \text{noise integral}$$

$$a_2^\dagger(t_n) = \left[i(-1)^n \sinh(r) a_1(0) + \cosh(r) a_2^\dagger(0)\right] + f_2(a_1(0), a_2^\dagger(0)) + y_2 b_m(0) + \text{noise integral}$$

Ideal terms
zero damping

Eigenmode
damping
$O(\kappa_i/g_0)$

Bath
fluctuations
$O(\kappa_i/g_0)$
$O(\gamma_m/g_0)n_{th}$

Effect of initial mechanical noise is eliminated to leading order!

$O(\kappa_i^2/g_0^2)n_0$ First-order mixing
with mechanical mode

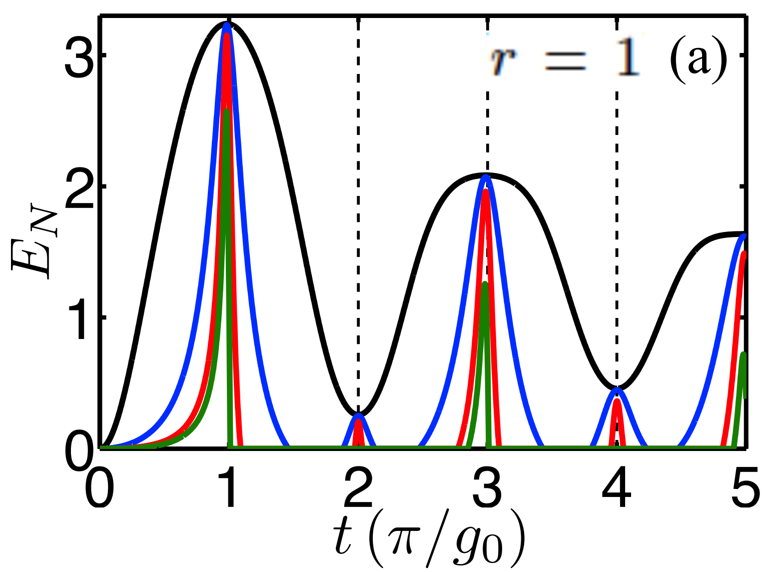
Robust Entanglement Generation

Entanglement of cavity states – time domain

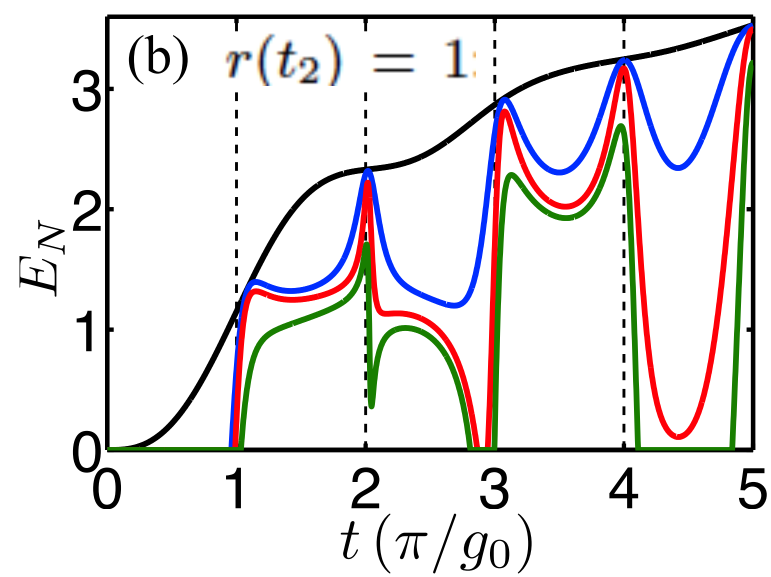
Numerical simulation of the covariance matrix n_{th} =0, 10, 100, 1000

- Resonances appear for finite n_{th} $t_n = n\pi/g_0$
- Peak height slowly varies with n_{th} - first order

Constant couplings



Adiabatic scheme



$g_1(t) = g_0 \cosh(\lambda t)$ $g_2(t) = g_0 \sinh(\lambda t)$

Robust Entanglement Generation

Entanglement of cavity states – time domain

Numerical simulation of the covariance matrix n_{th}

Entanglement at peaks robust against thermal noise

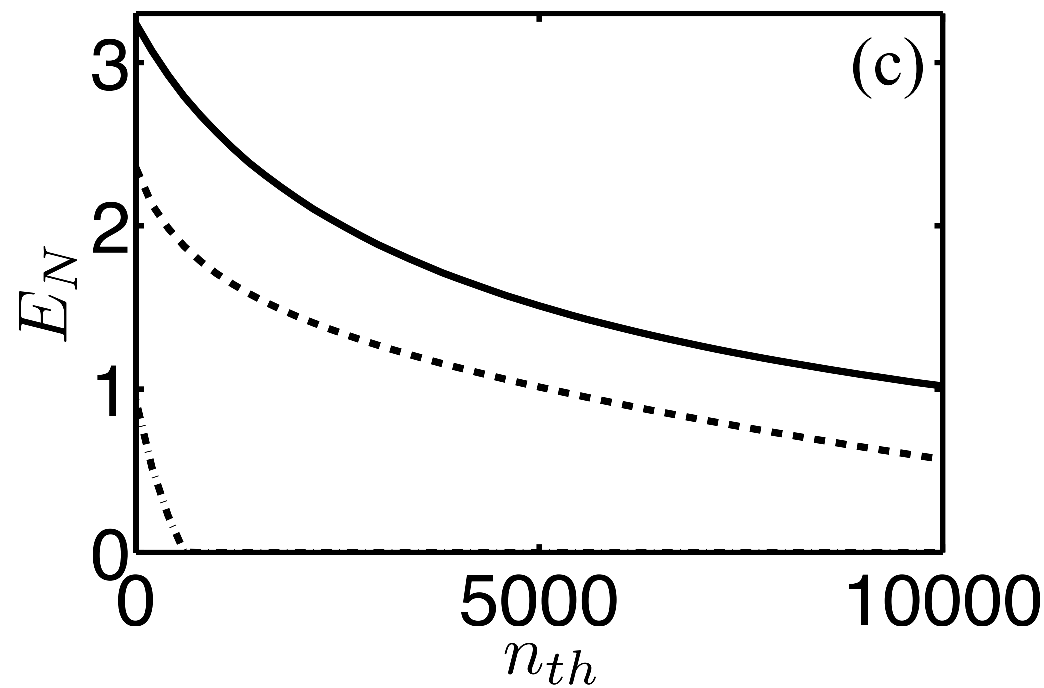
solid: constant couplings

$r = 1$

dashed: adiabatic

$r(t_2) = 1$

dotted: stationary scheme



Robust Entanglement Generation

Entanglement of output photons – frequency domain

Define mode with appropriate commutation relation – x = in, out

$$a_x^{(i)}(\omega_n) = \int d\omega g(\omega - \omega_n) a_x^{(i)}(\omega) \qquad \left[a_x^{(i)}(\omega_m), a_x^{(j)\dagger}(\omega_n)\right] = \delta_{mn}\delta_{ij}$$

Solving Langevin equation for eigenmode excitation at given frequency

$$\vec{\alpha}(\omega_n) = i(I\omega_n - \Lambda)^{-1} U^{\mathrm{T}} \sqrt{K} \vec{v}_{in}(\omega_n)$$

Project cavity modes to output $\vec{v}(\omega_n) = U\vec{\alpha}(\omega_n)$, similarly $\vec{v}_{out}(\omega_n)$

Strong excitation when ω_n near eigenvalues

At $\omega_n=0$, dark mode strongly excited ~$1/\delta\lambda_1$,

bright modes weakly excited ~ $1/g_0$

At $\omega_n=g_0$, one bright mode strongly excited $1/\delta\lambda_2$, (similarly at $-g_0$)

dark mode weakly excited ~ $1/g_0$

other bright mode weakly excited ~ $1/2g_0$

Entanglement can be strong at these frequencies

Robust Entanglement Generation

Entanglement of output photons – frequency domain

At $\omega_n=0$, dark mode strongly excited ~ $1/\delta\lambda_1$,

$$\alpha_1(\omega_0) = \left(\frac{\sinh(r)}{\delta\lambda_1} \quad \frac{ix_1}{\delta\lambda_1} \quad \frac{i\cosh(r)}{\delta\lambda_1} \right) \cdot \sqrt{K}\vec{v}_{in}(\omega_0)$$

bright modes weakly excited ~ $1/g_0$

$$\alpha_{2,3}(\omega_0) = \left(\mp\frac{\cosh(r)}{\sqrt{2}g_0} \quad -\frac{1}{\sqrt{2}g_0} \quad \mp\frac{i\sinh(r)}{\sqrt{2}g_0} \right) \cdot \sqrt{K}\vec{v}_{in}(\omega_0)$$

Interesting feature

$$\beta_1(\omega_0) \approx (\alpha_2(\omega_0) + \alpha_3(\omega_0))/\sqrt{2} = -\sqrt{\gamma_m} b_{in}(\omega_0)/g_0$$

Again, in cavity modes, mechanical input ~ $1/g_0$; cavity inputs ~$1/\delta\lambda_1$

At $\omega_n=g_0$, one bright mode strongly excited $1/\delta\lambda_2$, (similarly at $-g_0$)

dark mode weakly excited ~ $1/g_0$

other bright mode weakly excited ~ $1/2g_0$

Robust Entanglement Generation

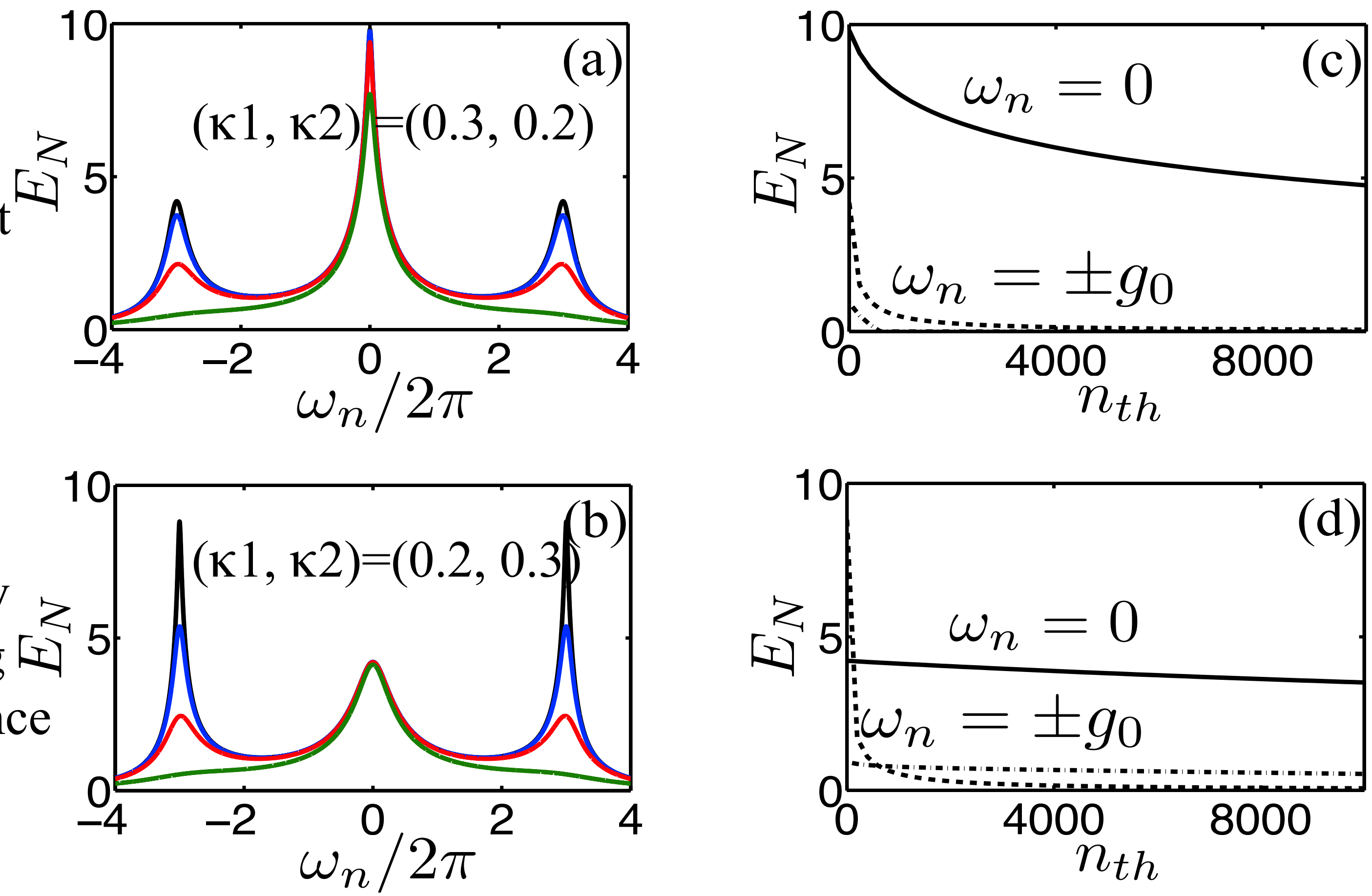
Entanglement of output photons – frequency domain

Entanglement for n_{th}=0, 10, 100, 1000

Strong entanglement at 0, g_0, $-g_0$

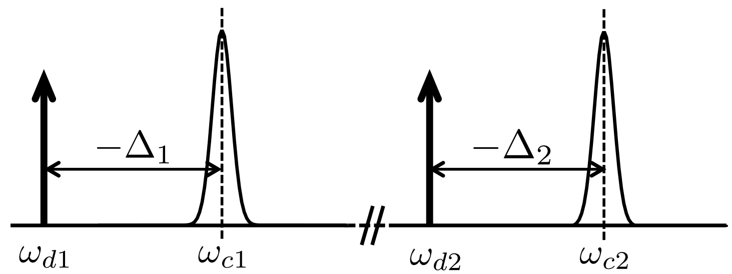
At 0, robust against thermal noise

Response differently to different damping rates – $\delta\lambda_i$ dependence



Robust Entanglement Generation

Discrete state entanglement: beam-splitter operations



Cavity mode a1 – red-detuned drive $-\Delta_1 = \omega_m$
Cavity mode a2 – red-detuned drive $-\Delta_2 = \omega_m$
Both coupling with mechanical mode bm

System Hamiltonian in the strong coupling regime with RWA

$$H_I = \sum_{i=1,2} \hbar g_i (a_i^\dagger b_m + b_m^\dagger a_i) + H_{I,diss}$$

Hamiltonian used for quantum state transfer

Robust Entanglement Generation

Discrete state entanglement: beam-splitter operations

Adiabatic scheme $g_1(t) = g_0 \sin(\lambda t)$ and $g_2(t) = -g_0 \cos(\lambda t)$

At time $t_f = \pi/4\lambda$, with $\lambda = g_0/4n$

$$a_1(t_f) = \frac{1}{\sqrt{2}} a_1(0) + \frac{(-1)^{n+1}}{\sqrt{2}} a_2(0)$$

$$a_2(t_f) = \frac{1}{\sqrt{2}} a_1(0) + \frac{(-1)^{n+2}}{\sqrt{2}} a_2(0)$$

Initial state $|1_1 0_2\rangle$, final state $|\psi_{en}\rangle = (|1_1 0_2\rangle + |0_1 1_2\rangle)/\sqrt{2}$

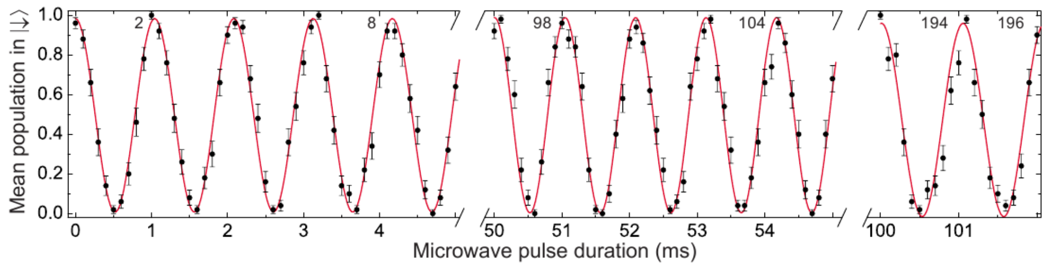
$|0_1 1_2\rangle$ $\qquad |\psi_{en}\rangle = (|1_1 0_2\rangle - |0_1 1_2\rangle)/\sqrt{2}$

Similar arguments for robustness against thermal noise

Trapped Particle and Superconducting Circuits

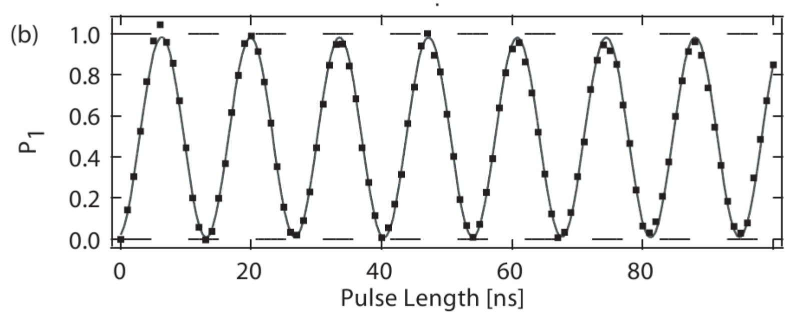
N. Daniilidis, D. J. Gorman, L. Tian, H. Haeffner, preprint (2013)

- Hybrid system for scalable quantum machines – best of two worlds
- Coherence of atomic systems



Rabi flops on $^{43}Ca^{+}$ hyperfine manifold (J. Benmhelm *et al.*, PRA **77** 062306)

- Speed of solid-state systems

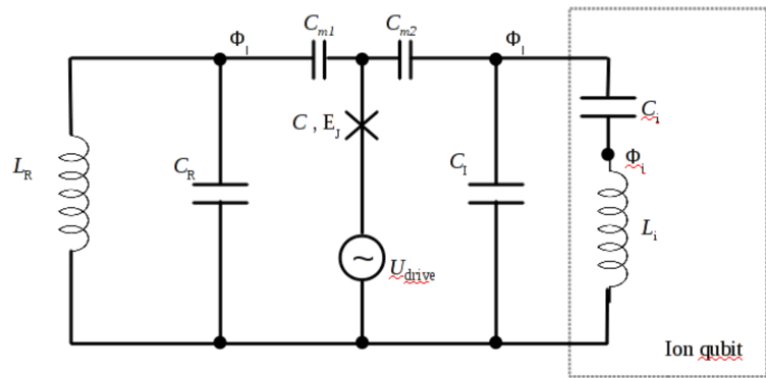


J. Chow, PhD thesis (2010)

Trapped Particle and Superconducting Circuits

- Challenges

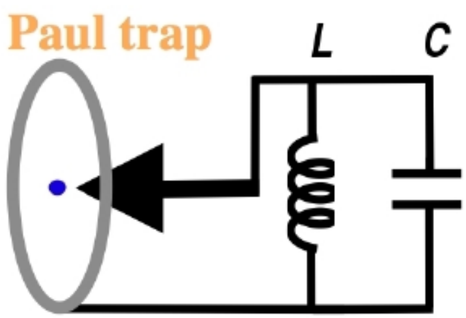
Coupling between systems needs to be stronger than noise picked up from environment. Initial idea – charge noise comparable to signal of trapped particle: (circuit)

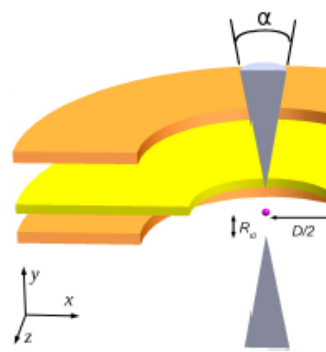


Frequency mismatch between particle motion and superconducting circuits

Trapped Particle and Superconducting Circuits

- Solution – driven electron motion in nonlinear potential





Particle (electron) trapped by effective harmonic potential
careful trap simulation was done
Coupling to pick-up electrode connected with superconducting circuit
Parametric driving on nonlinear potential to achieve energy conversion

$$U_{eff} = g x^2 \dot{\varphi}$$

No extra circuit noise

Trapped Particle and Superconducting Circuits

- Solution – driven electron motion in nonlinear potential

Parametric driving on motion of trapped particle – large classical component to provide energy difference between quantum motion and superconducting circuit

$$x_i = A_d \cos(\Omega_d t) + \hat{x}_i$$

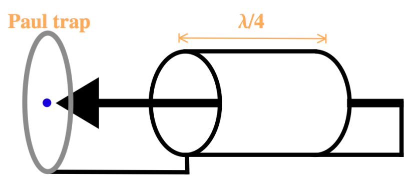
Effective coupling: beam-splitter operation, parametric amplifier operation

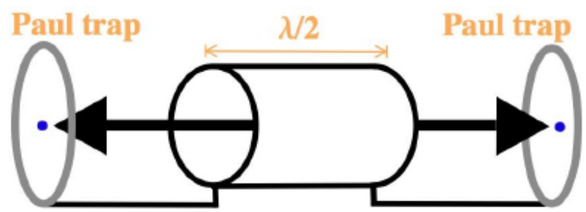
$$H_{\mathrm{er}} = \hbar g \cos(\Omega_{\mathrm{d}} t) \left(e^{i(\Omega - \omega_y)t} a_\phi^\dagger a_y + e^{i(\Omega + \omega_y)t} a_\phi^\dagger a_y^\dagger + h.c. \right)$$

Trapped Particle and Superconducting Circuits

- Protocols can be implemented

Transfer electron motion with superconducting LC oscillators

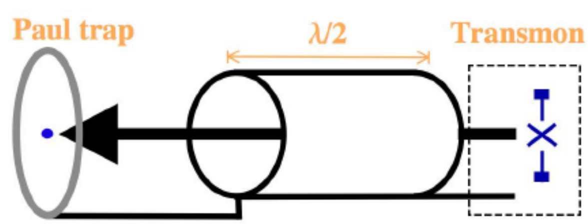




Connecting distant electrons via transmission line

Electron transmon coupling – with 3D transmon (long coherence time)

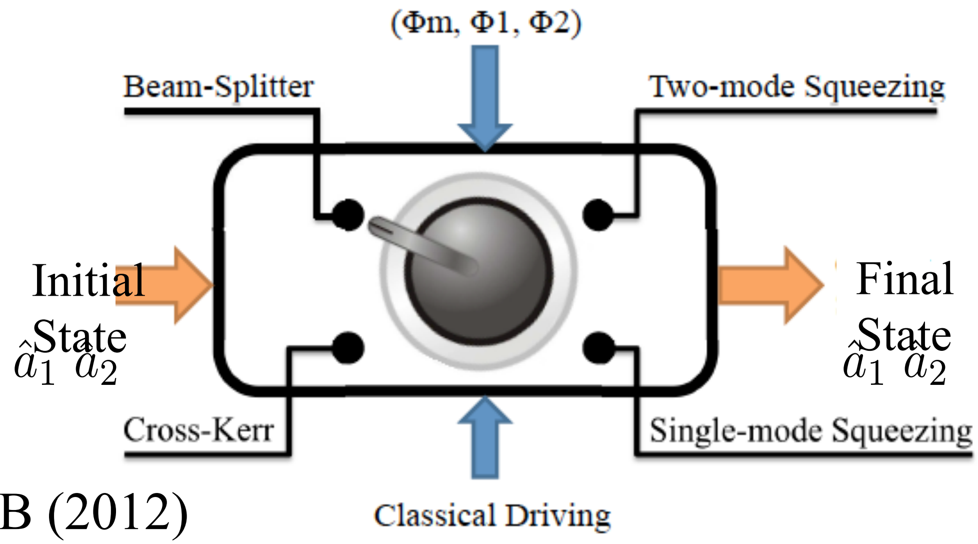
Numerical simulation:
Quantum state transfer (F = 0.992)
and entanglement (F = 0.997)



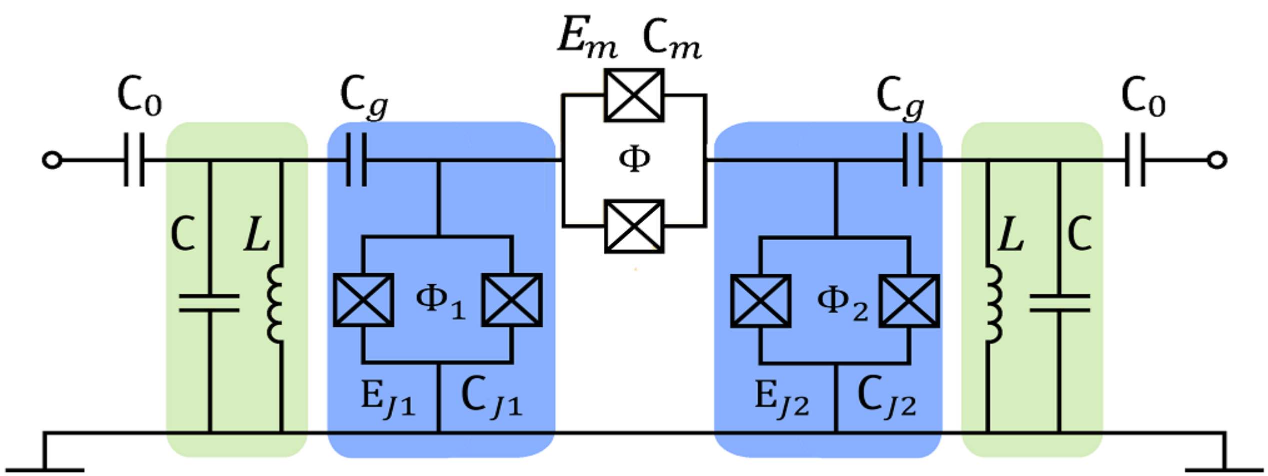
Architecture for large scale quantum computer …

4-wave Mixing Toolbox for Superconducting Resonators

Resonators couple to toolbox
Dispersive 4-wave mixing scheme
Generate all basic Q operations



A. Sharypov, X. Deng, L. Tian, PRB (2012)



Conclusions

- Optomechanical quantum interface for high fidelity state conversion
- Optomechanical quantum interface for robust entanglement generation
- Parametric conversion of trapped particle motion to superconducting cicuits

People Involved

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