Angular and Linear Momentum of Excited Ferromagnets

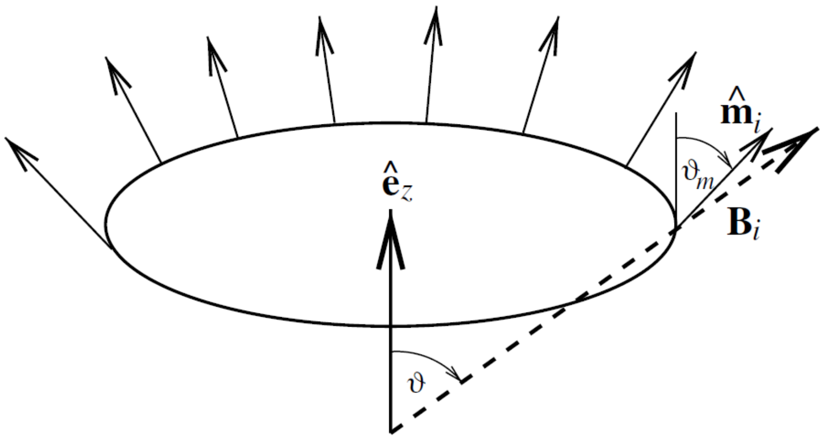
Peng Yan

Kavli Institute of NanoScience, Delft University of Technology, The Netherlands

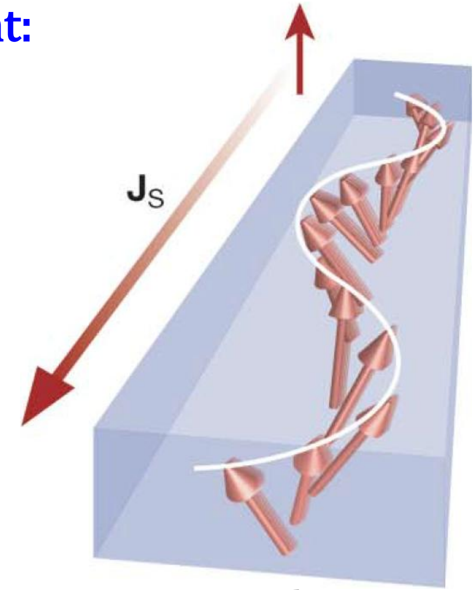
Collaborators: Akashdeep Kamra (Delft)
Yunshan Cao (Delft)
Gerrit E.W. Bauer (Sendai & Delft)

arXiv:1307.3432; Phys. Rev. B (2013) in press.

Spin-wave spin current:

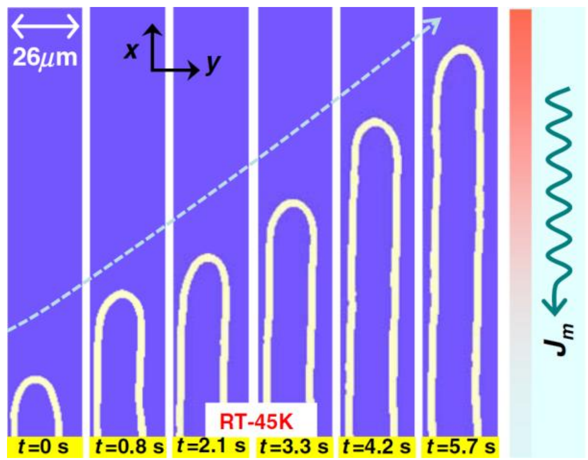


F. Schutz *et al.*, PRL **91**, 017205 (2003).



Y. Kajiwara *et al.*, Nature **464**, 262 (2010).

Magnonic spin transfer torque:



Experiment:W. Jiang *et al.*, PRL **110**, 177202 (2013);

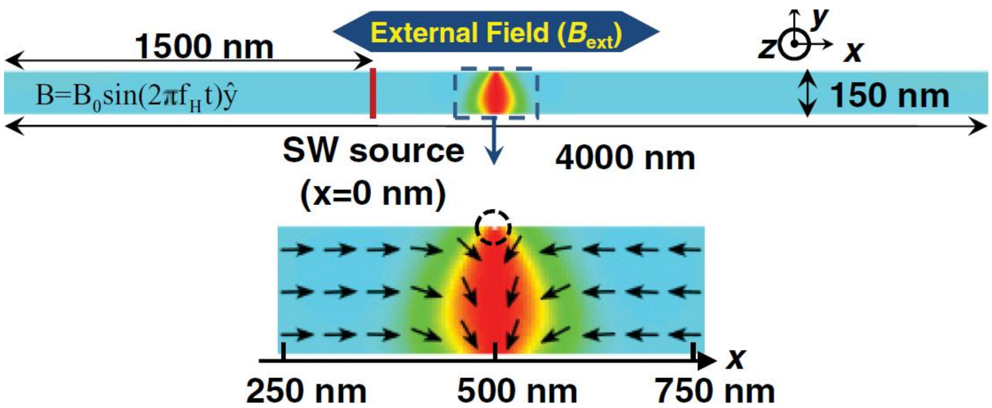
Theory:D. Hinzke and U. Nowak, PRL **107**, 027205 (2011).

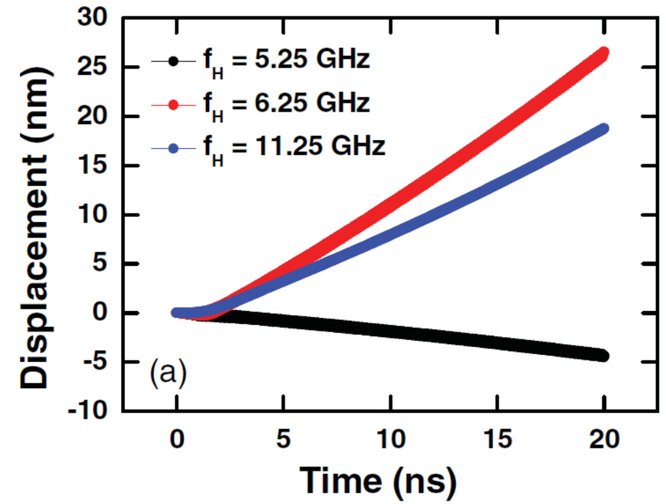
P. Yan *et al.*, PRL **107**, 177207 (2011);

A. Kovalev and Y. Tserkovnyak, EPL **97**, 67002 (2012); cf. Yaroslav's tutorial.

Issues:

1. Positive Wall Velocity





J.-S. Kim *et al.*, PRB **85**, 174428 (2012).

Linear momentum transfer: Gen Tatara and Hiroshi Kohno, PRL **92**, 086601 (2004).

2. Paradox about Linear Momentum in Ferromagnets:

G.E. Volovik, J Phys. C **20**, L83 (1987);
F.D.M. Haldane, PRL **57**, 1488 (1986);
C.H. Wong and Y. Tserkovnyak, PRB **80**, 184411 (2009);
……

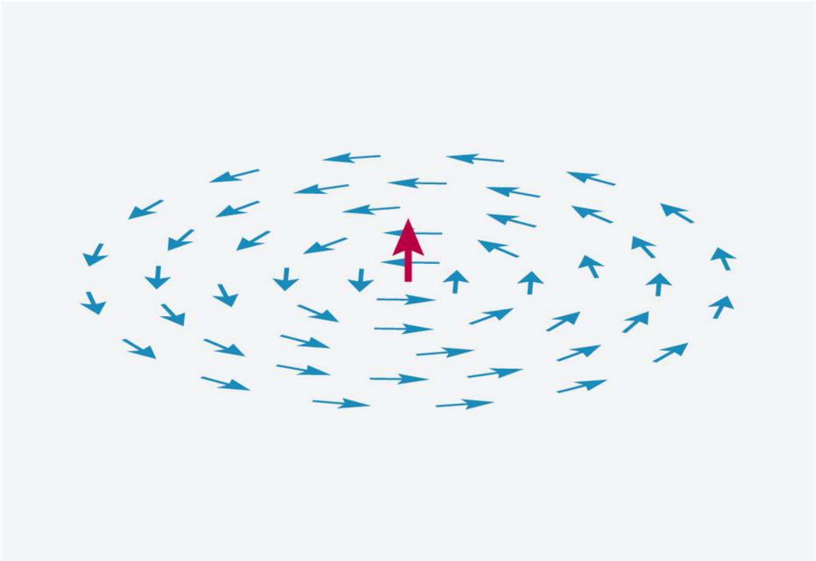
Outline

- Introduction: Symmetry and conservation laws
- Classical magnetization dynamics
- Linear momentum and its transfer in magnetic textures
- Angular momentum of insulating ferromagnets: spin and orbit
- Conclusions & Outlook

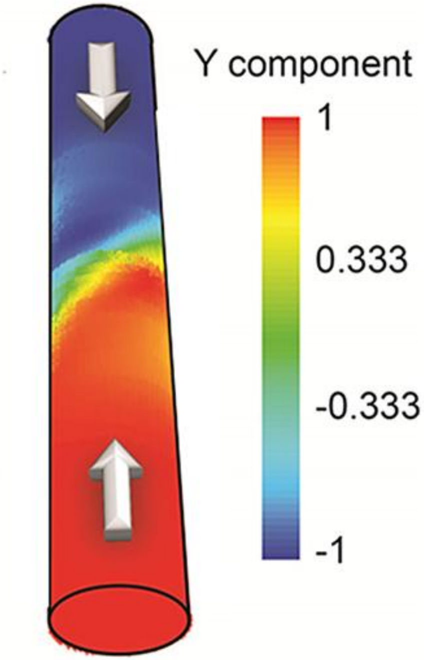
Conservation laws

- Angular momentum conservation← *Rotational symmetry*
- Linear momentum conservation← *Translational symmetry*

Emmy Noether (1882-1935)

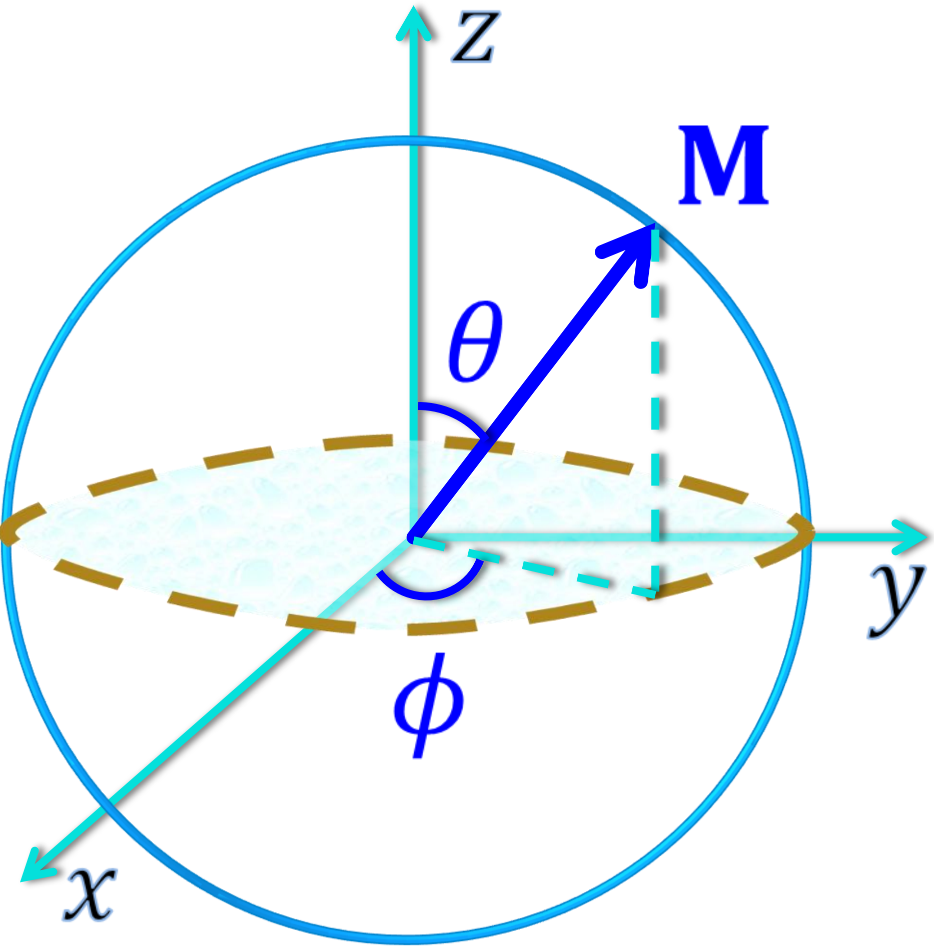


From SwissFEL



Biziere *et al.*, Nano Lett. **13**, 2053 (2013).

Classical description



$$\frac{\partial \mathbf{M}}{\partial t} = -\mathbf{M} \times (-\delta \mathcal{H} / \delta \mathbf{M}),$$

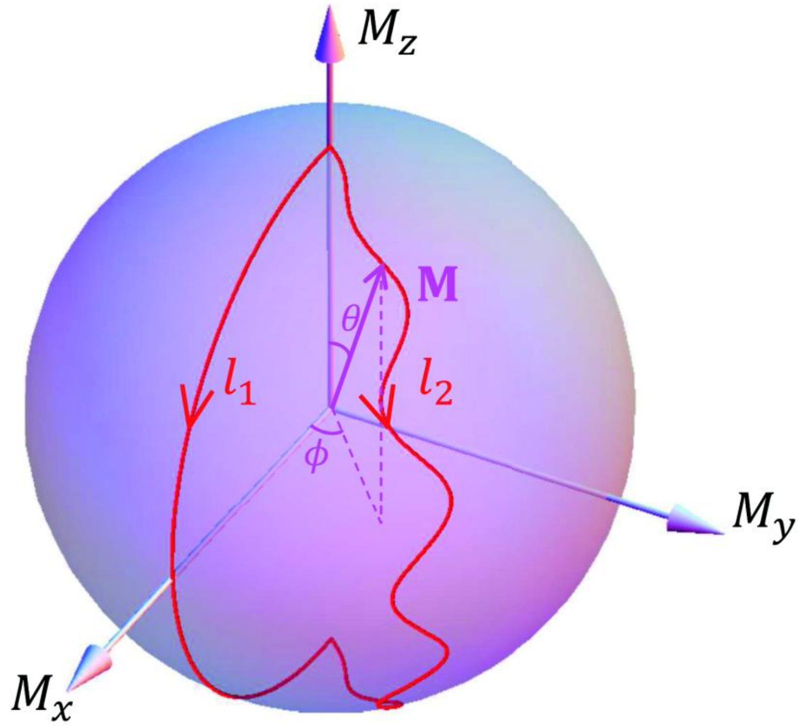
$\mathcal{H}$: Hamiltonian

$$\int \left\{ \frac{J}{2} (\nabla \mathbf{M})^2 + f(M_z) - \mathbf{M} \cdot \mathbf{h} - \frac{\mathbf{h}^2}{8\pi} \right\} d^3\mathbf{r},$$

Dipolar field:

$$\nabla \times \mathbf{h} = 0, \nabla \cdot (\mathbf{h} + 4\pi \mathbf{M}) = 0.$$

The Lagrangian



- Canonical variables:

$$M_z, \phi, \psi$$

with $\phi = \arctan(M_y/M_x)$, $\nabla\psi = \mathbf{h}$.

- Lagrangian density:

$$\mathbb{L} = M_z\dot{\phi} + \mathbb{H},$$

with

$$\mathbb{H} = \frac{J}{2}\left\{\frac{M_0^2}{M_0^2 - M_z^2}(\nabla M_z)^2 + (M_0^2 - M_z^2)(\nabla\phi)^2\right\} + f(M_z) - \left\{\sqrt{M_0^2 - M_z^2}\left(h_x\cos\phi + h_y\sin\phi\right) + M_z h_z\right\} - \frac{\mathbf{h}^2}{8\pi},$$

- Euler-Lagrangian equations:

$$\frac{\partial}{\partial x_i}\frac{\partial\mathbb{L}}{\partial(\partial q/\partial x_i)} - \frac{\partial\mathbb{L}}{\partial q} = 0,$$

with $q = M_z, \phi, \psi$; $i = 1,2,3,4$; $x_{1,2,3} = x, y, z$; $x_4 = t$.

Landau-Lifshitz equation

Maxwell's equation (magnetostatic approximation)

Linear momentum conservation

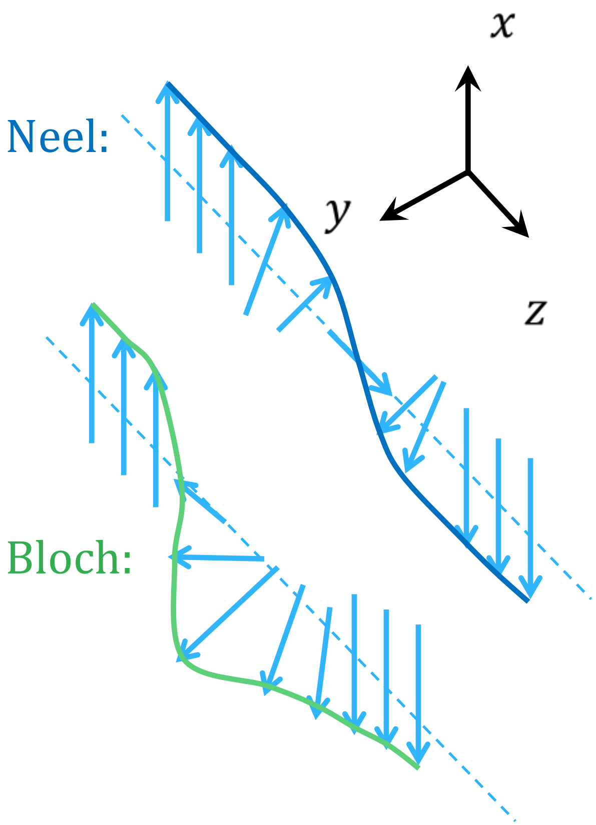
- Conservation law

$$\frac{\partial p_z}{\partial t} + \nabla \cdot \mathbb{T} = 0,$$

where

$$p_z = M_z \frac{\partial \phi}{\partial z}.$$

$\mathbb{T}$: Momentum current density



- Total linear momentum

$$P_z = \int p_z d^3\mathbf{r}$$

is conserved.

Complication

- Singularities at two poles

$$\mathbb{L} = (C + M_z)\dot{\phi} + \mathbb{H}, \quad \xrightarrow{C = -M_0} \quad p_z = (M_z - M_0)\frac{\partial \phi}{\partial z}.$$

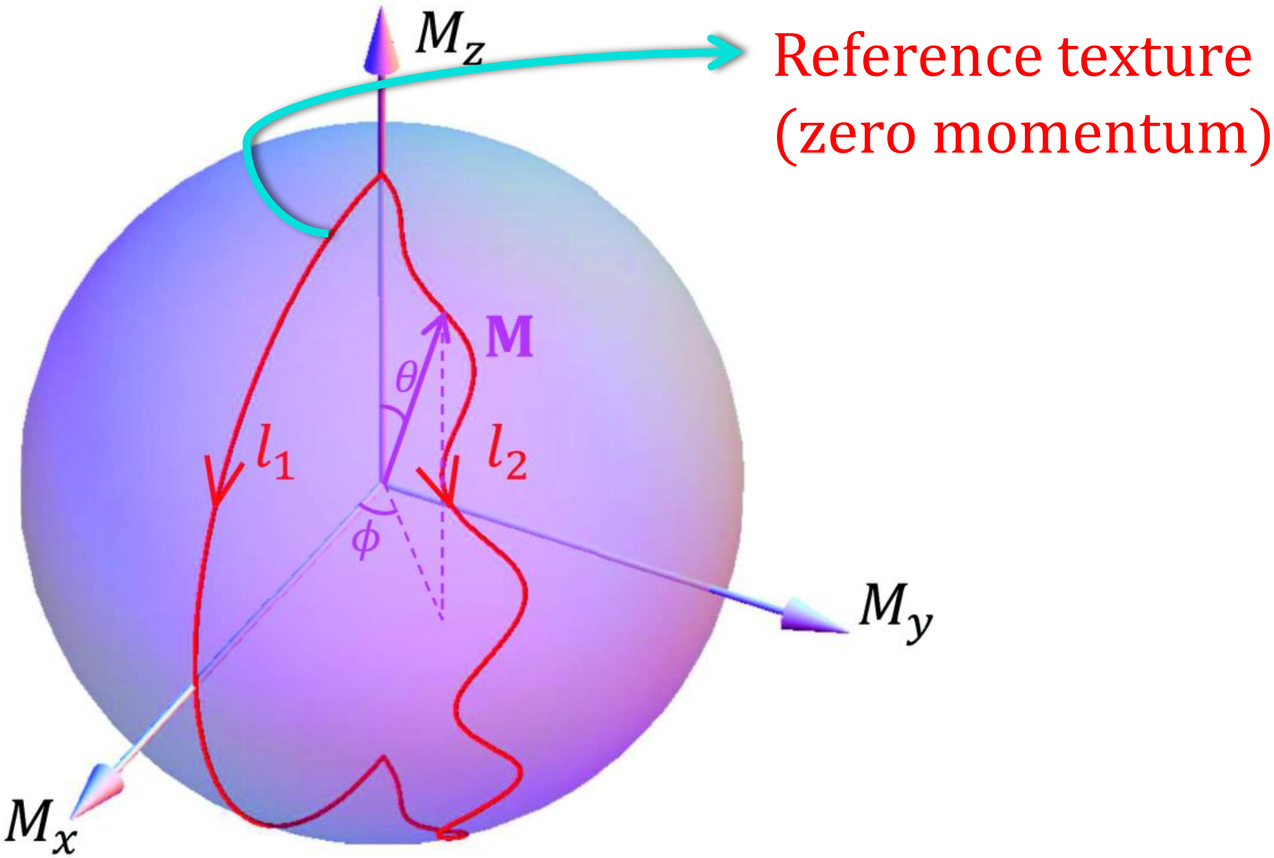
$$p_z = \mathbf{A} \cdot \frac{\partial \mathbf{M}}{\partial z},$$

with $\mathbf{A} = (M_y \mathbf{e}_x - M_x \mathbf{e}_y)/M_0(M_0 + M_z)$.

- The linear momentum of magnetic textures

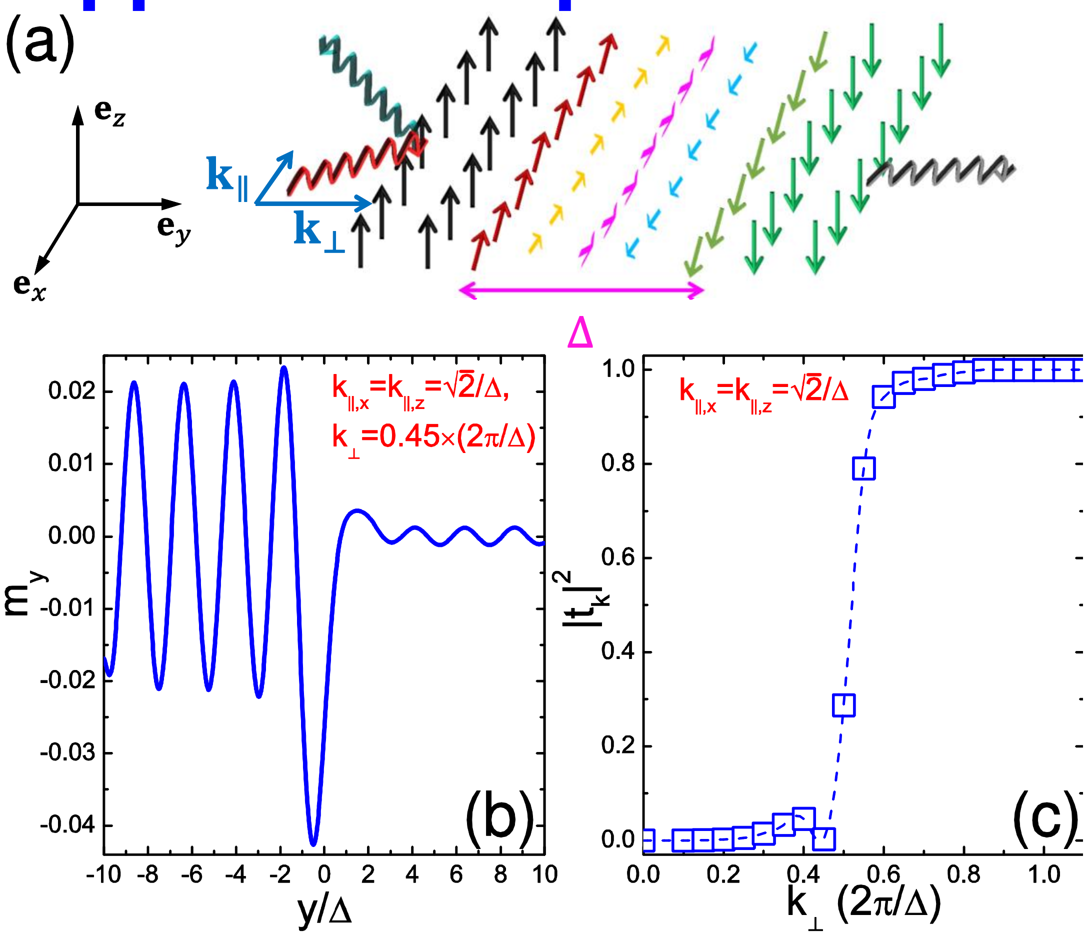
$$P_z = \int \mathbf{A} \cdot d\mathbf{M}$$

Domain wall (DW) momentum



$$\mathbf{P}_{\mathrm{DW}} = M_0 \iint_{S_{2-1}} sin\theta d\theta d\phi \, \mathbf{e}_z \xrightarrow{\text{Planar wall}} \mathbf{P}_{\mathrm{DW}} = 2\phi M_0 \mathbf{e}_z$$

Application: Spin wave scattering by DW



No lattice effect here.

Linear Momentum Conservation:

$$0 = \frac{d\mathbf{P}}{dt} = \frac{d}{dt}(\mathbf{P}_{\mathrm{SW}} + \mathbf{P}_{\mathrm{DW}}).$$

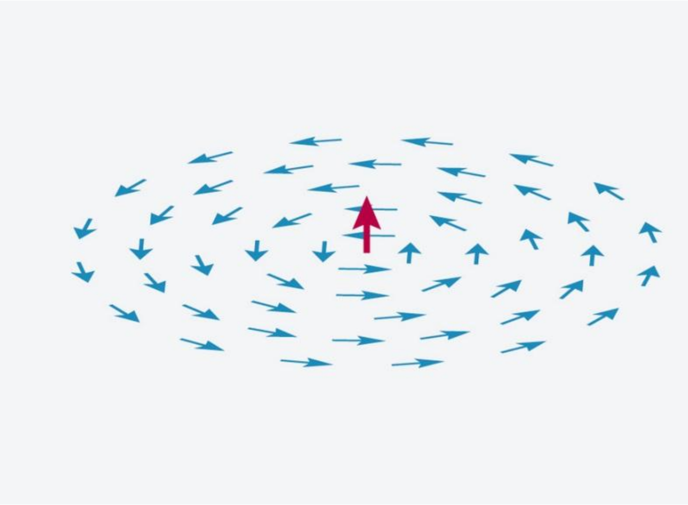
$$\mathbf{F}_{\mathrm{DW}} = 2M_0\dot{\phi}\mathbf{e}_z.$$

DW velocity in the presence of damping:

$$\mathbf{v}_{\mathrm{DW}} = \alpha\Delta\dot{\phi}\mathbf{e}_z$$

P. Yan and G.E.W. Bauer, PRL **109**, 087202 (2012);
IEEE Trans. Magn. **49**, 3109 (2013);
P. Yan, A. Kamra, Y.S. Cao, and G.E.W. Bauer, arXiv:1307.3432; Phys. Rev. B (2013) in press.

Angular momentum conservation



- Conservation law

$$\frac{\partial j_z}{\partial t} + \nabla \cdot \mathbb{J} = 0,$$

where

$$j_z = -M_z + M_z(\mathbf{r} \times \nabla\phi)_z$$

Spin angular momentum   Orbital angular momentum

$\mathbb{J}$: Angular momentum current

- Total angular momentum

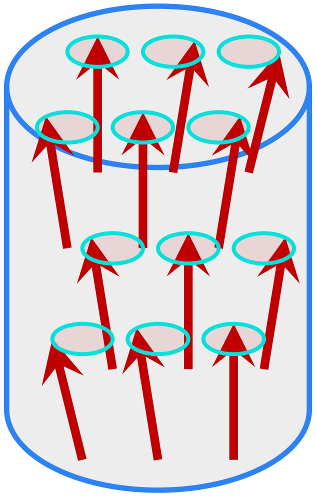
$$J_z = \int j_z d^3\mathbf{r}$$

is conserved.

Angular momentum of EM field:

$$\vec{j} = \mathbf{r} \times (\mathbf{e} \times \mathbf{h})$$

Illustration



- A simple case: Uniform ferromagnet ($\mathbf{M} = M_0\mathbf{z}$) with spin wave excitations ($\mathbf{M} = M_0\mathbf{z} + \mathbf{m}$ with $\mathbf{m} \cdot \mathbf{z} = 0$):

$$m_+ = m_x + im_y = \sum_{\mathbf{k}} \{u_{\mathbf{k}} a_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)} + v_{\mathbf{k}}^* a_{\mathbf{k}}^+ e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)}\}.$$

The total angular momentum is

$$J_z - J_{z0} = -\hbar \sum_{\mathbf{k}} a_{\mathbf{k}}^+ a_{\mathbf{k}} - i\hbar \sum_{\mathbf{k}} a_{\mathbf{k}}^+ (\mathbf{k} \times \nabla_{\mathbf{k}})_z a_{\mathbf{k}},$$

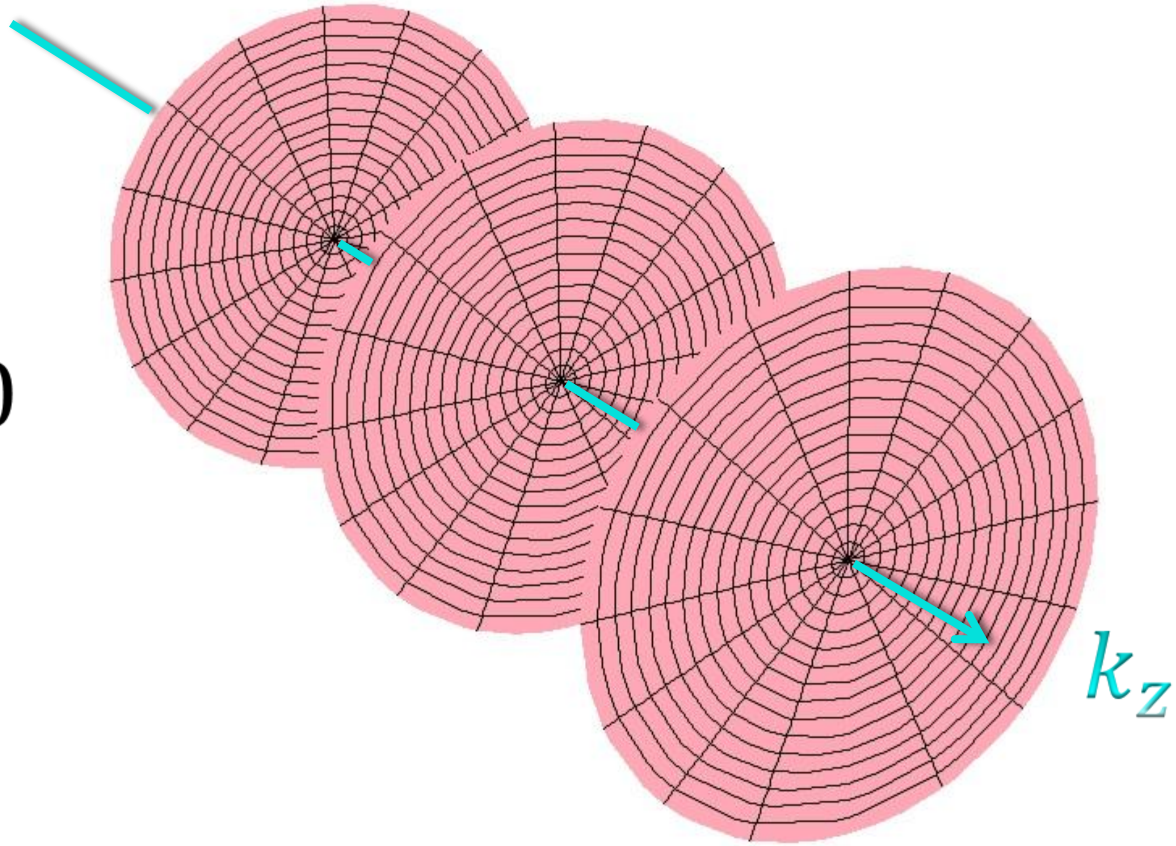
The angular momentum is diagonalized by the transformation $a_{\mathbf{k}} = \sum_m a_{k_\perp,k_z,m}\, e^{im\phi_{\mathbf{k}}}$:

$$J_z - J_{z0} = \hbar \sum_m (m-1) a_{k_\perp,k_z,m}^+ a_{k_\perp,k_z,m}.$$

Helical spin wave mode

- Wave-front shape

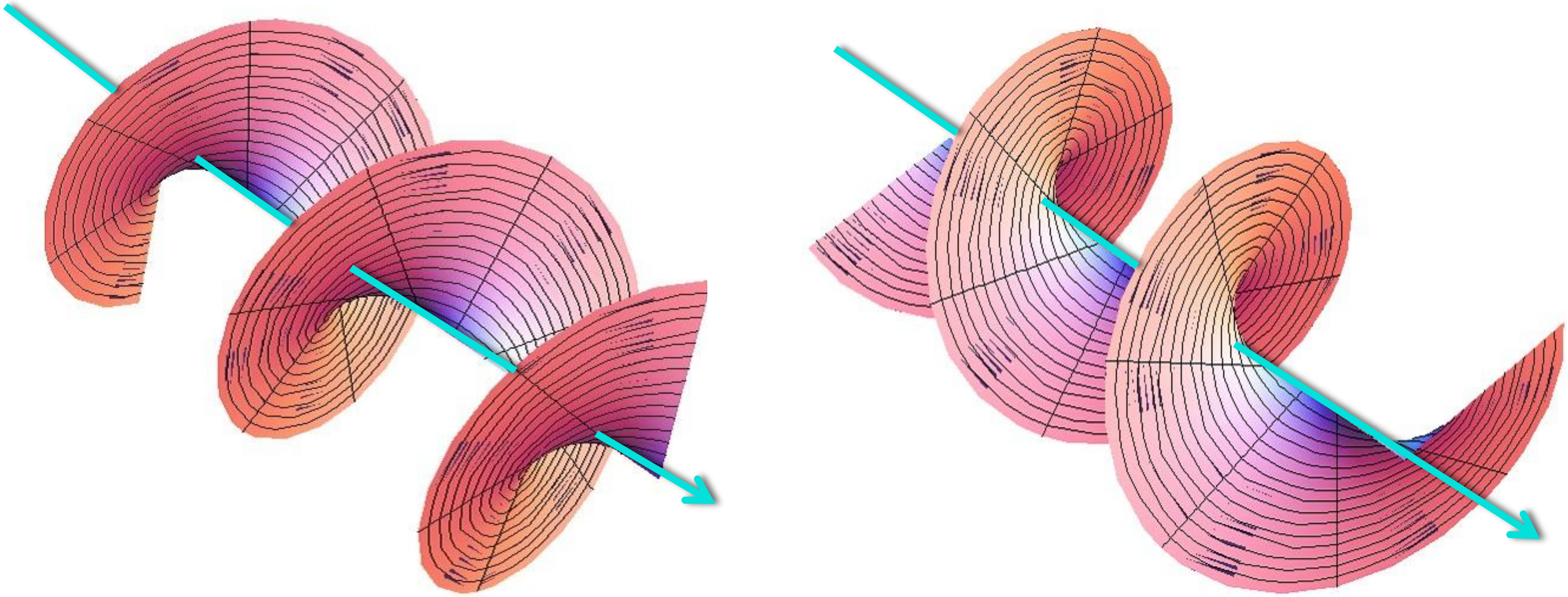
$$m = 0$$



Helical spin wave mode

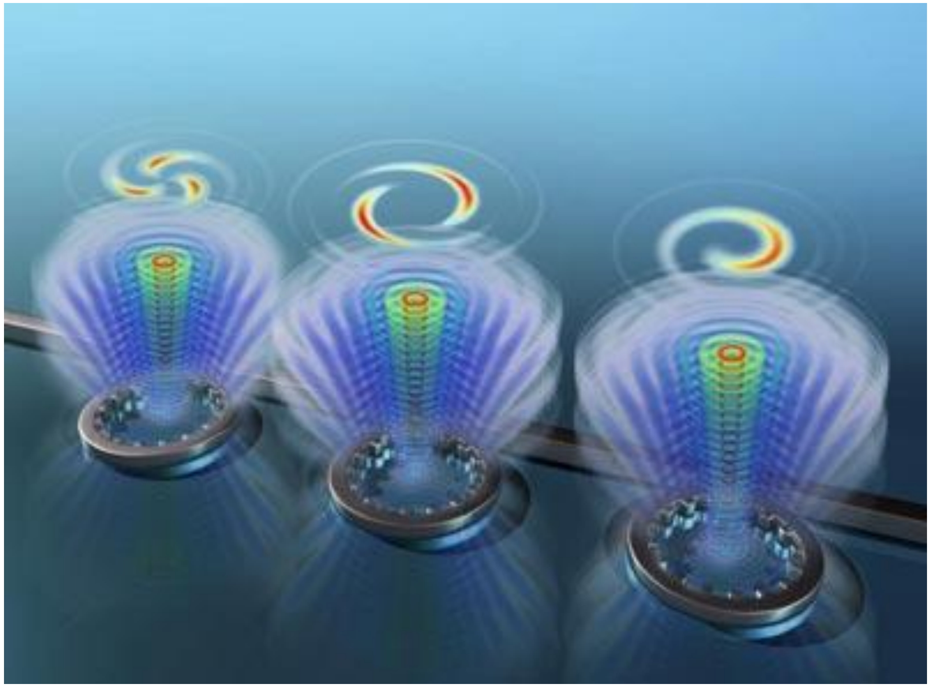
- Wave-front shape

$m = 1$ $\qquad$ $m = -1$



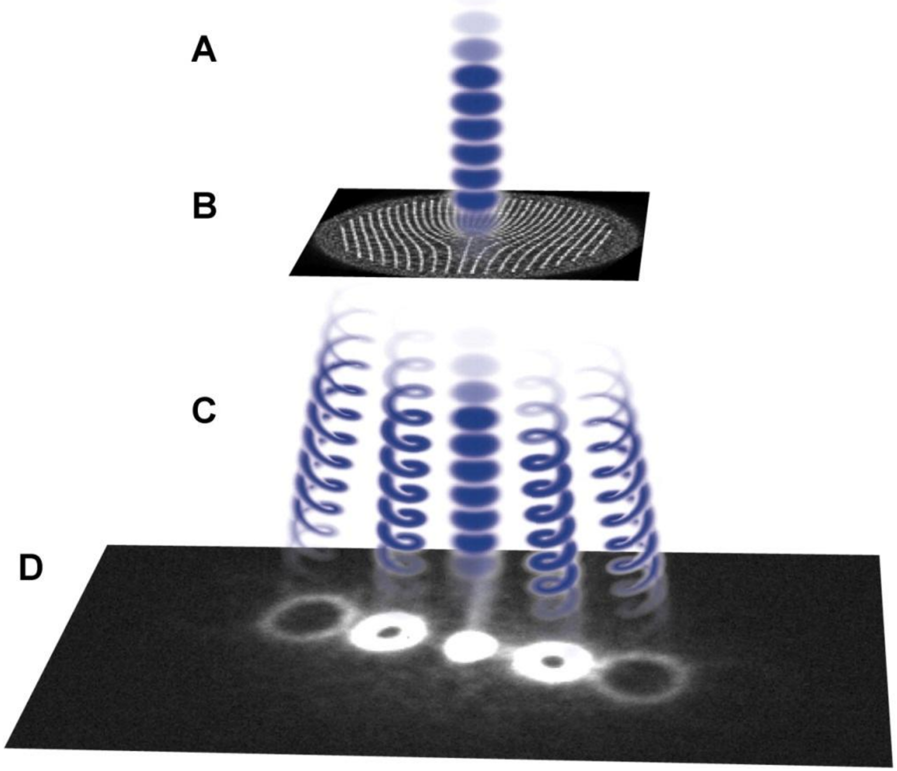
Its photonic and electronic counterparts

- Photons



Allen et al., Phys. Rev. A **45**, 8185 (1992);
Cai *et al.,* Science **338**, 363 (2012).

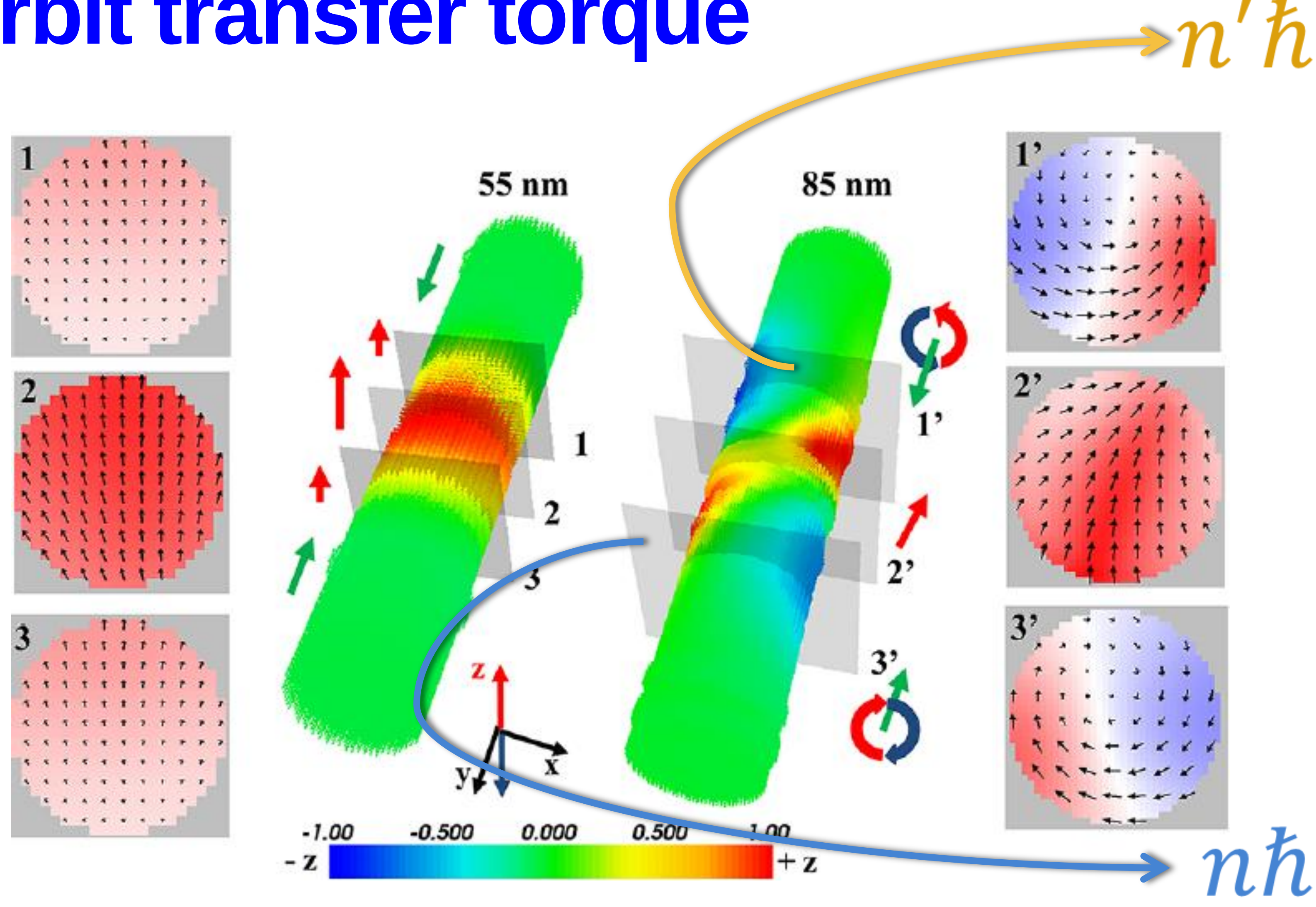
- Electrons



McMorran *et al.,* Science **331**, 192 (2011).

- Spiral phase plate
- Computer-generated holograms
- q-plate, …

Orbit transfer torque



Biziere *et al.,* Nano Lett. **13**, 2053 (2013).

Conclusions & Outlook

- We formulate the conservation laws for linear and angular momentum in ferromagnets.

- We identify a non-adiabatic torque on domain wall due to spin wave reflection.

- We predict an orbit angular momentum transfer which is highly efficient to drive domain wall motion.