# Symmetry protected topological phase for interacting bosons with Rydberg atoms

**Hans Peter Büchler**

Institute for theoretical physics III,
and Center for Integrated Quantum Science and Technology

Universität Stuttgart, Germany

**Experimental collaboration:**

**S. de Léséleuc, V. Lienhard, P. Scholl,**
**D. Barredo, T. Lahaye, and A. Browaeys**

Research group:

Jan Kumlin, **Nicolai Lang**, Tobias Ilg,
Kevin Kleinbeck, **Sebastian Weber**

Strongly interacting
Rydberg slow light
polaritons

Integrated Quantum Science and Technology

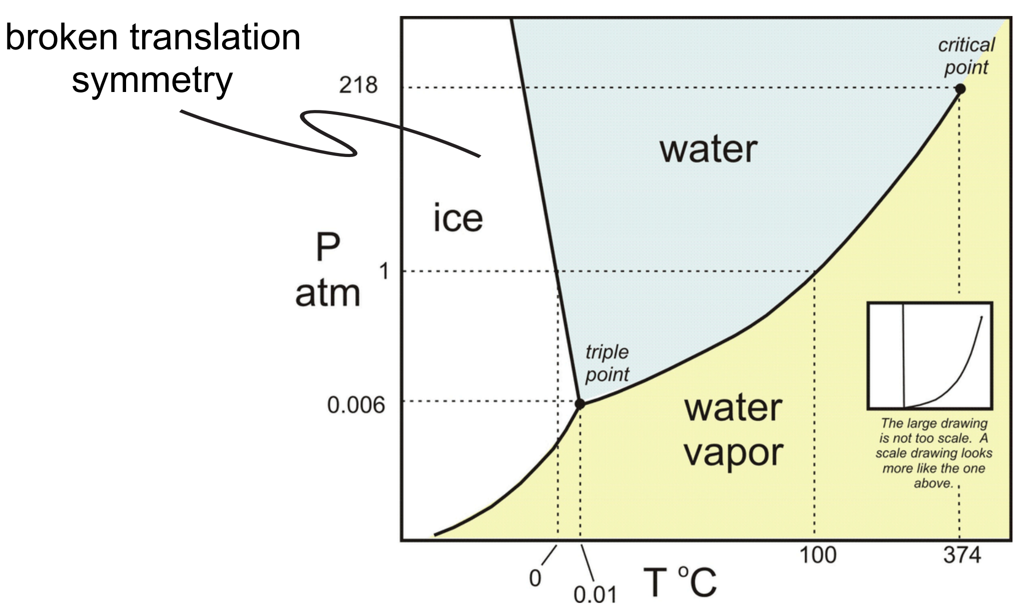
From few to many-body
physics with dipolar quantum
gases

# How to characterize states of matter?

## Characterize phases of matter

- based on Landau paradigm
- symmetry breaking
- order parameter and long range order
- thermal and quantum phase transitions

phase diagram of water




## Extremely successful

- band insulator/Fermi liquids
- crystals
- superfluids
  - Bose-Einstein condensate
  - superfluid Helium
- superconductors
- ferromagnets and anti-ferromagnets

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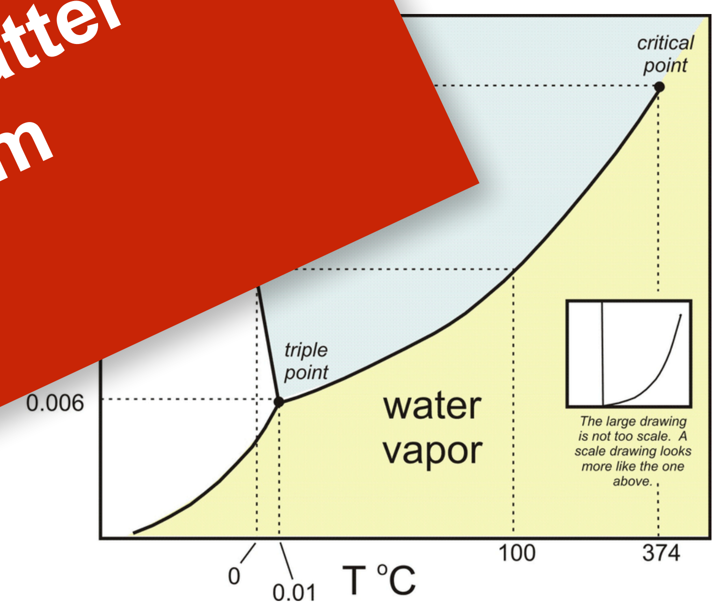



## Extremely su

- banc
- crystals
- superfluic
  - Bose-Ein
  - superfluid
- superconducto
- ferromagnets and anti-ferromagnets

**But, not all states of matter follow this paradigm**

# Topological phases

Characterizing ground states of quantum many-body systems at T=0

**- absence of symmetry breaking**

**- gapped phases**

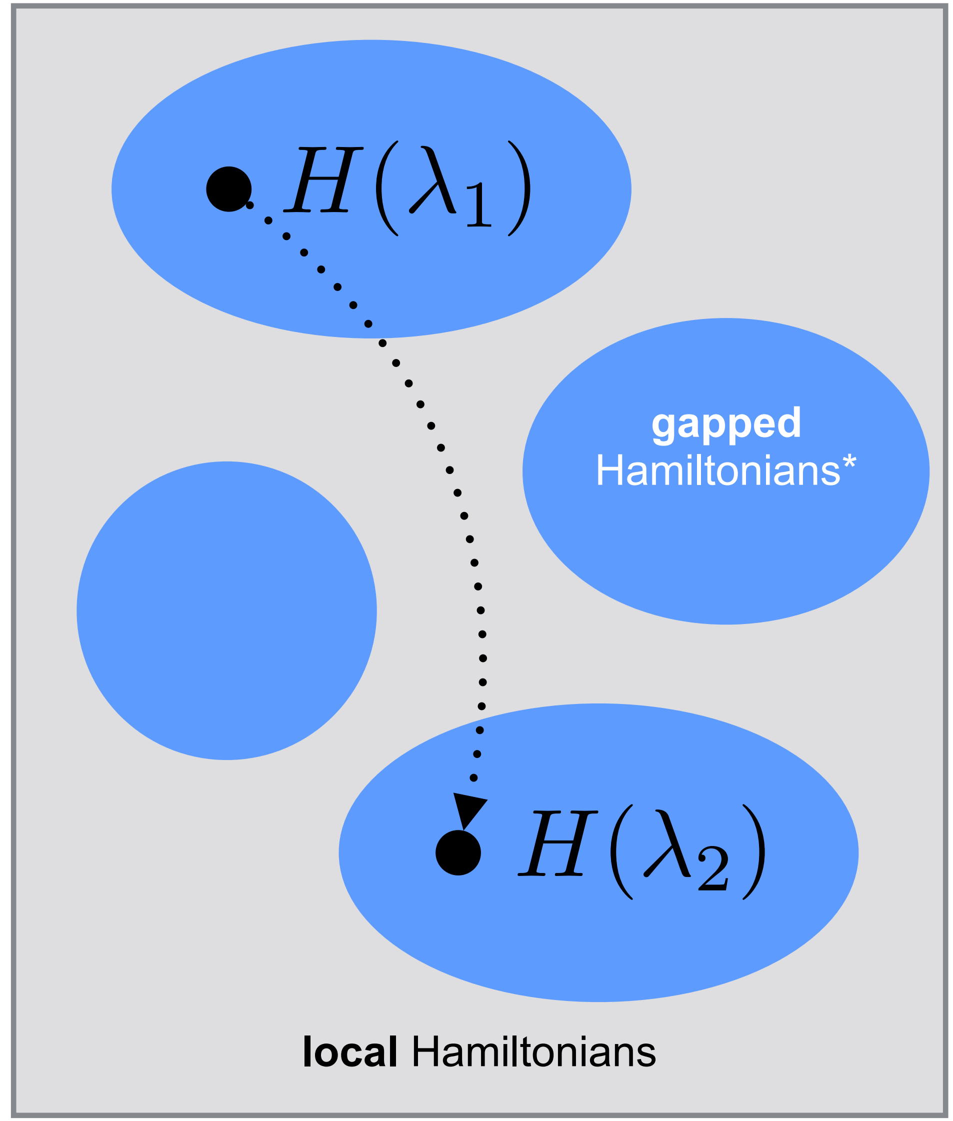
## Definition:

**two states are in the same topological phase if they can be smoothly transformed into each other without closing the gap**

Example in 2D:
integer quantum Hall states, fractional quantum Hall states, toric code,

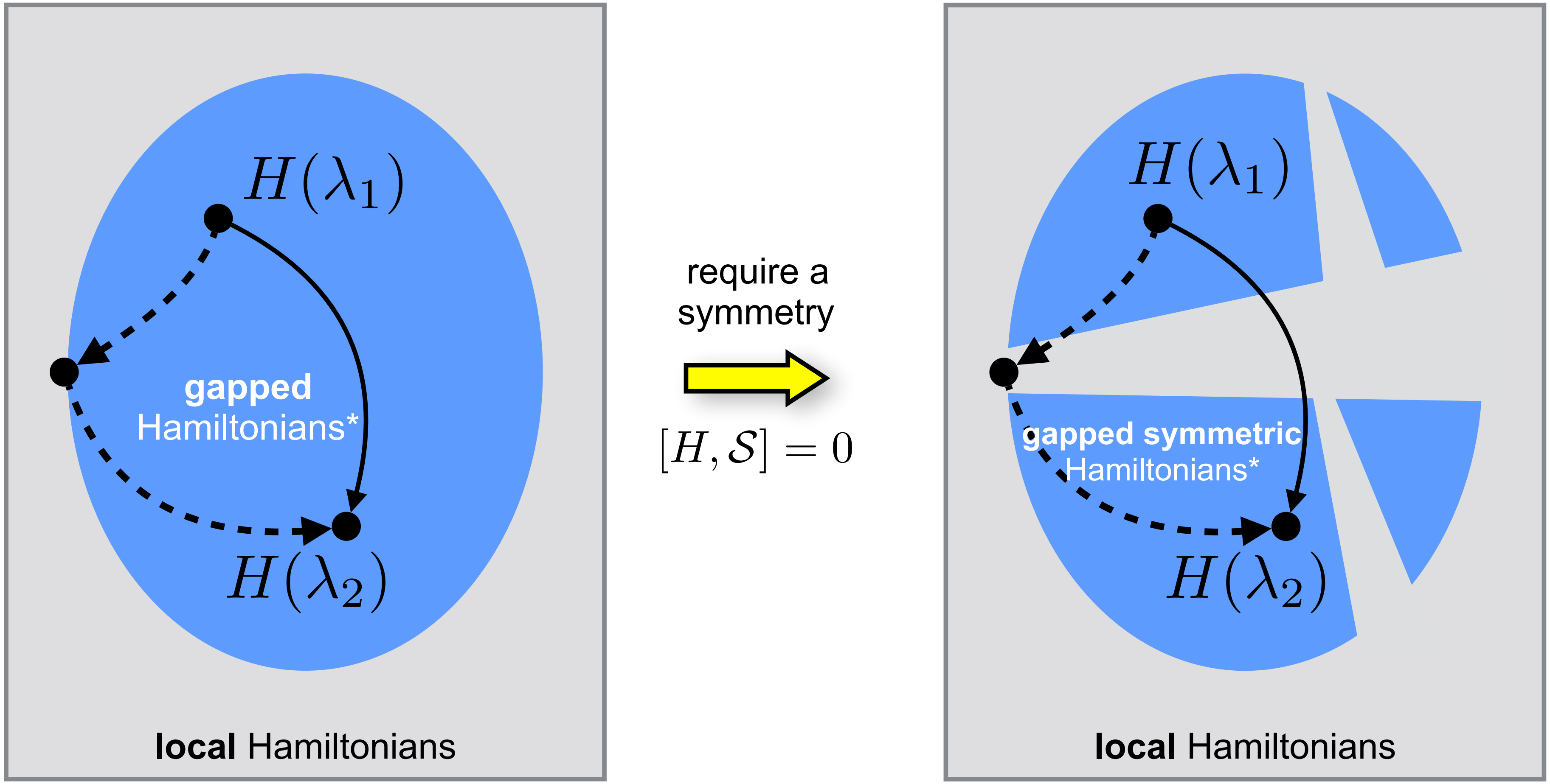
**Are there other topological phases also in 1D?**

**Two dimensions**



# Symmetry protected topological phases

- restrict to systems, which satisfy a certain symmetry
- symmetry group $\mathcal{S}$ with $[H, \mathcal{S}] = 0$




**One dimension**

# Topological insulators: fermionic SSH model

- non-interacting Fermions

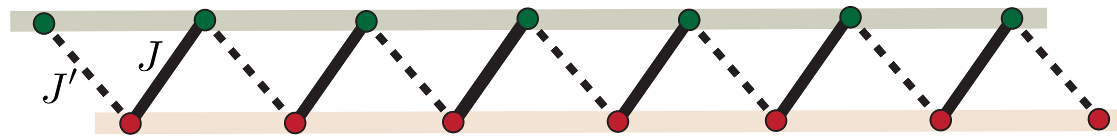
- Hamiltonian

$$H = -J \sum_{i \in \text{even}} c_{i+1}^\dagger c_i - J' \sum_{i \in \text{odd}} c_{i+1}^\dagger c_i + h.c.$$

$$= \sum_{ij} c_i^\dagger \hat{H}_{ij} c$$

- gapped ground state at half-filling

- 10-fold way classifications of topological insulators → AIII, Z-index
(Chiu, et al., RMP 2016; Ludwig, 2016)




- sub-lattice (chiral) symmetry (anti-unitary operator)

$$\mathcal{S}_S = \prod_i \left[ c_i + (-1)^i c_i^\dagger \right] K$$

complex conjugation

- on single particle Hamiltonian

$$U_S \hat{H} U_S^\dagger = -\hat{H} \qquad (U_S)_{ij} = (-1)^j \delta_{ij}$$

**Trivial phase** $J' > J$

- conventional band insulator
- excitation gap
- filling $n = 1/2$
- decoupled limit $J = 0$

$$|\psi\rangle = \prod_{i \in \text{odd}} (c_i^\dagger + c_{i+1}^\dagger)|0\rangle$$

topological phase transition

⟷ $J = J'$

gapless state

**Topological phase** $J > J'$

- bulk excitation gap
- half filling $n = 1/2$
- four-fold degenerate ground state for open chain

→ zero energy modes localized at the edges

# Topological insulators: fermionic SSH model

## SPT phase

- properties of the SSH model are robust under any perturbation commuting with the symmetry

- four-fold degenerate ground state for open chain
- zero energy modes localized at the edge

## Perturbations respecting the symmetry

- random variations of the hopping
- longer range hoppings between the chains

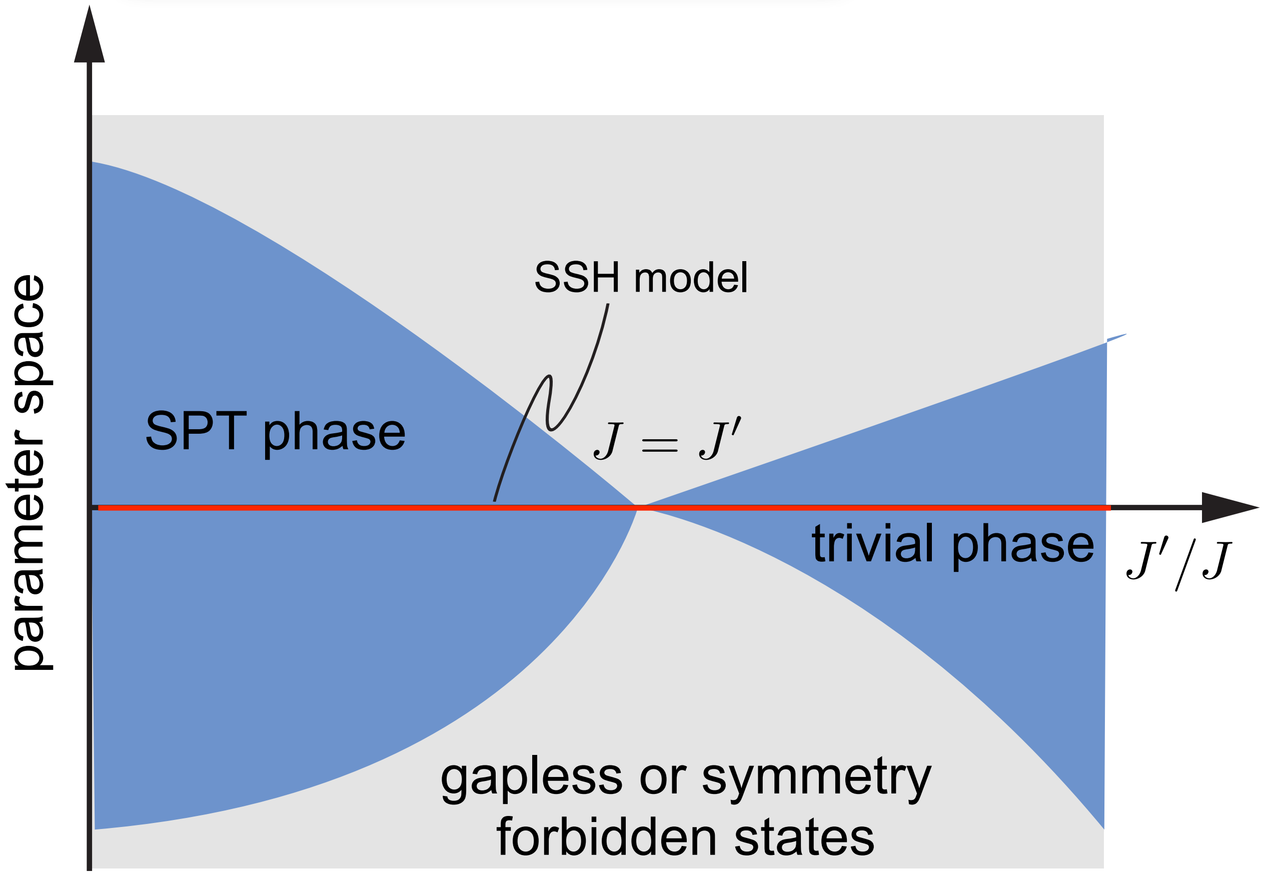
$$c^\dagger_{i+3} c_i + c^\dagger_i c_{i+3}$$

## Perturbations breaking the symmetry

- next-nearest neighbor hopping

$$c^\dagger_{i+2} c_i + c^\dagger_i c_{i+2}$$

- on-site shifts $c^\dagger_i c_i$



# Topological phases: condensed matter

**Integer quantum Hall effect** (1980):

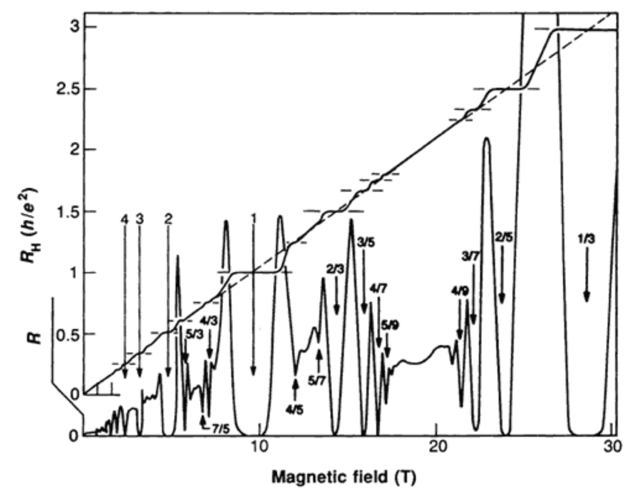
- 2D electron gas in a strong magnetic field

- present for non-interacting fermions

**Fractional quantum Hall effect** (1982):

- 2D electron gas in a strong magnetic field

- strong interactions between the particles




**Spin-1 anti-ferromagnets in 1D** (1990):

- realization of a bosonic SPT phase

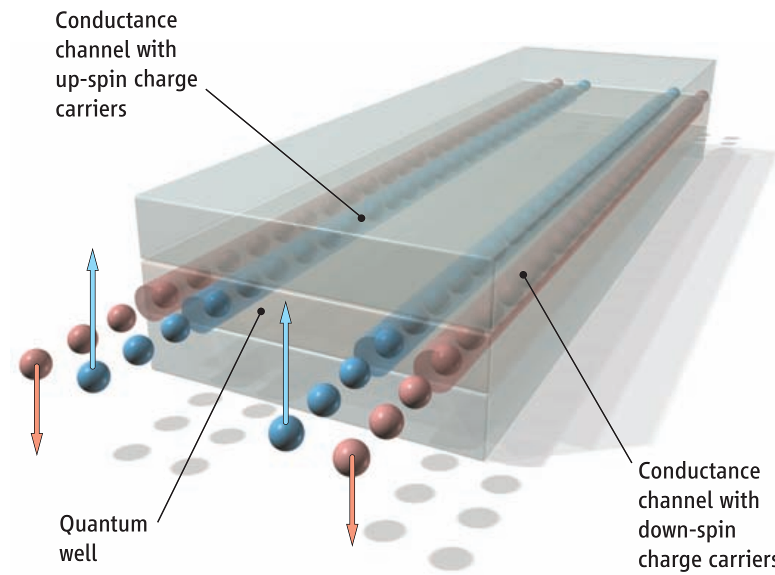
- protecting symmetry SO(3)

- NENP materials [$Ni(C_2H_8N_2)_2(NO_2)]ClN_4$

**Topological insulators** (2007):

- spin-Hall effect in HgTe quantum wells

- time-reversal symmetry



# Topological phases: artificial matter

## Artificial matter

- cold atomic gases in optical lattices
- Ion traps
- Photonic circuits
- …..

# Topological phases: artificial matter

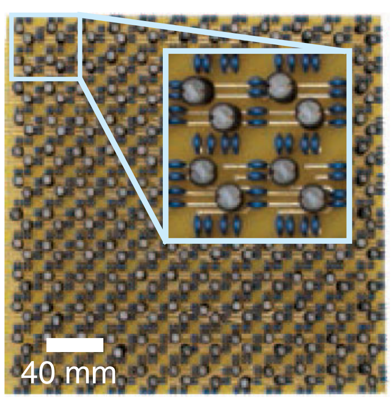
Topological band structures
and edge modes

- motivated by topological phases for non-interacting fermions
- probing spectrum of a single particle Hamiltonian
- property of the coupling matrix

→ accessible in classical and quantum systems

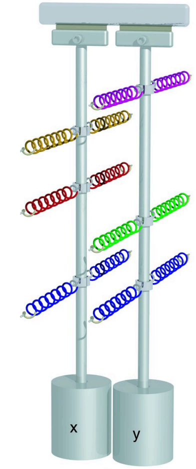
**Edge modes in photonic systems:**

- optical regime
  *Hafezi, et al, Nat Phot. 2013*
  *Rechtsman et al, Nature 2013*
- radio-frequencies
  *Ningyuan et al. PRX 2015*

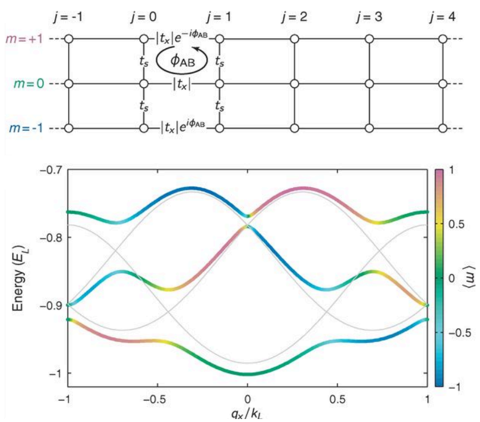



**Classical coupled pendulums:**

- time reversal invariant
  *Süsstrunk Huber, Science 2015*
- driven rotating pendulums
  *Nash et al., PNAS 2015*




**Cold atomic gases**



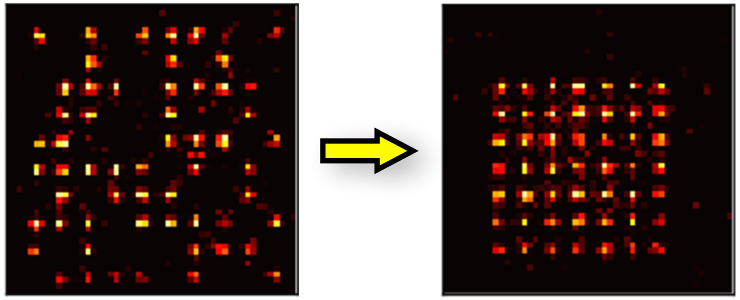

- artificial gauge fields
  *Stuhl et al, Science, 2015*
- artificial dimensions
  *Mancini et al, Science, 2015*
- optical lattices and lattice shaking
  *Jotzu et al. Nature 2014, Aidelsburger et al. Nat. Phys. 2014*
  *Lohse et al. Nat. Phys. 2015, Flaschner et al. Science 2016*

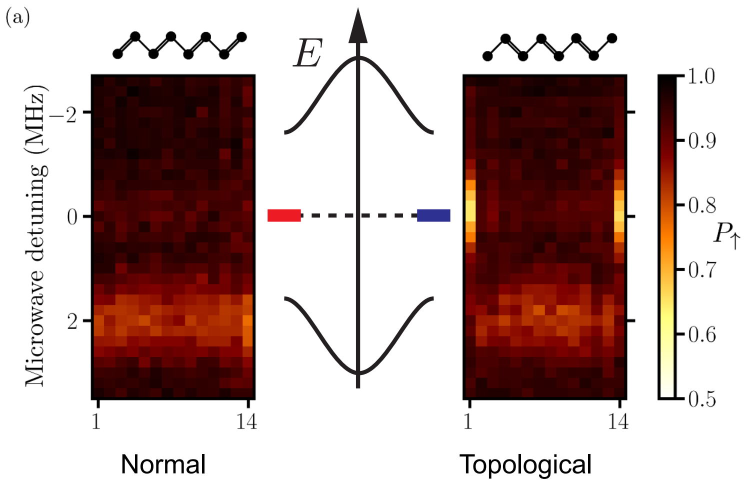
# Outline

## Experimental setup with Rydberg atoms

- single atoms in optical tweezers

- assembly of arbitrary structures






## Edge modes in the SSH model

**(S. de Léséleuc, *et al*, arXiv:1810.13286)**

- single particle physics (independent on statistics)

- observation of localized edge modes

## Symmetry protected topological phase

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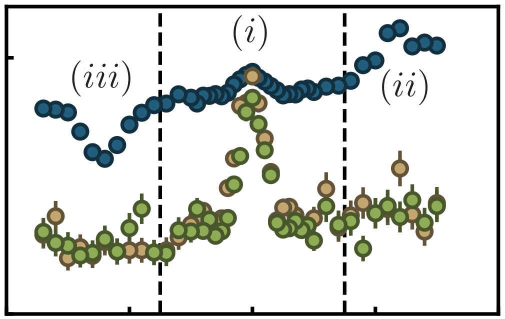
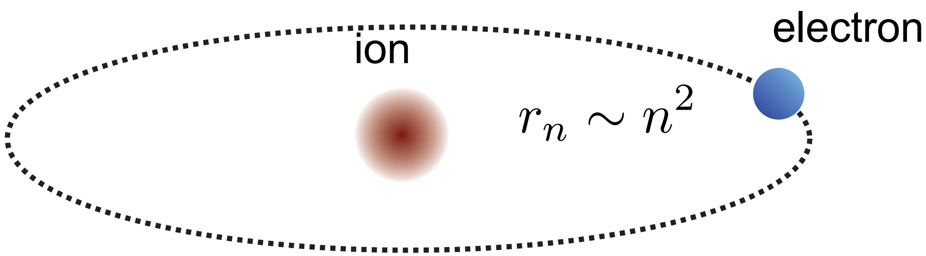


# Experimental setup with Rydberg atoms

## Rydberg atoms

- one electron excited into a state with high principal quantum number *n*

- here, Rubidium atoms *n*~40 -100, excited into s-states and p-states

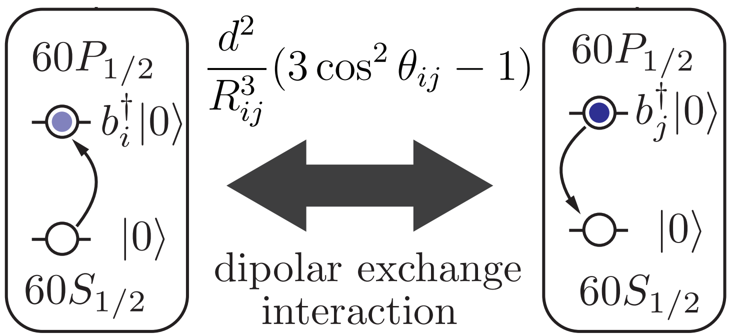



$n^{\text{th}}$ : principal quantum number

## Rydberg-Rydberg interaction

- strong van der Waals interactions between Rydberg states
  - attractive or repulsive
  - $C_6 \sim n^{11}$

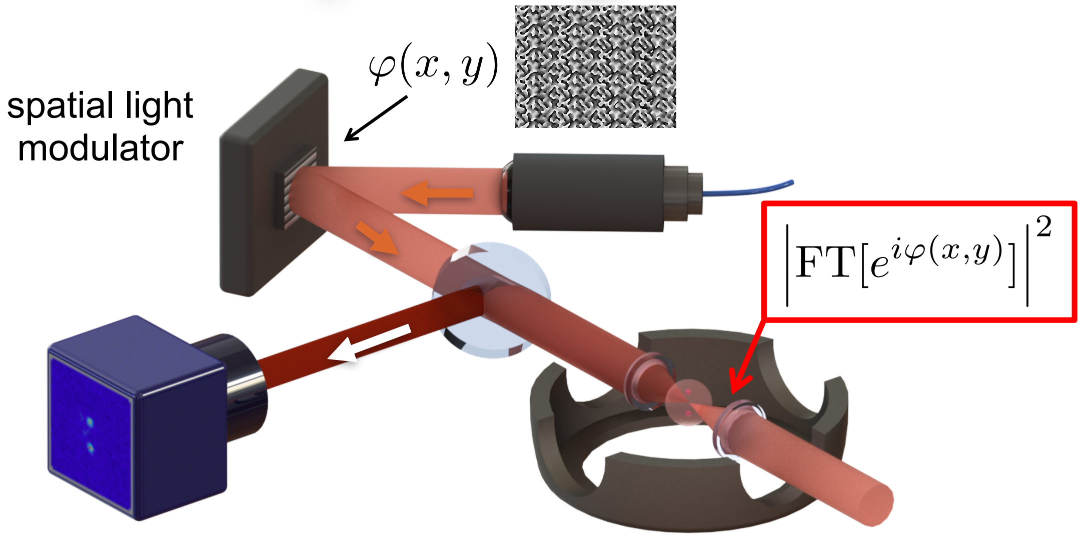
- dipolar exchange interactions
  - exchange of excitation between two different Rydberg states
  - $d \sim n^2$



# Experimental setup with Rydberg atoms

## Single atoms trapped in optical tweezers

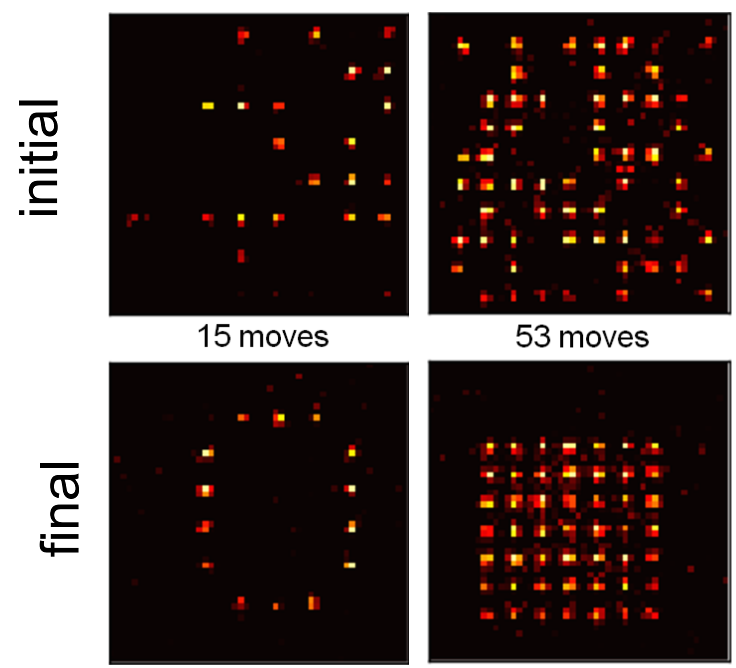
- individual traps for a single atom
- not in ground state of the trapping potential
- single site resolution




## Deterministic assembly in arbitrary structures and lattices

- loading from a cold thermal cloud
    - stochastic loading
- prepare lattice structure by moving the filled traps
- prepare arbitrary 2D as well as 3D structures
- achieved by different groups:
  Paris (2D) Science **354**, 1021 (2016)
  Harvard (1D), Science **354**,1024 (2016)
  Korea (2D), Nat. Comm. **7**, 13317 (2016)

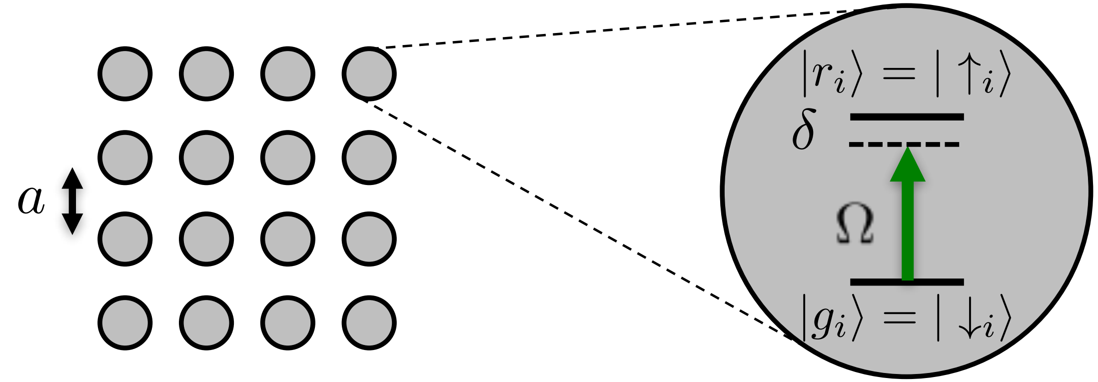
Barredo, *et al.*, Science **354**, 1021 (2016)



# Experimental setup with Rydberg atoms

## Quantum Ising like models

- all atoms coupled to a Rydberg S-state
- van der Waals interaction between Rydberg states




$$H = \Omega \sum_i \sigma_i^x + \sum_i \Delta_i \sigma_i^z + \sum_{i \neq j} \frac{C_6}{|\mathbf{r}_i - \mathbf{r}_j|^6} n_i n_j$$

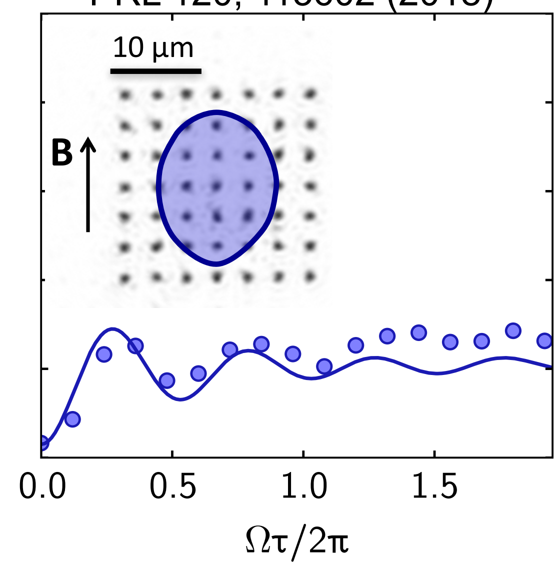
transverse field | longitudinal field | Ising type interaction

$$n_j = \frac{1 + \sigma_j^z}{2}$$

## Quantum simulation of spin models

- non-equilibrium quench dynamics
- time dependent driven and disordered systems
- dissipative systems by including spontaneous decay
- Labuhn, *et. al.*, Nature 534, 667 (2016)
- Bernien, *et al.*, Nature 551, 579 (2017)
- …

PRL 120, 113602 (2018)

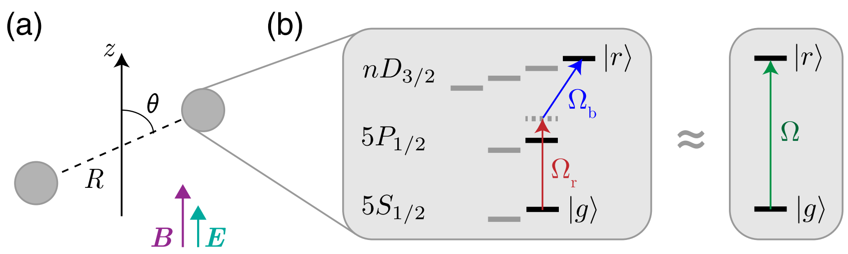


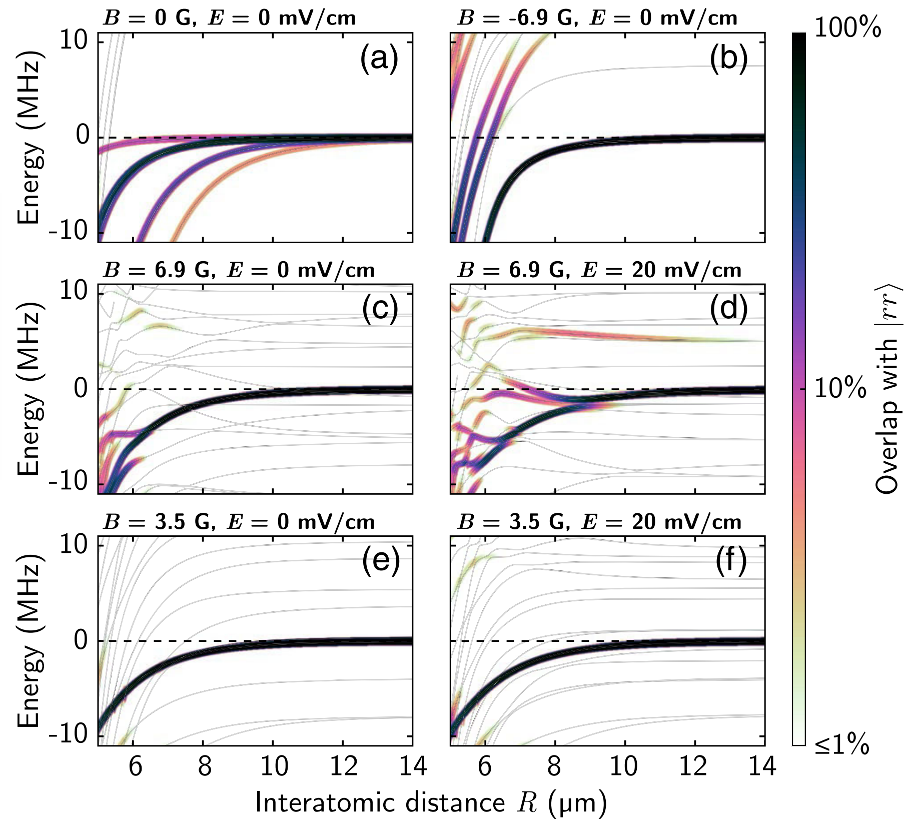
# Experimental setup with Rydberg atoms

Importance of microscopic details of interaction potentials

- pair-interactions are in general much more complex than pure van der Waals

- many crossings with additional Rydberg pair states




## A Rydberg interaction calculator

- open source software

- S. Weber, *et al.,* J. Phys. B **50**, 133001 (2017)

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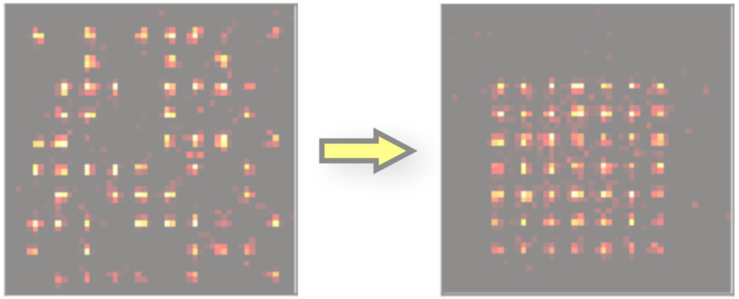

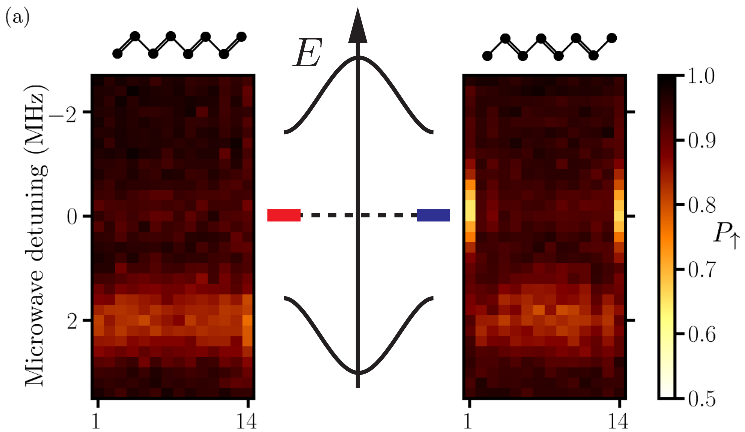


## Edge modes in the SSH model

**(S. de Léséleuc, *et al*, arXiv:1810.13286)**

- single particle physics (independent on statistics)

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## Symmetry protected topological phase

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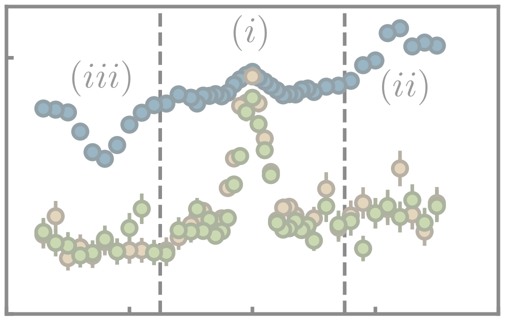


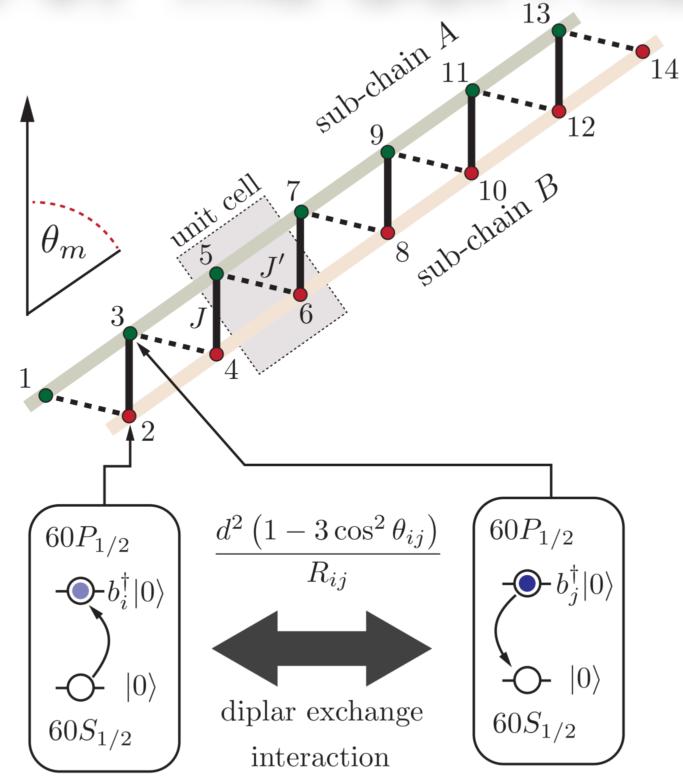
# Single particle properties of the SSH model

Hamiltonian of the single particle
SSH model for bosons

- all atoms in a Rydberg S-state

- bosonic excitation: Rydberg P-state

- hopping by dipolar exchange
  on a dimerized chain




$$H = \sum_{ij} b_i^\dagger \hat{H}_{ij} b_j = \sum_{i\in A, j\in B} J_{ij}\left[b_i^\dagger b_j + b_j^\dagger b_i\right]$$

## Chiral symmetry

- unitary matrix with

$$U_S \hat{H} U_S^\dagger = -\hat{H}$$

- here, we obtain

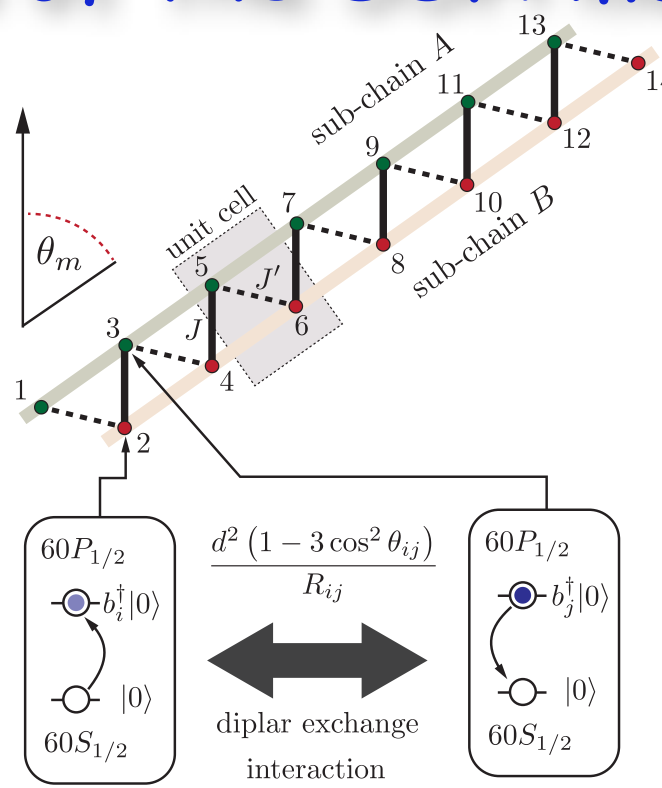
$$(U_S)_{ij} = (-1)^j \delta_{ij}$$

- chiral symmetry does not allow for hopping within the same sub-chain

- dipolar hopping naturally gives rise to longer range hopping

Here, we satisfy the symmetry by employing the anisotropy of the dipole-dipole interaction

# Single particle properties of the SSH model




Hamiltonian of the single particle SSH model for bosons

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**Chiral symmetry**

- unitary matrix with

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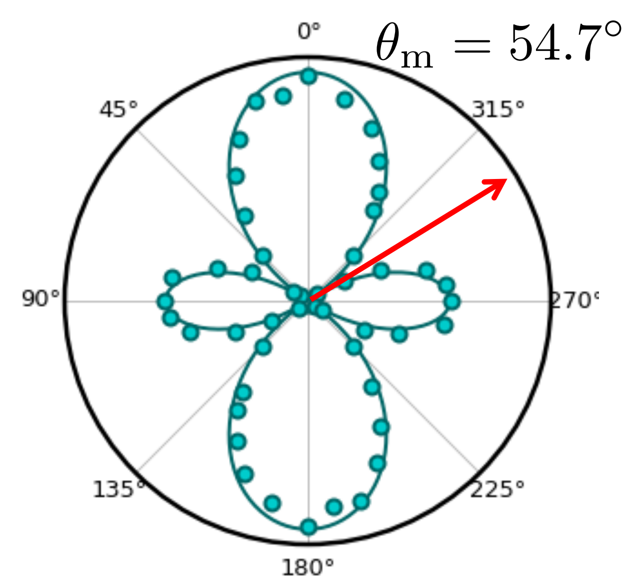
- here, we obtain

$$(U_S)_{ij} = (-1)^j \delta_{ij}$$

experimentally measured

angular dependence



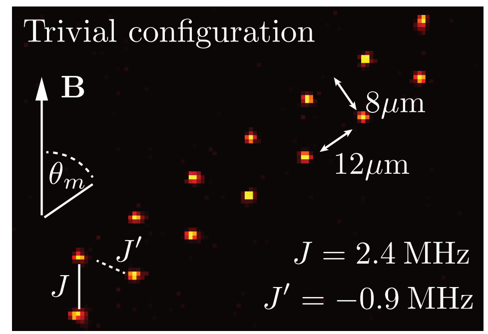
# Edge modes of the SSH model

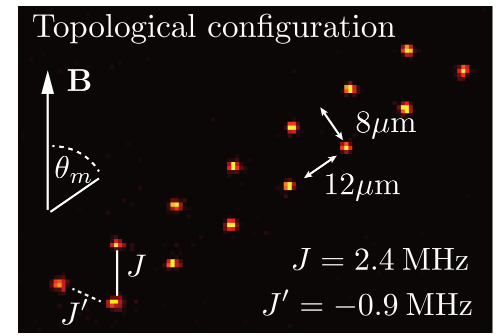
## Spectroscopy of the spectrum

- microwave field coupling the S-state to to the Rydberg P-state

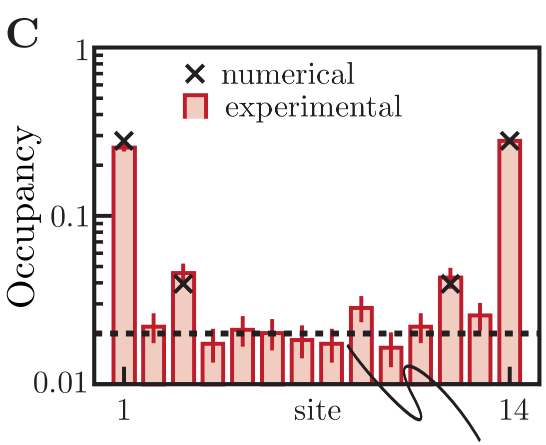
- no-momentum transfer: only couple to half of the states

- observation of zero energy states localized at the edge of the system

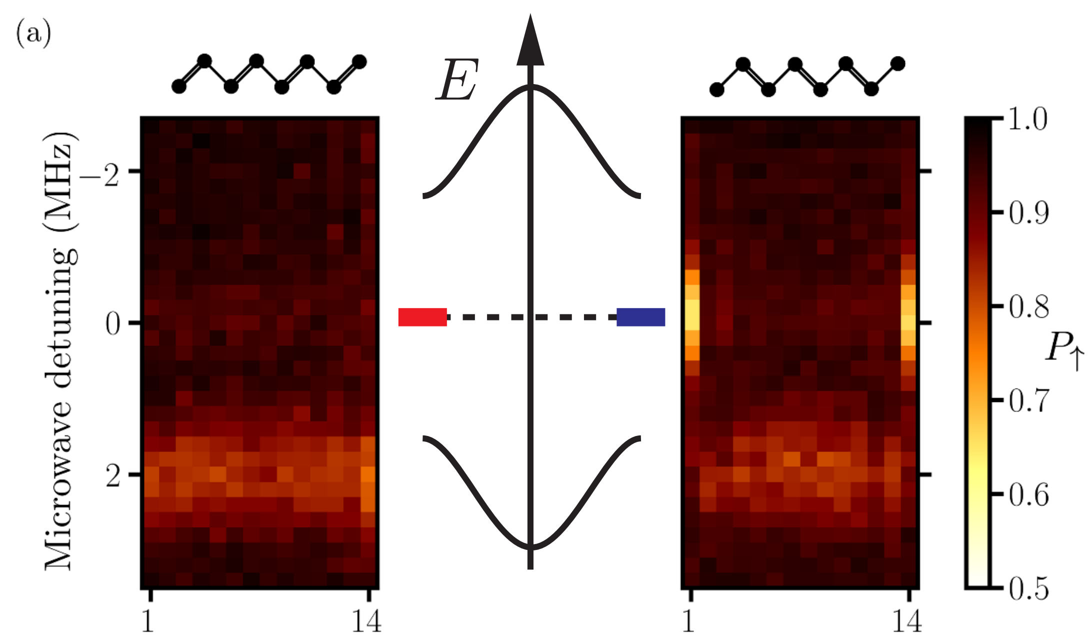



## Edge state localization




imperfections of the experiments




Trivial    Topological

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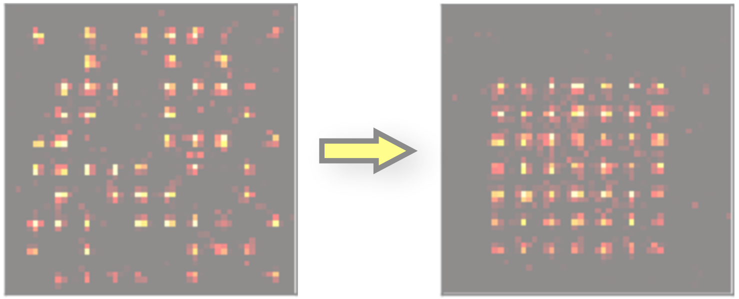

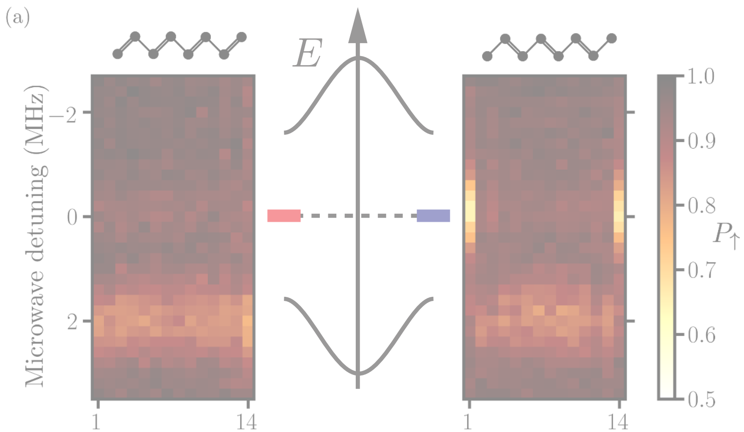


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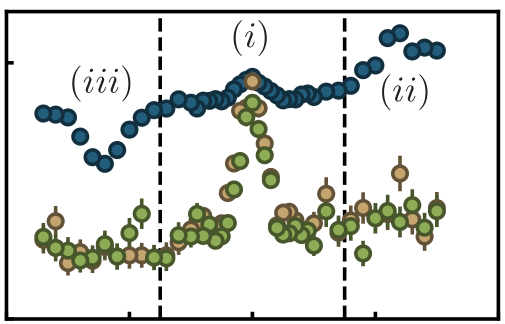


# Symmetry protected topological phase

Topological phases with Bosons

- non-interacting Bosons always
  from a BEC

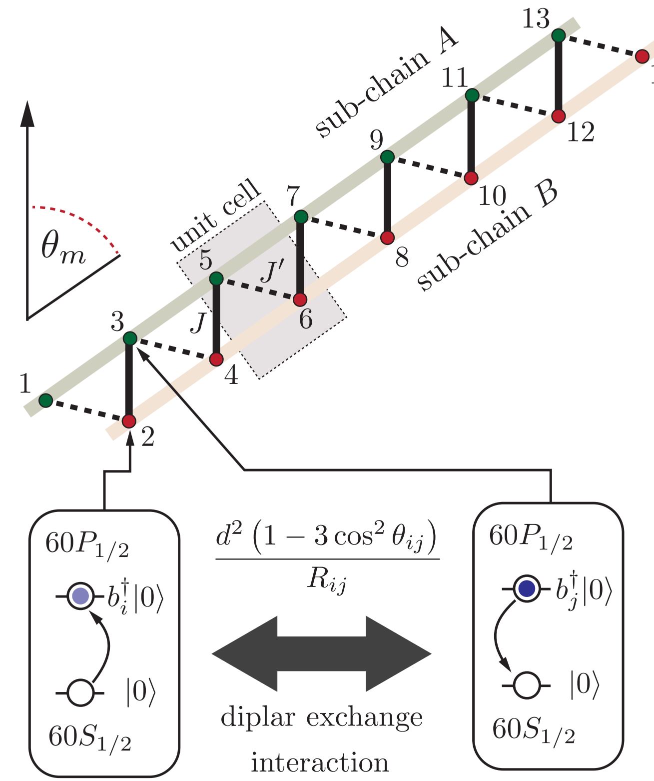
**topological phases with bosons require strong interactions**

**Here: hard-core bosons**

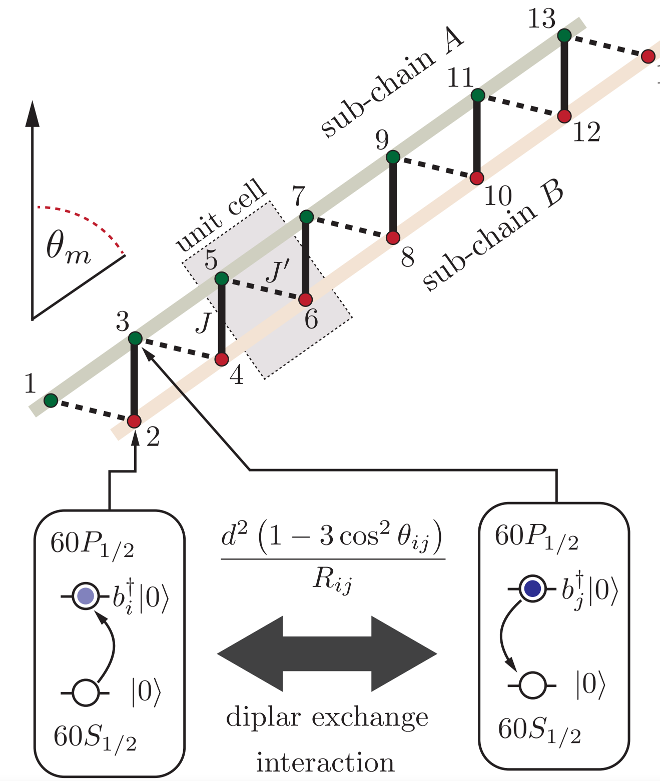
- full classification of SPT phases for
  interacting Bosons in one-dimension
  F. Pollman, *et al.*, PRB 81, 064439 (2010)
  X.-G. Wen, *et al*, Science (2012).

- different SPT phases carry different
  projective symmetry on the edge

- classification in terms of twisted
  cohomology group



# Symmetry protected topological phase




## Bosonic version of the SSH model

- hardcore bosons in one dimension
- Jordan Wigner transformation: $b_j = e^{i\pi \sum_{k<j} c_k^\dagger c_k} c_j$
    - non-local transformation
    - longer range hopping give rise to interactions

non-interacting fermions ⇔ hard-core bosons

**Fermions**

- Hamiltonian

$$H = -J \sum_{i\in\text{even}} c_{i+1}^\dagger c_i - J' \sum_{i\in\text{odd}} c_{i+1}^\dagger c_i + h.c.$$

- chiral symmetry

$$\mathcal{S}_S = \prod_i \left[c_i + (-1)^i c_i^\dagger\right] K$$

**Bosons**

- Hamiltonian

$$H = -J \sum_{i\in\text{even}} b_{i+1}^\dagger b_i - J' \sum_{i\in\text{odd}} b_{i+1}^\dagger b_i + h.c.$$

- symmetry

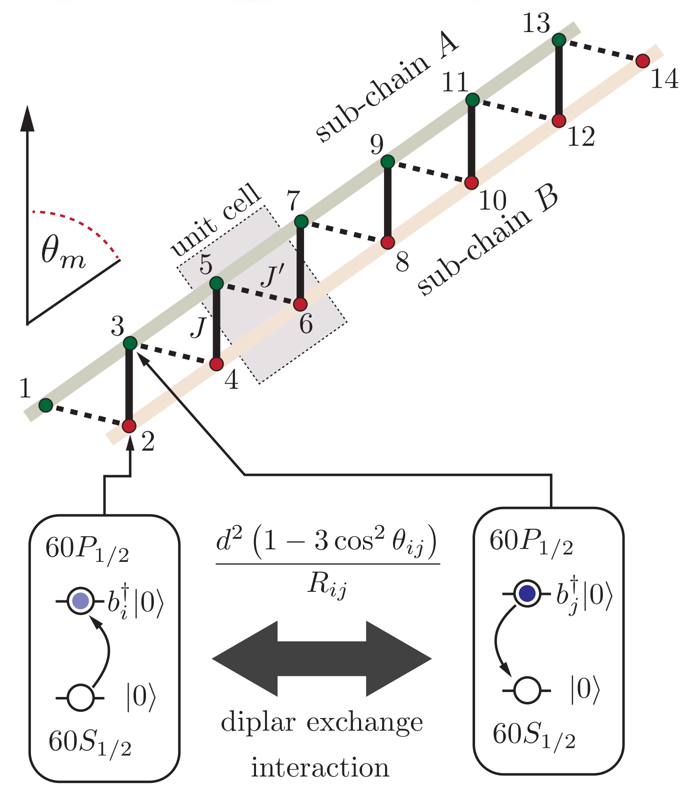
$$\mathcal{S}_B = \prod_i \left[b_i^\dagger + b_i\right] K$$

complex conjugation

# Symmetry protected topological phase

## Protecting symmetries

- particle conservation
- discrete operation $\mathcal{S}_B = \prod_i \left[b_i^\dagger + b_i\right] K$
- symmetry group $U(1) \times Z_2^T$
- Hamiltonian $[H, \mathcal{S}_B] = 0$
- allows for 4 different SPT phases
  X.-G. Wen, *et al*, Science (2012).




## SPT phase

- gapped ground state at half-filling
- four-fold ground state degeneracy
- zero energy edge states

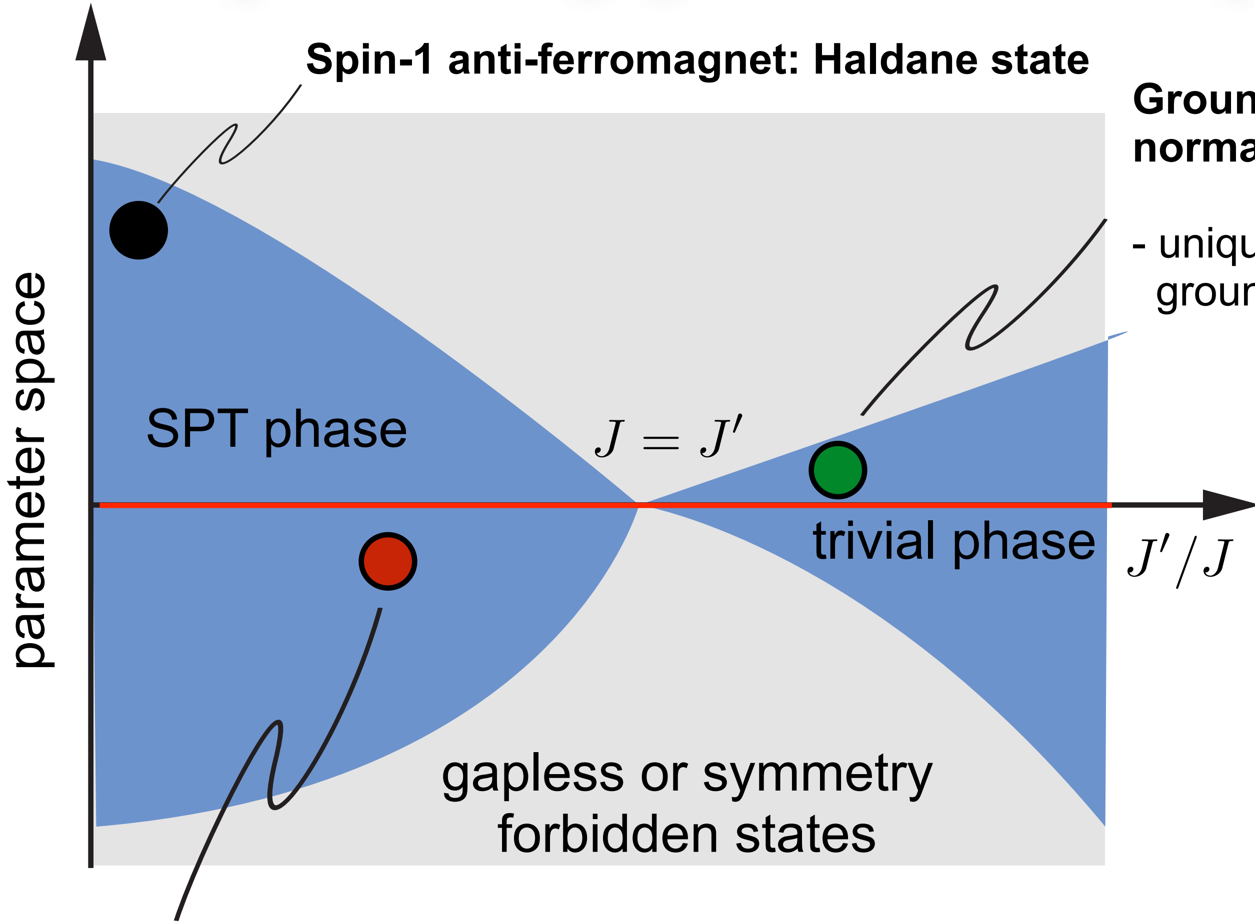
**Special point:** $J' = 0$

$$|m, m'\rangle = \left(b_1^\dagger\right)^m \left(b_L^\dagger\right)^{m'} \prod_{i \in \text{even}} \frac{1}{\sqrt{2}} \left(b_i^\dagger + b_{i+1}^\dagger\right) |0\rangle$$

**Perturbations respecting the symmetry:**

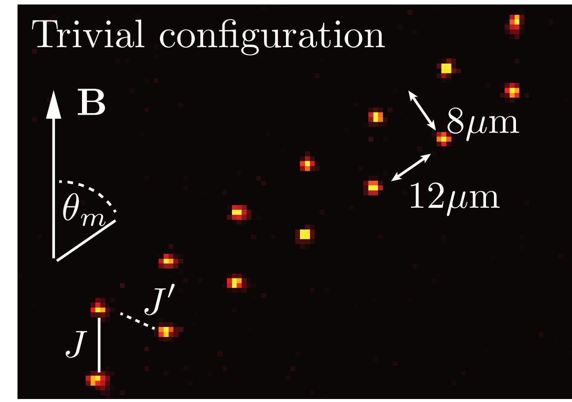
- arbitrary hoppings (also complex) $b_i^\dagger b_j + b_j^\dagger b_i$
- interactions $\left(b_i^\dagger b_i - 1/2\right)\left(b_j^\dagger b_j - 1/2\right)$

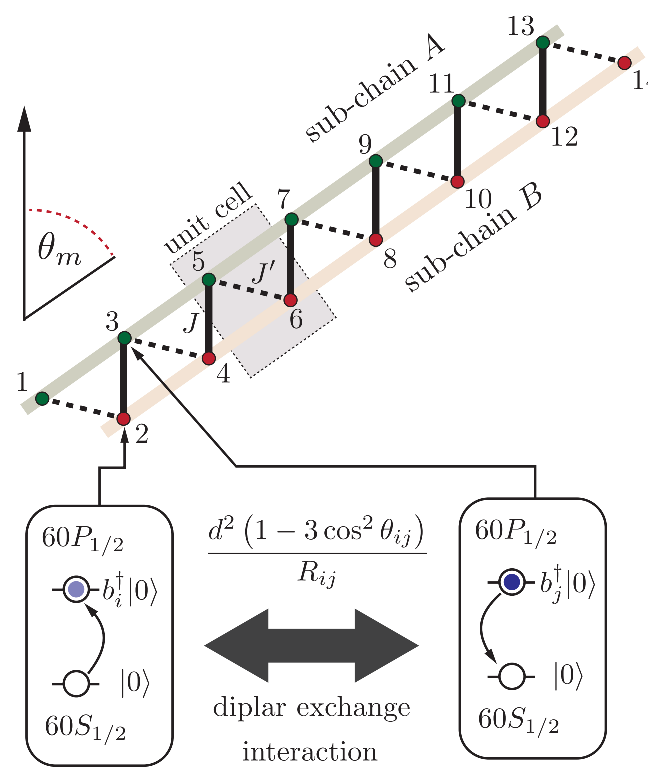
# Symmetry protected topological phase




**Ground state in normal phase**

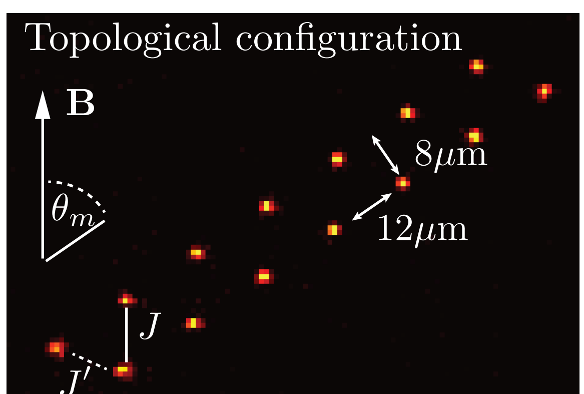
- unique gapped ground state




**Ground state in SPT phase**

- ground state degeneracy
- edge states



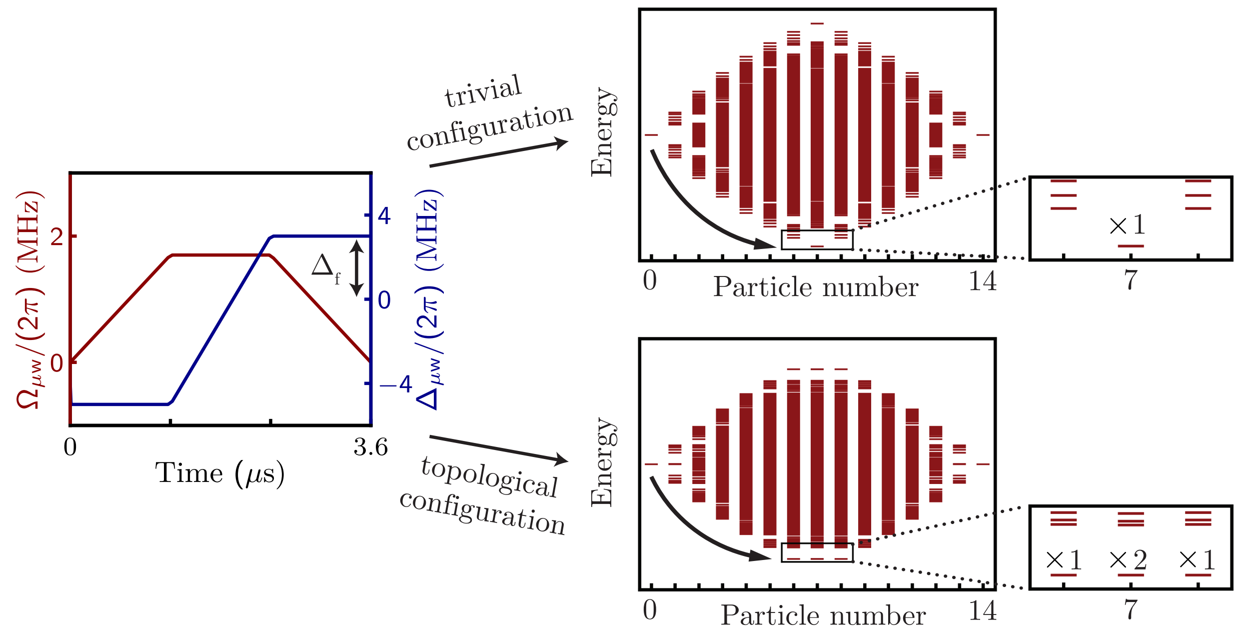
# Preparation of ground state at half-filling

Adiabatic preparation of quantum many-body ground state

- prepare the ground state at half-filling

- highly efficient due to large gap

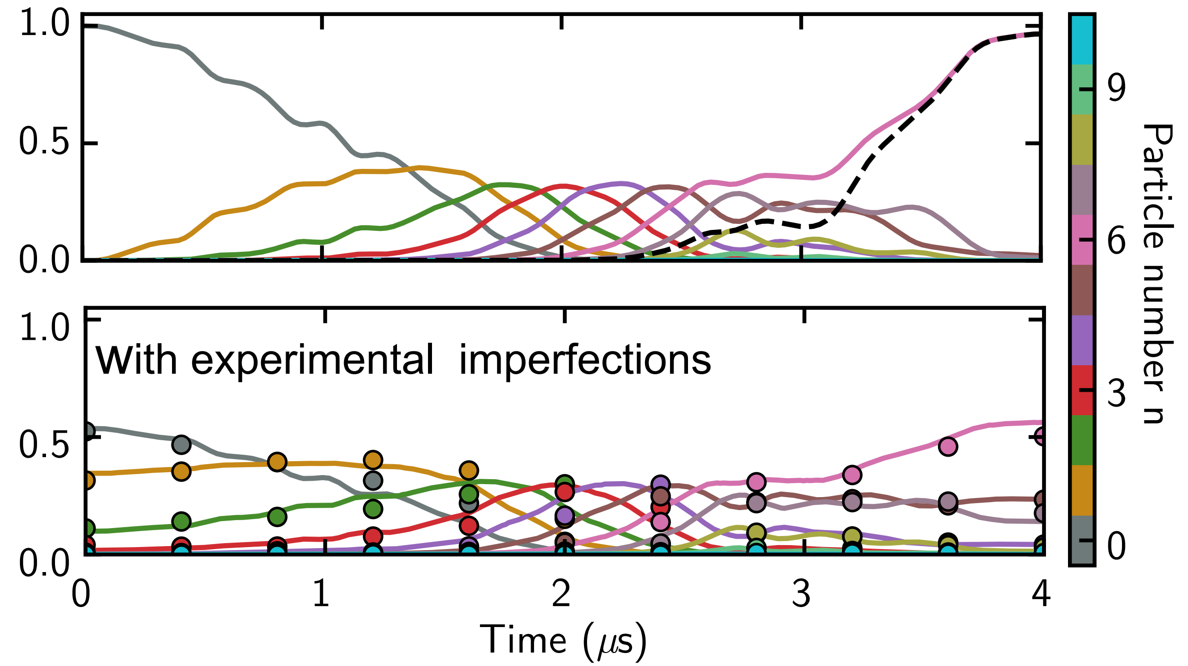
- ramping scheme motivated for prefect dimerization




Full numerical simulation

- efficient preparation of ground state

- high fidelity to prepare exactly ground state with N/2 excitations



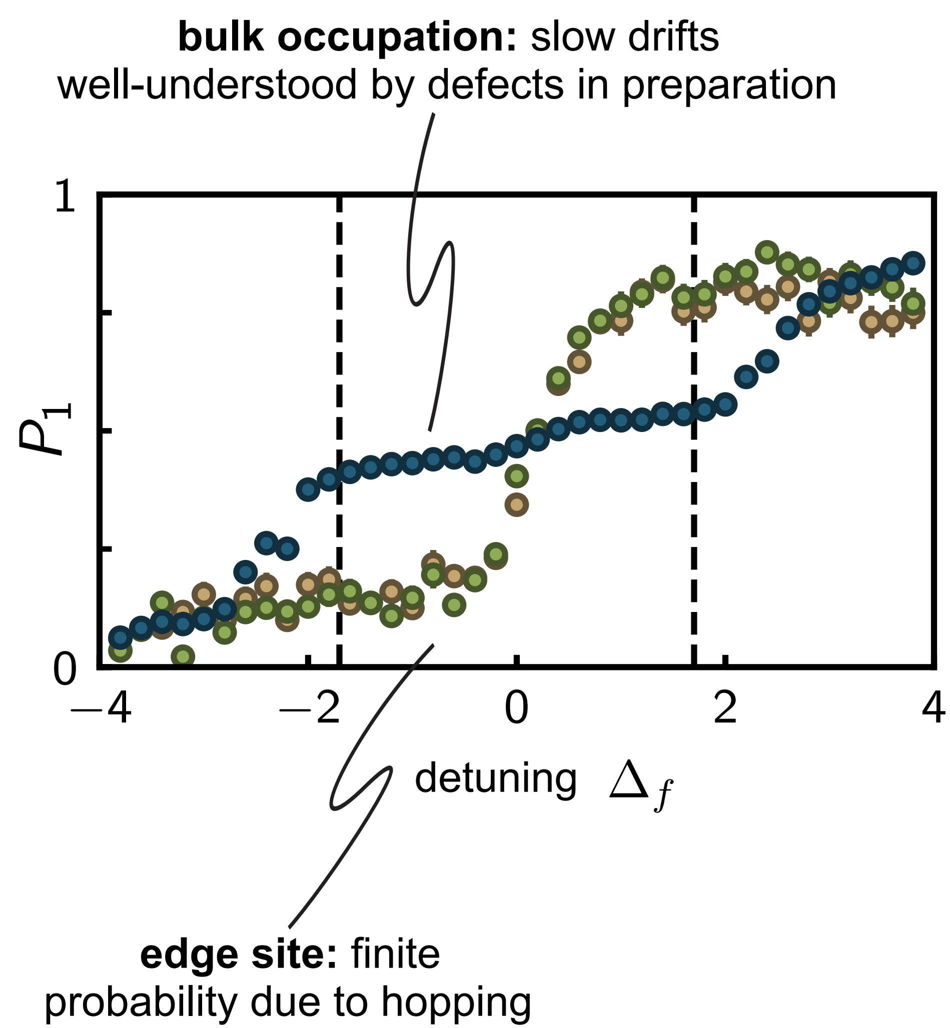
# Symmetry protected topological phase

Characterization of topological phase

- half-filling in the bulk
- occupation of edge states depending on ramping procedure

→ ground state degeneracy

- zero-energy modes in spectroscopy
- robust to perturbation
- absence of any spontaneous symmetry breaking
- experimental detection of string order parameter



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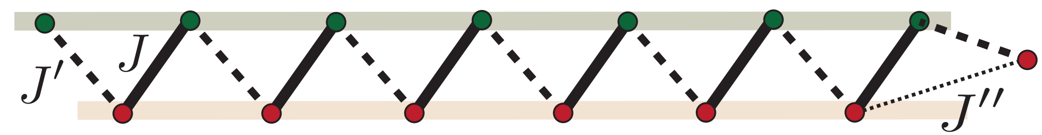


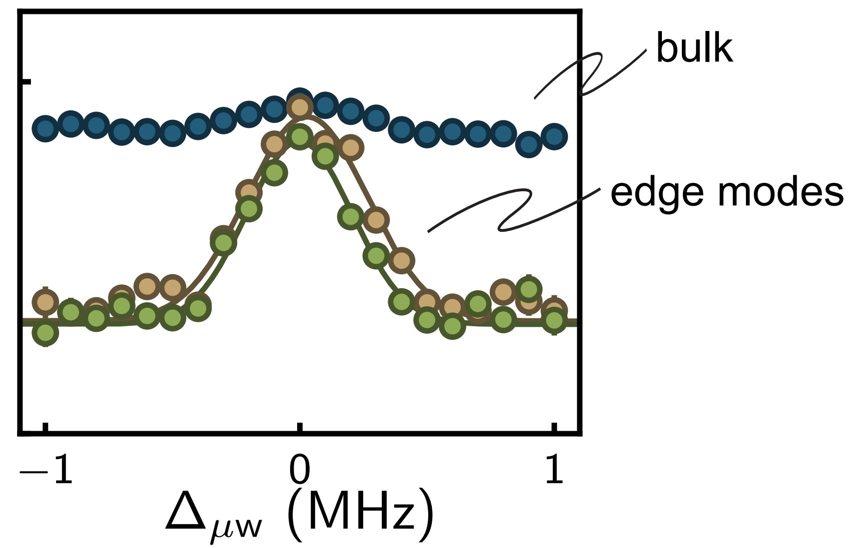


- edge modes excitation is at zero detuning
- in perturbed and unperturbed setup
- line-width well accounted for by finite pulse shape

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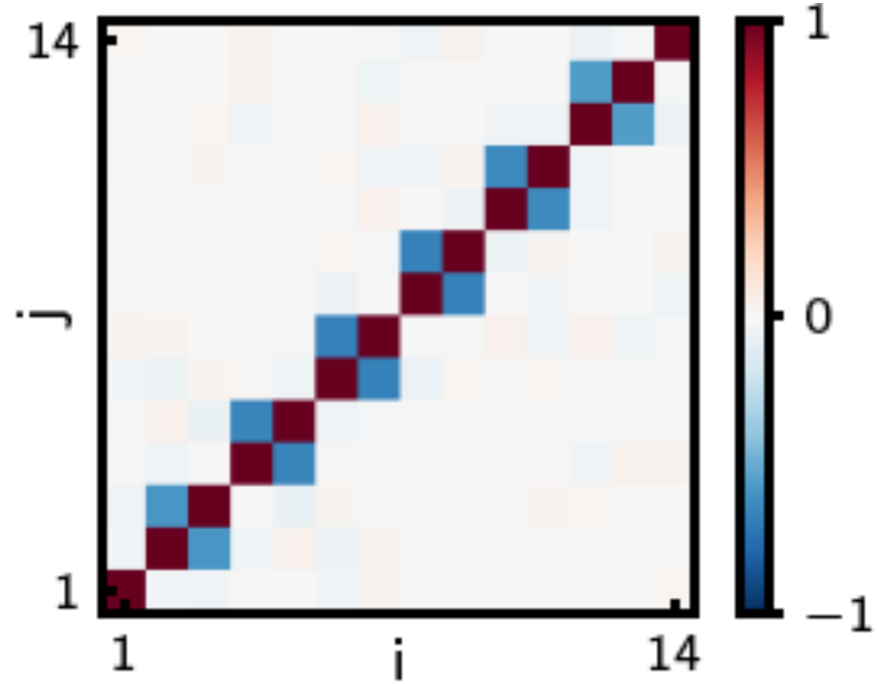
- correlation function

$$C_{ij} = \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle$$




- one excitation shared between two dimers

# Symmetry protected topological phase

Characterization of topological phase

- half-filling in the bulk
- occupation of edge states depending on ramping procedure

→ ground state degeneracy

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- robust to perturbation
- absence of any spontaneous symmetry breaking
- experimental detection of string order parameter

- string order parameter $Z_i = 1 - b_i^\dagger b_i$

$$C^z_{\text{string}} = -\left\langle Z_2 \, e^{i\frac{\pi}{2}\sum_{k=3}^{N-2} Z_k} \, Z_{N-1} \right\rangle$$

| | $C^z$ | $C^x$ | $C^z_{\text{string}}$ | $C^x_{\text{string}}$ |
|---|---|---|---|---|
| Th. (no errors) | -0.96 | 0.98 | 0.78 | 0.88 |
| Full simulation | -0.69(1) | 0.68(2) | 0.11(2) | 0.10(2) |
| Experiments | -0.67(1) | 0.48(2) | 0.11(2) | 0.05(2) |

# Symmetry protected topological phase

Does the different symmetry between bosons and fermions play a role?

### Non-interacting Fermions

- SSH chain requires chiral symmetry

$$\mathcal{S}_S = \prod_i \left[c_i + (-1)^i c_i^\dagger\right] K$$

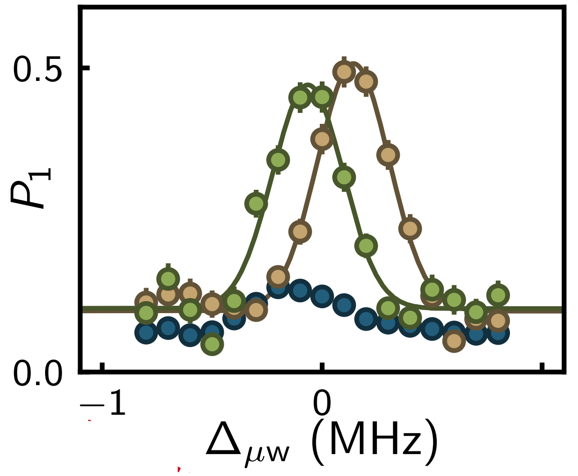
- only hopping between the sub-chains
- also required for single particle Hamiltonian

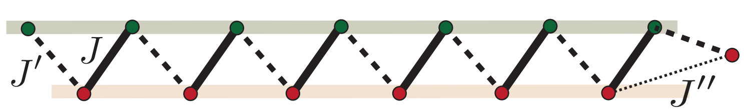
### Interacting bosons

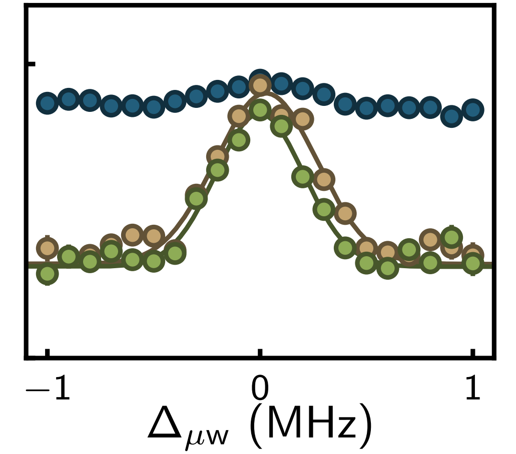
- protecting symmetry

$$\mathcal{S}_B = \prod_i \left[b_i^\dagger + b_i\right] K$$

- allows arbitrary hopping
- robust ground state degeneracy by shifting the edge atom



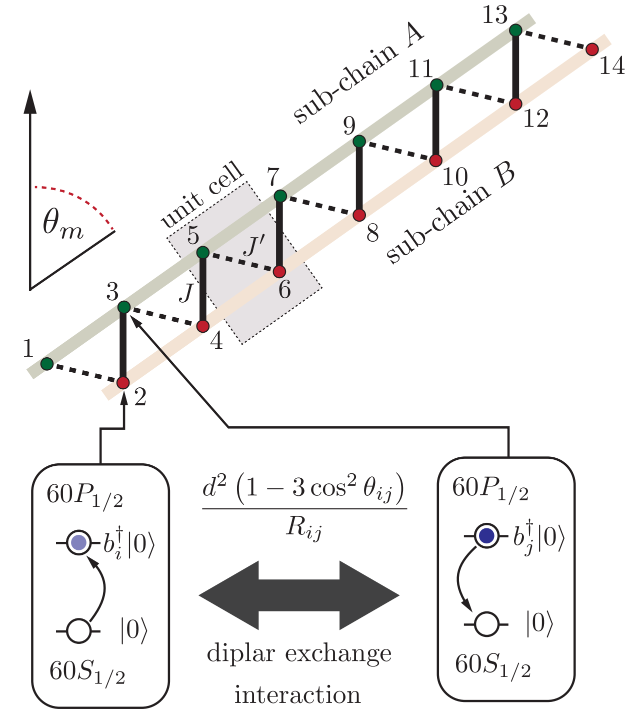



# Summary

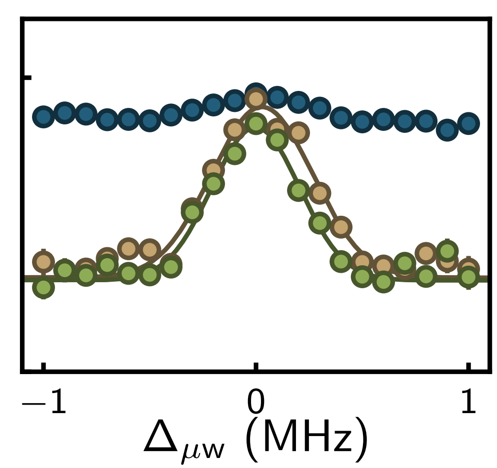
## Edge modes in the SSH model

- implementation of the chiral symmetry
- observation of localized edge modes
- verification of exponential splitting of energy with system size
- benchmarking the experimental results with theoretical predictions




## Symmetry protected topological phase

- ground state of the interacting many-body system at half filling
- first realization of a symmetry protected topological phase in artificial
- quantum simulation of novel states of matter



# Outlook

Topological band structure

- dipolar exchange interactions

- spin-orbit coupling due to anisotropy of dipole-dipole interaction

- Chern number C=1 or C=2

- probing dynamics of edge states

quantum many-body states?

fractional bosonic Chern insulators?

dynamical preparation?

competition with losses?