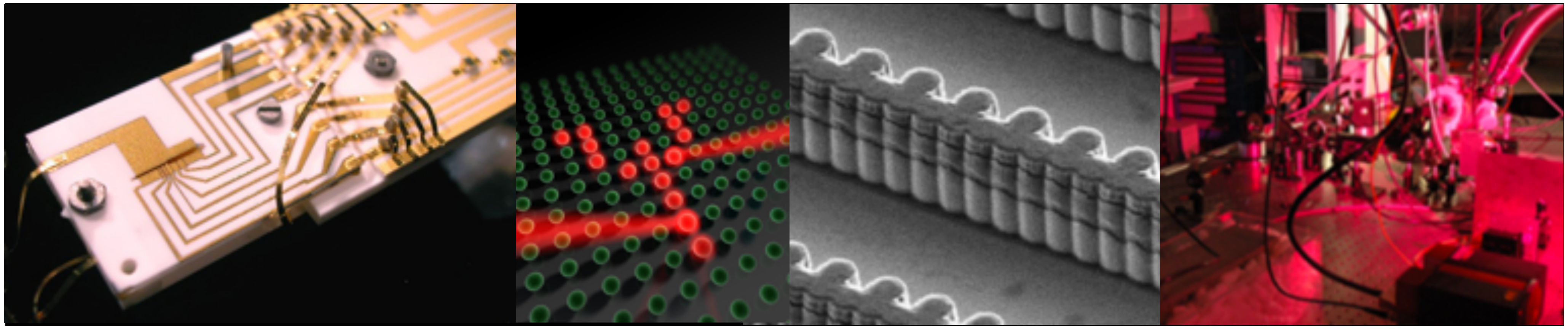
Synthetic quantum systems out of equilibrium

and the quest for quantum supremacy for quantum simulators

Jens Eisert, Freie Universität Berlin
Synthetic quantum matter, KITP, September 2016

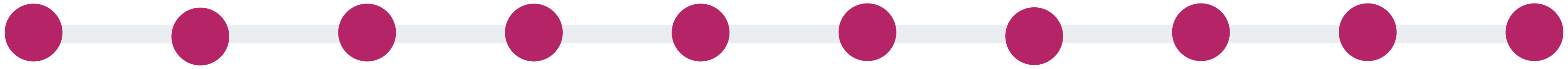
Quantum simulators

- Synthetic quantum systems as quantum simulators

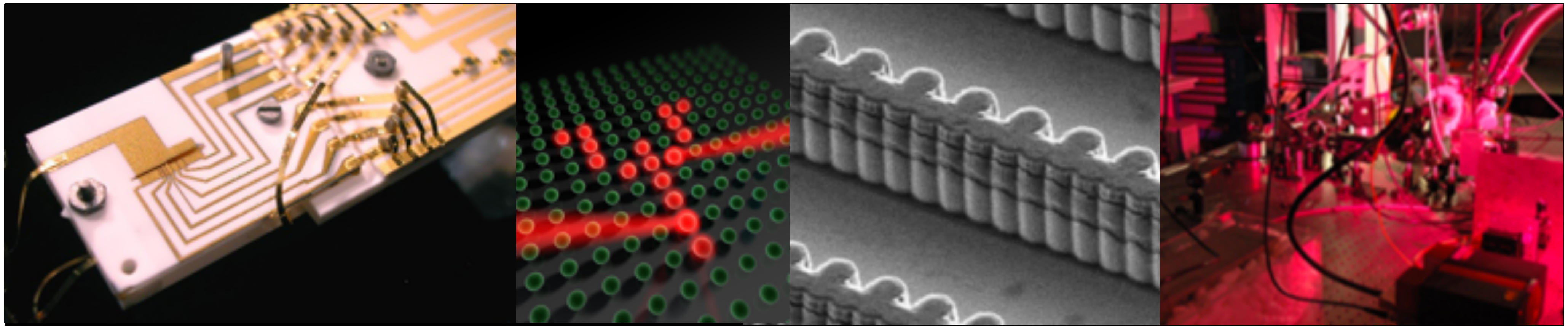


- Cold atoms in optical lattices, trapped ions, etc

Quantum simulators



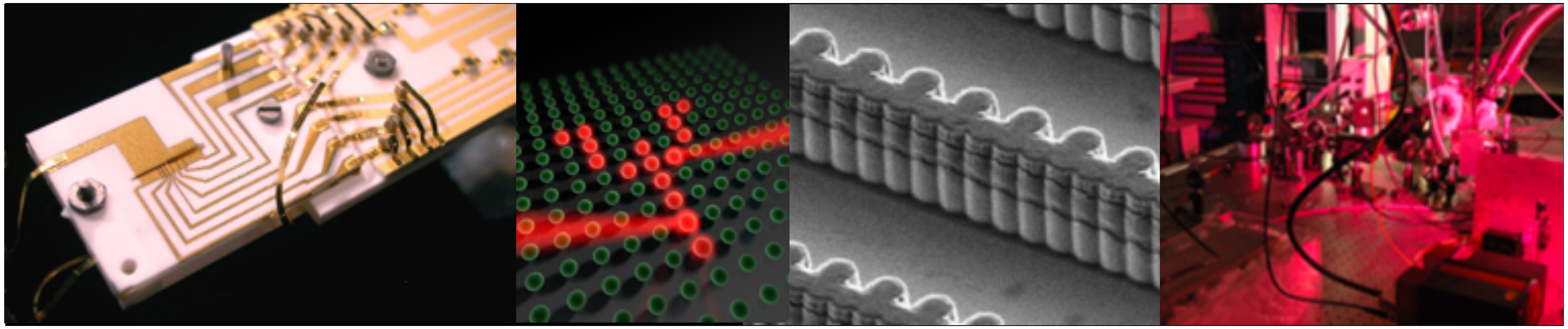
- Synthetic quantum systems as quantum simulators



- Allow to probe physics of non-equilibrium
    - Equilibration
    - Thermalisation and generalised thermalisation
    - Dynamics of quantum phase transitions
    - Many-body localisation

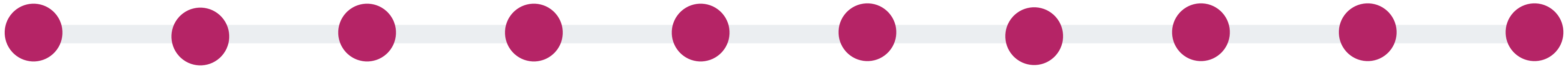
Quantum simulators

- Synthetic quantum systems as quantum simulators

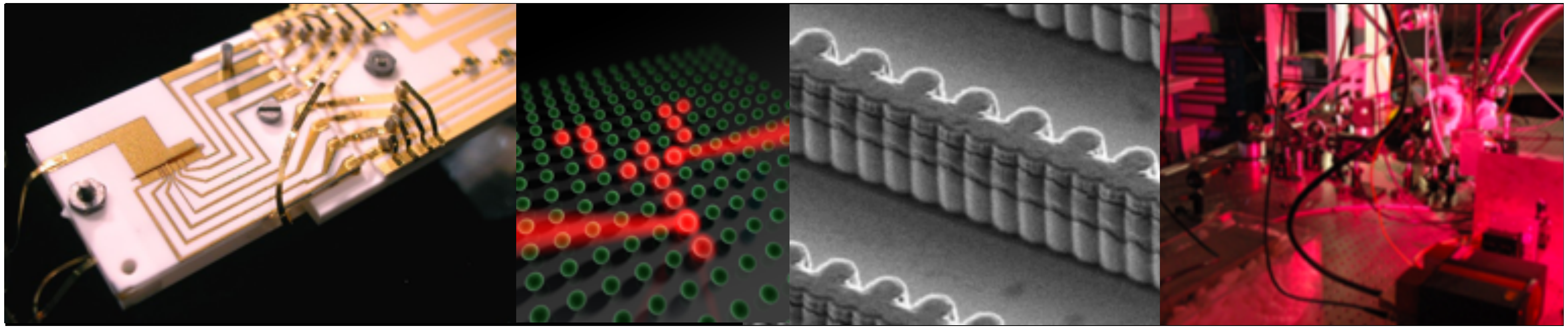


- Should solve problems inaccessible to classical simulations

Quantum simulators

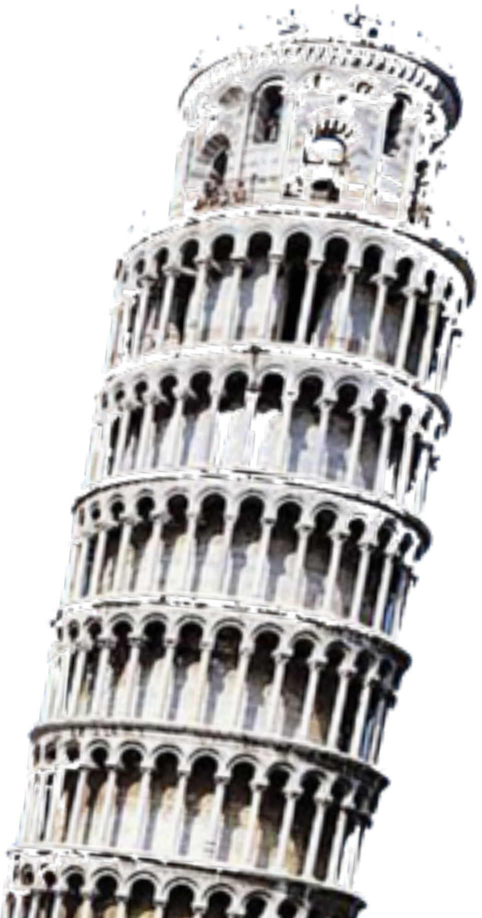


- Synthetic quantum systems as quantum simulators

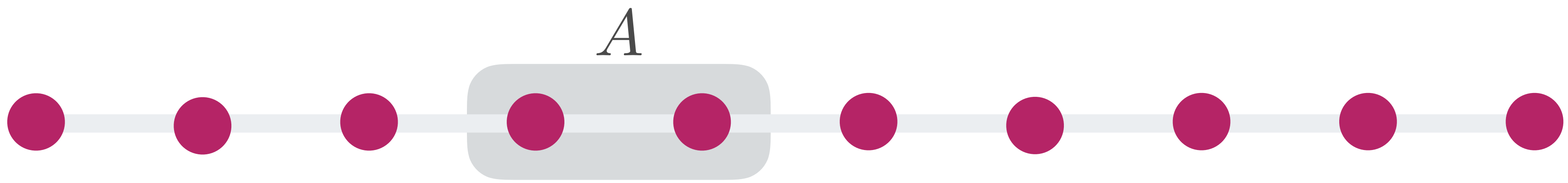


- Certification and verification?

Probing many-body systems out of equilibrium



Quantum information and condensed-matter physics



- Quantum many-body systems out of equilibrium

$$\rho(t) = e^{-itH}\rho(0)e^{itH}$$

- Do they *equilibrate*? Yes, locally

- Expectation values of time average

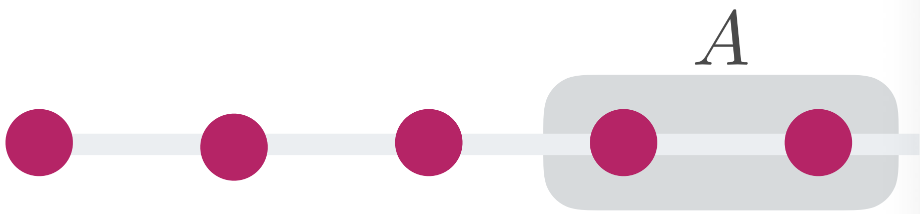
$$\omega = \lim_{T\to\infty}\int_0^T dt e^{-itH}\rho(t)e^{itH}$$

for overwhelming times

Eisert, Friesdorf, Gogolin, Nature Physics 11, 124 (2015)
Gogolin, Eisert, Rep Prog Phys 79, 056001 (2016)
Polkovnikov, Sengupta, Silva, Vengalattore, RMP 83, 863 (2011)
Calabrese, Cardy, Phys Rev Lett 96, 136801 (2006)
Cramer, Dawson, Eisert, Osborne, PRL100, 030602 (2008)
Linden, Popescu, Short, Winter, Phys Rev E 79, 061103 (2009)

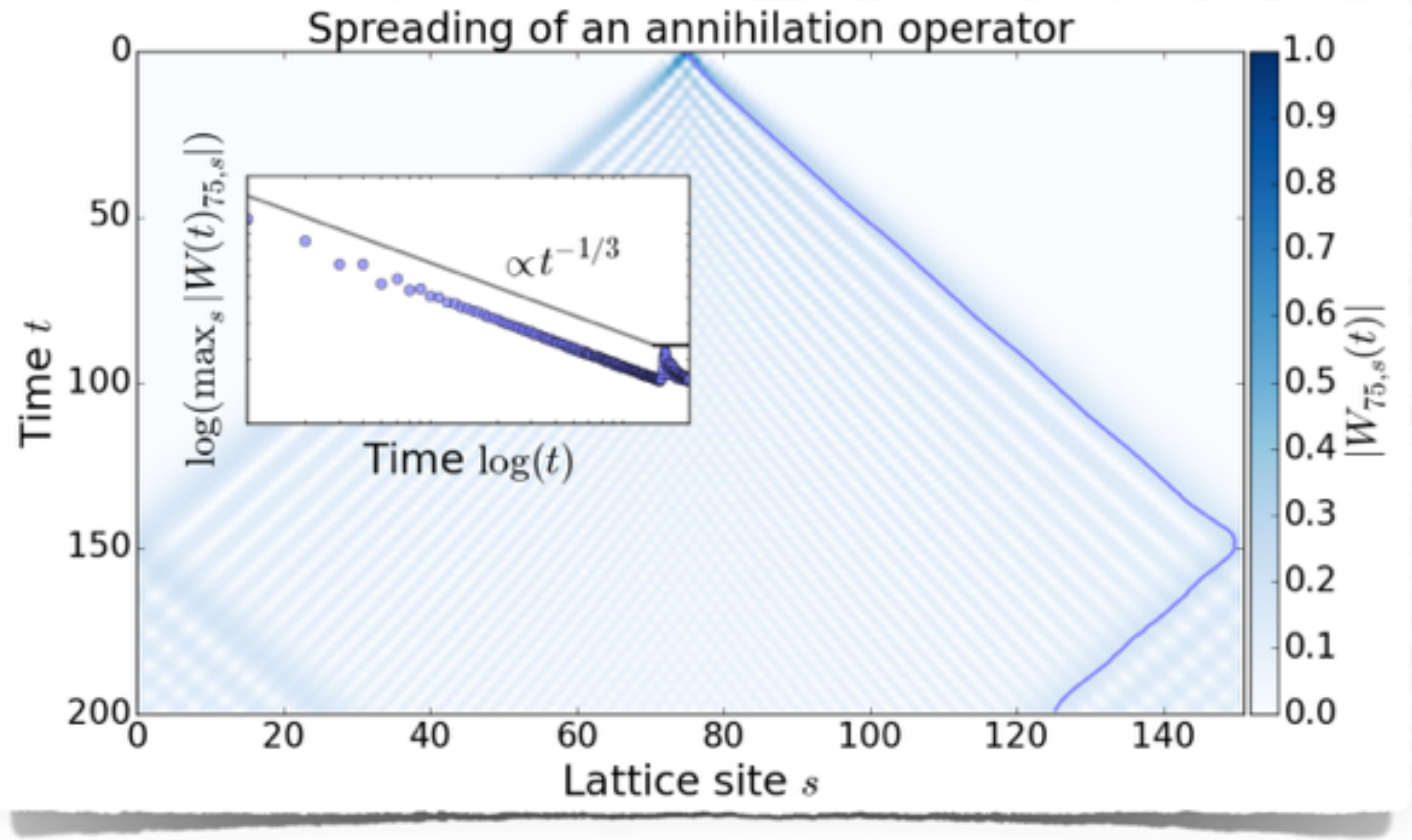
- True in several specific senses

Quantum information and conc



- Quantum many-body system

$$\rho(t) = e^{-itH}\rho(0)e$$



- **Theorem:**

  1. For initial states with **exponentially decaying correlations**

  $$|\mathrm{tr}(\rho AB) - \mathrm{tr}(\rho A)\mathrm{tr}(\rho B)| \leq C|A|\,|B|e^{-d(A,B)/\xi}$$

  2. non-interacting Hamiltonians exhibiting **transport, one can** prove local convergence to a **generalised Gibbs ensemble**

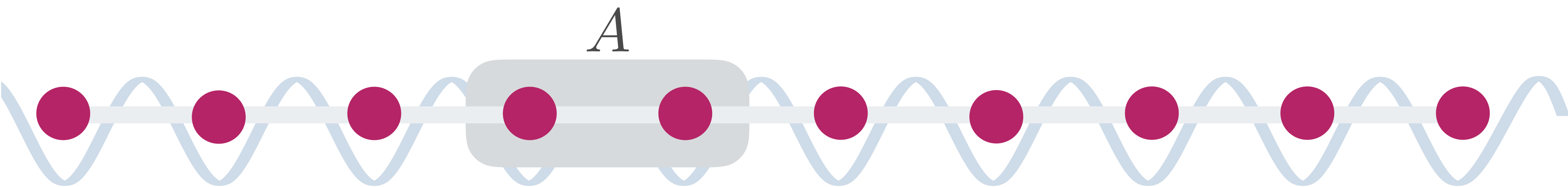
  $$|\mathrm{tr}(A(t)\rho) - \mathrm{tr}(A(t)\rho_G)| < \varepsilon$$

  (Gaussian states for same second moments) with **power law in time**

Gluza, Krumnow, Friesdorf, Gogolin, Eisert, PRL 117 (2016)
Cramer, Dawson, Eisert, Osborne, PRL100, 030602 (2008)

Compare Calabrese, Essler, Fagotti, PRL 106, 227203 (2011)

Thermalisation



- Quantum many-body systems out of equilibrium

$$\rho(t) = e^{-itH}\rho(0)e^{itH}$$

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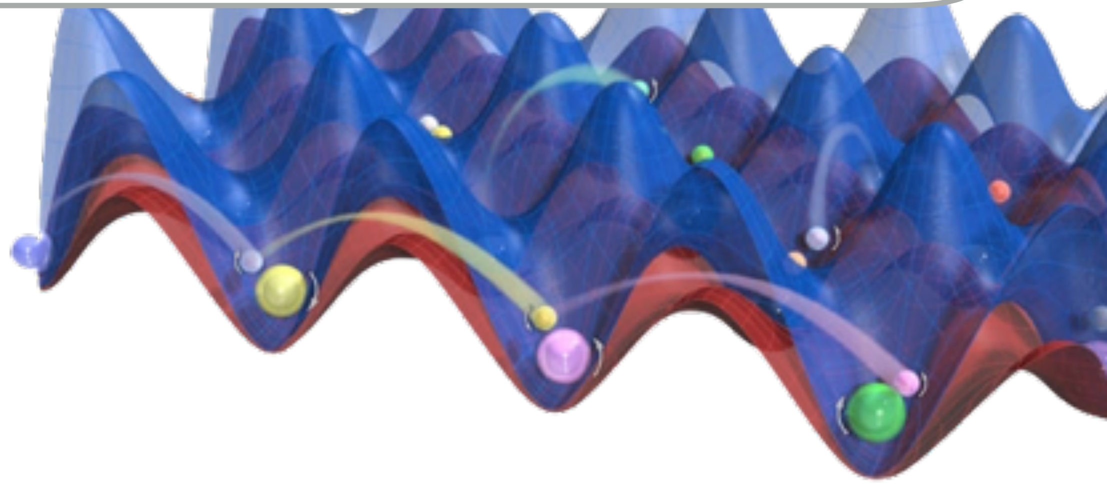
- Do they *thermalise*? Presumably yes, for non-integrable models



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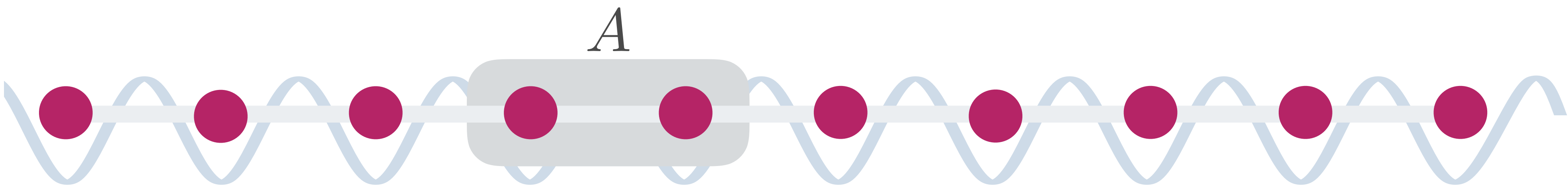
Thermalisation



- Quantum many-body systems out of equilibrium

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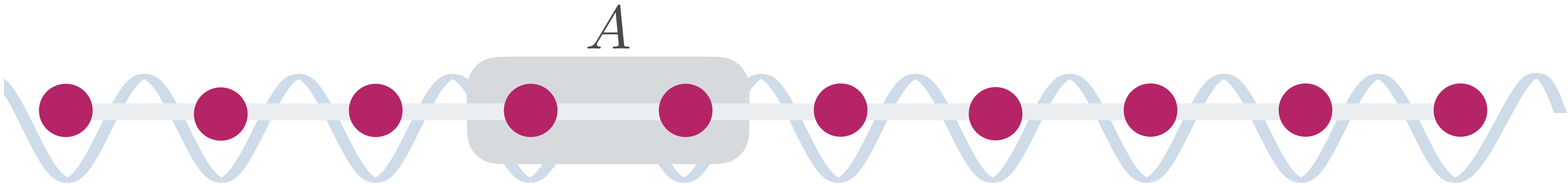
- Eigenstate thermalisation hypothesis

$$\mathrm{tr}_{A^c}(|k\rangle\langle k|) \sim \mathrm{tr}_{A^c}(e^{-\beta H})$$

Eisert, Friesdorf, Gogolin, Nature Physics 11, 124 (2015)
Gogolin, Eisert, Rep Prog Phys 79, 056001 (2016)
Deutsch, Phys Rev A 43, 2046 (1991)
Srednicki, Phys Rev E 50, 888 (1994)
Polkovnikov, Sengupta, Silva, Vengalattore, RMP 83, 863 (2011)

Lots of open questions



- Quantum many-body systems out of equilibrium

$$\rho(t) = e^{-itH}\rho(0)e^{itH}$$

- Time scales of equilibration?

- Slow quenches and dynamics of quantum phase transitions?

- Do all non-integrable models thermalise?

Recent reviews →

Eisert, Friesdorf, Gogolin, Nature Physics 11, 124 (2015)
Gogolin, Eisert, Rep Prog Phys 79, 056001 (2016)
Deutsch, Phys Rev A 43, 2046 (1991)
Srednicki, Phys Rev E 50, 888 (1994)
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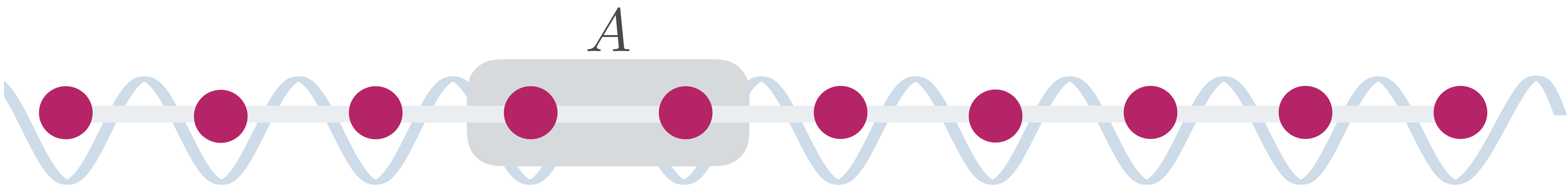
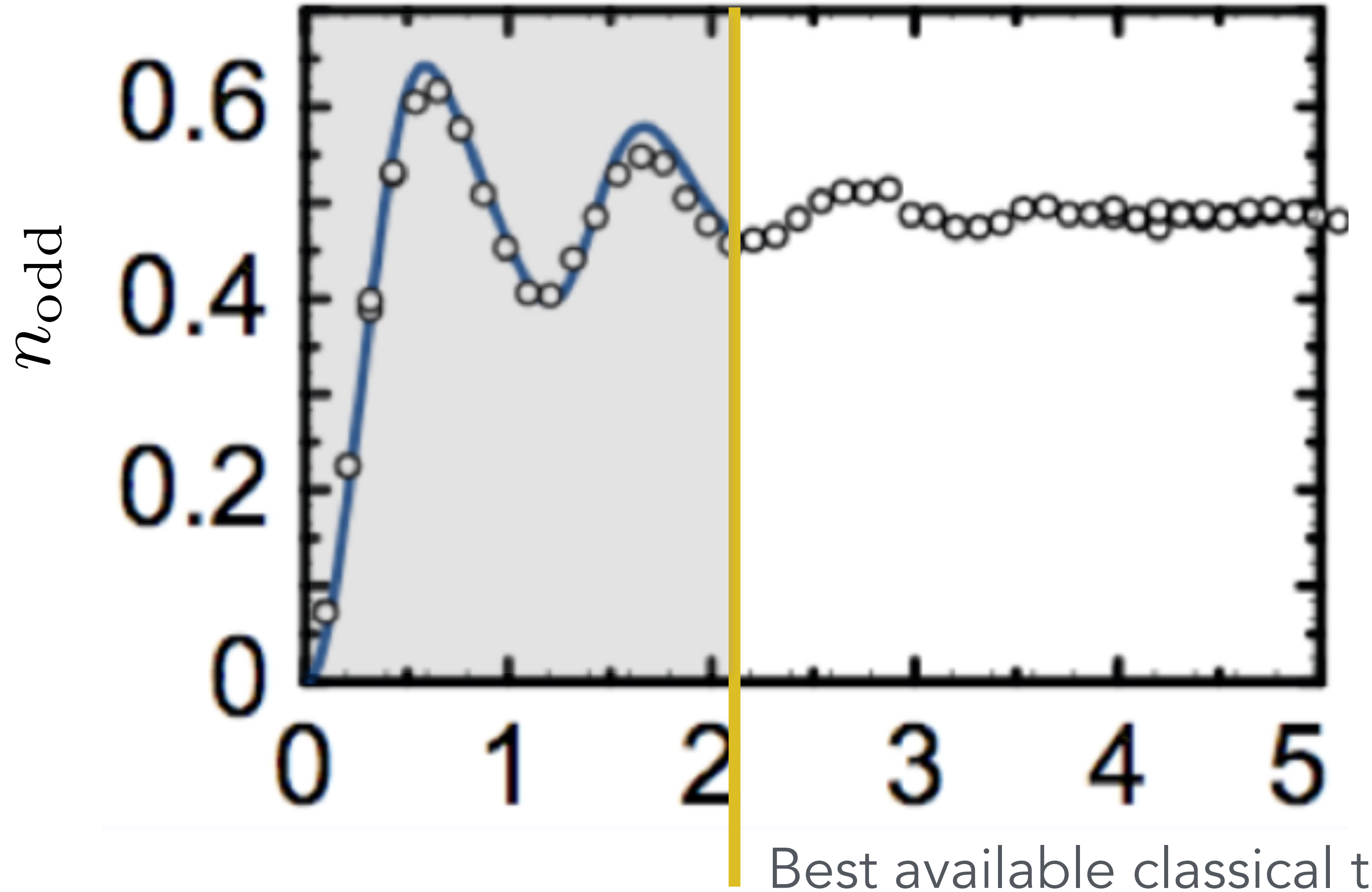
Experimental analog(ue) quantum simulators

Experimental Bose-Hubbard dynamics



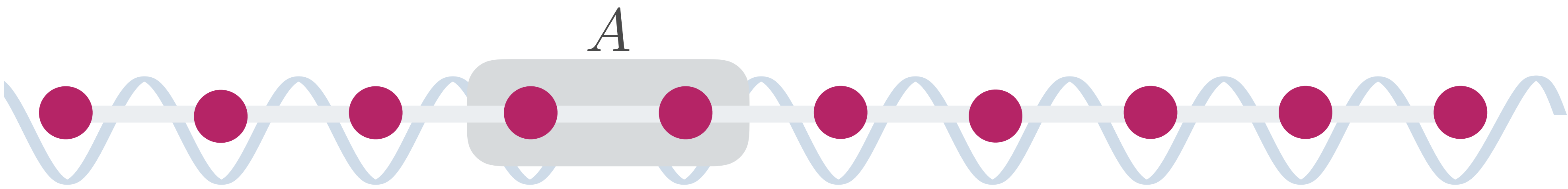
- Probing questions with ultra-cold atoms in optical super-lattices (MPQ)
- Imbalance as function of time for $|\psi(0)\rangle = |0,1,\ldots,0,1\rangle$



Best available classical tensor network simulation, bond dimension 5000

Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, Nature Phys 8, 325 (2012)

Experimental Bose-Hubbard dynamics



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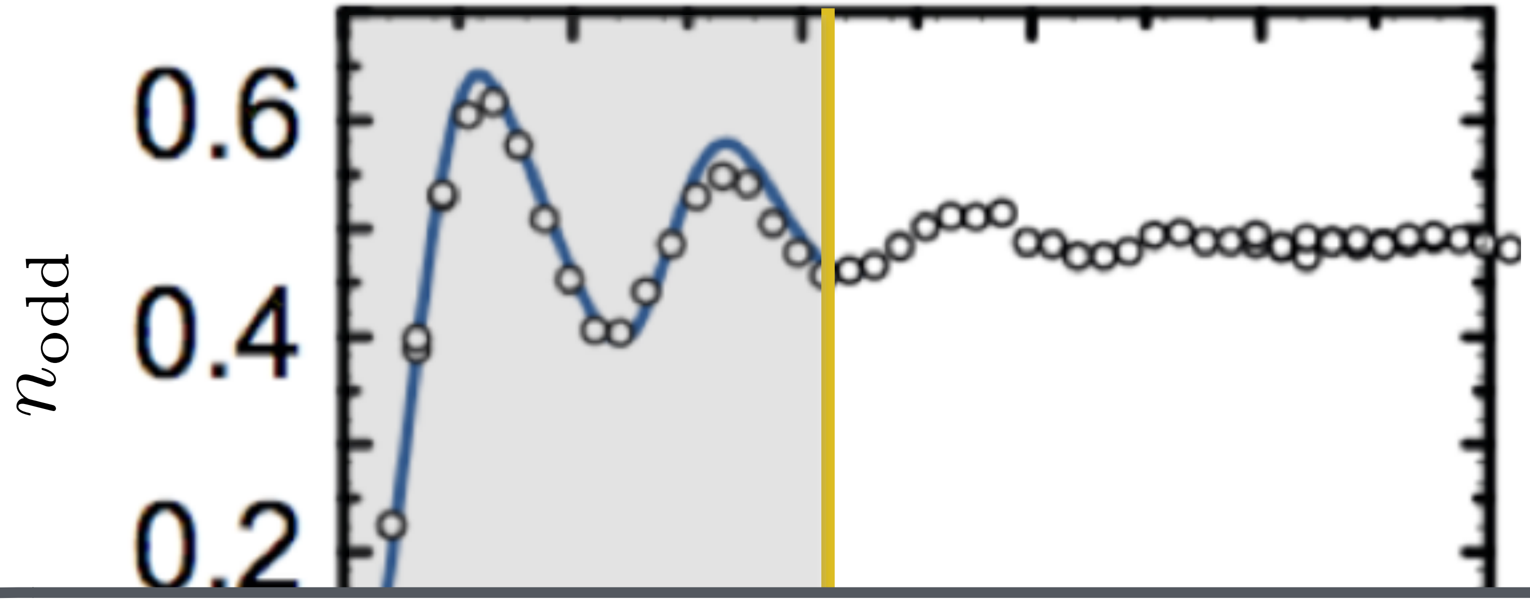
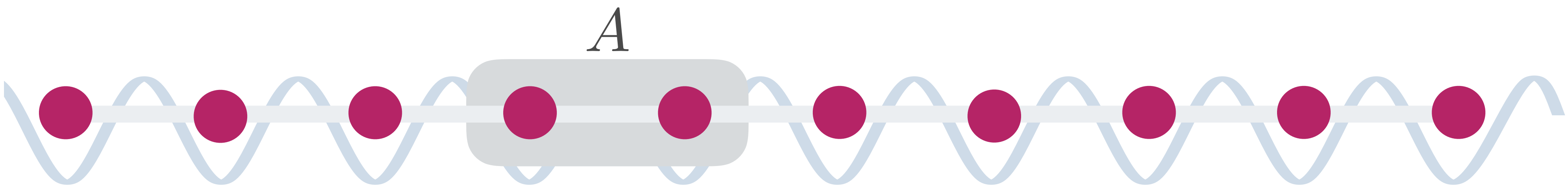


- Can be seen as evidence that dynamical quantum simulators outperform classical computers

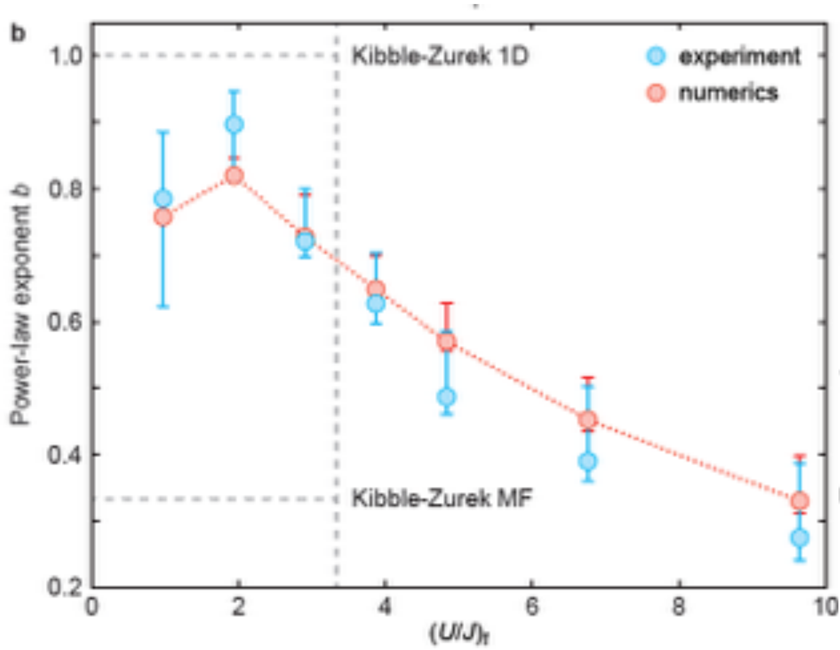
Best available classical tensor network simulation, bond dimension 5000

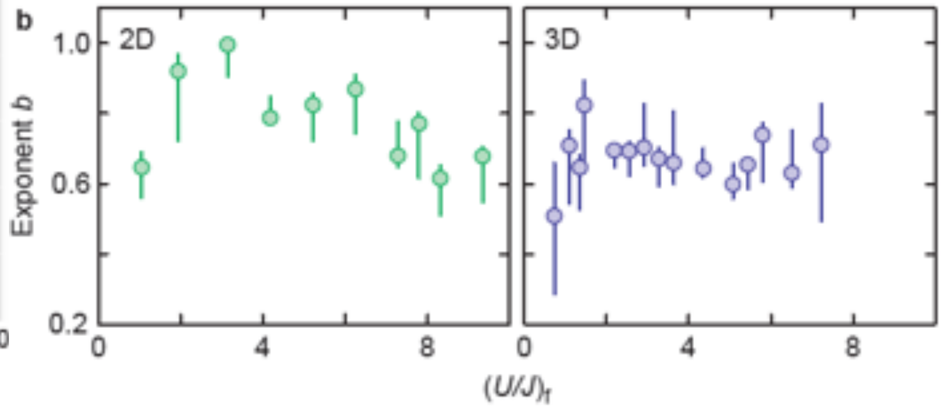
Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, Nature Phys 8, 325 (2012)

Dynamics of quantum phase transitions



- Slow quenches (MPQ)
  - Gapped phase: Adiabatic theorem ensures equilibrium
  - Crossing critical line: Never sufficiently slow
- Kibble-Zurek predicts power laws for correlation lengths





KZM description $\alpha = 1$ (1D)

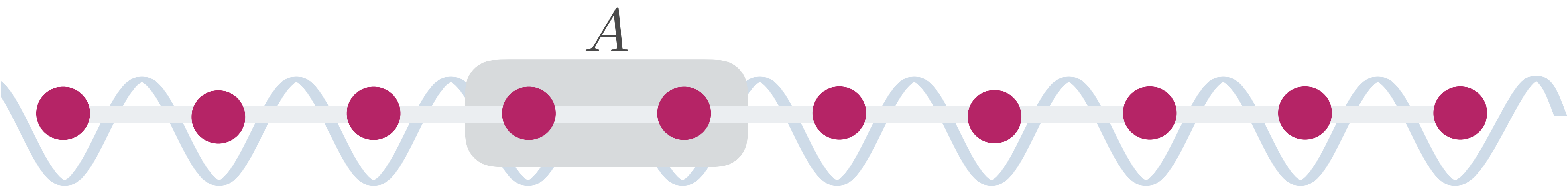
(possibly log corrections)

$\alpha = 0.4$ (2D)

$\alpha = 1/3$(3D)

Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, Schneider, Proc Natl Acad Sci 112 3641 (2015)

Dynamics of quantum phase transitions



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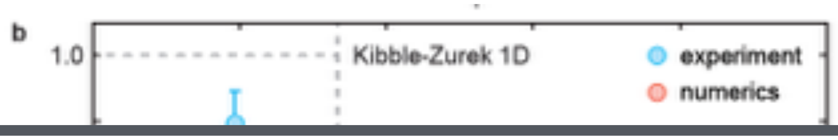
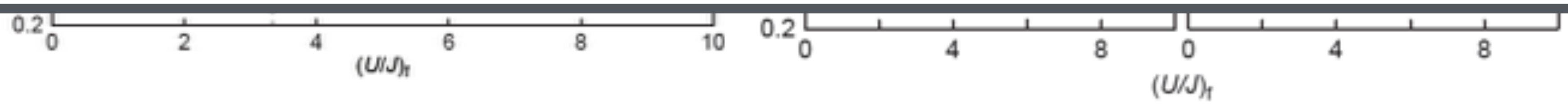


- Slow quenches give rise to rich behavior
- **Quantum simulation:** "Certify correctness" of simulation in 1D, experiment allows for assessment of 2D, 3D, alternative schedules, etc



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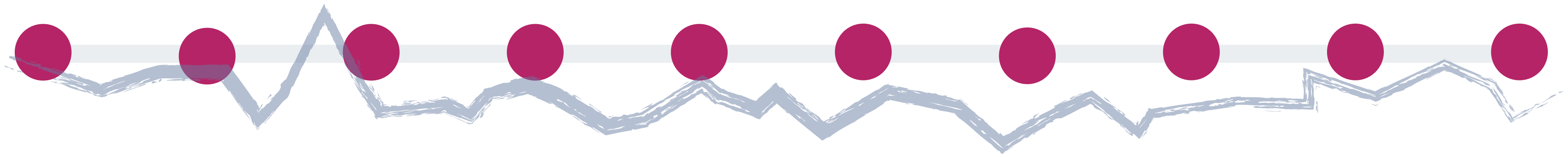
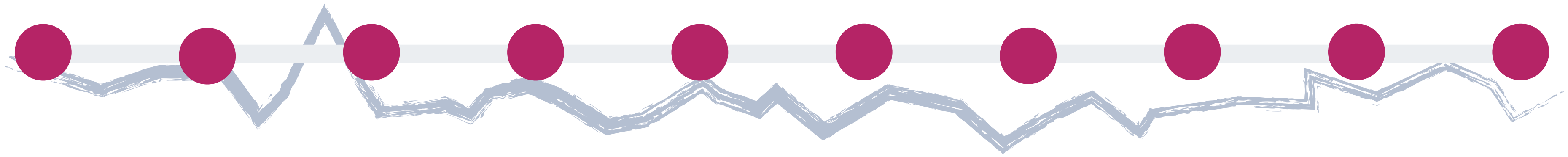
The many flavours of many-body localisation

Violation of thermalisation



- Stubbornly not thermalising systems - do not ‘serve as own heat bath’
- **Many-body localisation (MBL)** of disordered models key incarnation

Anderson localisation



- Single particle hopping on a line subject to i.i.d. random potential

$$H = \sum_j \left(|j\rangle\langle j+1| + |j+1\rangle\langle j| + f_j |j\rangle\langle j| \right)$$

- **Static reading:** ‘All’ eigenfunctions exponentially decaying correlations

- **Dynamical reading:** $\mathbb{E}(\sup_t |\langle n|e^{-itH}|m\rangle|) \leq c_1 e^{c_2 \mathrm{dist}(n,m)}$

- Does localisation survive finite interactions? **MBL: explosion of interest**

Plethora of readings of many-body localisation

- Violation of eigenstate thermalisation
- System exhibits MBL if ETH violated

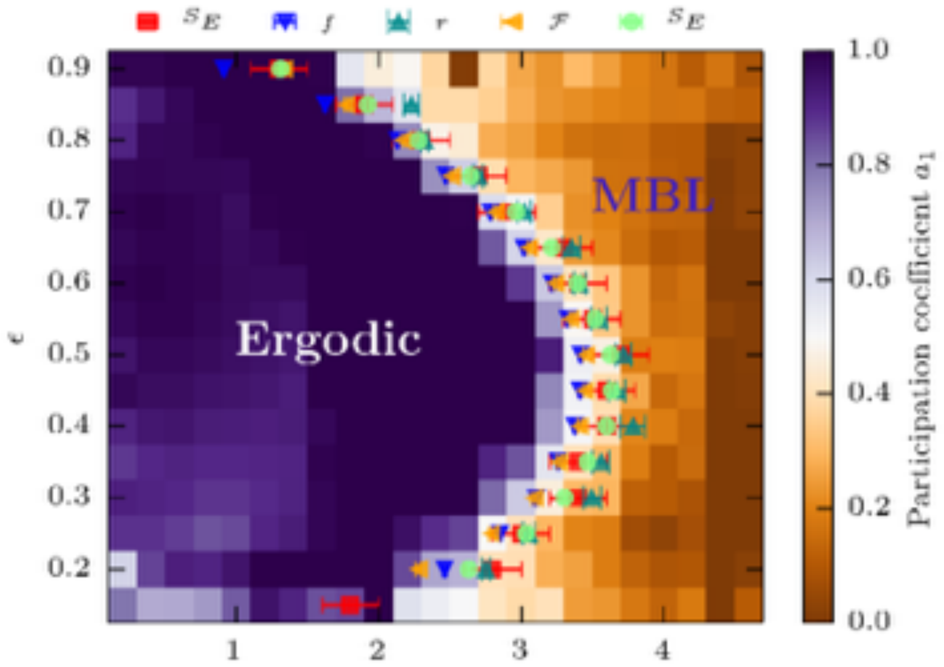
Pal, Huse, Phys Rev B 82, 174411 (2010)
Ogenesyan, Huse, Phys Rev B 75, 155111 (2007)
Gogolin, Mueller, Eisert, Phys Rev Lett 106, 040401 (2011)

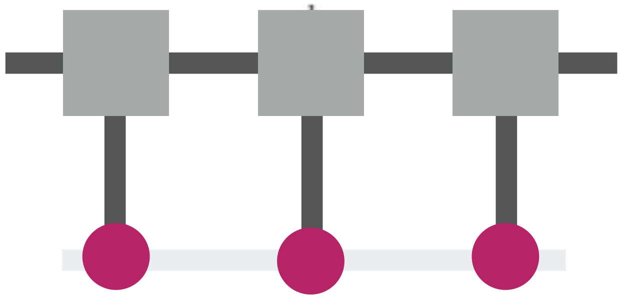
- Static reading
- Dynamical reading

Plethora of readings of many-body localisation

- Violation of eigenstate thermalisation

- Entanglement area laws and matrix-product eigenstates





Bauer, Nayak, J Stat Mech P09005 (2013)
Basko, Aleiner, Altshuler, Ann Phys 321, 1126 (2006)
Luitz, Laflorencie, Alex, arXiv:1411.0660
Eisert, Cramer, Plenio, Rev Mod Phys 82, 277 (2010)

- Dynamical reading

Plethora of readings of many-body localisation

- Violation of eigenstate thermalisation

- Entanglement area laws and matrix-product eigenstates

- Extensively many commuting approx. local constants of motion

$\mathcal{Z}^{(j)}$

$$[\mathcal{Z}^{(j)}, H] = 0,\ [\mathcal{Z}^{(j)}, \mathcal{Z}^{(k)}] = 0$$

- Static reading

- Dynamical reading

Plethora of readings of many-body localisation

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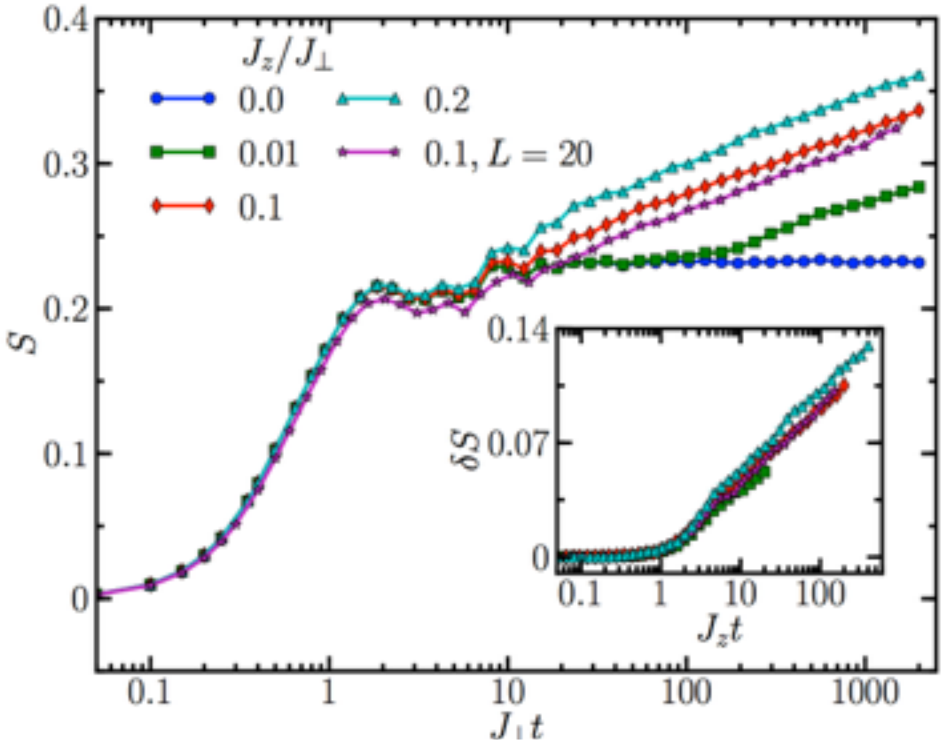
- Log growth of entanglement entropies



Znidaric, Prosen, Prelovsek, PRB 77, 064426 (2008)
Badarson, Pollmann, Moore, PRL 109, 017202 (2012)

- Static reading

Plethora of readings of many-body localisation

- Violation of eigenstate thermalisation
- Entanglement area laws and matrix-product eigenstates
- Dynamical reading: Absence of (particle) transport
- Log growth of entanglement entropies
- Extensively many commuting approx. local constants of motion

- Many-body localisation

Bringing pictures together

- Entanglement area laws and matrix-product eigenstates

- Dynamical reading: Absence of (particle) transport

- Zero-velocity Lieb-Robinson bound

$$\|A(t) - e^{itH^l_A} A e^{-itH^l_A}\| \leq c_{\text{loc}} e^{-\mu l} t^{\alpha}$$

allowing for power laws in time

- Absence of information propagation with mobility edge

$$\forall \rho \in \{|l\rangle\langle k| : E_l, E_k \leq E_{\text{mob}}\} :$$

$$|\text{tr}(\rho[A(t), B])| \leq \min(t, 1) c_{\text{mob}} e^{-\mu d(A,B)}$$

Bringing pictures together

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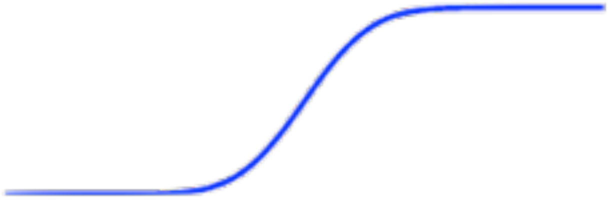
- Dynamical reading: Absence of (particle) transport

Machinery of filter functions

$$I_f^H(A) = \int_{-\infty}^{\infty} dt\, f(t) A(t)$$

Lieb-Robinson bounds

$$I_\alpha^H(A) = \frac{\alpha^{1/2}}{\pi^{1/2}} \int_{-\infty}^{\infty} dt e^{-\alpha t^2} A(t)$$



$$I_\alpha^H(A) = \lim_{\epsilon \to 0} \frac{i}{2\pi} \int_{-\infty}^{\infty} dt \frac{e^{-\alpha t^2}}{t + i\epsilon} A(t)$$

Bringing pictures together

- Entanglement area laws and matrix-product eigenstates

- Dynamical reading: Absence of (particle) transport

- **Theorem:** Absence of information propagation with mobility edge implies exponentially decaying correlations

$$
\begin{aligned}
&|\langle k|AB|k\rangle - \langle k|A|k\rangle\langle k|B|k\rangle| \\
&\leq \left(12\pi 2^N \mathcal{N}(E_k+\kappa)c_{\mathrm{mob}} + \ln\frac{\pi\mu d(A,B)e^{4+2\pi}}{\kappa^2}\right)\frac{e^{-\mu d(A,B)/2}}{2\pi}
\end{aligned}
$$

Friesdorf, Werner, Scholz, Brown, Eisert, Phys Rev Lett 114, 170505 (2015)

Bringing pictures together

- Entanglement area laws and matrix-product eigenstates

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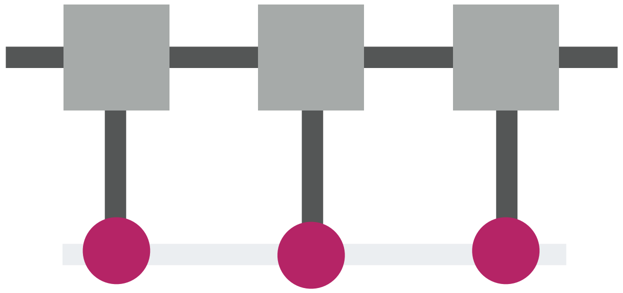

- In 1D matrix-product eigenstates

Friesdorf, Werner, Scholz, Brown, Eisert, PRL 114, 170505 (2015)
Verstraete, Cirac, Phys Rev B 73, 094423 (2006)

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Friesdorf, Werner, Scholz, Brown, Eisert, Phys Rev Lett 114, 170505 (2015)

Bringing pictures together

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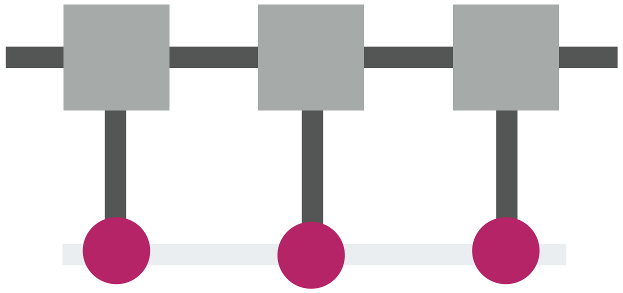

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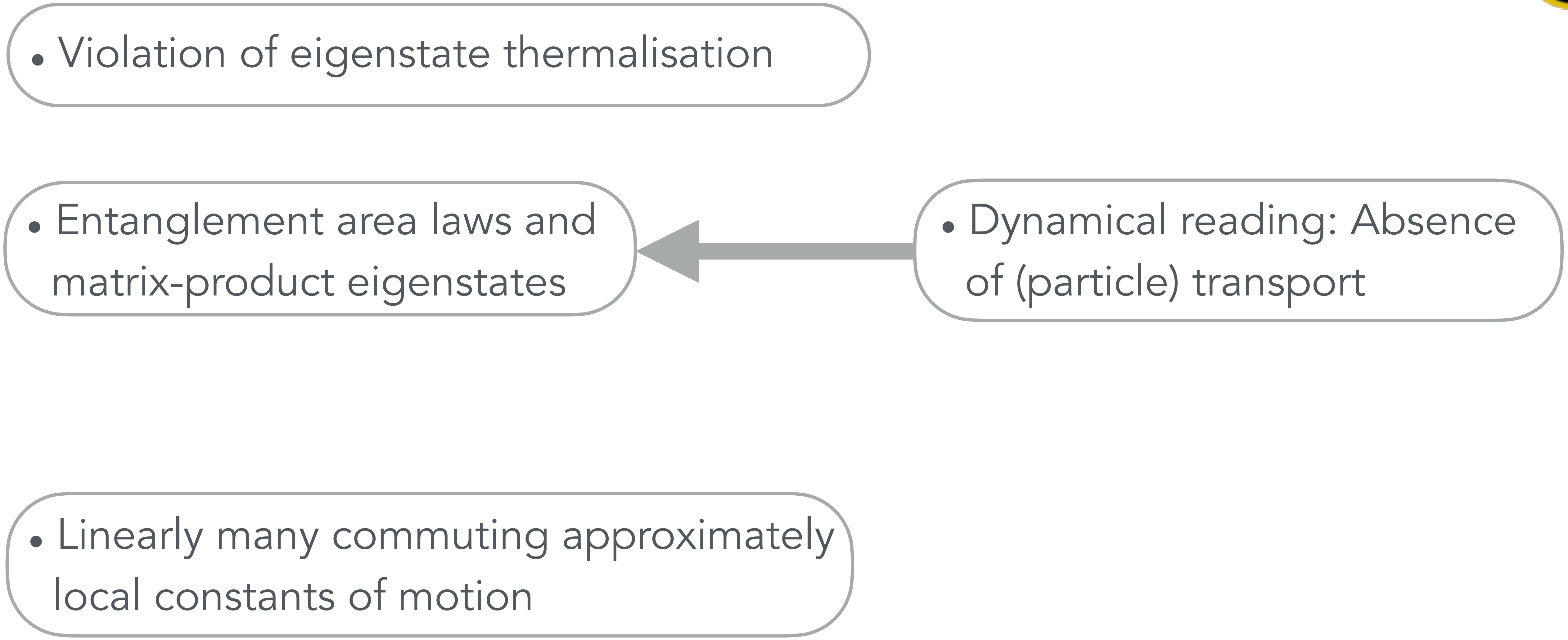


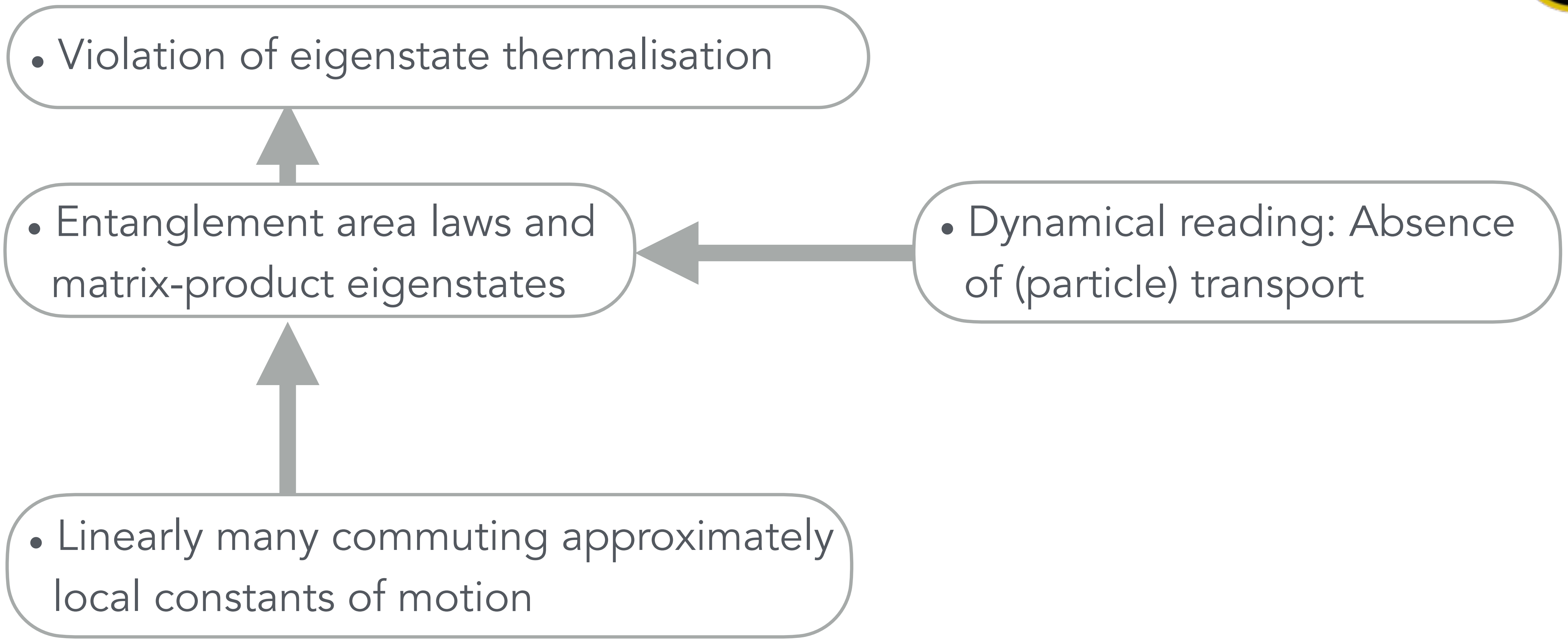
- Violation of eigenstate thermalisation

- Entanglement area laws and matrix-product eigenstates

- Dynamical reading: Absence of (particle) transport

- Linearly many commuting approximately local constants of motion

Bringing pictures together



- Violation of eigenstate thermalisation

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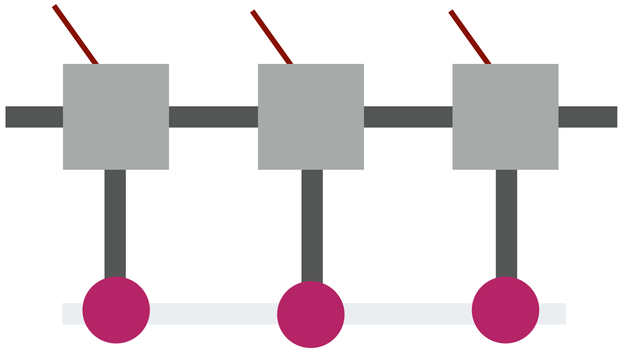
- **Theorem:** … give rise to efficient spectral tensor network

Chandran, Carresquilla, Kim, Abanin, Vidal, Phys Rev B 92, 024201 (2015)
Friesdorf, Werner, Goihl, Eisert, Brown, New J Phys 17, 113054 (2015)



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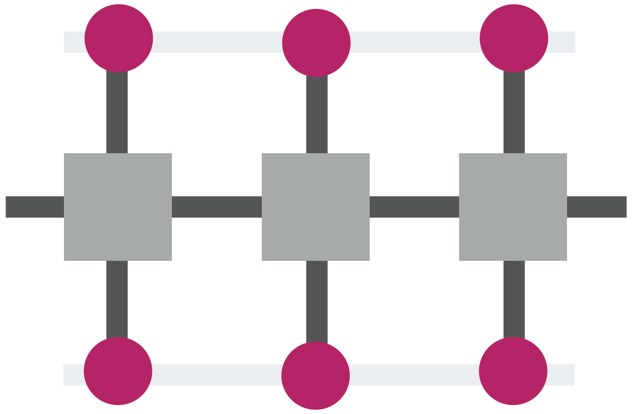
Kim,
Brown
Eisert

- Develop **matrix-product operator (MPO)** based numerical algorithms, complementing DMRG-X

Compare Pal, Pekker, Yu, Clark, Serbyn, Karrasch, Pollmann

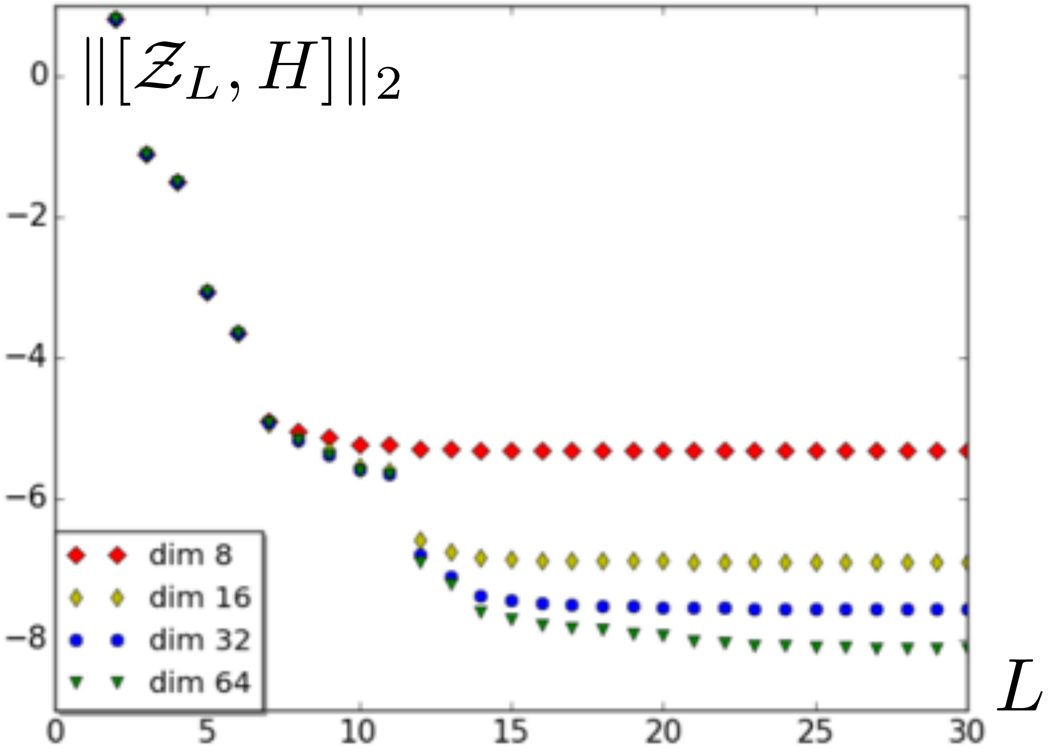


- Finding **local constants of motion**:

Minimise $\|[\mathcal{Z}, H]\|_2$

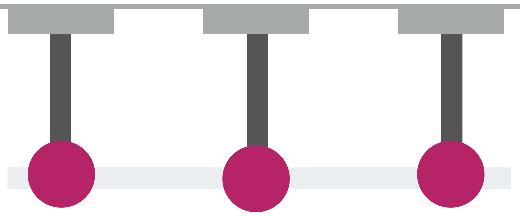
subject to MPO bond-dimension and support constraints

Nebendahl, Goihl, Brown, Werner, Eisert, in preparation (2016)
Kim, Banuls, Cirac, Hastings, Huse, Phys Rev E 92, 012128 (2015)



- Combine **Wegner-flow** and **MPO-simulations**

Orus, Schmidt, Eisert, in preparation (2016)



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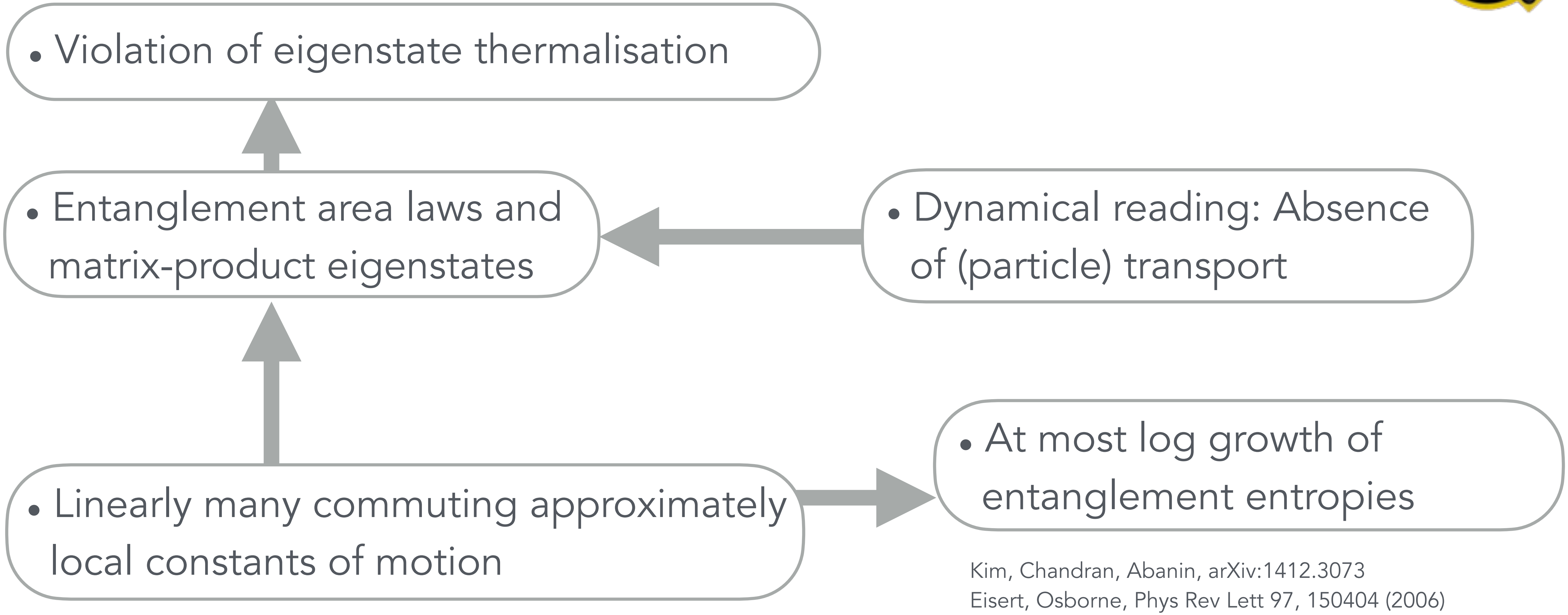
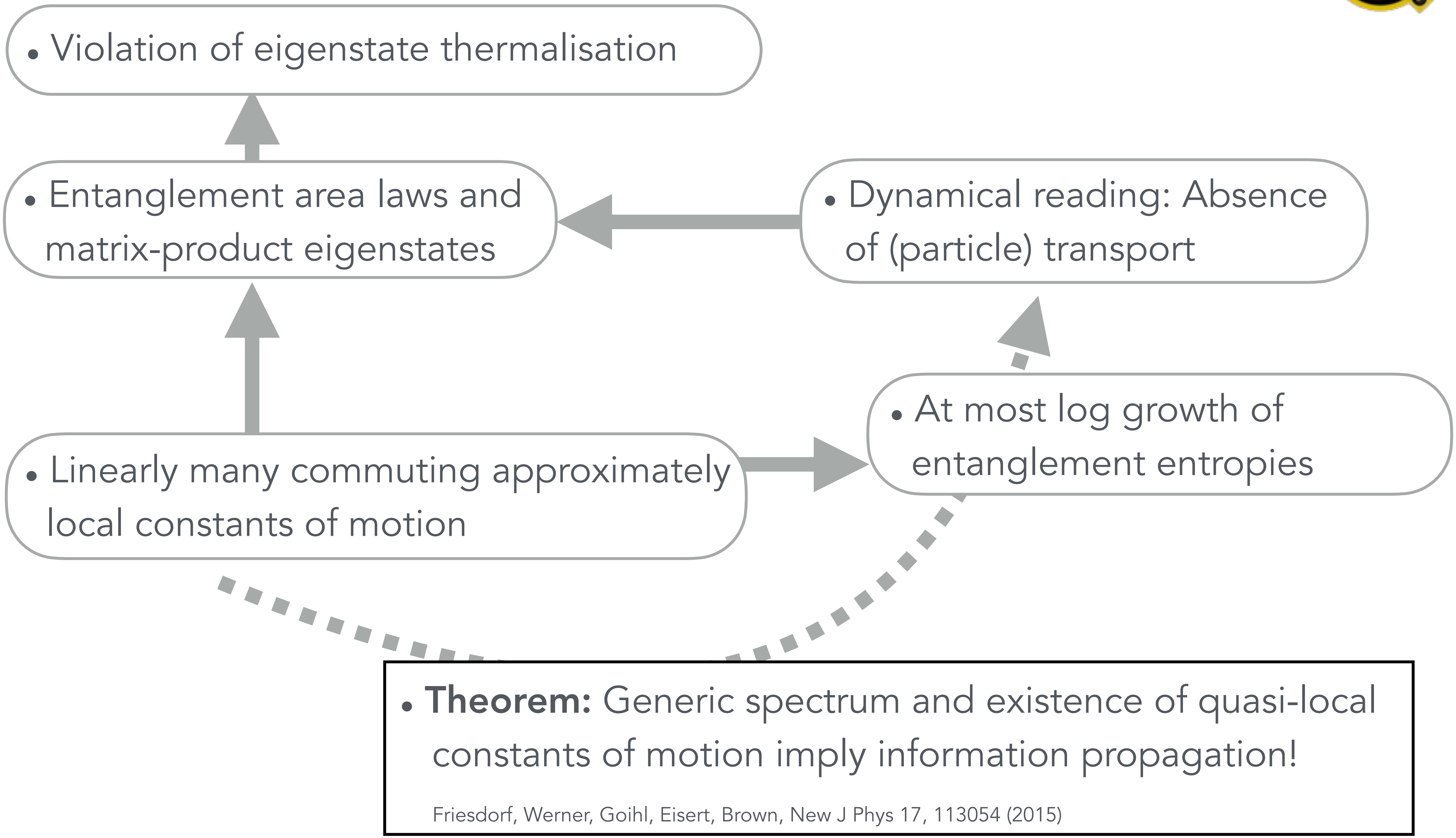


- Violation of eigenstate thermalisation
- Entanglement area laws and matrix-product eigenstates
- Dynamical reading: Absence of (particle) transport
- Linearly many commuting approximately local constants of motion
- At most log growth of entanglement entropies

Kim, Chandran, Abanin, arXiv:1412.3073
Eisert, Osborne, Phys Rev Lett 97, 150404 (2006)

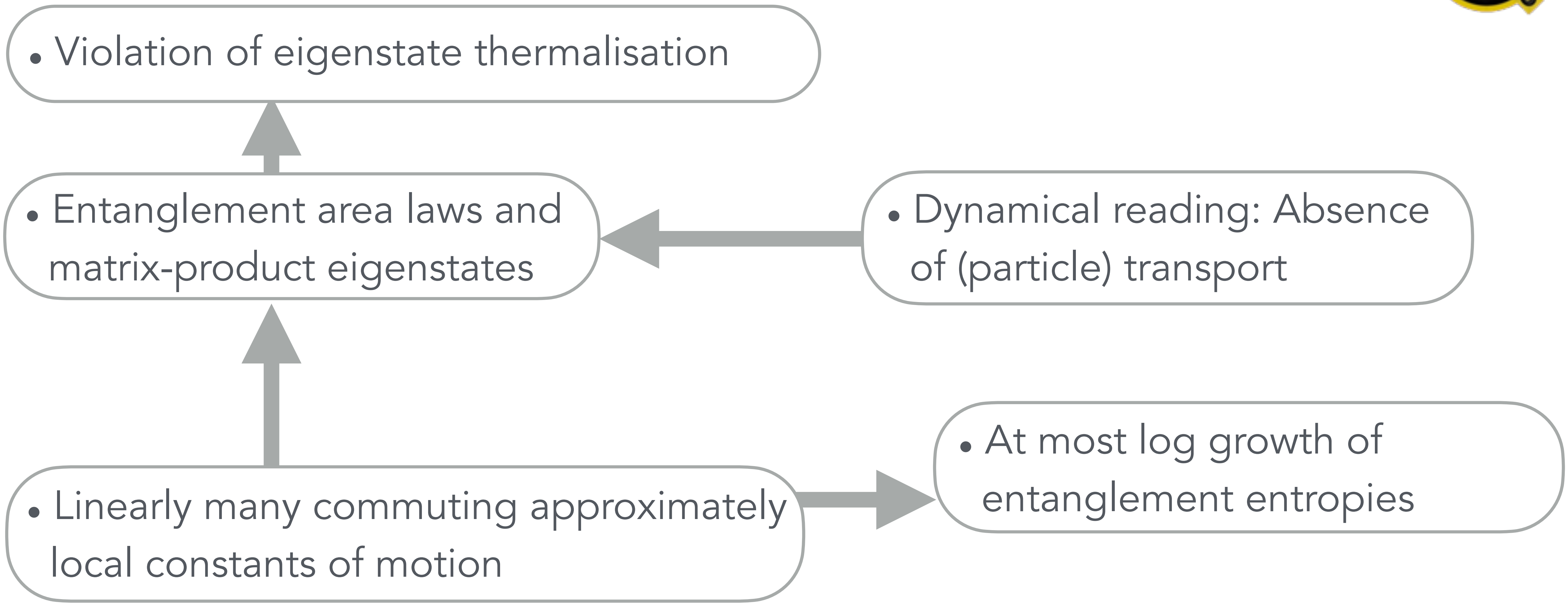
Bringing pictures together



- Violation of eigenstate thermalisation
- Entanglement area laws and matrix-product eigenstates
- Dynamical reading: Absence of (particle) transport
- At most log growth of entanglement entropies
- Linearly many commuting approximately local constants of motion
- **Theorem:** Generic spectrum and existence of quasi-local constants of motion imply information propagation!

Friesdorf, Werner, Goihl, Eisert, Brown, New J Phys 17, 113054 (2015)

Bringing pictures together



- Violation of eigenstate thermalisation
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- At most log growth of entanglement entropies
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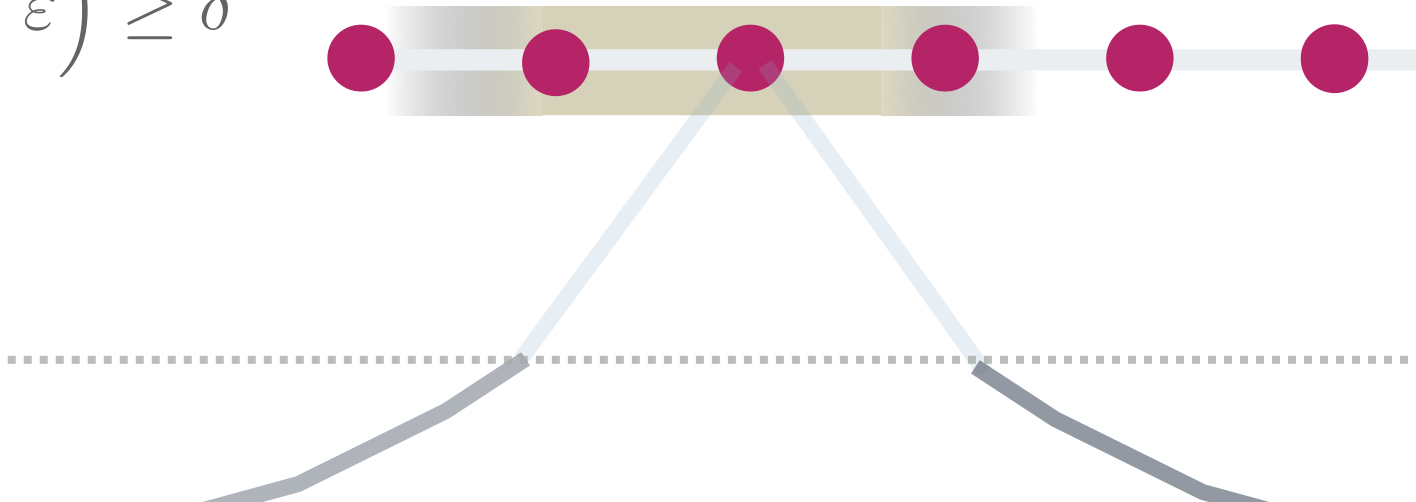
- Show equilibration for Hamiltonians for l-bit Hamiltonians $H = \sum_{\mu} \omega_\mu \mathcal{Z}(\mu)$

$$\mathbb{P}\left(\forall \nu \in S : \|\Gamma_S(\chi_t(\nu))) - \overline{\chi(\nu)}^{\infty}\| < \varepsilon\right) \geq \delta$$

Brown, Goihl, Werner, Friesdorf, Eisert, in preparation (2016)



Towards a unified view

- Violation of eigenstate thermalisation

- Entanglement area laws and matrix-product eigenstates

- Dynamical reading: Absence of (particle) transport

- Log growth of entanglement entropies

- Linearly many commuting approximately local constants of motion

- Quantum information ideas give fresh perspective to rich phenomenon, beyond what can be learned from numerical tools

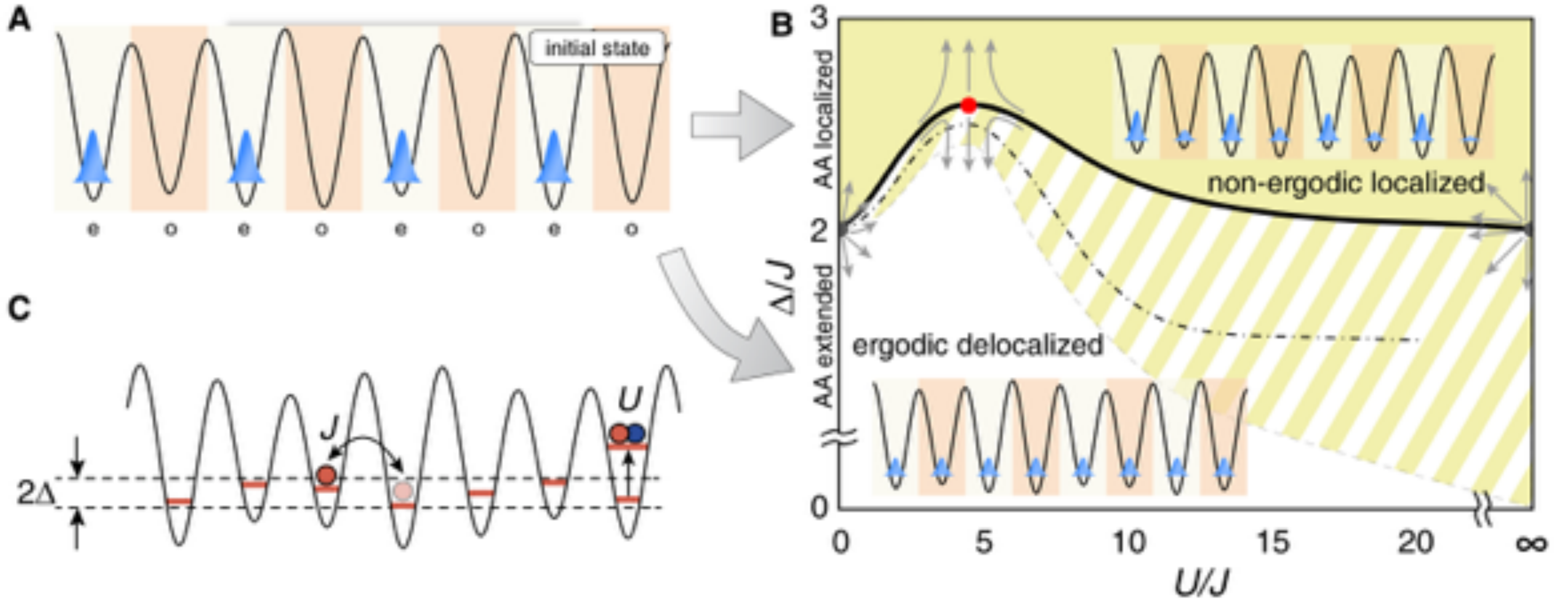
Towards a unified view

- **Towards a unified view of MBL**

MBL witnesses from feasible measurements

- Beautiful **first experiment** on MBL (MPQ)

$$|\psi(0)\rangle = |0, 1, \ldots, 0, 1\rangle$$

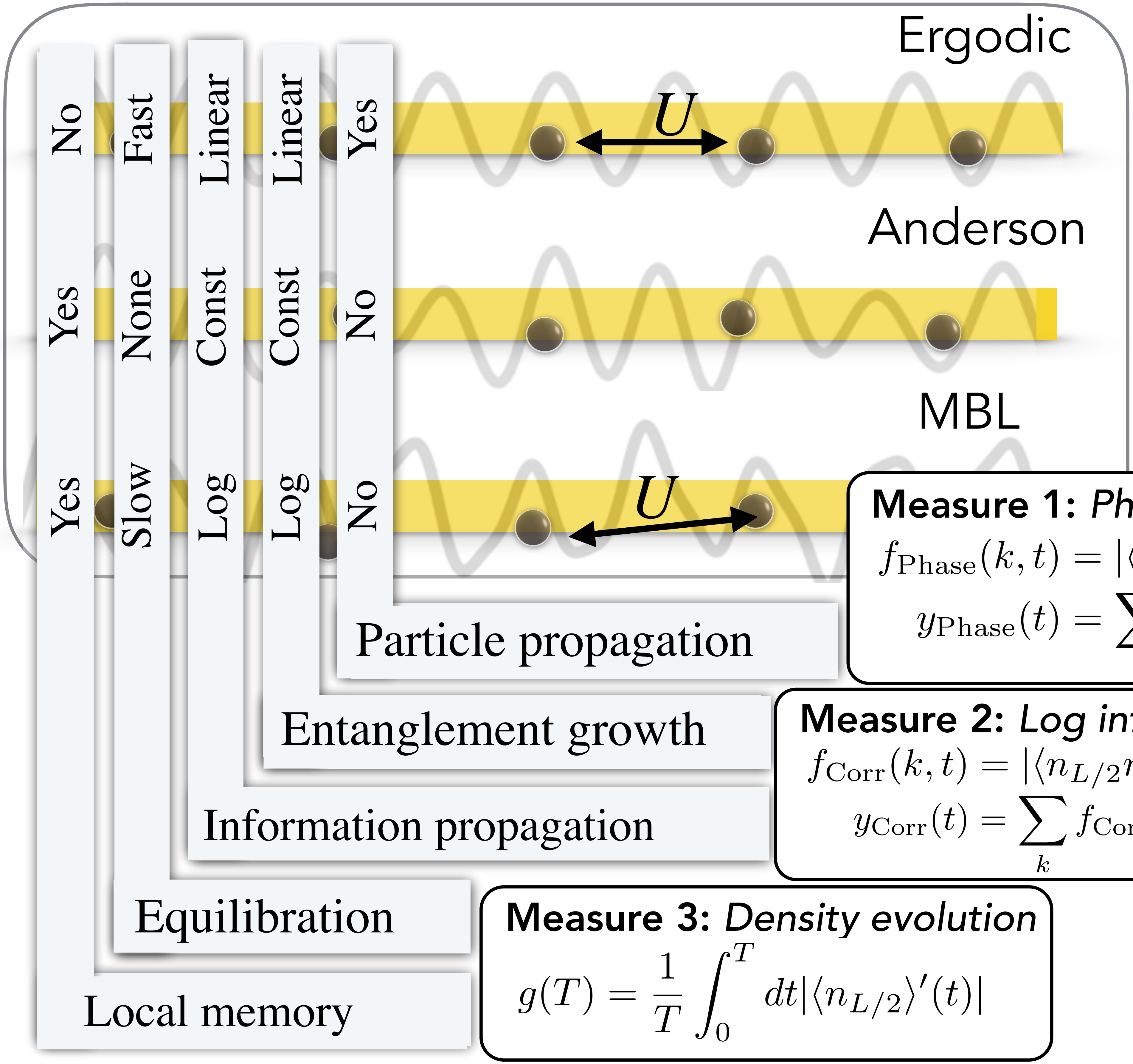


Schreiber, Hodgman, Bordia, Lueschen, Fischer, Vask, Altman, Schneider, Bloch, Science 349, 842 (2015)

- Discriminate Anderson localisation from MBL from (i) time of flight, (ii) parity density-density correlators and (iii) in situ

Compare Serbyn, Knap, Gopalakrishnan, Papic, Yao, Laumann, Abanin, Lukin, Demler, Phys Rev Lett 113, 147204 (2014)
Vasseur, Parameswaran, Moore, Phys Rev B 91, 140202 (2015)

MBL witnesses from feasible measurements



	Local memory	Equilibration	Information propagation	Entanglement growth	Particle propagation
Ergodic	No	Fast	Linear	Linear	Yes
Anderson	Yes	None	Const	Const	No
MBL	Yes	Slow	Log	Log	No

Measure 1: ***Phase correlation***

$$f_{\mathrm{Phase}}(k,t) = |\langle f^{\dagger}_{L/2} f_{L/2+k}(t)\rangle|$$
$$y_{\mathrm{Phase}}(t) = \sum_k f_{\mathrm{Phase}}(k,t)k^2$$

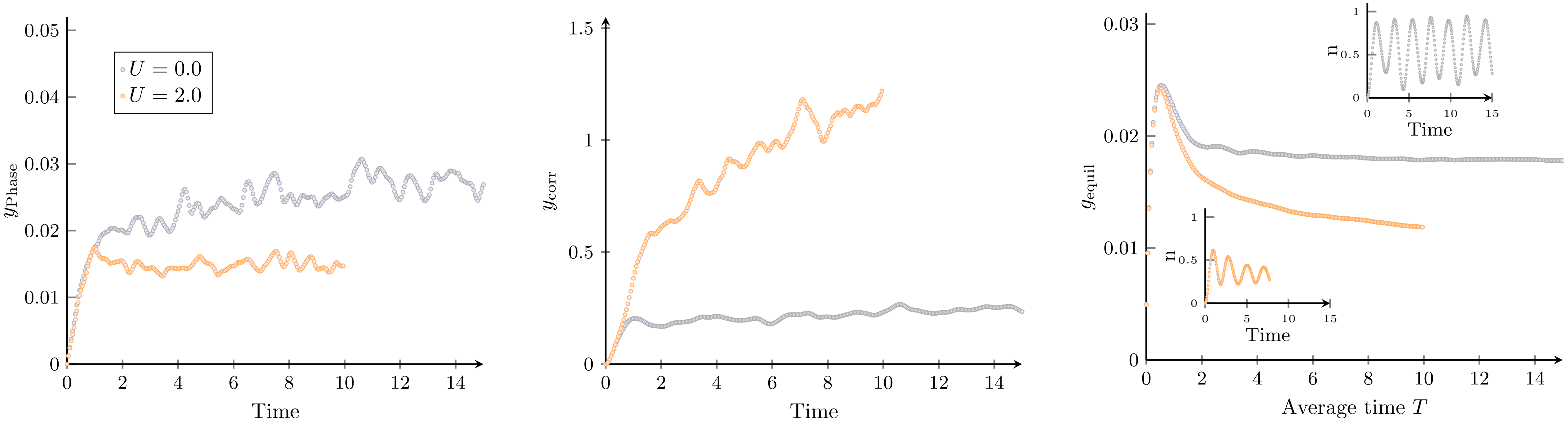
Measure 2: ***Log information***

$$f_{\mathrm{Corr}}(k,t) = |\langle n_{L/2} n_{L/2+k}\rangle - \langle n_{L/2}\rangle\langle n_{L/2+k}\rangle|$$
$$y_{\mathrm{Corr}}(t) = \sum_k f_{\mathrm{Corr}}(k,t)k^2$$

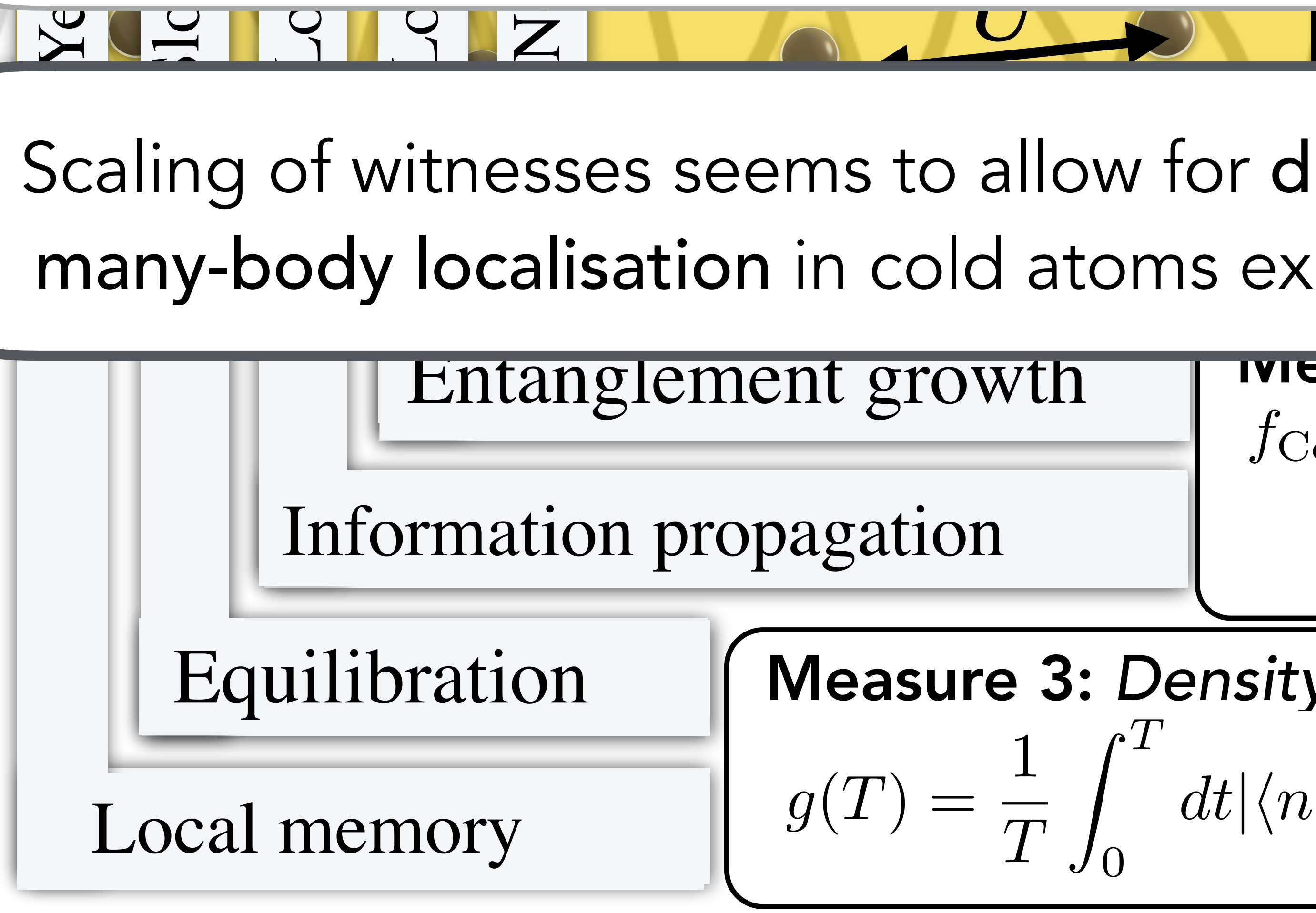
Measure 3: ***Density evolution***

$$g(T) = \frac{1}{T}\int_0^T dt|\langle n_{L/2}\rangle'(t)|$$

MBL witnesses from feasible measurements



- Scaling of witnesses seems to allow for **discrimination of Anderson from many-body localisation** in cold atoms experiments



Measure 1: *Phase correlation*

Measure 2: *Log information*

$$f_{\mathrm{Corr}}(k,t) = |\langle n_{L/2} n_{L/2+k} \rangle - \langle n_{L/2} \rangle \langle n_{L/2+k} \rangle|$$

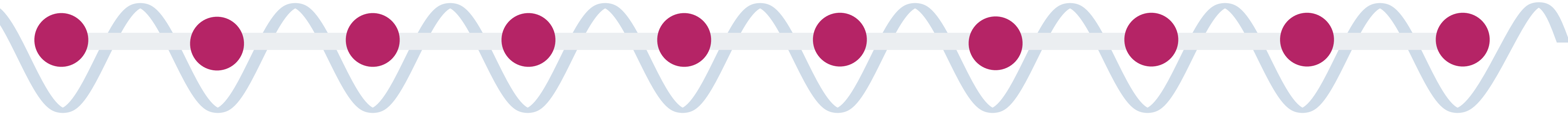
$$y_{\mathrm{Corr}}(t) = \sum_k f_{\mathrm{Corr}}(k,t) k^2$$

Measure 3: *Density evolution*

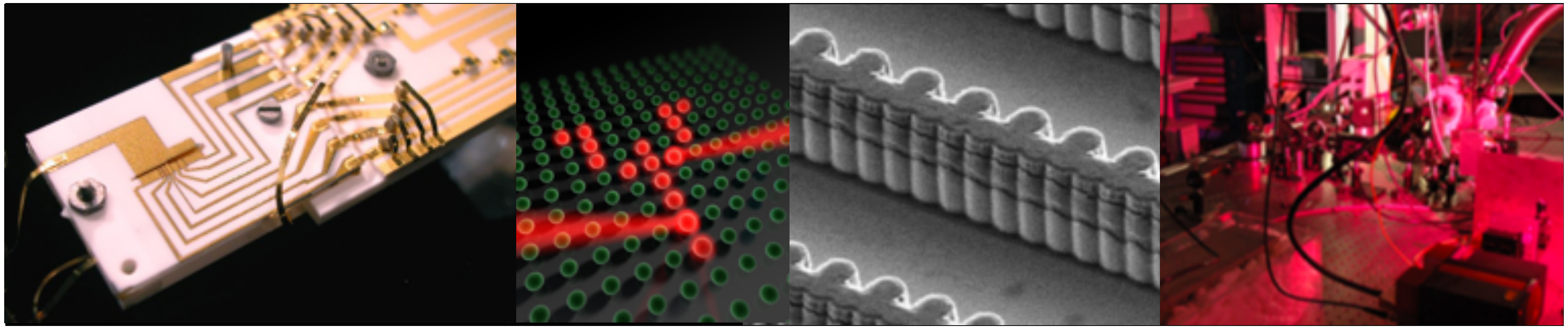
$$g(T) = \frac{1}{T} \int_0^T dt |\langle n_{L/2} \rangle'(t)|$$

Final musings

Towards supremacy of quantum simulators



- Analog quantum simulators outperforming classical devices?



- "Quantum supremacy"

  Preskill, Quantum supremacy now?, blog entry on July 22, 2012, in Quantum Frontiers

- Error correction out of scope - simulate robust features?
- Presumably not BQP complete, what is computational power?

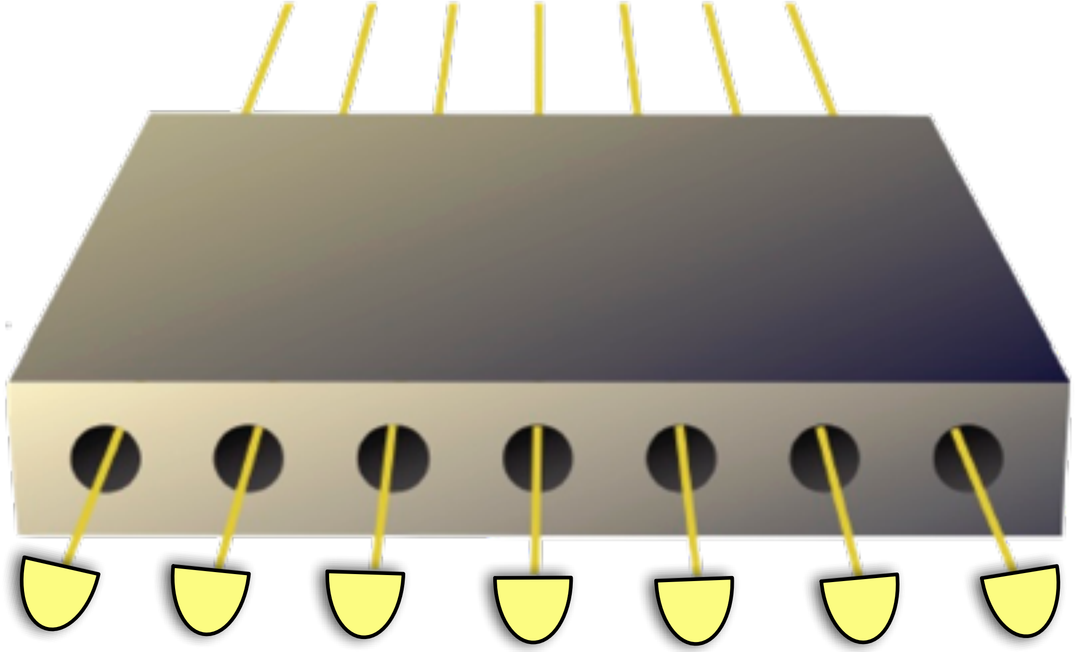
- Markovian noise renders Bose-Hubbard dynamics classically simulable

  DiCandia, Bermejo-Vega, Hangleiter, Eisert, in preparation (2016)
  Mari, Eisert, PRL 109, 230503 (2012)

Towards supremacy of quantum simulators

- **Boson sampling with photons:** Sampling from a distribution close in 1-norm to boson sampling distribution, leads [...] to collapse of poly hierarchy

Aaronson, Arkhipov, Proceedings of ACM Symposium on the Theory of Computing, STOC (2011)



- Output distribution can with overwhelming probability not be distinguished from efficiently preparable distribution

Gogolin, Kliesch, Aolita, Eisert, arXiv:1306.3995
Trevisan, Tulsiani, Vadhan, Proc IEEE Conf Comp Complex, 126 (2009)
Aaronson, Arkhipov, arXiv:1309.7460

Towards supremacy of quantum simulators

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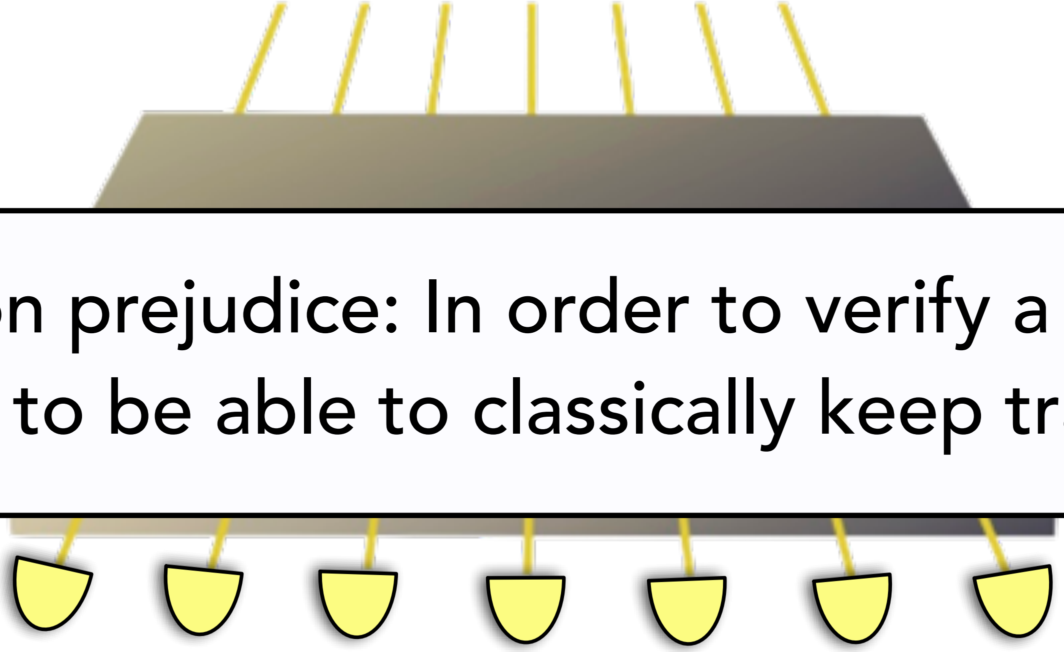

- Common prejudice: In order to verify a quantum simulation, one has to be able to classically keep track of it

- Output distribution can with overwhelming probability not be distinguished from efficiently preparable distribution

Gogolin, Kliesch, Aolita, Eisert, arXiv:1306.3995
Trevisan, Tulsiani, Vadhan, Proc IEEE Conf Comp Complex, 126 (2009)
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Towards supremacy of quantum simulators

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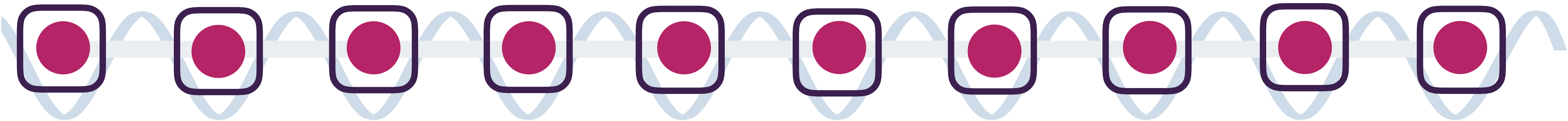
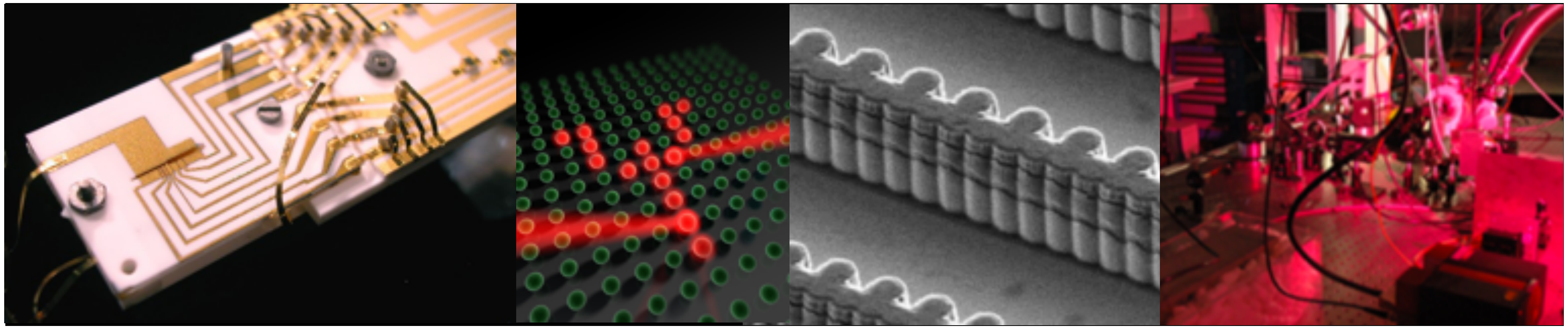


- Supremacy for analog quantum simulators?



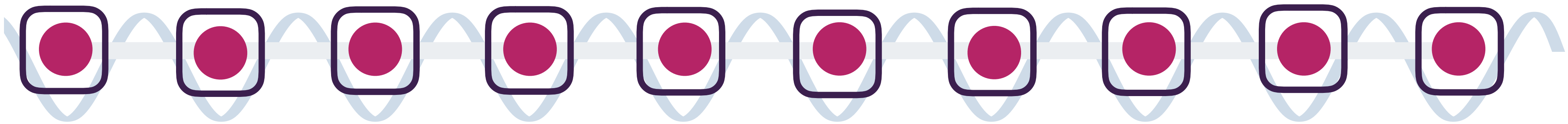
  - (i) With disordered **entangled initial state** and
  - (ii) **non-adaptive local measurements**

one can sample from **IQP circuits (“hard problem”)**, but now one can also

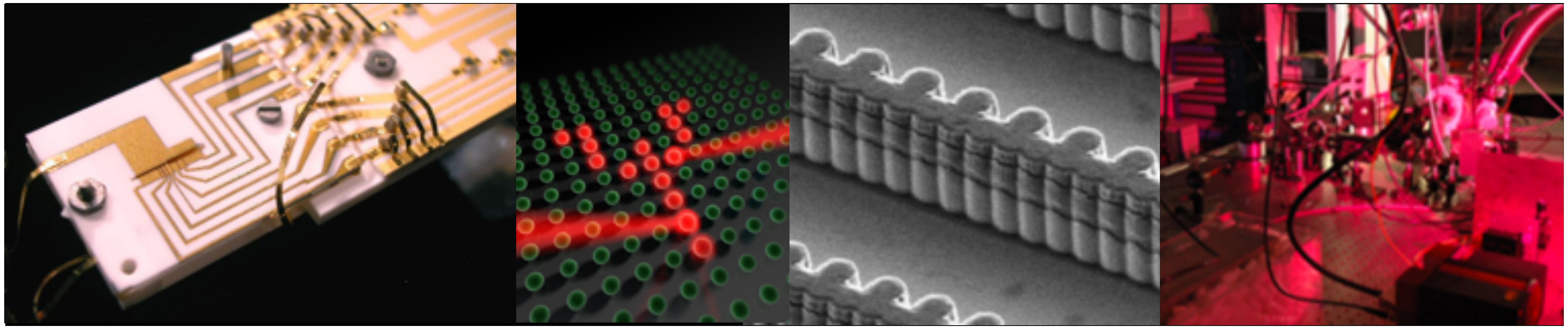
  - (iii) efficiently certify correctness of prepared state

Bermejo-Vega, Hangleiter, Schwarz, Raussendorf, Eisert, in preparation (2016)
Gao, Wang, Duan, arXiv:1607.04947
Bremner, Montanaro, Shepherd, PRL 117, 080501 (2016)

Towards supremacy of quantum simulators



- Supremacy for analog quantum simulators?



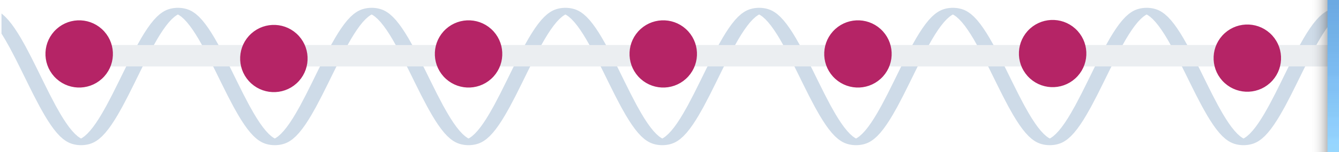
  - (i) With disordered **entangled initial state** and
  - (ii) **non-adaptive local measurements**

so

- The correctness of quantum simulations can sometimes be certified, even if one cannot predict the outcome!

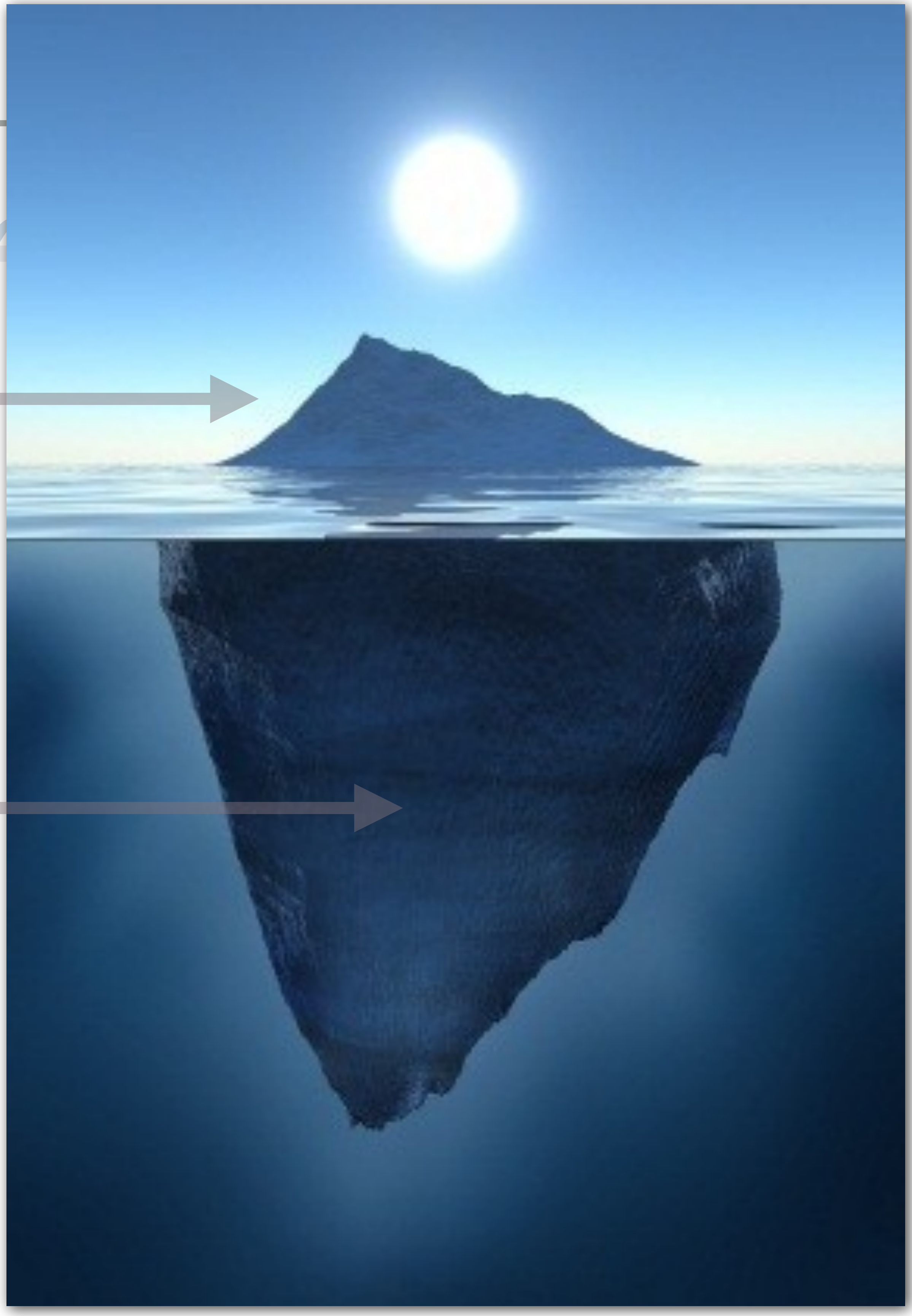
Bermejo-Vega, Hangleiter, Schwarz, Raussendorf, Eisert, in preparation (2016)
Gao, Wang, Duan, arXiv:1607.04947
Bremner, Montanaro, Shepherd, PRL 117, 080501 (2016)

Summary



- This talk

- Synthetic dynamical quantum simulators



Thanks for your attention!