The Abdus Salam
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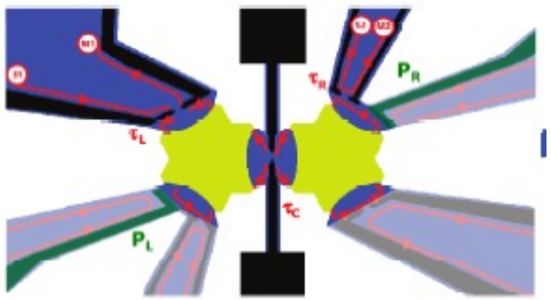
M.N.Kiselev

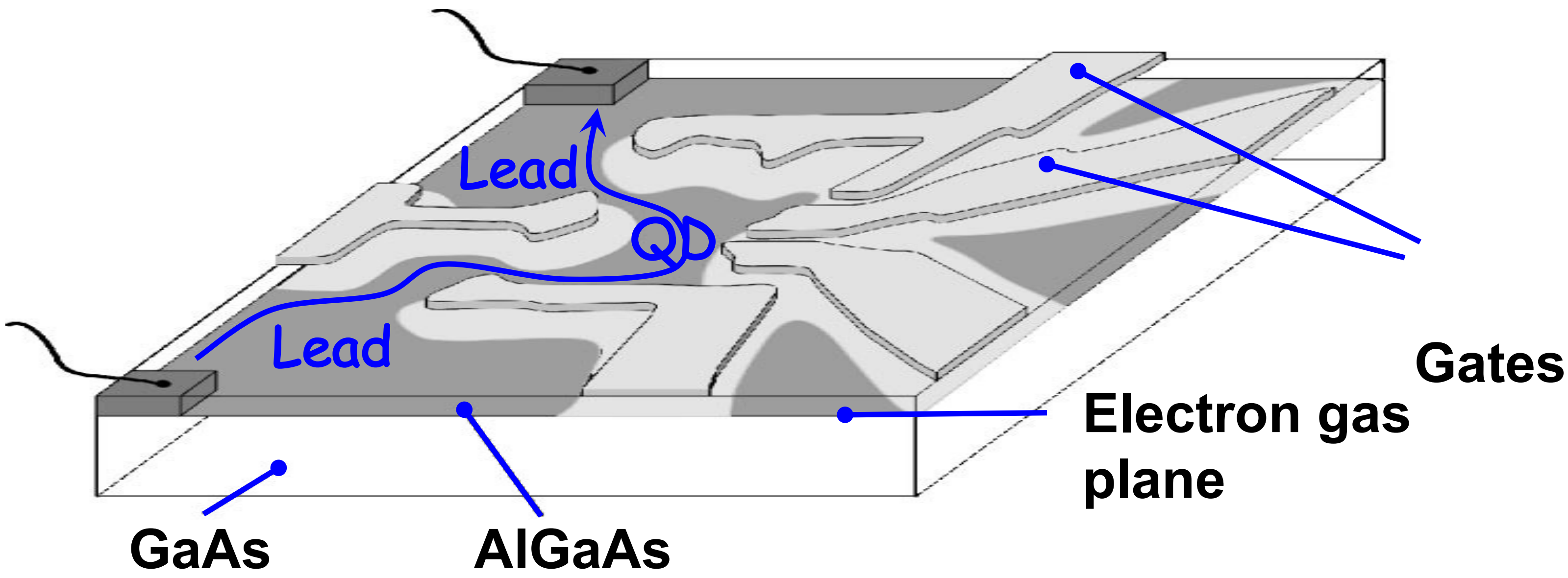
Magic Lorenz ratios
and Anderson orthogonality catastrophe
in quantum simulators

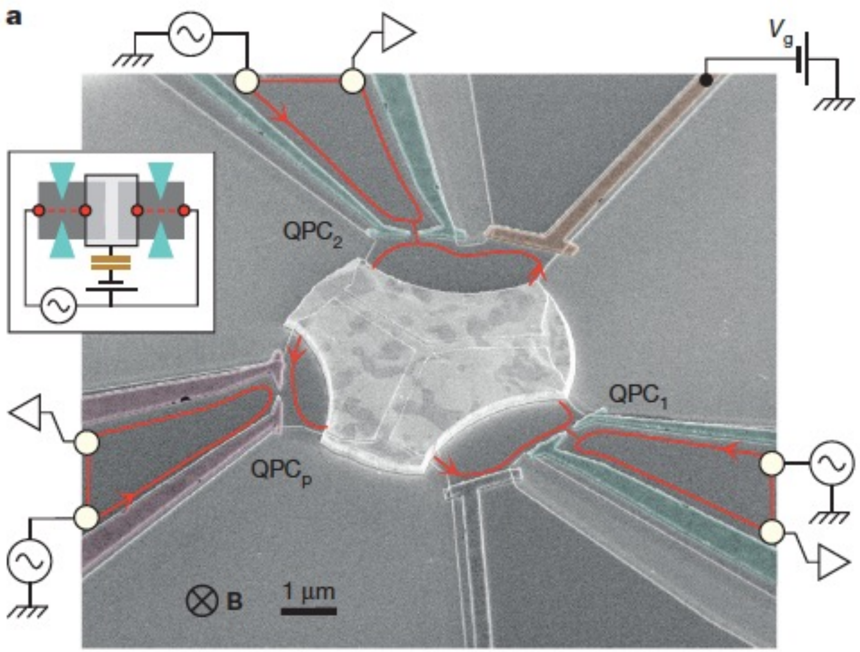


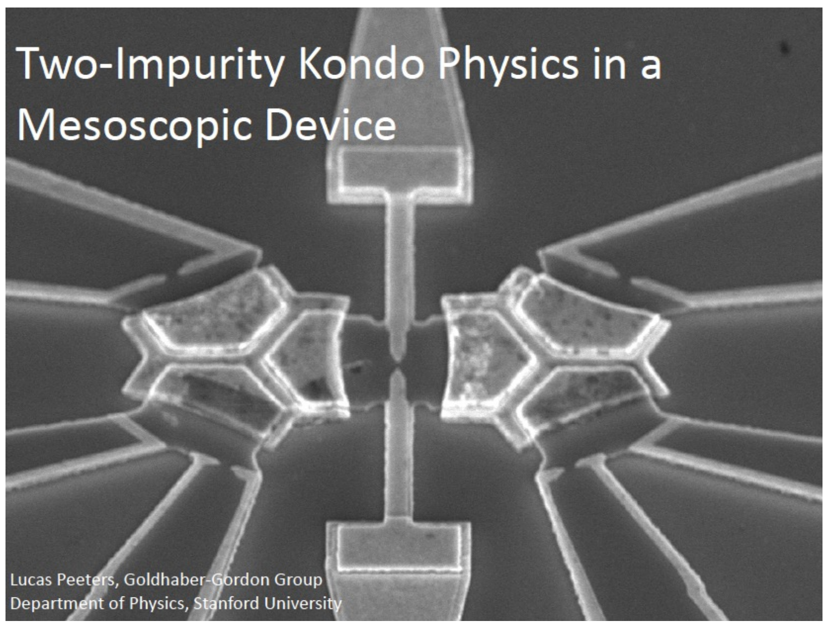
MK, Phys. Rev. B 108, L081108 (2023)

QUASIPART23, KITP, October 12, 2023

Quantum simulators: real physics of artificial structures







Quantum simulators: what are they good for?

Controllable quantum criticality

Engineering non-Fermi liquids ✓

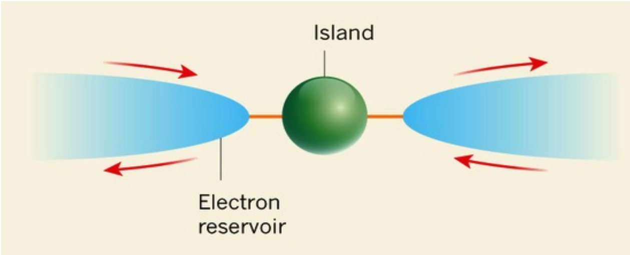
Exotic quasiparticles ✓

Many other interesting facets of SCES

QUASIPART23, KITP, October 12, 2023

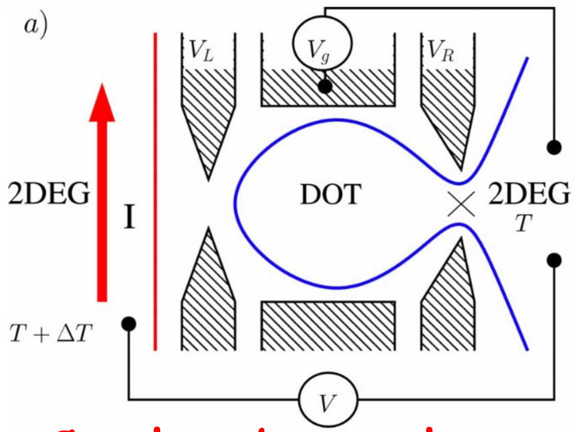
Spin vs Charge Kondo effect in quantum simulators

Spin Kondo

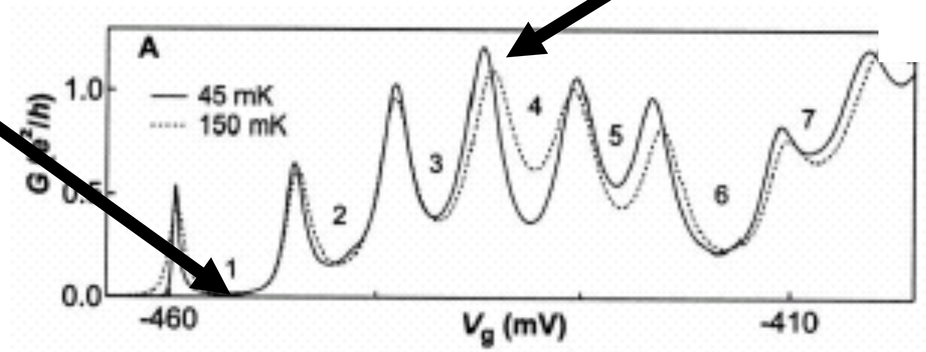


$$H_K = J\,\vec{s}\cdot\vec{S}$$

Charge Kondo



Coulomb valley
(odd N)



Coulomb peaks
N, N+1 degeneracy
"S"=1/2

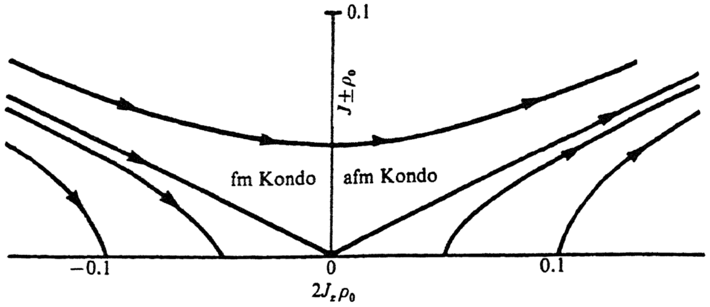
$$H = \sum_{k\alpha\sigma} \xi_k c^\dagger_{\alpha k\sigma} c_{\alpha k\sigma} + \sum_{\sigma} \epsilon_0 d^\dagger_\sigma d_\sigma + \sum_{\sigma\neq\sigma'} U d^\dagger_\sigma d_\sigma d^\dagger_{\sigma'} d_{\sigma'} + \sum_{k\alpha\sigma} t_\alpha d^\dagger_\sigma c_{\alpha k\sigma} + c.c.$$

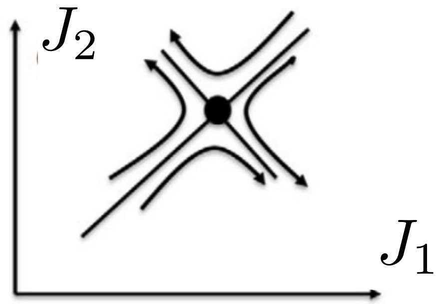
$$H_0 = \sum_{k\alpha} \epsilon_k c^\dagger_{k,\alpha} c_{k,\alpha} + \sum_{\alpha} \epsilon_0 d^\dagger_\alpha d_\alpha + \sum_{\alpha} \frac{v_{F\alpha}}{2\pi} \int_{-\infty}^{+\infty} dx \left\{ [\Pi_\alpha(x)]^2 + [\partial_x \phi_\alpha(x)]^2 \right\}$$

$$H_{\rm BS} = -\frac{D}{\pi} \sum_{\alpha} |r_\alpha| \cos[2\phi_\alpha(0)] \qquad H_C = E_C \left[\hat{n} + \frac{1}{\pi} \sum_{\alpha} \phi_\alpha(0) - N(V_g) \right]^2$$

$$H_{\rm tun} = \sum_{k,\alpha} \left[t_\alpha c^\dagger_{k,\alpha} d_\alpha + h.c. \right]$$

Fermi Liquid vs Non-Fermi Liaiud





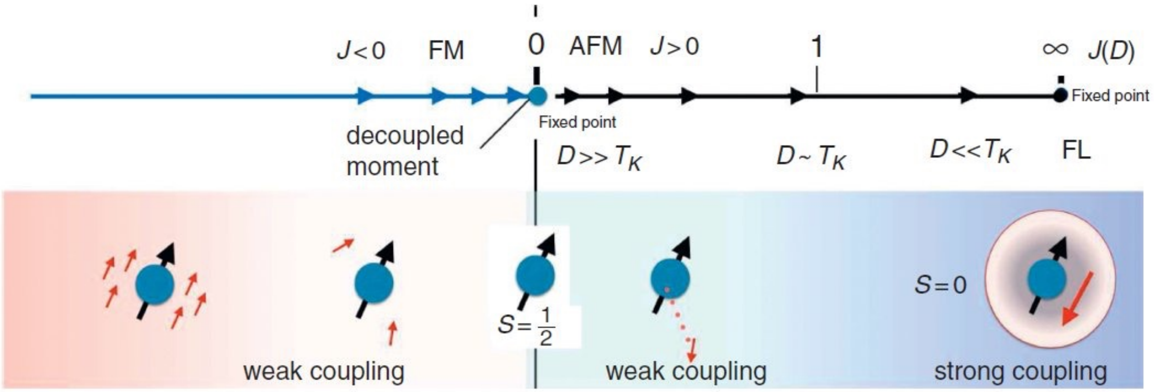
Strong coupling fixed point

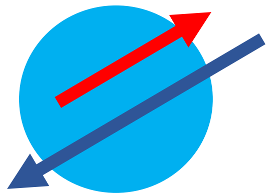
Intermediate coupling fixed point

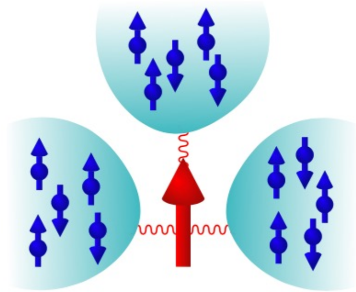
$N = 2S$ Fully screened- FL

$N > 2S$ Overscreened - NFL

Example: 2CK, 3CK







Effective strong coupling (or intermediate coupling) fixed point Hamiltonian

$$H_{\text{fixed point}} = \sum_{ks} \xi_k \varphi^\dagger_{ks}\varphi_{ks} - \sum_{kk's} \frac{\xi_k + \xi_{k'}}{2\pi\nu T_K} \varphi^\dagger_{ks}\varphi_{k's} + \frac{1}{\pi\nu^2 T_K}\rho_\uparrow\rho_\downarrow .$$

$U(1)$ Fermi Liquid

$$H_{\text{fixed point}} = \int_{-\infty}^{+\infty} dk \left\{ \xi_k c^\dagger_k c_k - \lambda(d^\dagger + d)(c_k - c^\dagger_k) \right\}$$

$U(1) \rightarrow Z_2$ Non- Fermi Liquid

Wilson Ratio as a measure of e-e correlations

$$R_W = \frac{4}{3}\left(\frac{\pi}{g\mu_B}\right)^2 \frac{\chi}{C_v/T}$$

Free fermions: $\chi = \frac{1}{4}(g\mu_B)^2\,\nu(\epsilon_F)$ $\quad C_v = \frac{\pi^2}{3}\nu(\epsilon_F)\cdot T$ $\quad R_W = 1$

Wilson Ratio for a quantum impurity problem N=2S

$$R_W = \frac{\delta\chi/\chi}{\delta C_v^{\text{spin}}/C_v^{\text{spin}}} = \boxed{\frac{\delta\chi/\chi}{\delta C_v^{\text{tot}}/C_v^{\text{tot}}}}\,\frac{\delta C_v^{\text{tot}}/C_v^{\text{tot}}}{\delta C_v^{\text{spin}}/C_v^{\text{spin}}} = \frac{\delta C_v^{\text{tot}}/C_v^{\text{tot}}}{\delta C_v^{\text{spin}}/C_v^{\text{spin}}} = \frac{2(2+N)}{3}$$

Wilson Ratio measures the ratio of the total specific heat to that coming from the spin degrees of freedom (Nozieres-Blandin 1980; Affleck-Ludwig 1993)

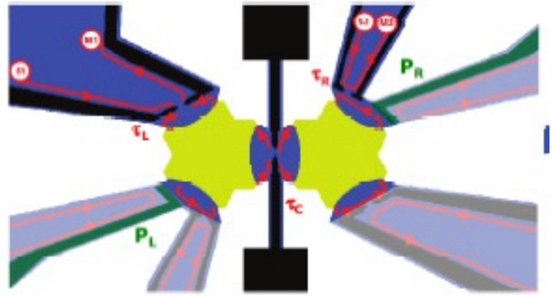
Single impurity Kondo (FL) N=1 $\quad R_W = 2$

What about NFL regime?

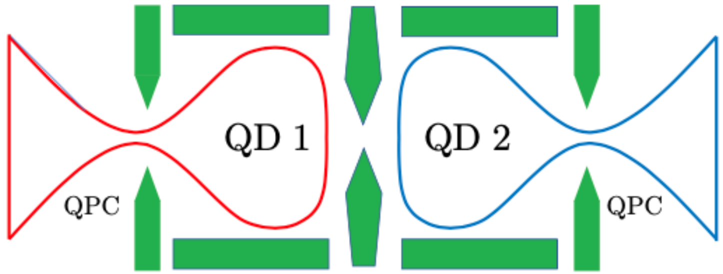
Charge and Heat transport through quantum simulators

$$I_e = -2\pi e|t|^2 \int_{-\infty}^{\infty} d\epsilon \nu_1(\epsilon) \cdot \nu_2(\epsilon) \left[f_1(\epsilon) - f_2(\epsilon) \right],$$

$$I_h = 2\pi|t|^2 \int_{-\infty}^{\infty} d\epsilon\epsilon \cdot \nu_1(\epsilon) \cdot \nu_2(\epsilon) \left[f_1(\epsilon) - f_2(\epsilon) \right]$$



$$\begin{pmatrix} I_e \\ I_h \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta V \\ \Delta T \end{pmatrix}$$



$$I_e = 2\pi e^2|t|^2 \frac{\Delta V}{4T} \int_{-\infty}^{\infty} d\epsilon \frac{\nu_1(\epsilon) \cdot \nu_2(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)} - 2\pi e|t|^2 \frac{\Delta T}{4T^2} \int_{-\infty}^{\infty} d\epsilon \frac{\epsilon \cdot \nu_1(\epsilon) \cdot \nu_2(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)}$$

$$I_h = -2\pi e|t|^2 \frac{\Delta V}{4T} \int_{-\infty}^{\infty} d\epsilon \frac{\epsilon\nu_1(\epsilon) \cdot \nu_2(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)} + 2\pi|t|^2 \frac{\Delta T}{4T^2} \int_{-\infty}^{\infty} d\epsilon \frac{\epsilon^2 \cdot \nu_1(\epsilon) \cdot \nu_2(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)}$$

Transport coefficients

$$G = \left.\frac{\partial I_e}{\partial \Delta V}\right|_{\Delta T=0} = \frac{\pi e^2 |t|^2}{2T}\int_{-\infty}^{\infty} d\epsilon \frac{\nu_1(\epsilon)\cdot\nu_2(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)}$$

Charge conductance

$$G_T = \left.\frac{\partial I_e}{\partial \Delta T}\right|_{\Delta V=0} = \frac{1}{T}\left.\frac{\partial I_h}{\partial \Delta V}\right|_{\Delta T=0} = -\frac{\pi e |t|^2}{2T^2}\int_{-\infty}^{\infty} d\epsilon \frac{\epsilon\cdot\nu_1(\epsilon)\cdot\nu_2(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)}$$

$$G_H = \left.\frac{\partial I_h}{\partial \Delta T}\right|_{\Delta V=0} = \frac{\pi |t|^2}{2T^2}\int_{-\infty}^{\infty} d\epsilon \frac{\epsilon^2\nu_1(\epsilon)\cdot\nu_2(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)}$$

$$S = -\left.\frac{\Delta V}{\Delta T}\right|_{I_e=0} = \frac{G_T}{G}$$

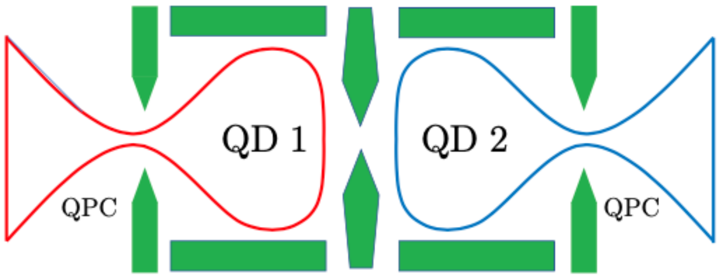
Seebeck coefficient - thermopower

$$K = \left.\frac{\partial I_h}{\partial \Delta T}\right|_{I_e=0} = G_H - \frac{G_T^2}{G}\cdot T = G\cdot T\left[\frac{G_H}{G\cdot T} - S^2\right]$$

Thermal conductance

Connection between transport integrals and transmission coefficient

$$\mathcal{L}_n(T) = \frac{1}{4T}\int_{-\infty}^{+\infty} d\epsilon \frac{\epsilon^n}{\cosh^2(\epsilon/2T)} \mathcal{T}(T,\epsilon), \qquad \text{n} = 0, 1, 2.$$

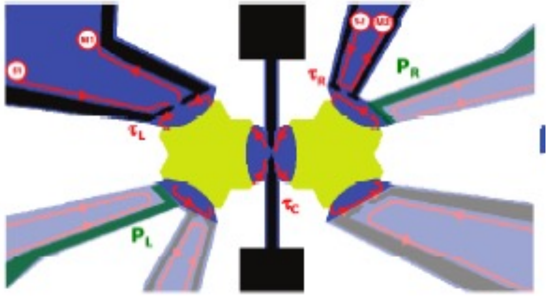


Transmission coefficient

$$\mathcal{T}(T,\epsilon) = 2\pi |t|^2 \nu_1(\epsilon, T)\nu_2(\epsilon, T)$$

$$K = \left.\frac{\partial I_h}{\partial \Delta T}\right|_{I_e=0} = L_{22} - \frac{L_{12}\cdot L_{21}}{L_{11}} = \frac{\det \mathbf{L}}{L_{11}}$$

$$L_{11} = e^2\mathcal{L}_0,\ L_{12} = -e\mathcal{L}_1/T,\ L_{22} = \mathcal{L}_2/T.$$



Wiedemann-Franz law

$$R_L \cdot L_0 = \frac{\mathcal{K}}{GT} \qquad L_0 = \left(\frac{k_B}{e}\right)^2 \frac{\pi^2}{3}$$

$$R_L(T, \mathcal{N}_1, \mathcal{N}_2) = \frac{3}{(\pi T)^2}\left[\frac{\mathcal{L}_2}{\mathcal{L}_0} - \left(\frac{\mathcal{L}_1}{\mathcal{L}_0}\right)^2\right]$$

Free fermions: $R_L = 1$

What does WF law say? $R_L = 1$?

How universal is WF law?

Shall we say “violation” or “generalization”?

What does Lorenz ratio measure?

Upper bound for the Lorenz ratio in FL regime

FL: Transmission coefficient is an analytic function of energy

$$\mathcal{T}(\epsilon) = \mathcal{T}_0 + \mathcal{T}_1 \frac{\epsilon}{\Gamma} + \mathcal{T}_2 \left(\frac{\epsilon}{\Gamma}\right)^2 + \ldots$$

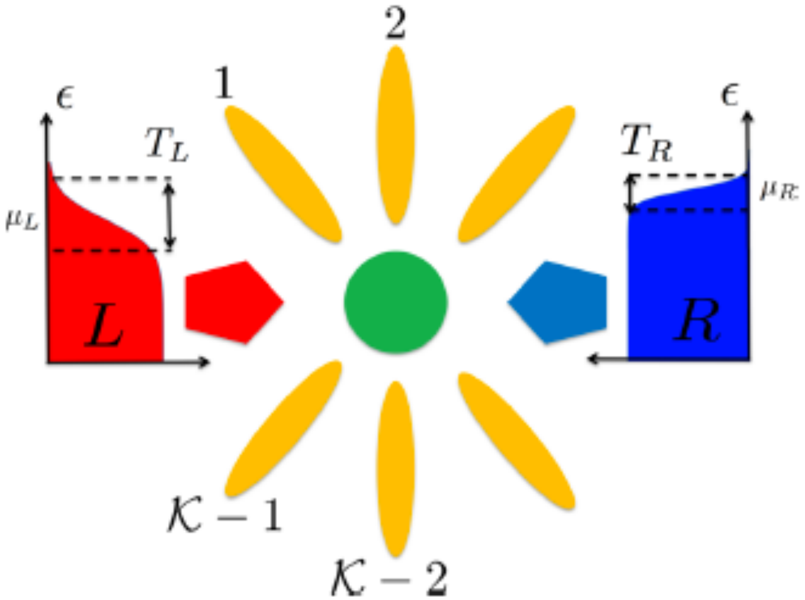
$$R_L = \frac{\mathcal{T}_0 + \mathcal{T}_2 \frac{7\pi^2}{5} \left(\frac{T}{\Gamma}\right)^2 + \ldots}{\mathcal{T}_0 + \mathcal{T}_2 \frac{\pi^2}{3} \left(\frac{T}{\Gamma}\right)^2 + \ldots} \rightarrow \begin{cases} 1, & \mathcal{T}_0 \neq 0 \\ \frac{21}{5}, & \mathcal{T}_0 = 0 \end{cases}$$

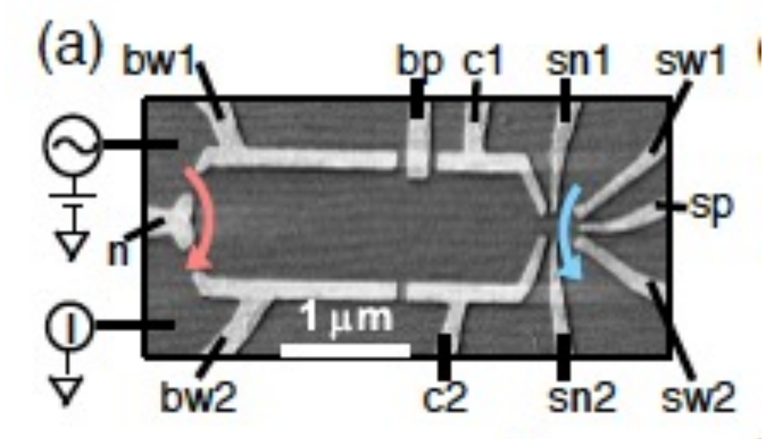
FL upper bound

What if transmission coefficient is not an analytic function of energy, for example, due to Anderson orthogonality catastrophe?

D. Karki PRB (2020)

NFL transport: Spin Kondo effect





Experiment: Stanford University

$$\mathscr{H}=\sum_k \varepsilon_k \left(\psi_k^{\alpha,i}\right)^\dagger \psi_{k\alpha,i} + J\mathcal{S}\sum_{kk'}\left(\psi_k^{\alpha,i}\right)^\dagger \frac{\sigma_\alpha^\beta}{2}\psi_{k'\beta,i},$$

$$\delta\mathscr{H}_P = \frac{v\,\delta_P}{\pi}\mathcal{J}_L(0),$$

$$\Delta \equiv \frac{2}{\mathcal{K}+2}.$$

$$\mathbb{S} = \cos\left[\frac{\pi(2\mathcal{S}+1)}{\mathcal{K}+2}\right]\Big/\cos\left[\frac{\pi}{\mathcal{K}+2}\right],$$

$$G(T) = G_0\left[1-\frac{\mathscr{M}\mathscr{C}_0\cos 2\delta_P}{1-\mathbb{S}\cos 2\delta_P}\left(\frac{\pi T}{T_K}\right)^\Delta\right],$$

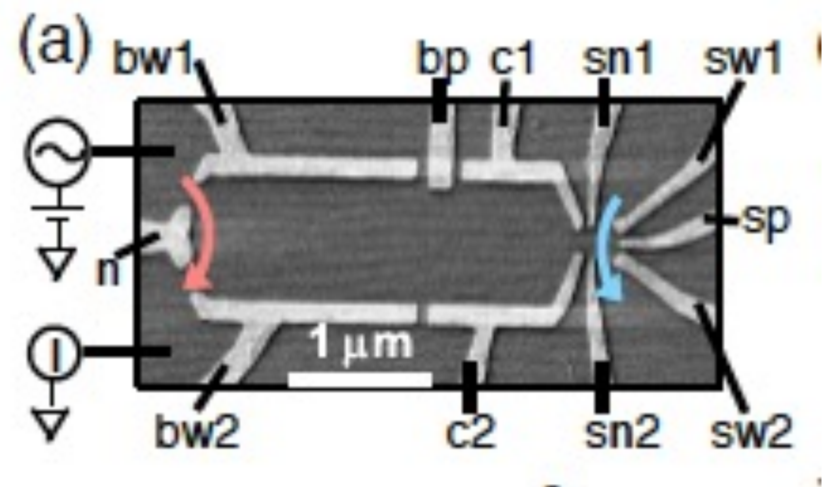
Large $\mathcal{K}$ limit

$$\mathbb{S} = 1-\frac{3\pi^2}{2}\frac{1}{\mathcal{K}^2}+\mathcal{O}\left(\frac{1}{\mathcal{K}^3}\right),$$

$$\mathcal{S}_{\text{th}}(T) = -\frac{\pi\mathscr{M}\,\mathscr{C}_1\,\sin 2\delta_P\left(\frac{\pi T}{T_K}\right)^\Delta}{1-\mathbb{S}\cos 2\delta_P-\mathscr{M}\mathscr{C}_0\cos 2\delta_P\left(\frac{\pi T}{T_K}\right)^\Delta}.$$

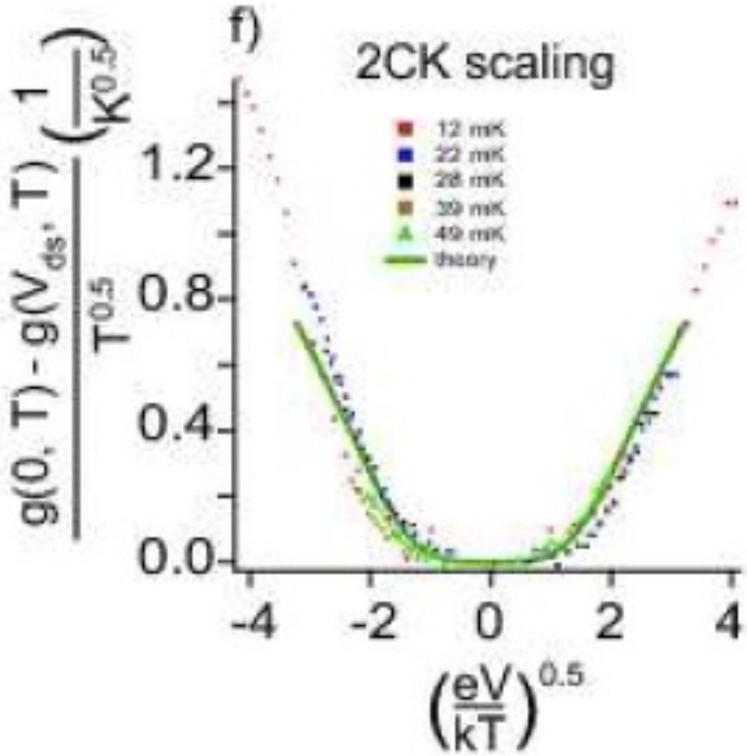
D.B. Karki and MK, PRB 102, 241402(R) (2020)
D.B. Karki and MK, PRB 103, L201402 (2021)

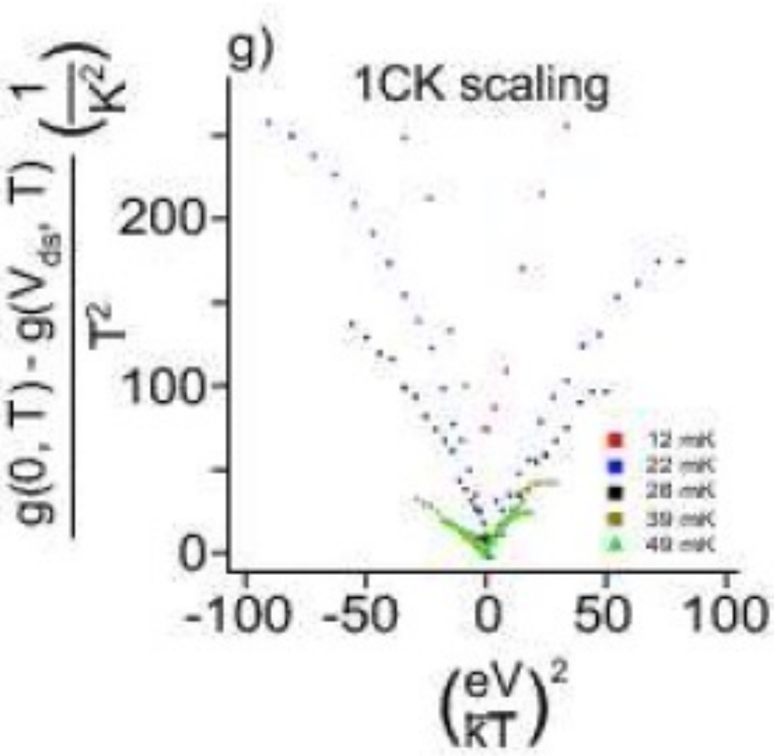
NFL transport: Spin Kondo effect



$$J \sum_j c^+_{j\sigma} \vec{\sigma}_{\sigma\sigma'} c_{j\sigma'} \vec{S}$$

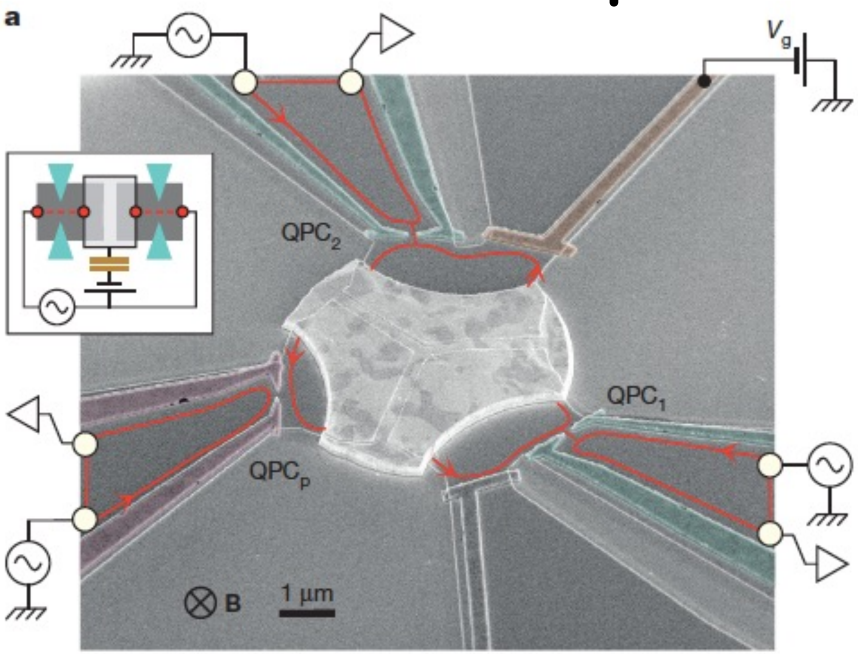
$$g(V_{\rm ds}) \sim {\rm const} - \sqrt{eV_{\rm ds}}$$



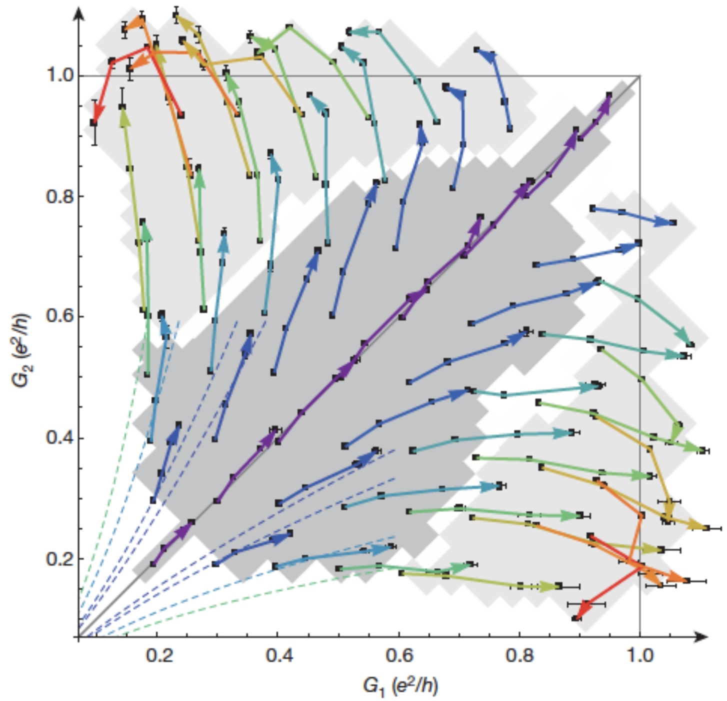


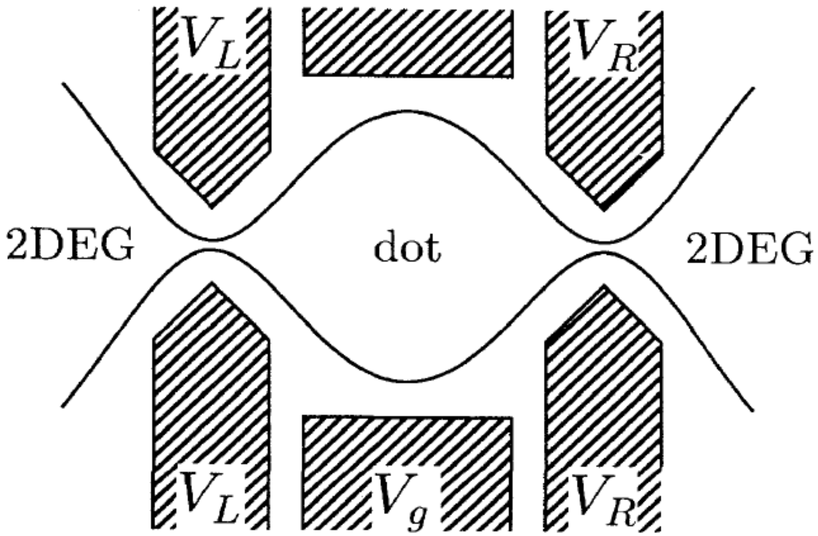
R.M. Potok et al, Nature, 2007

NFL transport: Charge Kondo effect

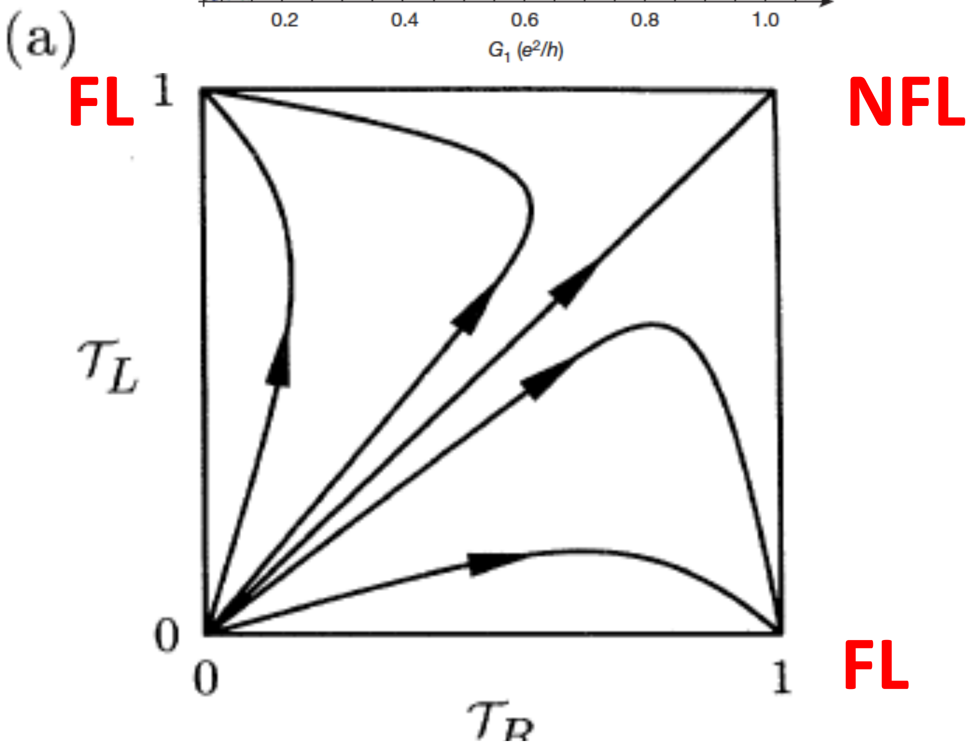


Z. Iftikhan et al, Nature 526 (2015)

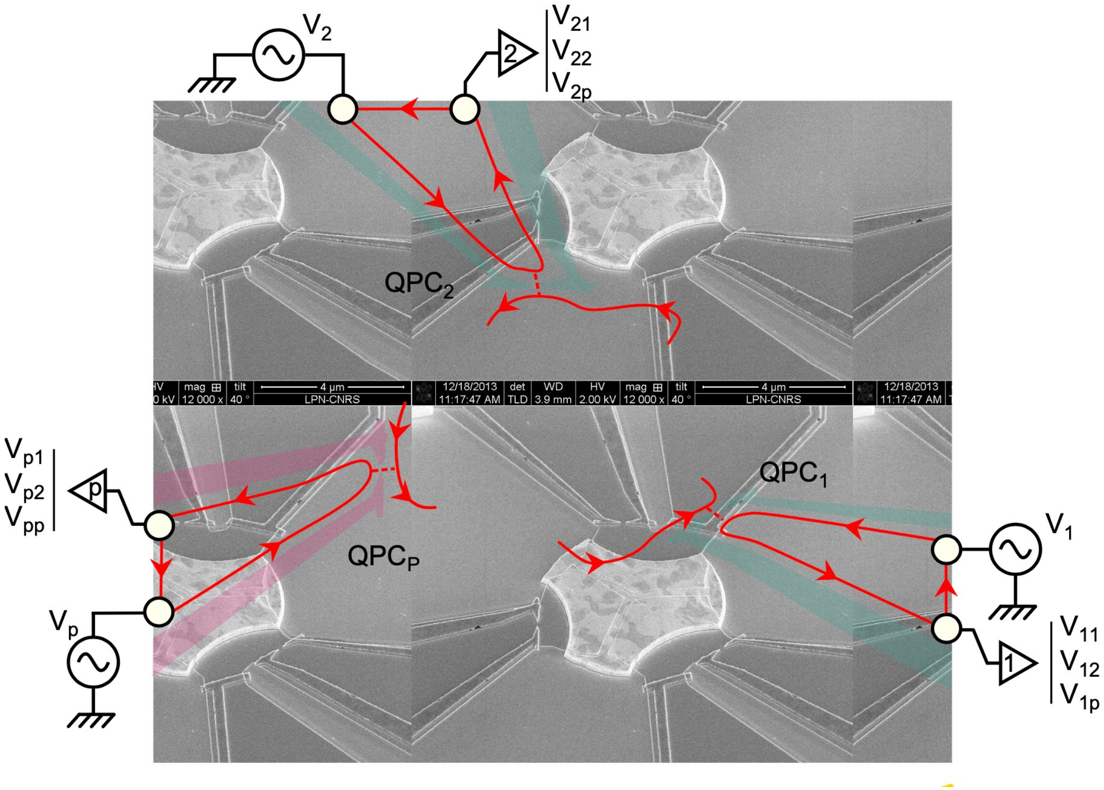


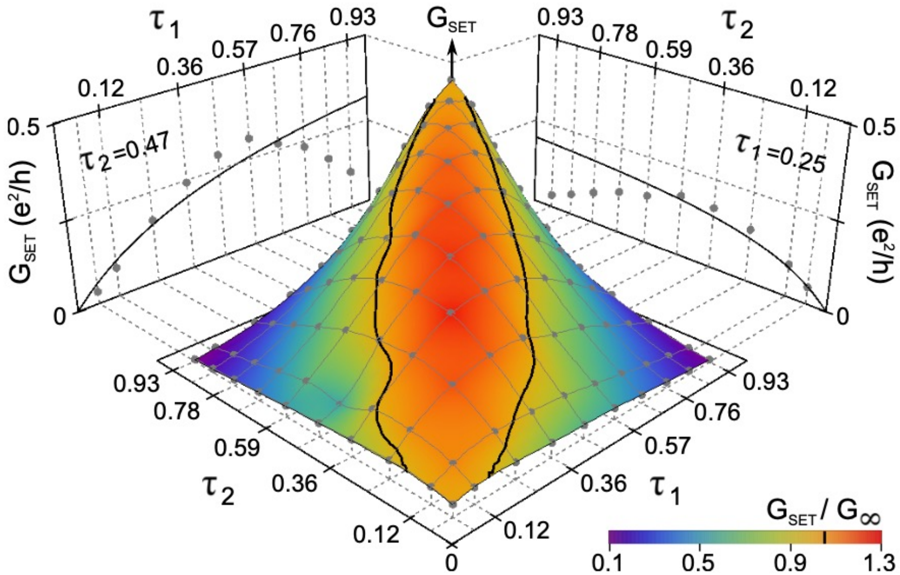


A.Furusaki and K.A.Matveev, PRB 52 (1995)



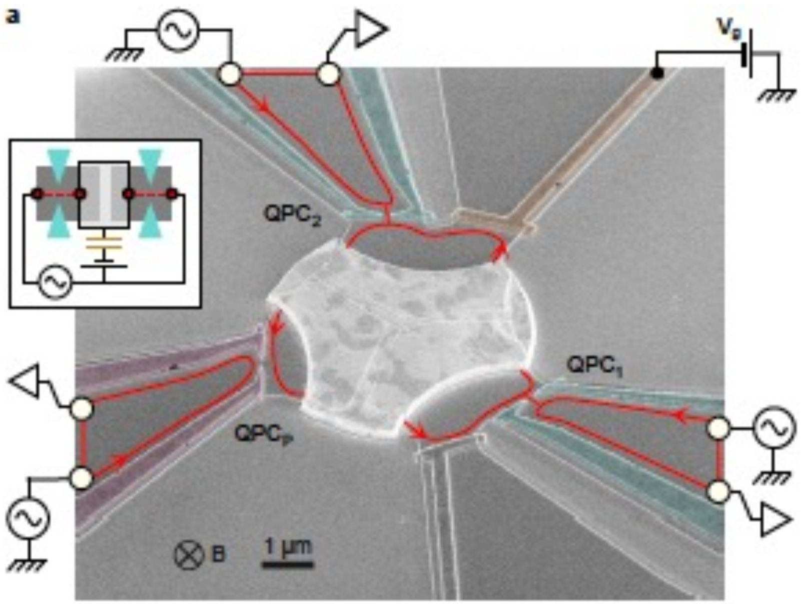
NFL transport: Charge Kondo effect

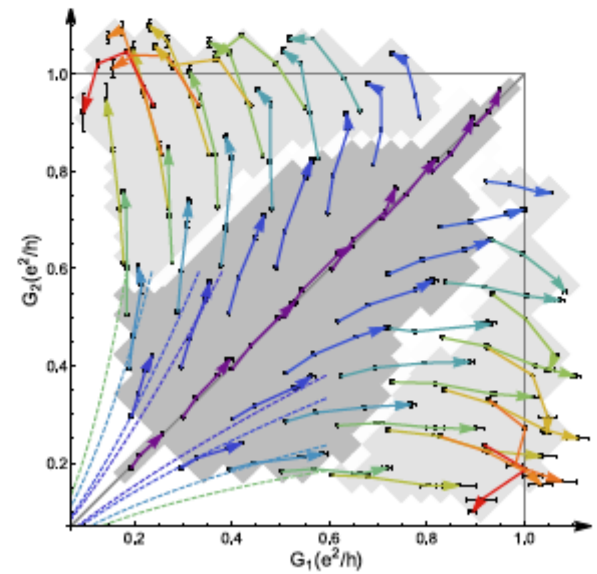




Z. Iftikhan et al, Nature 526 (2015)

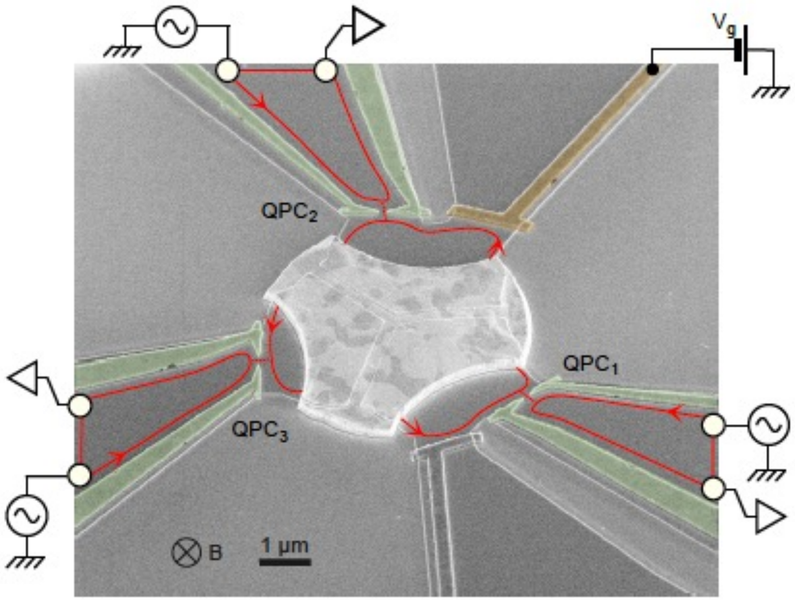
Parafermion states with multi-channel charge Kondo effect

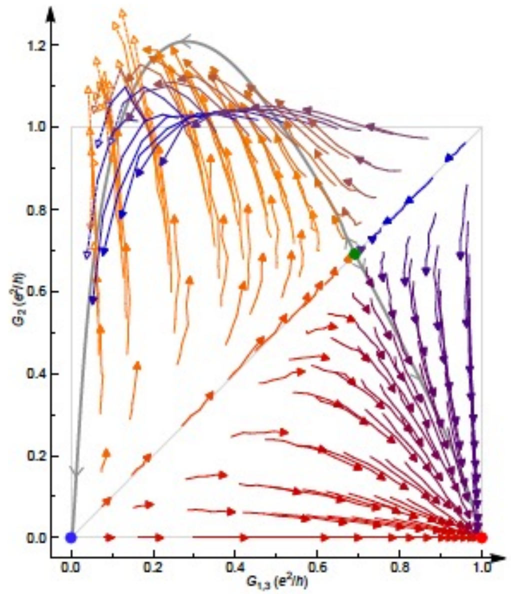




Z_2

Z. Iftikhan et al, Nature 526 (2015)

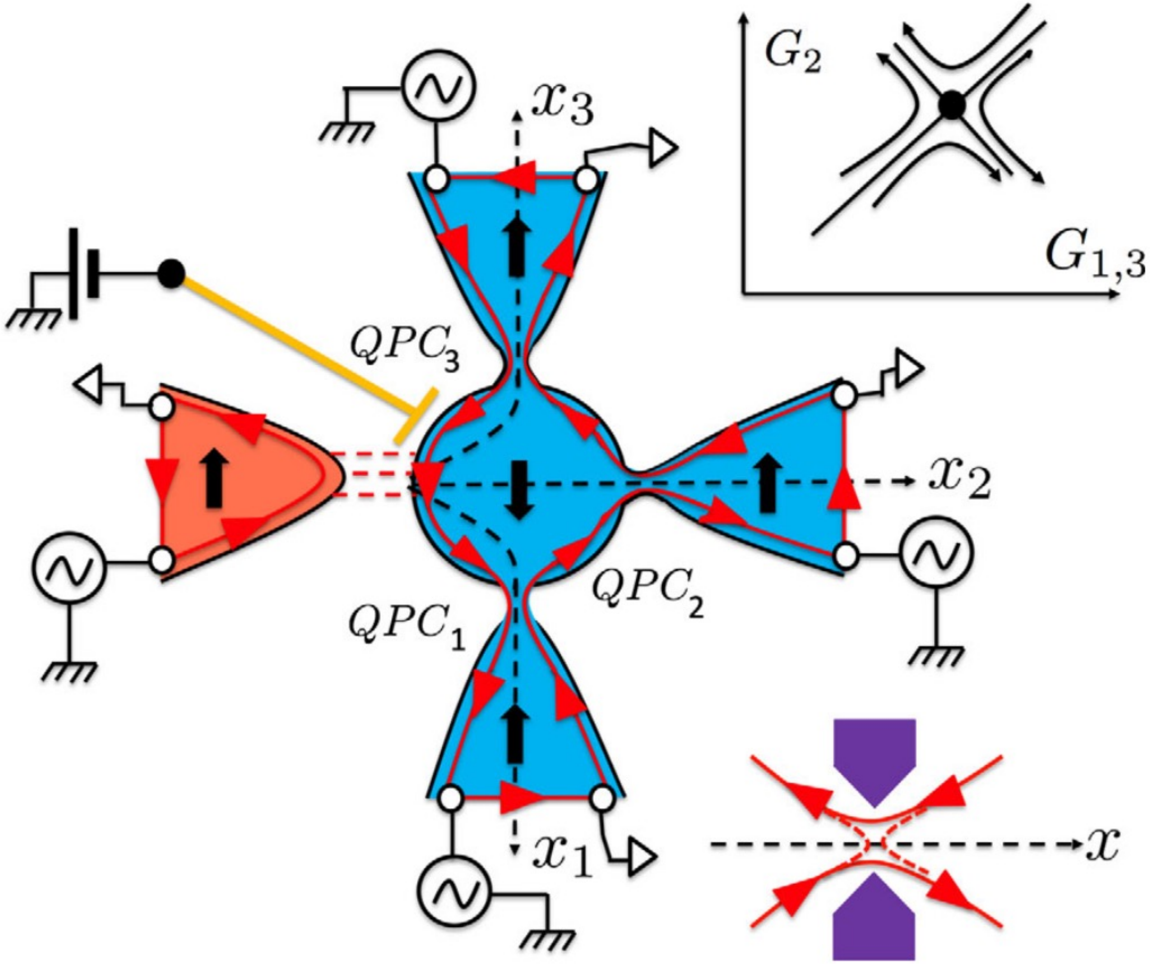


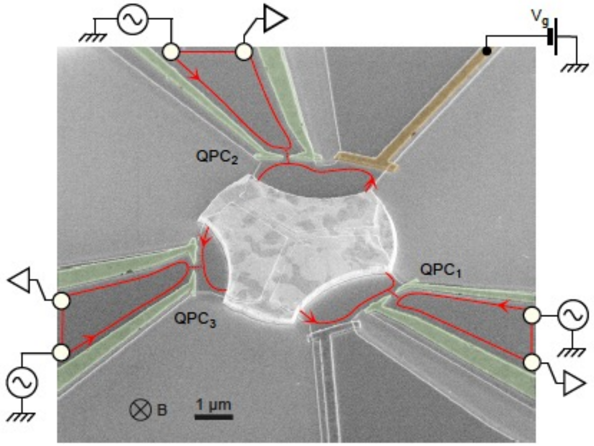


Z_3

Z. Iftikhan et al, Science 360 (2018)

Thermoelectrics of 3-channel charge Kondo effect





$$G \sim \left(\frac{T}{E_c}\right)^{2/3}$$

$$S \sim \frac{1}{e}|r|^2 \sin\left(2\pi N\right)\left[1+2a\cos\left(2\pi N\right)\right]\left[\frac{T}{E_C}\right]^{\frac{1}{3}} \ln\left[\frac{E_C}{T}\right]$$

T. K. T. Nguyen and MK. PRL 125, 026801 (2020)

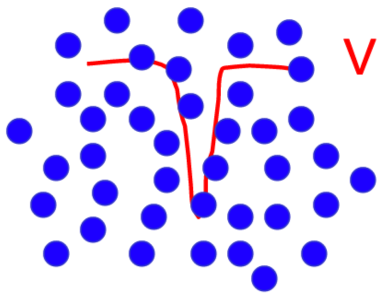
Anderson Orthogonality Catastrophe

作庭記

Make sure that all the stones, right down to the front of the arrangement, are placed with their best sides showing. If a stone has an ugly-looking top you should place it so as to give prominence to its side. Even if this means it has to lean at a considerable angle, no one will notice. There should always be more horizontal than vertical stones. If there are "running away" stones there must be "chasing" stones. If there are "leaning" stones, there must be "supporting" stones.

Sakuteiki ("Records of Garden Keeping")

Sudden perturbation

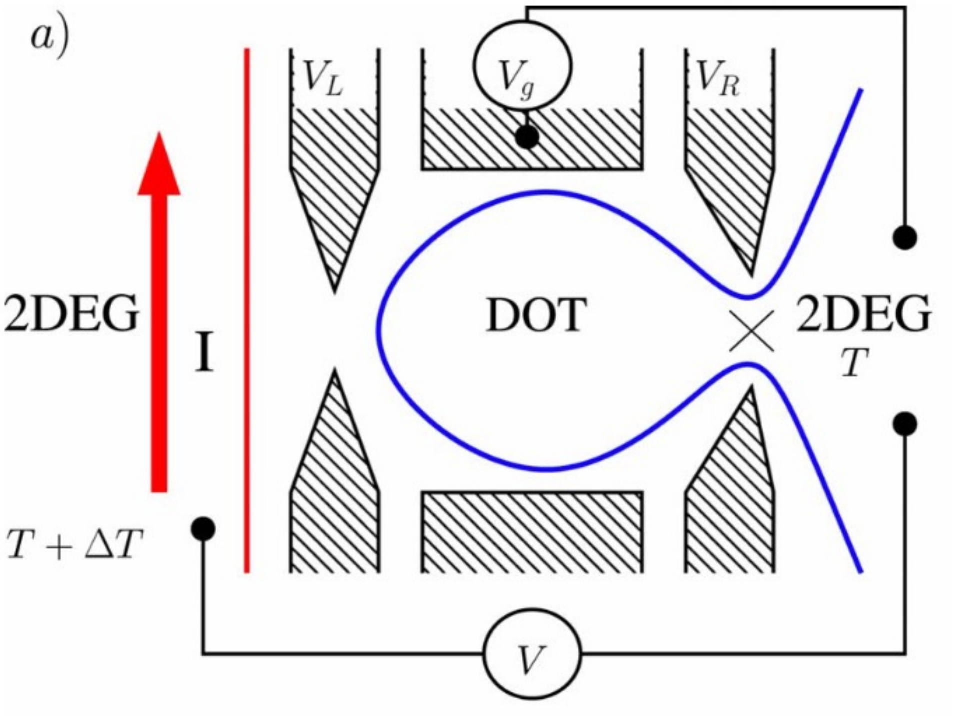


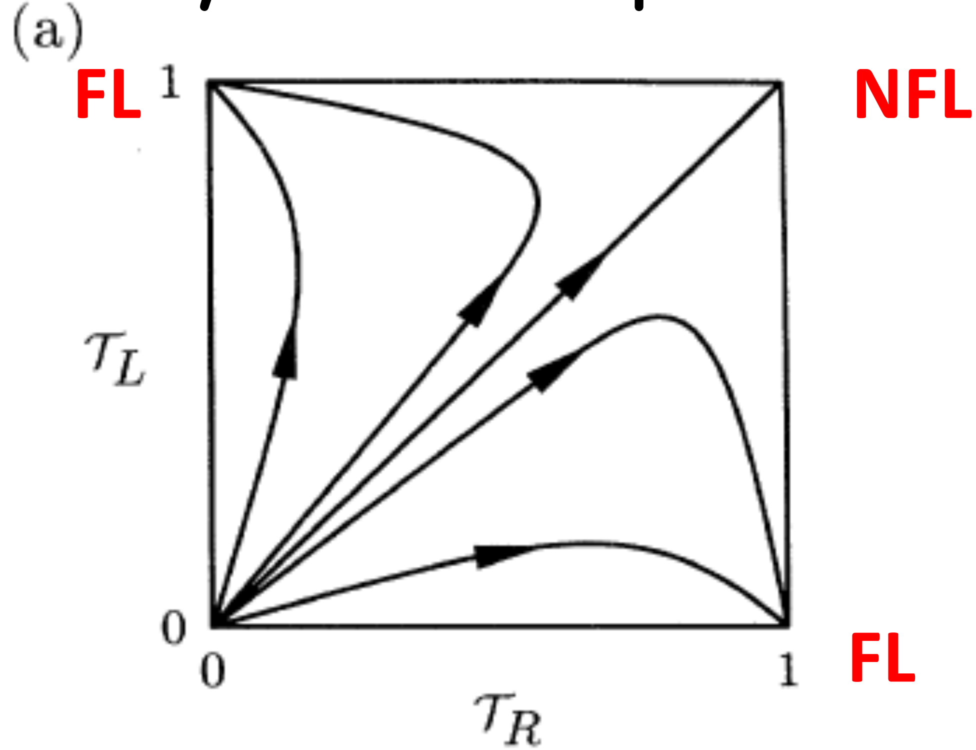
Anderson Orthogonality Catastrophe

It relates to the introduction of a magnetic impurity in a metal. When a magnetic impurity is introduced into a metal, the conduction electrons will tend to screen the potential $V(r)$ that the impurity creates. The N-electron ground state for the system when $V(r) = 0$, which corresponds to the absence of the impurity and $V(r) \neq 0$, which corresponds to the introduction of the impurity are orthogonal in the thermodynamic limit $N \to \infty$.

- P. W. Anderson (1967). "Infrared Catastrophe in Fermi Gases with Local Scattering Potentials".

Anderson Orthogonality Catastrophe



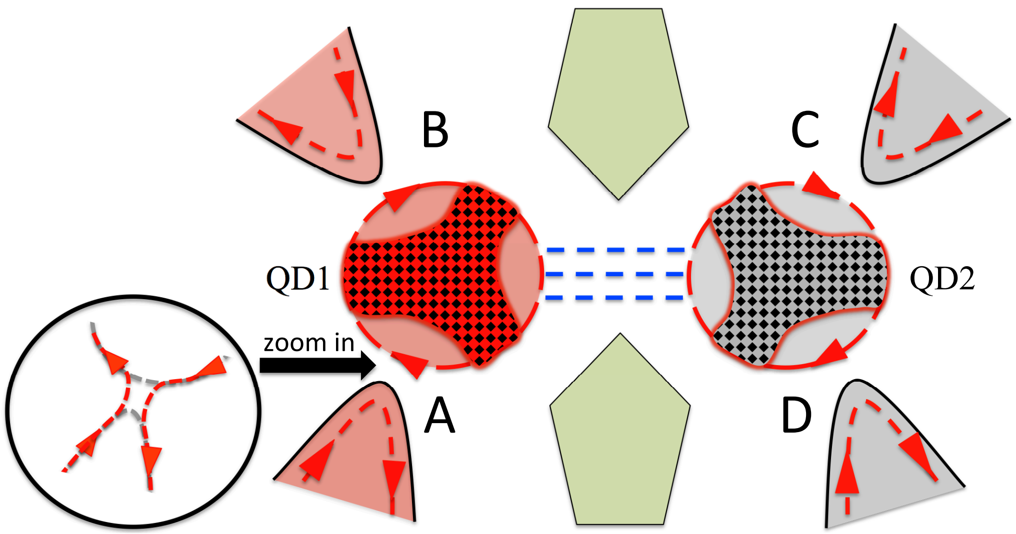


$$G \propto \frac{T}{E_C}$$

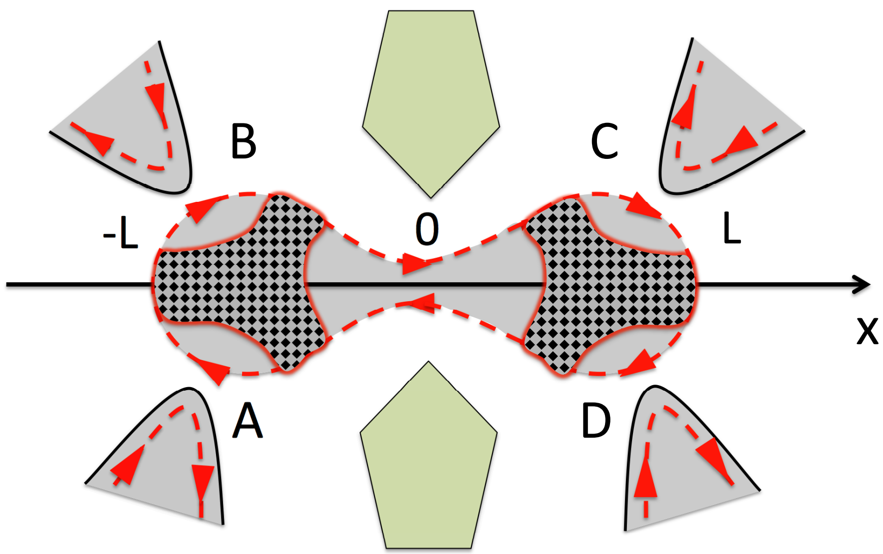
$$\nu(\epsilon) \propto \epsilon^{\chi} \qquad \chi = \sum_{n=1}^{4} \left(\frac{\delta_n}{\pi}\right)^2 \qquad |\delta_n| = \frac{\pi}{2}$$

A.Furusaki and K.A.Matveev, PRB 52 (1995)

Weak link between two Kondo circuits

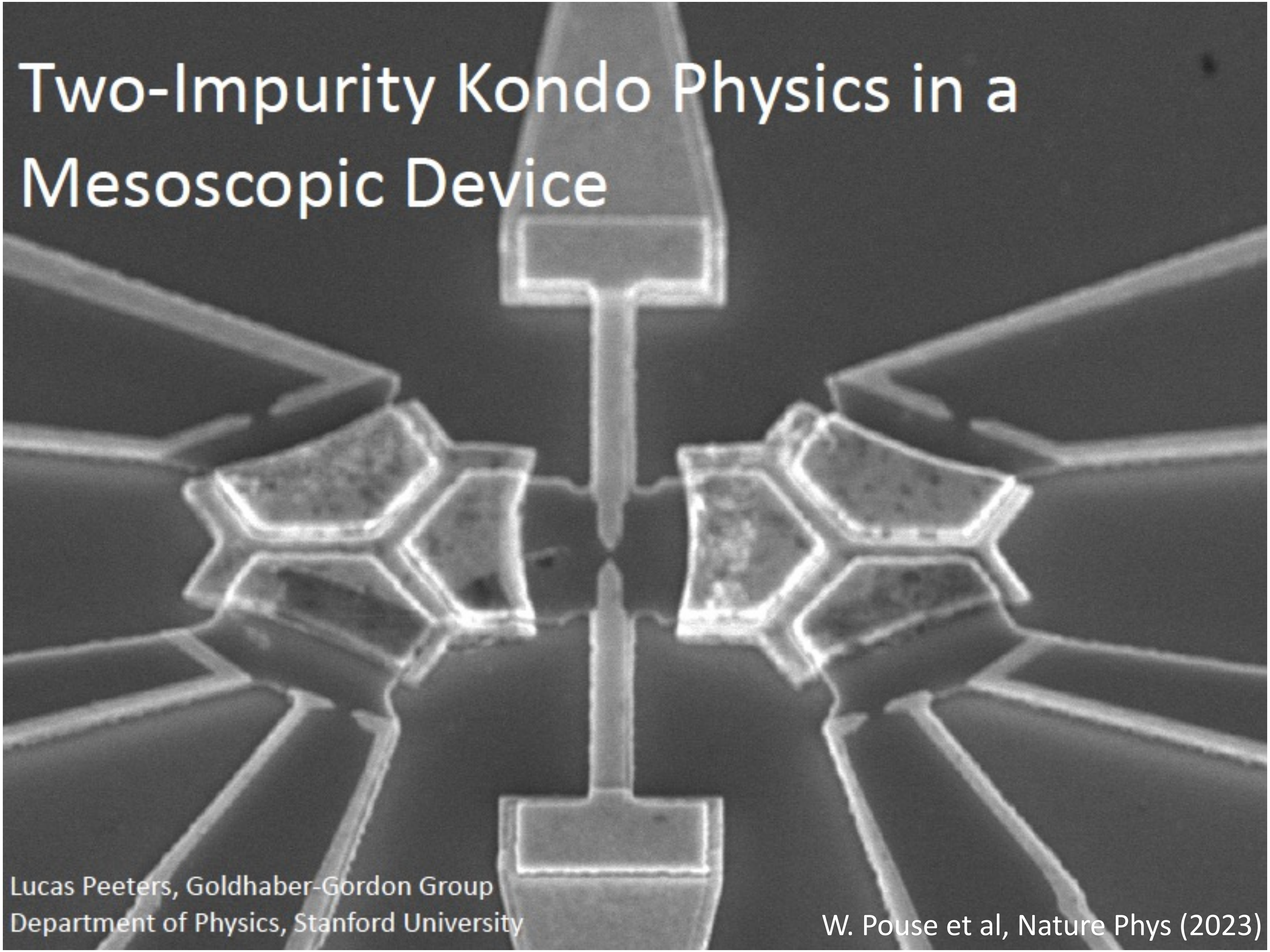


Two-Impurity non-Fermi liquid driven by Kondo



T. K. T. Nguyen and MK, PRB 97, 085403 (2018)

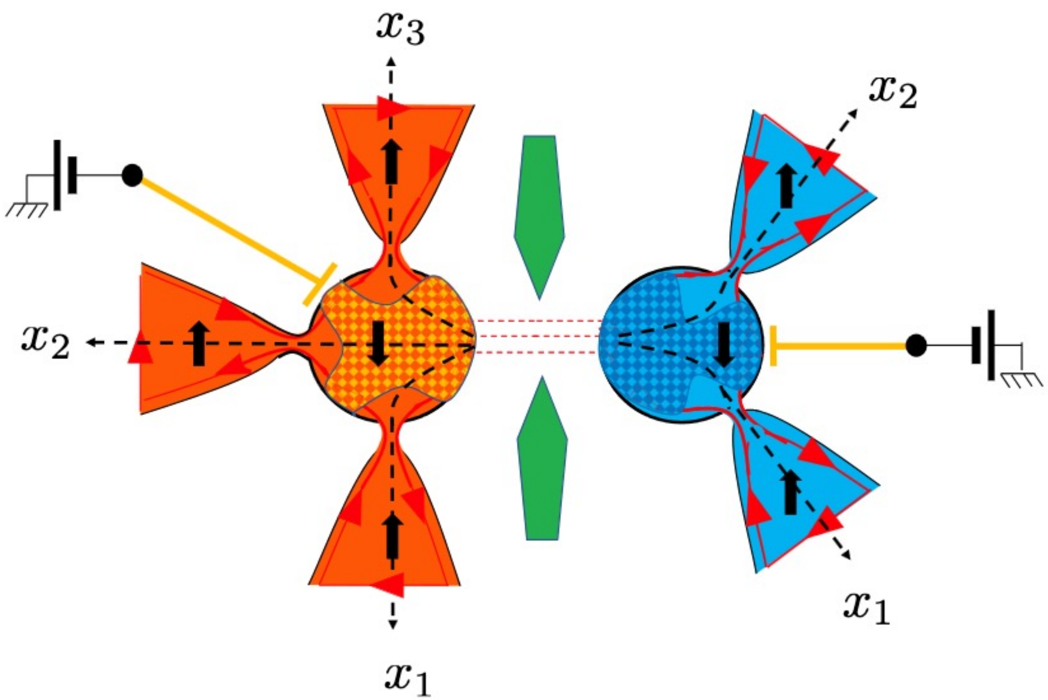
Two-Impurity Kondo Physics in a Mesoscopic Device

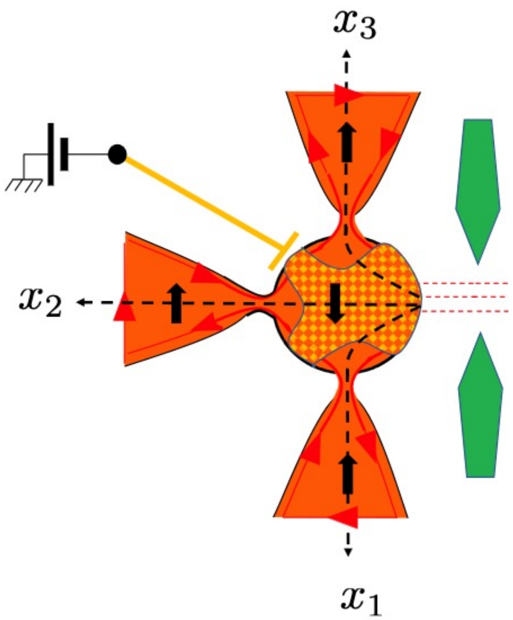


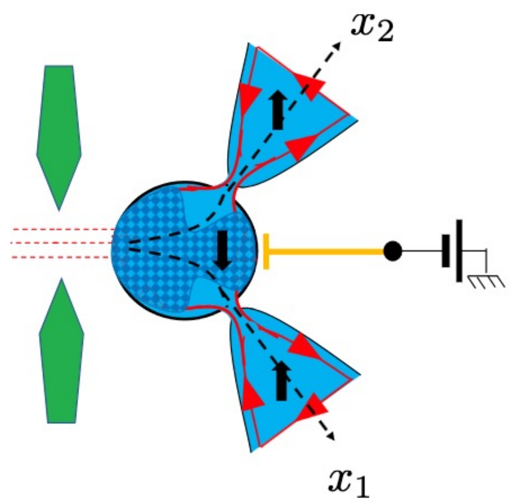
Lucas Peeters, Goldhaber-Gordon Group
Department of Physics, Stanford University

W. Pouse et al, Nature Phys (2023)

Lorenz Ratio as a measure of orthogonality catastrophe







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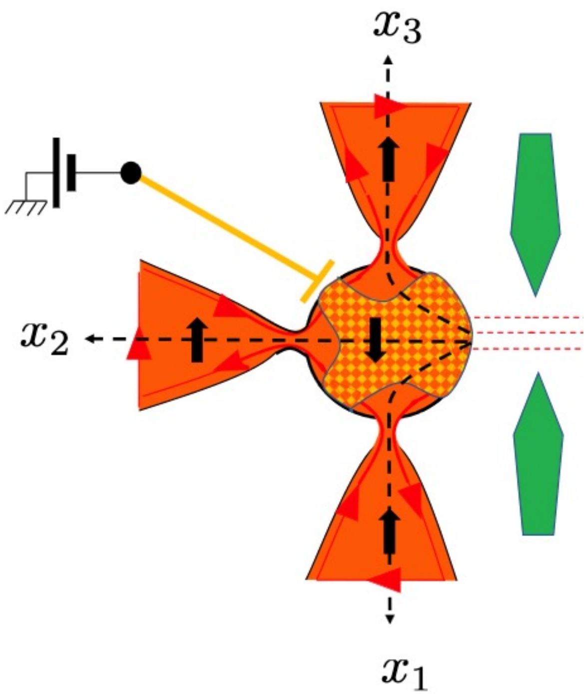
Anderson Orthogonality Catastrophe

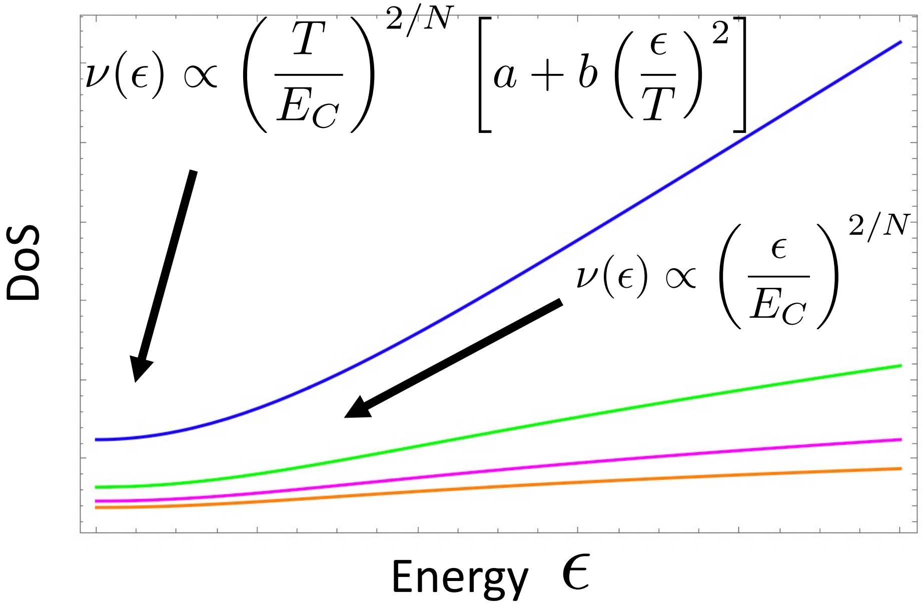


$$\nu_i(\epsilon, T) = -\frac{1}{\pi}\cosh\left(\frac{\epsilon}{2T}\right)\int_{-\infty}^{\infty}\mathcal{G}_i\left(\frac{1}{2T}+it\right)e^{i\epsilon t}dt.$$

$$\mathcal{G}_i(\tau) = -\frac{\nu_0^{(i)}\pi T}{\sin(\pi T\tau)}K_i(\tau),$$

$$K_i^{\pm}(z, T)|_{r=0} = \frac{A_i(T)}{\cosh^{2/m_i}[z]}, \quad A_i(T) = \left(\frac{\pi^2 T}{\gamma E_C^{(i)} m_i}\right)^{2/m_i}.$$

$$\mathcal{T}(T, \epsilon) = 2\pi|t_{12}|^2\nu_1(\epsilon, T)\nu_2(\epsilon, T).$$



$\propto \epsilon$ $\quad N = 2$

$\propto \epsilon^{2/3}$ $\quad N = 3$

$\propto \epsilon^{1/2}$ $\quad N = 4$

$\propto \epsilon^{2/5}$ $\quad N = 5$

$$\nu \propto \epsilon^{\chi}$$

$$\chi = \sum_{n=1}^{2N}\left(\frac{\delta}{\pi}\right)^2 = \frac{2}{N}$$

$$\delta = \frac{\pi}{N}$$

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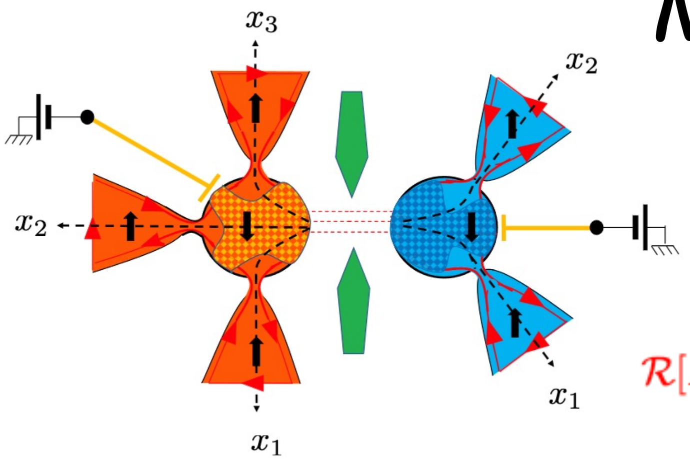
“Magic” Lorenz numbers @ $T \to 0$



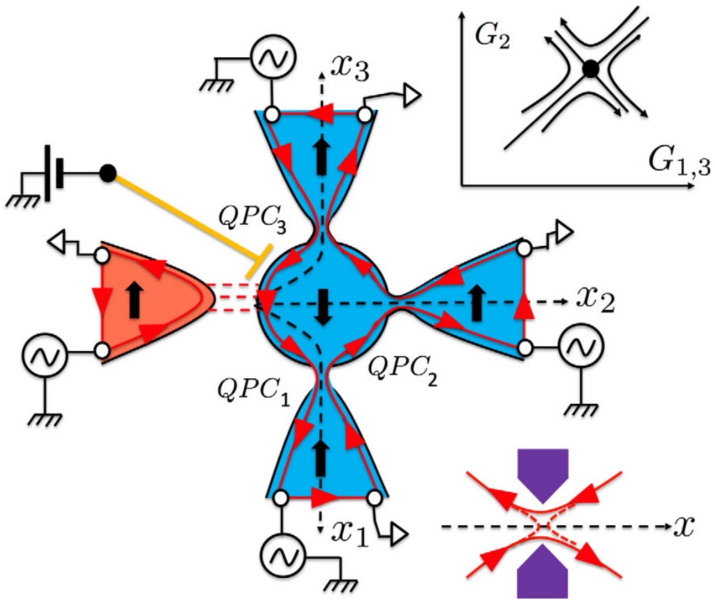
$$R_L(T, \mathcal{N}_1, \mathcal{N}_2) = \frac{3}{(\pi T)^2}\left[\frac{\mathcal{L}_2}{\mathcal{L}_0} - \left(\frac{\mathcal{L}_1}{\mathcal{L}_0}\right)^2\right].$$

non-universal part depending on |r| and T vanishes at Coulomb peaks

$$\mathcal{R}[N,M] = 3\int_{-\infty}^{\infty}\frac{(2-\cosh^2(x)+4\sinh^2(x)/(NM))}{\cosh^{4+2/N+2/M}(x)}dx \Big/ \int_{-\infty}^{\infty}\frac{1}{\cosh^{2+2/N+2/M}(x)}dx$$

$$\mathcal{R}_{N,M} = 1+4\left(\frac{1}{M}+\frac{1}{N}+\frac{3}{MN}\right)\frac{\left({}_2F_1\left(1,-2-\frac{1}{N}-\frac{1}{M},2+\frac{1}{N}+\frac{1}{M},-1\right)-1\right)\Gamma\left(\frac{3}{2}+\frac{1}{N}+\frac{1}{M}\right)}{\sqrt{\pi}\,\Gamma\left(3+\frac{1}{N}+\frac{1}{M}\right)}$$

Large {N,M} expansion



$$\mathcal{R}_{N,M}\big|_{N\gg1,M\gg1} = 1+\frac{4}{3}\left(\frac{1}{N}+\frac{1}{M}\right)-\frac{8}{9}\left(\frac{1}{N}-\frac{1}{M}\right)^2+\frac{4}{9}\frac{1}{NM}+\ldots$$

FL-NFL tunnel contact

$$\mathcal{R}_{N,\infty} = 1+\frac{4}{N}\frac{\left({}_2F_1\left(1,-2-\frac{1}{N},2+\frac{1}{N},-1\right)-1\right)\Gamma\left(\frac{3}{2}+\frac{1}{N}\right)}{\sqrt{\pi}\,\Gamma\left(3+\frac{1}{N}\right)}$$

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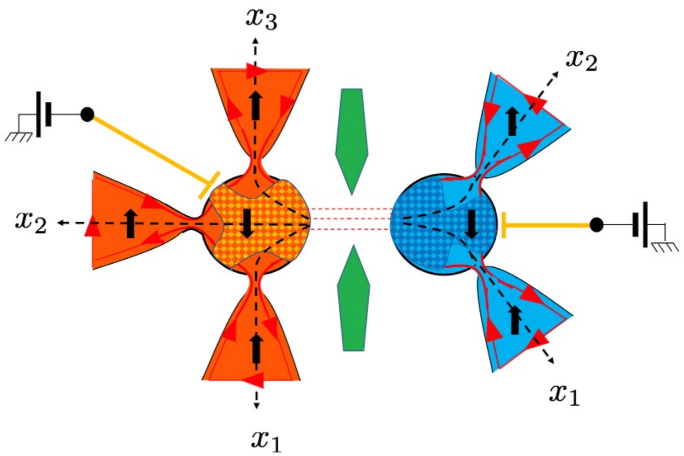
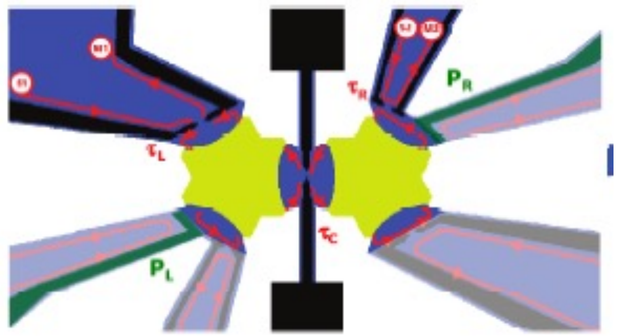
"Magic" Lorenz numbers





N	M	$\mathcal{R}$	$\mathcal{L}_0$	$\mathcal{L}_2$	$\mathcal{S}_{\max}$
1	1	27/7	T^4	T^6	T
1	2	3	T^3	T^5	$\sqrt{T}\ln T$
1	3	45/17	$T^{8/3}$	$T^{14/3}$	$\sqrt[3]{T}\ln T$ ♦
1	∞	9/5★	T^2	T^4	T
2	2	12/5	T^2	T^4	T
2	3	15/7	$T^{5/3}$	$T^{11/3}$	$\sqrt[3]{T}\ln T$ ♦
2	∞	3/2★	T	T^3	$\sqrt{T}\ln T$
3	3	25/13	$T^{4/3}$	$T^{10/3}$	$\sqrt[3]{T}\ln T$ ♦
3	∞	15/11	$T^{2/3}$	$T^{8/3}$	$\sqrt[3]{T}\ln T$ ♦
∞	∞	1	T^0	T^2	0

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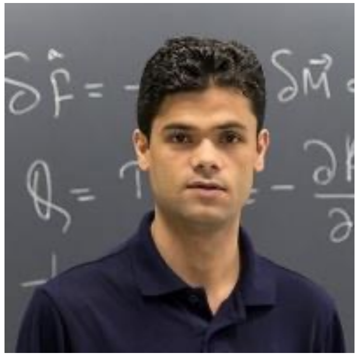
Take home message

What does Lorenz ratio measure?

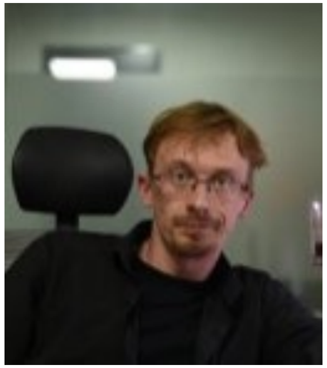
- property of a transmission coefficient
- transmission coefficient of the multi-channel charge Kondo simulators is a non-analytic function of energy in the NFL regime @ $T \to 0$ close to the Coulomb peaks
- LR provides unique benchmarking of the NFL universality class for a tunnel contact low temperature quantum simulator's measurements

Collaborators

Thanh Nguyen



Deepak Karki



Anton Parafilo

Thermoelectrics of SU(N) quantum impuruty

D. B. Karki and MK, PRB **96**, 121403(R) (2017)

D. B. Karki and MK, PRB **100**, 125426 (2019)

D. B. Karki and MK. PRB 100, 195425 (2019)

Charge Kondo effect in a double-site quantum circuit

T. K. T. Nguyen and MK, PRB 97, 085403 (2018)

T. K. T. Nguyen and MK. Comm. in Phys., 32, 331 (2022)

T. K. T. Nguyen, H. Q. Nguyen,and MK, ArXiv: 2306.12229 (2023)

CFT for thermoelectric transport through a quantum impurity

D.B. Karki and MK, PRB 102, 241402(R) (2020)

D.B. Karki and MK, PRB 103, L201402 (2021)

Z_3 parafermions in Kondo simulators

T. K. T. Nguyen and MK. PRL 125, 026801 (2020)

Charge Kondo effect as a detector for e-e interactions in Luttinger Liquid

T. K. T. Nguyen and MK. Comm. in Phys., 30, 1 (2020)

A.V. Parafilo, T.K.T. Nguyen and MK, PRB 105, L121405 (2022)

T. K. T. Nguyen, A. V. Parafilo, H. Q. Nguyen,and MK, PRB 107, L201402 (2023)