Anderson (de)localization in Z_2 topological insulators

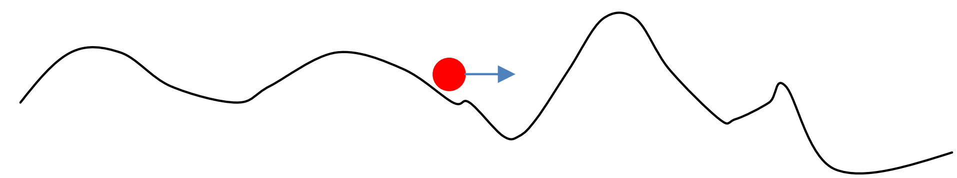
Akira Furusaki (RIKEN)

in collaboration with

Shinsei Ryu (UCSB ⟹ UC Berkeley)
Christopher Mudry (Paul Scherrer Institut)
Hideaki Obuse (Kyoto U.)
Andreas Ludwig (UCSB)
Andreas Schnyder (UCSB)
Kentaro Nomura (Tohoku U.)
Mikito Koshino (Tokyo Inst. Tech.)

Anderson localization

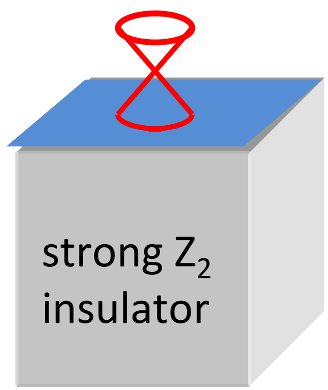
An electron in random potential



P.W. Anderson (1958)

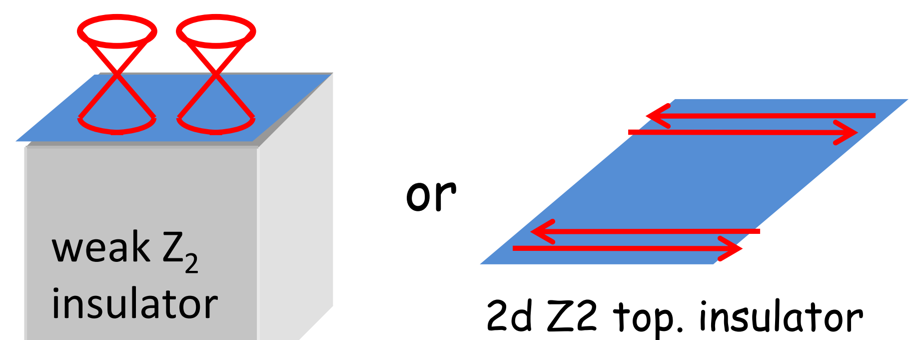
Let's put disorder into 3d Z2 topological insulators and study Anderson localization of surface Dirac fermions

(1) strong topological insulator



No localization!
(topological metal)

(2) weak topological insulator



Localization-delocalization transition (in the standard symplectic class)

Universality classes of disordered electron systems

- Dimensionality of space (= 2)

- Symmetry of Hamiltonian

  time-reversal symmetry

  SU(2) rotation symmetry in spin space

  3 standard classes (Wigner-Dyson random matrix theory)

	time reversal symmetry	spin rotation	
orthogonal	○	○	$T = K$
unitary	×	○ / ×	
symplectic	○	×	$T = i\sigma_y K$

↑

Z2 topological insulators

Hikami, Larkin, & Nagaoka

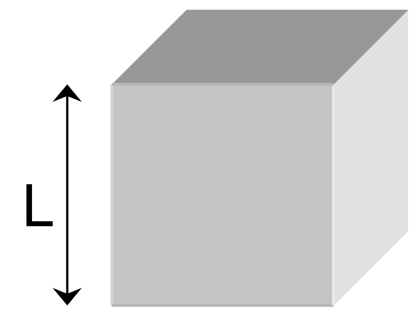
Prog. Theor. Phys. Vol. 63, No. 2, February 1980, Progress Letters

Spin-Orbit Interaction and Magnetoresistance in the Two Dimensional Random System

Shinobu HIKAMI, Anatoly I. LARKIN*) and Yosuke NAGAOKA

Research Institute for Fundamental Physics
Kyoto University, Kyoto 606

(Received November 5, 1979)

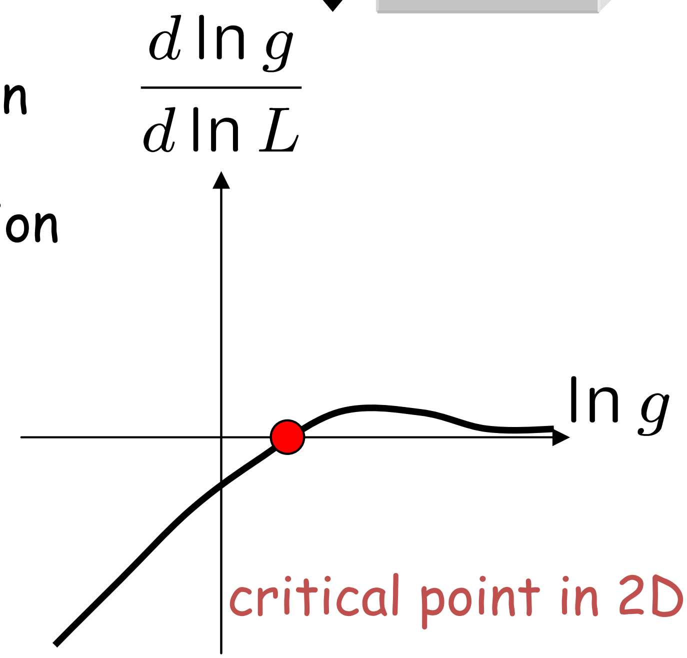


symplectic class: ○ time-reversal, × spin-rotation

disordered metal with spin-orbit interaction

anti-weak localization

$$\frac{d \ln g}{d \ln L} = d - 2 + \frac{c}{g} \qquad c > 0$$



Metal-insulator transition in 2D

Nonlinear sigma model: (Wegner, Efetov, Altshuler-Kravtsov-Lerner, …)

low-energy effective field theory for Nambu-Goldstone bosons

$$E = \int \left(\partial \vec{n}\right)^2 dx d\tau + i2\pi SN \qquad E = \int \mathrm{tr}\left(\partial Q\right)^2 d^2 r \ + i\theta N$$

	Antiferromagnets	Integer Quantum Hall effect
N-G bosons	magnons	Diffuson
Ordered phase	antiferromagnetic	metallic
Disordered phase	paramagnetic	insulating
Order parameter	$\vec{n} \in R^3 \qquad \vec{n}\cdot\vec{n} = 1$	$Q \in \mathrm{U}(2N) \qquad Q \approx \mathrm{diag}(1_N, -1_N)$
Target space	$G/H = \mathrm{O}(3)/\mathrm{O}(2)$	$G/H = U(2N)/U(N)\times U(N)$
	$\pi_2\left(G/H\right) = Z$	$\pi_2\left(G/H\right) = Z$
	Haldane	Pruisken

Topological terms lead to nonperturbative effects.

IQHE (and 1d Antiferromagnet)

$$\pi_2(G/H) = \pi_2(U(2N)/U(N) \times U(N)) = Z$$ (Pruisken, 1983)

topological sectors labeled by an integer

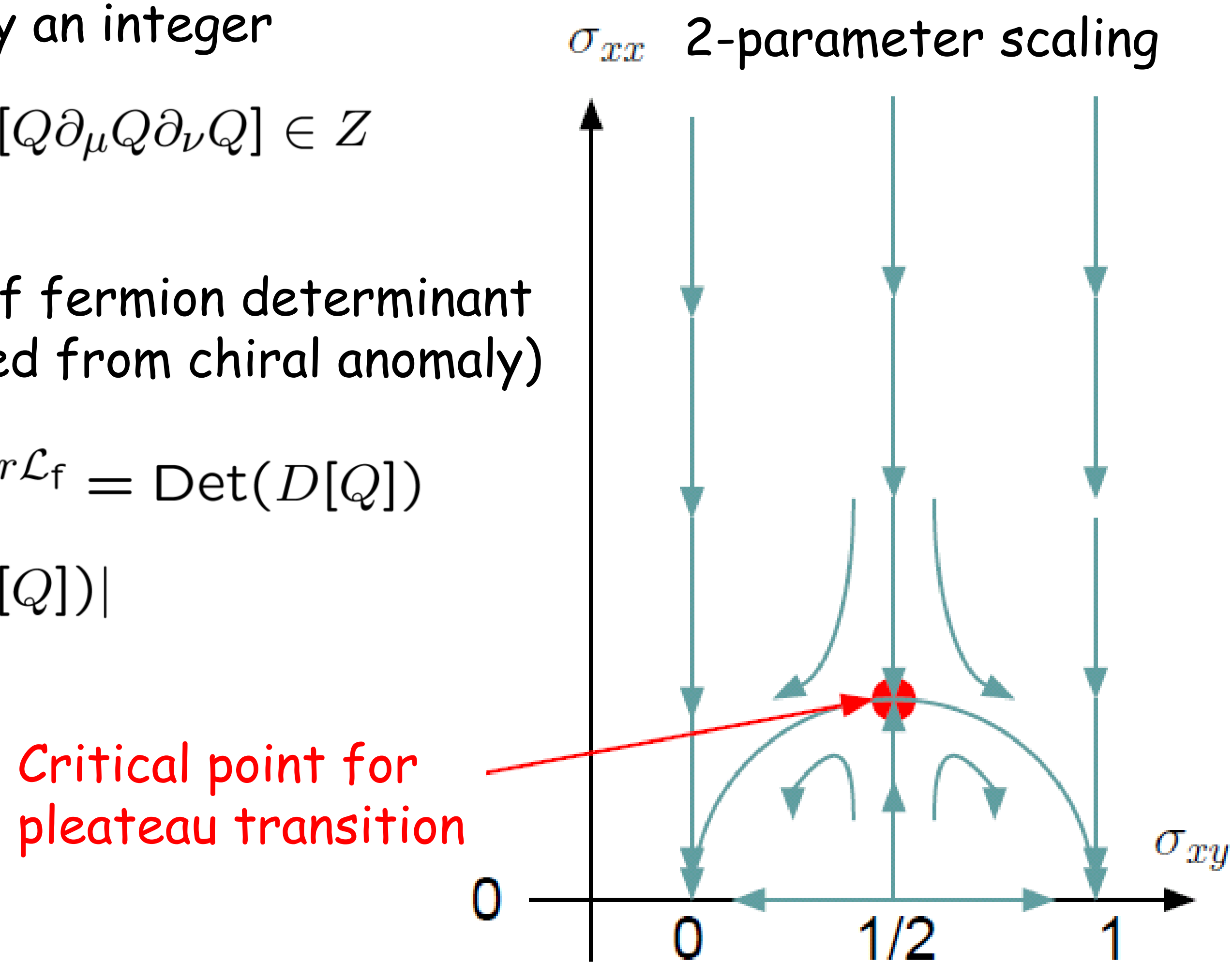
$$\mathrm{Ch}[Q] := \frac{1}{16\pi i}\int d^2 r \epsilon_{\mu\nu} \mathrm{tr}[Q\partial_\mu Q \partial_\nu Q] \in Z$$

topological term as a phase of fermion determinant
(can be obtained from chiral anomaly)

$$e^{-S_{\mathrm{eff}}[Q]} = \int \mathcal{D}[\bar{\psi},\psi] e^{-\int d^2 r \mathcal{L}_{\mathrm{f}}} = \mathrm{Det}(D[Q])$$
$$= e^{iS_{\mathrm{top}}[Q]} |\mathrm{Det}(D[Q])|$$

theta angle can be tuned

$$\theta = \sigma_{xy}/(e^2/h)$$



Nonlinear sigma model for the symplectic class

N-G bosons — Diffuson & Cooperon

Ordered phase — metallic

Disordered phase — insulating

Matrix fields $Q^2 = 1_{4N}, \quad Q^T = Q, \quad \mathrm{Tr}\, Q = 0$

Target space $G/H = \mathrm{O(4N)/O(2N) \times O(2N)}$

$$\pi_2(G/H) = Z_2$$

Fendley, PRB (2001)

2 distinct sectors in the space of field configurations

$$e^{-S_1} + e^{-S_2}$$ (no top. term)

or

$$e^{-S_1} - e^{-S_2}$$ (with Z_2 top. term)

Q: What is a minimal model whose transport is described by the nonlinear sigma model with a Z_2 topological term?

A: Single-flavor Dirac fermion in 2d.

(1) 2d surface of 3d strong Z_2 topological insulators

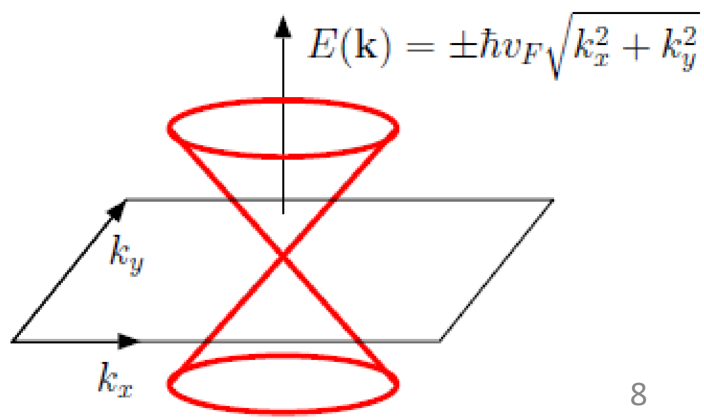
(2) graphene with smooth disorder potential

Dirac fermion in 2 dimensions

$$\mathcal{H} = -i\hbar v_F \partial_\mu \sigma_\mu = -i\hbar v_F \begin{pmatrix} 0 & \partial_x - i\partial_y \\ \partial_x + i\partial_y & 0 \end{pmatrix}$$

+disorder

2-component, massless, single flavor



Disorder effects on Dirac fermions in 2 dimensions

$$\mathcal{H} = -i(\sigma_x\partial_x + \sigma_y\partial_y) + V(x,y)\sigma_0$$

"time-reversal symmetry"

$$i\sigma_y \mathcal{H}^*(-i\sigma_y) = \mathcal{H}$$

$$\overline{V(r)V(r')} = g\delta^{(2)}(r-r'), \quad \overline{V(r)} = 0$$

fermionic replica trick

disorder model —disorder average→ interacting model

$$\bar{Z} = \int \mathcal{D}[\bar{\psi},\psi] \exp\left(-\int d^2r\, \mathcal{L}\right)$$

$$\mathcal{L} = \bar{\psi}_a \left(-i\sigma_\mu\partial_\mu + i\delta\Lambda_{ab}\right)\psi_b - \frac{g}{2}\bar{\psi}_a\psi_a\bar{\psi}_b\psi_b$$

$\bar{\psi}_a := \psi_a^T i\sigma_y$ Majorana fermion $\quad \Lambda = \mathrm{diag}\left(1_{2N}, -1_{2N}\right)$

$$\bar{Z}_{\mathrm{eff}} = \int \mathcal{D}[\bar{\psi},\psi]\mathcal{D}[Q] \exp\left(-\int d^2r \left[\mathcal{L}_{\mathrm{f}} + \frac{\Delta^2}{4g}\mathrm{tr}\left(QQ^T\right)\right]\right)$$

$$\mathcal{L}_{\mathrm{f}} := \bar{\psi}D[Q]\psi, \qquad (D[Q])_{ab} := \sigma_\mu p_\mu \delta_{ab} - i\Delta Q_{ab}$$

$$
\begin{aligned}
e^{-S_{\mathsf{eff}}[Q]} &= \int \mathcal{D}[\bar{\psi},\psi] e^{-\int d^2 r \mathcal{L}_{\mathsf{f}}} \\
&= \mathsf{Pf}(i\sigma_y D[Q]) \\
&= \pm |\mathsf{Pf}(i\sigma_y D[Q])|
\end{aligned}
$$

$$S_{\text{eff}} = \frac{1}{t}\int d^2r\,\mathrm{tr}(\partial Q)^2 + i\pi n[Q]$$

Z_2 topological term

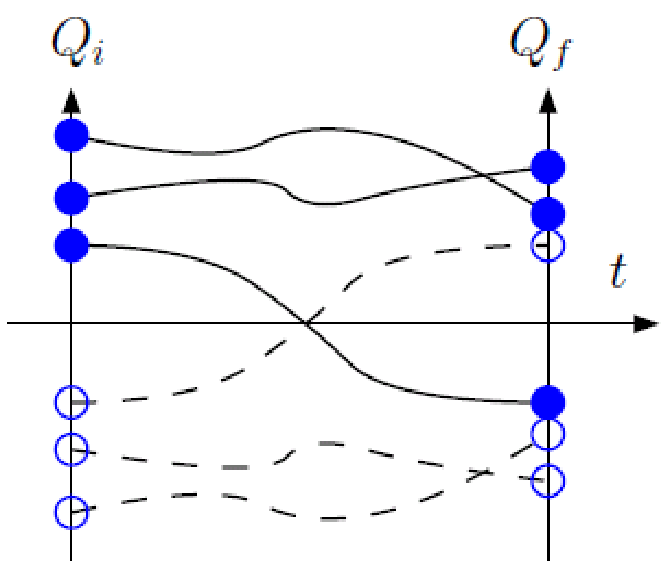
Sign of Pfaffian: $\pm 1 = e^{i\pi n[Q]}$

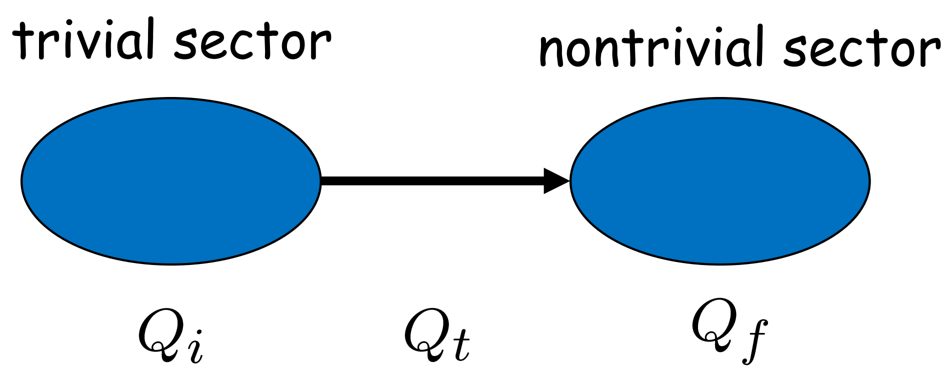
Ryu, Mudry, Obuse, & AF, PRL 99, 116601 (2007)

also by Ostrovsky, Gornyi, & Mirlin, PRL 98, 256801 (2007)

Spectral flow

$$Q_t := (1-t)Q_i + tQ_f$$





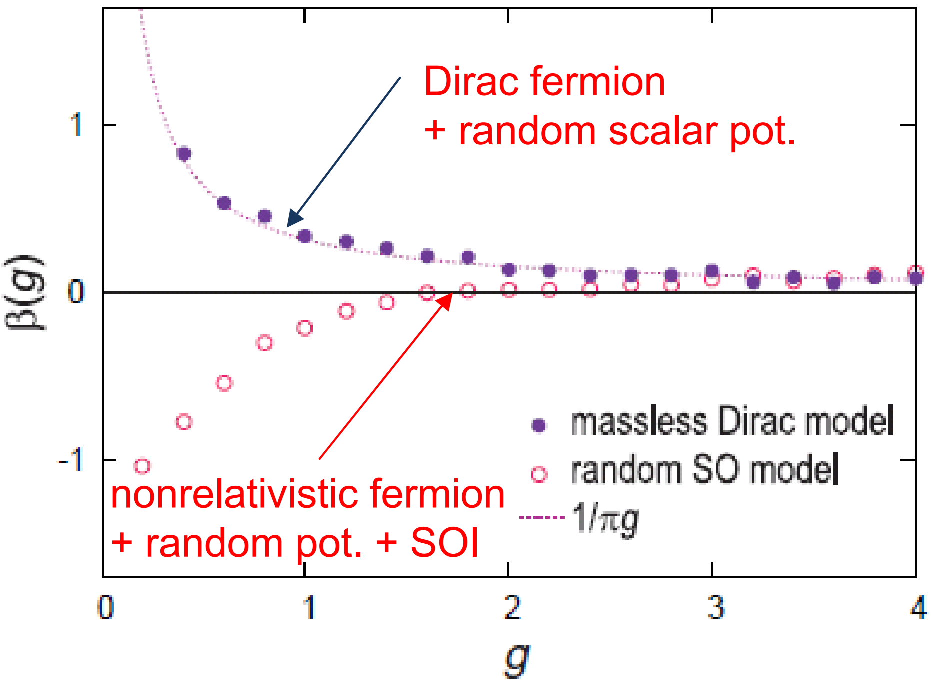
RG flow of the nonlinear sigma model with a Z_2 topological term

$$S_{\text{eff}} = \frac{1}{t}\int d^2r\, \mathrm{tr}(\partial Q)^2 + i\pi n[Q]$$

any critical points in RG ??

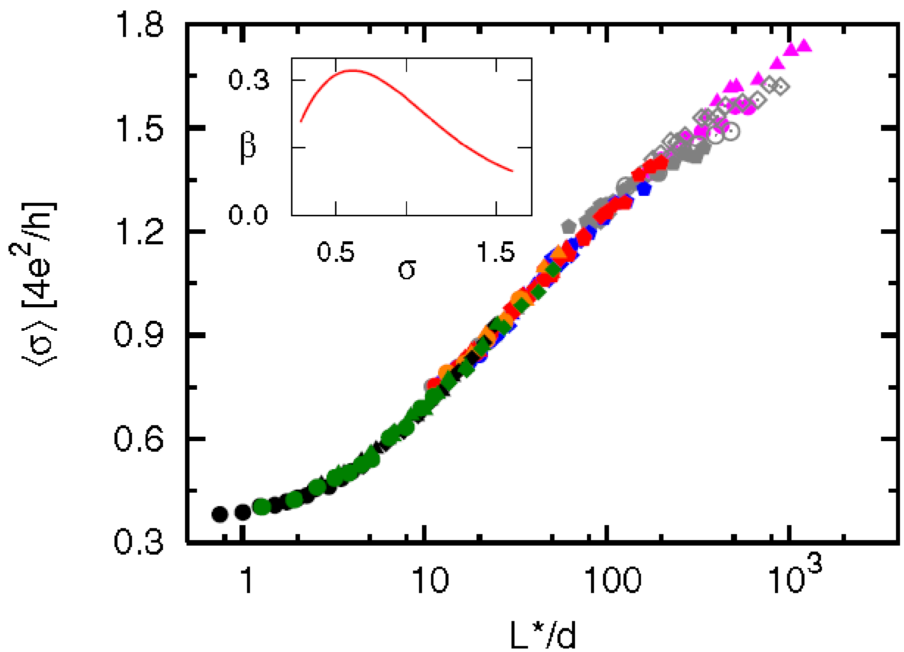
Direct numerical calculations of the beta function $\beta(g) = \dfrac{d \ln g}{d \ln L}$

Nomura, Koshino, & Ryu, PRL (2007)



The beta function is always positive!

Bhardarson, Tworzydlo, Brouwer & Beenakker PRL (2007)



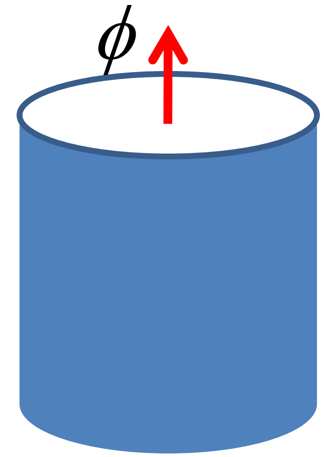
No localization at all

$$g \to \infty (L \to \infty)$$

Spectral flow induced by twisting boundary conditions

(Nomura, Koshino, & Ryu, PRL 2007)

$$\psi(x+L_x, y) = e^{i\phi}\psi(x,y), \quad \psi(x, y+L_y) = \psi(x,y)$$



$E_n(\phi)$ are flat → localization

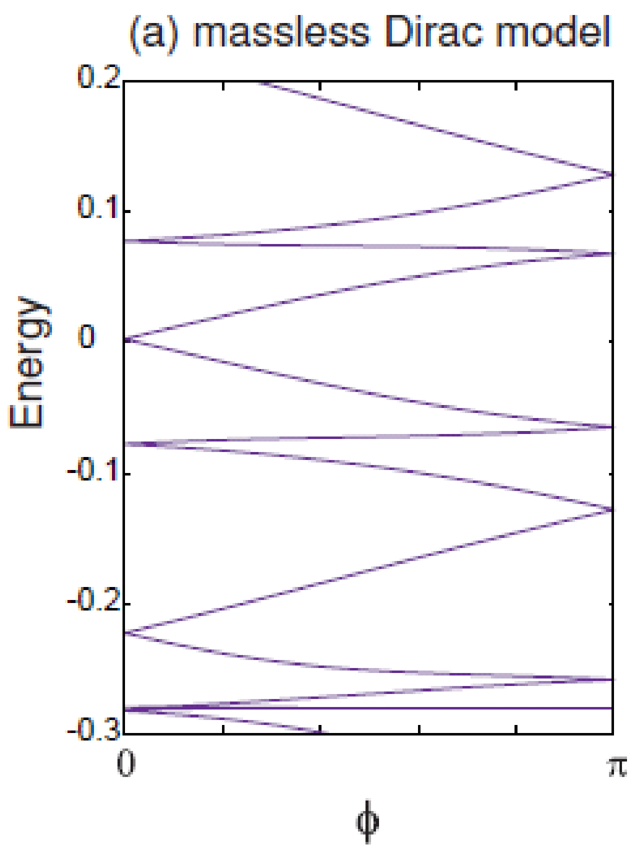
not flat → delocalization

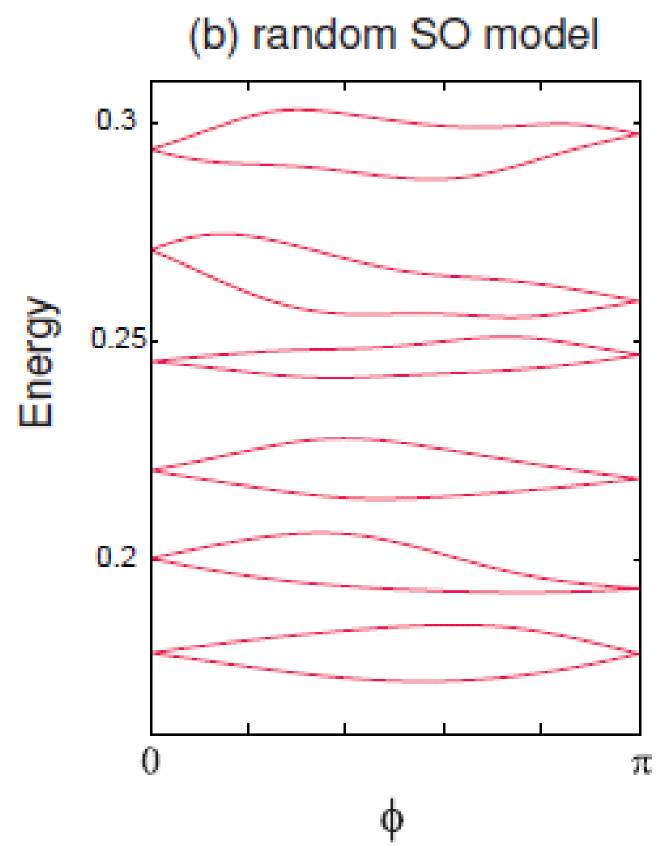
Time-reversal symmetry at $\phi = 0, \pi$

Kramers pairs switch their partners.

Thouless number

$$g = \frac{\delta E}{\Delta} \approx 1$$

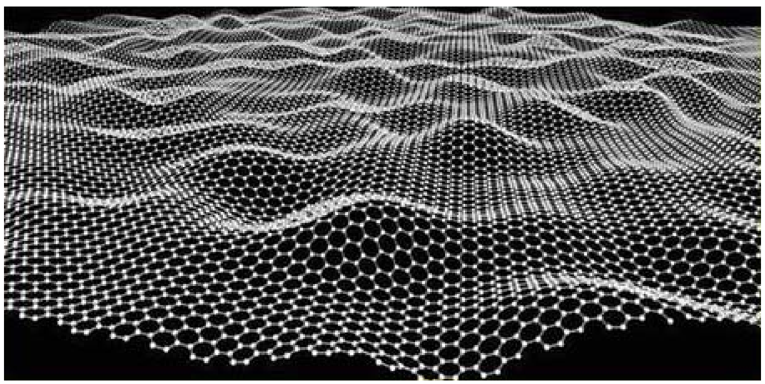


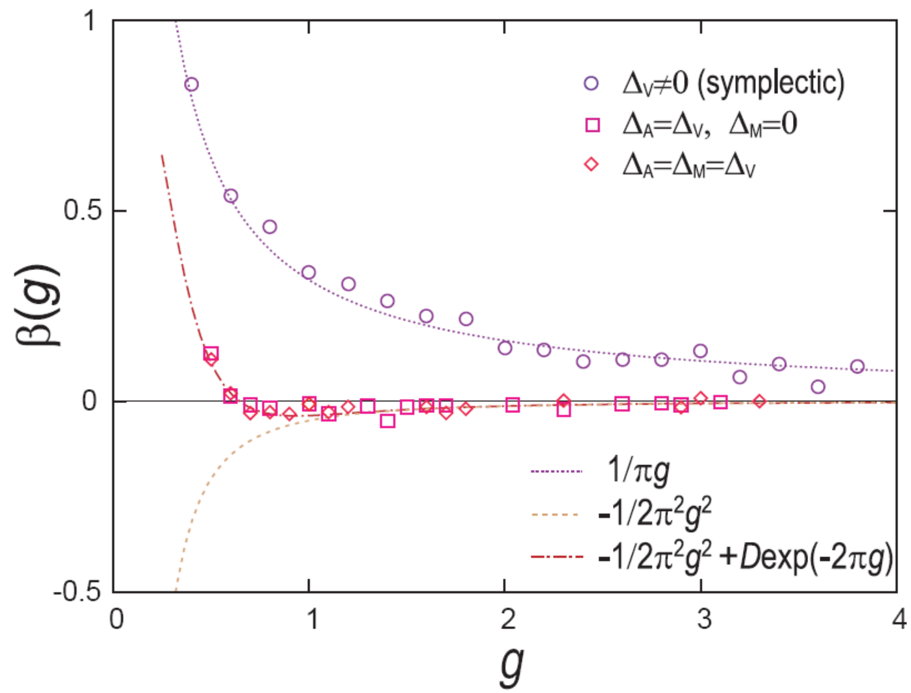


Including other disorder terms:

$$H = -i\hbar v_F \boldsymbol{\sigma}\cdot\nabla + V(\mathbf{x})$$

Ripples in corrugated graphene = a *random vector potential*





Nomura, Ryu, Koshino, Mudry, AF, PRL 100, 246806 (2008)

(i) Suppressed (anti)localization effect
(ii) Exactly at the IQH plateau transition point between $\sigma_{xy} = \pm \frac{1}{2}$

IQHE

$$\pi_2(G/H) = \pi_2(U(2N)/U(N) \times U(N)) = Z$$ (Pruisken, 1983)

topological sectors labeled by an integer

$$\mathrm{Ch}[Q] := \frac{1}{16\pi i} \int d^2 r \epsilon_{\mu\nu} \mathrm{tr}[Q\partial_\mu Q \partial_\nu Q] \in Z$$

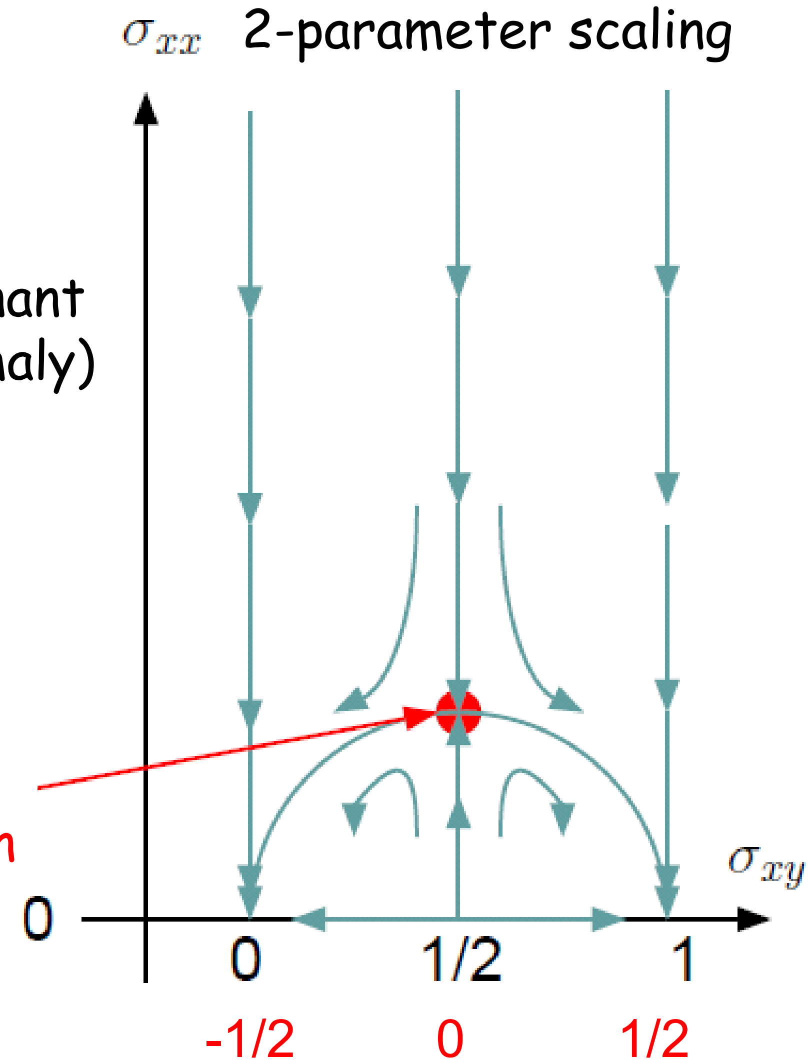
topological term as a phase of fermion determinant
(can be obtained from chiral anomaly)

$$e^{-S_{\mathrm{eff}}[Q]} = \int \mathcal{D}[\bar{\psi}, \psi] e^{-\int d^2 r \mathcal{L}_{\mathrm{f}}} = \mathrm{Det}(D[Q])$$
$$= e^{iS_{\mathrm{top}}[Q]} |\mathrm{Det}(D[Q])|$$

theta angle can be tuned

$$\theta = \sigma_{xy}/(e^2/h)$$

Critical point for
pleateau transition

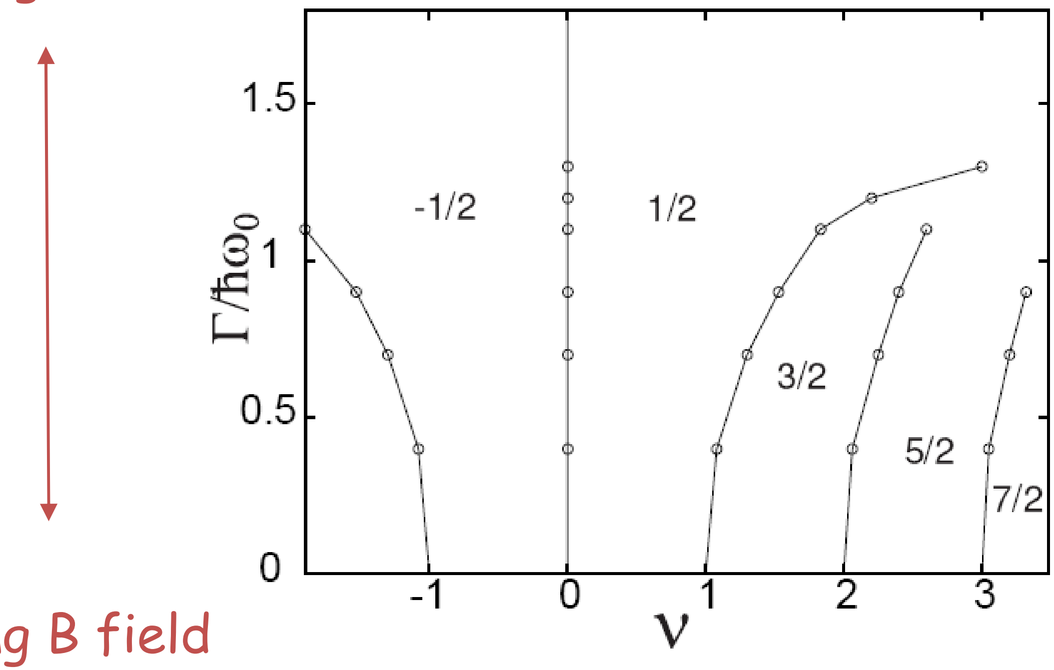


Dirac fermions with random V, m, A (Ludwig et al., PRB 1994)

Phase diagram in uniform magnetic field

Single Dirac cone

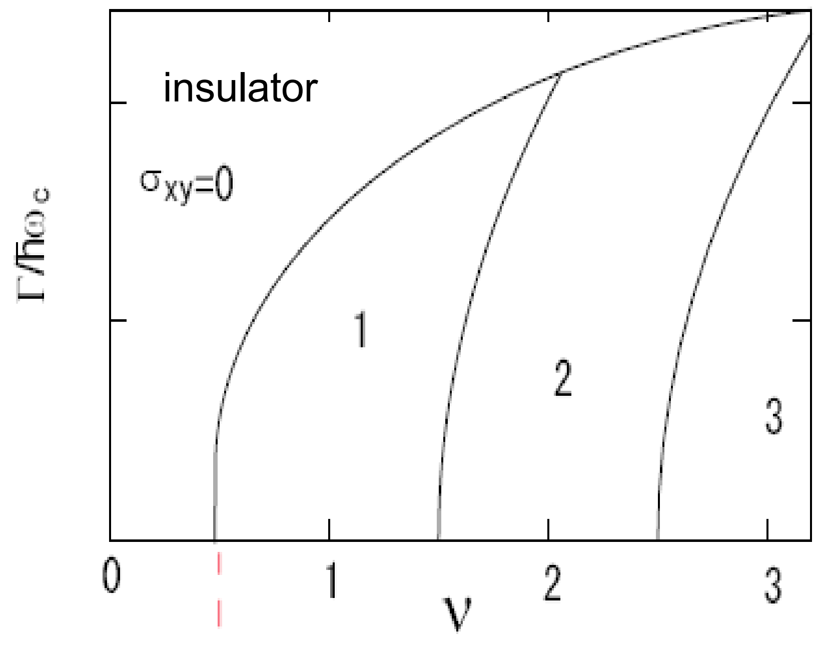
weak B field
(strong disorder)



strong B field
(weak disorder)

$\sigma_{xy} = 0$ is a critical line (QH transition)

"Non-relativistic" case



Summary

- 2d surface states of 3d strong Z_2 insulators:

an odd number of Dirac fermions (graphene with smooth disorder)

$$\mathcal{H} = -i(\sigma_x \partial_x + \sigma_y \partial_y) + V(x,y)\sigma_0$$

nonlinear sigma model with a Z_2 topological term

$$t^{-1}\int (\partial Q)^2 + i\pi n[Q]$$ → Always metallic (no localization at all)

$$\beta(g) = \frac{d \ln g}{d \ln L} > 0$$

- 2d surface of 3d weak Z_2 insulators

an even number of Dirac fermions (Kane-Mele model + disorder)

nonlinear sigma model has no topological term

$$t^{-1}\int tr(\partial Q)^2$$ → conventional symplectic class

$$\nu \approx 2.7$$

2 sets of boundary multifractal exponents

(metal/top. insulator vs. metal/insulator)